Some new classes of neutrosophic minimal open sets

Selvaraj Ganesan1, Florentin Smarandache2

1PG & Research Department of Mathematics, Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India. (Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India). Orchid iD: 0000-0002-7728-8941
2Mathematics & Science Department, University of New Mexico, 705 Gurley Ave, Gallup, NM 87301, USA. ORCID iD: 0000-0002-5560-5926

Received: 14 Feb 2021 • Accepted: 19 Mar 2021 • Published Online: 28 Apr 2021

Abstract: This article focuses on $N_m$-$\beta$-open, $\beta$-interior and $\beta$-closure operators using neutrosophic minimal structures. We investigate properties of such concepts and we introduced the concepts of $N_m$-$\beta$-continuous, $N_m$-$\beta$-closed graph, $N_m$-$\beta$-compact and almost $N_m$-$\beta$-compact. Finally, we introduced the concepts of $N_m$-regular-open sets and $N_m$-$\pi$-open sets and investigate some properties.

Key words: $N_m$-$\beta$-continuous, $N_m$-$\beta$-closed graph, $N_m$-$\beta$-compact, almost $N_m$-$\beta$-compact, $N_m$-regular-open and $N_m$-$\pi$-open

1. Introduction
Zadeh’s [17] Fuzzy set laid the foundation of many fields such as intuitionistic fuzzy, neutrosophic set, rough sets. Later, researchers developed K. T. Atanassov’s [4] intuitionistic fuzzy set theory in many fields such as differential equations, topology, computer science and so on. F. Smarandache [15, 16] found that some objects have indeterminacy or neutral other than membership and non-membership. So he coined the notion of neutrosophy. V. Popa & T. Noiri [12] introduced the notions of minimal structure which is a generalization of a topology on a given nonempty set. We introduced the concepts of $\mathcal{M}$-continuous maps. M. Karthika et al [11] studied neutrosophic minimal structure spaces. S. Ganesan and F. Smarandache [9] studied $N_m$-semi-open in neutrosophic minimal structure spaces. S. Ganesan et al [10] studied $N_m$-pre-continuous maps. This article focuses on $N_m$-$\beta$-open, $\beta$-interior and $\beta$-closure operators using neutrosophic minimal structures. We investigate properties of such concepts and we introduced the notions of $N_m$-$\beta$-continuous, $N_m$-$\beta$-closed graph, $N_m$-$\beta$-compact and almost $N_m$-$\beta$-compact and investigate some properties for such concepts. Finally, we introduced $N_m$-regular-open, $N_m$-$\pi$-open sets and investigate fundamental properties.

2. Preliminaries
Definition 2.1. [15, 16] Neutrosophic set (in short ns) $K$ on a set $G \neq \emptyset$ is defined by $K = \{ \prec a, P_K(a), Q_K(a), R_K(a) : a \in G \}$, where $P_K : G \to [0,1]$, $Q_K : G \to [0,1]$ and $R_K : G \to [0,1]$ denotes the membership of an object, indeterminacy and non-membership of an object, for each $a$ on $G$ to $K$, respectively and $0 \leq P_K(a) + Q_K(a) + R_K(a) \leq 3$ for each $a \in G$.

Proposition 2.1. [13] For any ns $S$, then the following conditions are holds:

© Asia Mathematika, DOI: 10.5281/zenodo.4724804
*Correspondence: sgsgsgsgsg77@gmail.com
1. \( 0 \leq S, 0 \leq 0 \).

2. \( S \leq 1 \), \( 1 \leq 1 \).

Definition 2.2. [13] Let \( K = \{ \prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in G \} \) be a ns.

1. A ns \( K \) is an empty set i.e., \( K = 0 \) if 0 is membership of an object and 0 is an indeterminacy and 1 is an non-membership of an object respectively. i.e., \( 0 = \{ g, (0, 0, 1) : g \in G \} \)

2. A ns \( K \) is a universal set i.e., \( K = 1 \) if 1 is membership of an object and 1 is an indeterminacy and 0 is an non-membership of an object respectively. \( 1 = \{ g, (1, 1, 0) : g \in G \} \)

3. \( K_1 \cup K_2 = \{ a, \max\{P_{K_1}(a), P_{K_2}(a)\}, \max\{Q_{K_1}(a), Q_{K_2}(a)\}, \min\{R_{K_1}(a), R_{K_2}(a)\} : a \in G \} \)

4. \( K_1 \cap K_2 = \{ a, \min\{P_{K_1}(a), P_{K_2}(a)\}, \min\{Q_{K_1}(a), Q_{K_2}(a)\}, \max\{R_{K_1}(a), R_{K_2}(a)\} : a \in G \} \)

5. \( K_C = \{ \prec a, R_K(a), 1 - Q_K(a), P= P_K(a) \succ : a \in G \} \)

Definition 2.3. [13] Neutrosophic topology (nt) in Salama’s sense on a nonempty set \( G \) is a family \( \tau \) of ns in \( G \) satisfying three conditions:

1. Empty set (0) and universal set (1) are members of \( \tau \).
2. \( K_1 \cap K_2 \in \tau \) where \( K_1, K_2 \in \tau \).
3. \( \cup K_\delta \in \tau \) for every \( \{ K_\delta : \delta \in \Delta \} \subseteq \tau \).

Definition 2.4. [11] The neutrosophic minimal structure space over a universal set \( G \) be denoted by \( N_m \). \( N_m \) is said to be neutrosophic minimal structure space (in short, nms) over \( G \) if it satisfying following the axiom: \( 0, 1 \in N_m \). A family of neutrosophic minimal structure space is denoted by \( (G, N_mG) \). Note that neutrosophic empty set and neutrosophic universal set can form a topology and it is known as neutrosophic minimal structure space.


Definition 2.5. [11] A is \( N_m \)-closed if and only if \( N_m \text{cl}(A) = A \). Similarly, A is a \( N_m \)-open if and only if \( N_m \text{int}(A) = A \).

Definition 2.6. [11] Let \( N_m \) be any nms and A be any neutrosophic set. Then

1. Every A \( \in N_m \) is open and its complement is \( N_m \) closed.
2. \( N_m \)-closure of A = \( \min\{F : F \text{ is a nmc and } F \geq A\} \) and it is denoted by \( N_m \text{cl}(A) \).
3. \( N_m \)-interior of A = \( \max\{F : F \text{ is a nmo and } F \leq A\} \) and it is denoted by \( N_m \text{int}(A) \).

In general \( N_m \text{int}(A) \) is subset of A and A is a subset of \( N_m \text{cl}(A) \).

Proposition 2.2. [11] Let R and S are any ns of nms \( N_m \) over \( G \). Then

1. \( N_C^i = \{ 0, 1, R^i \} \) where \( R^i \) is a complement of ns \( R_i \).
2. \(G - N_m \text{int}(S) = N_m \text{cl}(G - S)\).

3. \(G - N_m \text{cl}(S) = N_m \text{int}(G - S)\).

4. \(N_m \text{cl}(R^C) = (N_m \text{cl}(R))^C = N_m \text{int}(R)\).

5. \(N_m\) closure of an empty set is an empty set and \(N_m\) closure of a universal set is a universal set. Similarly, \(N_m\) interior of an empty set and universal set respectively an empty and a universal set.

6. If \(S\) is a subset of \(R\) then \(N_m \text{cl}(S) \leq N_m \text{cl}(R)\) and \(N_m \text{int}(S) \leq N_m \text{int}(R)\).

7. \(N_m \text{cl}(N_m \text{cl}(R)) = N_m \text{cl}(R)\) and \(N_m \text{int}(N_m \text{int}(R)) = N_m \text{int}(R)\).

8. \(N_m \text{cl}(R \lor S) = N_m \text{cl}(R) \lor N_m \text{cl}(S)\).

9. \(N_m \text{cl}(R \land S) = N_m \text{cl}(R) \land N_m \text{cl}(S)\).

\textbf{Definition 2.7.} Let \((G, N_mG)\) be a nms and \(S \leq G\) is said to be

1. \(N_m\)-semi-open set (in short, \(N_m\)-so) \([9]\) if \(S \leq N_m \text{cl}(N_m \text{int}(S))\).

2. \(N_m\)-pre-open set (in short, \(N_m\)-po) \([10]\) if \(S \leq N_m \text{int}(N_m \text{cl}(S))\).

The complement of above \(N_m\)-open set is called an \(N_m\)-closed set.

\textbf{Definition 2.8.} \([11]\) Let \((G, N_mG)\) be nms.

1. Arbitrary union of \(N_m\) in \((G, N_mG)\) is \(N_m\) (Union Property).

2. Finite intersection of \(N_m\) in \((G, N_mG)\) is \(N_m\) (intersection Property).

\textbf{Definition 2.9.} \([11]\) A function \(f: (G, N_mG) \to (H, N_mH)\) is called neutrosophic minimal continuous map iff \(f^{-1}(V) \in N_mG\) whenever \(V \in N_mH\).

\textbf{Definition 2.10.} \([11]\) Let \(A\) be a ns in nms \((G, N_mG)\). Then \(Y\) is said to be neutrosophic minimal subspace if \((H, N_mH) = \{A \cap U : U \in N_mH\}\).

\textbf{3. \(N_m\)-\(\beta\)-open sets}

\textbf{Definition 3.1.} \((G, N_mG)\) be a nms \& \(S \leq G\) is said to be \(N_m\)-\(\beta\)-open set (in short, \(N_m\)-\(\beta\)o ) if \(S \leq N_m \text{cl}(N_m \text{int}(N_m \text{cl}(S)))\).

The complement of an \(N_m\)-\(\beta\)o is called an \(N_m\)-\(\beta\)-closed set (in short, \(N_m\)-\(\beta\)c)

\textbf{Remark 3.1.} \((G, T)\) be a nt \& \(S \leq G\) is said to be \(N\)-\(\beta\)-open set \([3]\) if \(S \leq N \text{cl}(N \text{int}(N \text{cl}(S)))\). If the nms \(N_mG\) is a topology, clearly an \(N_m\)-\(\beta\)o is \(N\)-\(\beta\)-open.

Above definition of 3.1, trivially the following statement are obtained.

\textbf{Lemma 3.1.} Consider \((G, N_mG)\) be a nms.

1. Every \(N_m\)-open is \(N_m\)-\(\beta\)o.

2. \(S\) is an \(N_m\)-\(\beta\)o iff \(S \leq N_m \text{cl}(N_m \text{int}(N_m \text{cl}(S)))\).
3. Every $N_m$-closed set is $N_m\beta$-closed.

4. $S$ is an $N_m\beta$-closed set iff $N_m\text{int}(N_m\text{cl}(N_m\text{int}(S))) \leq S$.

**Theorem 3.1.** $(G, N_mG)$ be a nms. Any union of $N_m\beta o$ is $N_m\beta o$.

**Proof.** Suppose $A_\delta$ be an $N_m\beta o$ for $\delta \in \Delta$. Above definition 3.1 and Proposition 2.2(6), $A_\delta \leq N_m\text{cl}(N_m\text{int}(N_m\text{cl}(A_\delta))) \leq N_m\text{cl}(N_m\text{int}(N_m\text{cl}(A_\delta))))$. This implies $\bigcup A_\delta \leq N_m\text{cl}(N_m\text{int}(N_m\text{cl}(A_\delta))))$. Hence $\bigcup A_\delta$ is an $N_m\beta o$.

**Remark 3.2.** Consider $(G, N_mG)$ be a nms. Intersection of any 2 $N_m\beta o$ may not be $N_m\beta o$.

**Example 3.1.** Consider $G = \{a\}$ with $N_m = \{0_\sim, P, Q, R, S, 1_\sim\}$ and $N_mC = \{1_\sim, I, J, K, L, 0_\sim\}$

- $P = \prec (0.5, 0.6, 0.6)\succ$
- $Q = \prec (0.4, 0.6, 0.8)\succ$
- $R = \prec (0.4, 0.7, 0.9)\succ$
- $S = \prec (0.5, 0.7, 0.6)\succ$
- $I = \prec (0.6, 0.4, 0.5)\succ$
- $J = \prec (0.8, 0.4, 0.4)\succ$
- $K = \prec (0.9, 0.3, 0.4)\succ$
- $L = \prec (0.6, 0.3, 0.5)\succ$

We know that $0_\sim = \{\prec g, 0, 0, 1 \succ : g \in G\}$, $1_\sim = \{\prec g, 1, 1, 0 \succ : g \in G\}$ and $0_C = \{\prec g, 1, 1, 0 \succ : g \in G\}$

Now we define the two $N_m\beta o$s as follows:

- $A = \prec (0.6, 0.7, 0.9)\succ$
- $B = \prec (0.5, 0.8, 0.4)\succ$

Here $N_mC(A) = 0_C$, $N_m\text{int}(N_mC(A)) = 1_\sim$, $N_m\text{cl}(N_m\text{int}(N_mC(A))) = 0_C$ and $N_mC(B) = 0_C$, $N_m\text{int}(N_mC(B)) = 1_\sim$, $N_m\text{cl}(N_m\text{int}(N_mC(B))) = 0_C$. But $A \wedge B = \prec (0.5, 0.7, 0.9)\succ$ is not a $N_m\beta o$ in $G$.

**Proposition 3.1.** Let $(G, N_mG)$ be a nms.

1. If $S$ is a $N_m$ then it is a $N_m\beta o$.
2. If $S$ is a $N_m$-po then it is a $N_m\beta o$.

**Proof.** (1) The proof is straightforward from the definitions.

(2) The proof is straightforward from the definitions.

**Definition 3.2.** Let $(G, N_mG)$ be a nms.

1. $N_m\beta$-closure of $A = \min \{ S : S \text{ is } N_m\beta \text{-closed set and } S \geq A \}$ and it is denoted by $N_m\beta \text{cl} (A)$.

2. $N_m\beta$-interior of $A = \max \{ V : V \text{ is } N_m\beta o \text{ and } V \leq A \}$ and it is denoted by $N_m\beta \text{int}(A)$.

**Theorem 3.2.** Suppose $(G, N_mC)$ be a nms and $R, S \leq G$. Then

1. $N_m\beta \text{int}(0_\sim) = 0_\sim$.
2. $N_m\beta \text{int}(1_\sim) = 1_\sim$.
3. $N_m\beta \text{int}(R) \leq R$.
4. If $R \leq S$, then $N_m\beta \text{int}(R) \leq N_m\beta \text{int}(S)$. 
5. $R$ is $N_m^{-\beta}o$ iff $N_m^{-\beta}\text{int}(R) = R$.

6. $N_m^{-\beta}\text{int}(N_m^{-\beta}\text{int}(R)) = N_m^{-\beta}\text{int}(R)$.

7. $N_m^{-\beta}\text{cl} (G - R) = G - N_m^{-\beta}\text{int}(R)$.

Proof. (1), (2) are Obvious.

(3), (4) are Obvious.

(5) It follows from Theorem 3.1.

(6) It follows condition from (5).

(7) For $R \leq G$, $G - N_m^{-\beta}\text{int}(R) = G - \max \{U : U \leq R, U is N_m^{-\beta}o\} = \min \{G - U : U \leq R, U is N_m^{-\beta}o\} = N_m^{-\beta}\text{cl} (G - R)$.

Theorem 3.3. Let $(G, N_mG)$ be a nms and $R, S \leq G$. Then

1. $N_m^{-\beta}\text{cl} (0_\sim) = 0_\sim$.
2. $N_m^{-\beta}\text{cl} (1_\sim) = 1_\sim$.
3. $R \leq N_m^{-\beta}\text{cl} (R)$.

4. If $R \leq S$, then $N_m^{-\beta}\text{cl} (R) \leq N_m^{-\beta}\text{cl} (S)$.

5. $R$ is $N_m^{-\beta}c$ iff $N_m^{-\beta}\text{cl} (R) = R$.

6. $N_m^{-\beta}\text{cl} (N_m^{-\beta}\text{cl} (R)) = N_m^{-\beta}\text{cl} (R)$.

7. $N_m^{-\beta}\text{int}(G - R) = G - N_m^{-\beta}\text{cl} (R)$.

Proof. It is similar to the proof of above Theorem 3.2.

Theorem 3.4. Let $(G, N_mG)$ be a nms and $S \leq G$. Then

1. $g \in N_m^{-\beta}\text{cl} (S)$ iff $S \cap V \neq \emptyset$ for every $N_m^{-\beta}o V$ containing $g$.

2. $g \in N_m^{-\beta}\text{int}(S)$ iff there exists an $N_m^{-\beta}o U$ such that $U \leq S$.

Proof. (1) Suppose there is an $N_m^{-\beta}o V$ containing $g$ such that $S \cap V = \emptyset$. Then $G - V$ is an $N_m^{-\beta}c$ such that $S \leq G - V$, $g \notin G - V$. This implies $g \notin N_m^{-\beta}\text{cl} (S)$.

The reverse relation is obvious.

(2) Obvious.

Lemma 3.2. Let $(G, N_mG)$ be a nms and $S \leq G$. Then

1. $N_m\text{int}(N_m\text{cl}(N_m\text{int}(S))) \leq N_m\text{int}(N_m\text{cl}(N_m\text{int}(N_m^{-\beta}\text{int}(S)))) \leq N_m^{-\beta}\text{int}(S)$.

2. $N_m^{-\beta}\text{cl} (S) \leq N_m\text{cl}(N_m\text{int}(N_m\text{cl}(N_m^{-\beta}\text{cl}(S)))) \leq N_m\text{cl}(N_m\text{int}(N_m\text{cl}(S)))$.

Proof. (1) For $S \leq G$, by Theorem 3.3, $N_m^{-\beta}\text{cl} (S)$ is an $N_m^{-\beta}c$ set. Hence from Lemma 3.1, we have $N_m\text{int}(N_m\text{cl}(N_m\text{int}(S))) \leq N_m\text{int}(N_m\text{cl}(N_m^{-\beta}\text{int}(S)))) \leq N_m^{-\beta}\text{int}(S)$.

(2) It is similar to the proof of (1).
4. $N_m$-$\beta$-continuous map

**Definition 4.1.** Map $f : (G, N_mG) \to (H, N_mH)$ is said to be $N_m$-$\beta$-continuous if $f^{-1}(O)$ is a $N_m$-$\beta$o in $G$, for each $N_m$-open $O$ in $H$.

**Theorem 4.1.** Every neutrosophic minimal continuous is $N_m$-$\beta$-continuous but not conversely.

2. Every $N_m$-semi-continuous is $N_m$-$\beta$-continuous but not conversely.

3. Every $N_m$-pre-continuous is $N_m$-$\beta$-continuous but not conversely.

**Proof.** (1) The proof follows from [Lemma 3.1 (1)].

(2) The proof follows from [Proposition 3.1 (1)].

(3) The proof follows from [Proposition 3.1 (2)].

**Theorem 4.2.** Map $f : G \to H$ be a function on 2 nms $(G, N_mG)$ and $(H, N_mH)$. Then the following statements are equivalent:

1. $f$ is $N_m$-$\beta$-continuous.

2. $f^{-1}(O)$ is an $N_m$-$\beta$o, for each $N_m$-open set $O$ in $H$.

3. $f^{-1}(S)$ is an $N_m$-$\beta$c set, for each $N_m$-closed $S$ in $H$.

4. $f(N_m$-$\beta cl (R)) \leq N_m cl(f(R))$, for $R \subseteq G$.

5. $N_m$-$\beta cl (f^{-1}(S)) \leq f^{-1}(N_m cl(S))$, for $S \subseteq H$.

6. $f^{-1}(N_m$-$\beta int(S)) \leq N_m$-$\beta int(f^{-1}(S))$, for $S \subseteq H$.

**Proof.** (1) $\Rightarrow$ (2) Let $O$ be an $N_m$-open in $H$ and $g \in f^{-1}(O)$. By hypothesis, there exists an $N_m$-$\beta$o $U_g$ containing $g$ such that $f(U) \leq O$. This implies $g \in U_g \leq f^{-1}(O)$ for all $g \in f^{-1}(O)$. Hence by Theorem 3.1, $f^{-1}(O)$ is $N_m$-$\beta$o.

(2) $\Rightarrow$ (3) Obvious.

(3) $\Rightarrow$ (4) For $R \subseteq G$, $f^{-1}(N_m cl(f(R))) = f^{-1}(\min \{F \subseteq H : f(R) \subseteq F \text{ and } F \text{ is } N_m\text{-closed}\}) = \min \{f^{-1}(F) \subseteq G : R \subseteq f^{-1}(F) \text{ and } F \text{ is } N_m\text{-closed} \} \geq \min \{K \subseteq G : R \subseteq K \text{ and } K \text{ is } N_m\text{-}\beta c \} = N_m \beta cl (R)$. Hence $f(N_m \beta cl (R)) \leq N_m cl(f(R))$.

(4) $\Rightarrow$ (5) For $R \subseteq G$, from (4), it follows $f(N_m \beta cl (f^{-1}(R))) \leq N_m cl(f(f^{-1}(R))) \leq N_m cl(R)$. Hence we get (5).

(5) $\Rightarrow$ (6) For $S \subseteq H$, from $N_m int(S) = Y - N_m cl(H - S)$ and (5), it follows: $f^{-1}(N_m int(S)) = f^{-1}(Y - N_m cl(H - S)) = G - f^{-1}(N_m cl(H - S)) \leq G - N_m \beta cl (f^{-1}(H - S)) = N_m \beta int(f^{-1}(S))$. Hence (6) is obtained.

(6) $\Rightarrow$ (1) Let $g \in G$ and $O$ an $N_m$-open set containing $f(g)$. Then from (6) and Proposition 2.2, it follows $g \in f^{-1}(O) = f^{-1}(N_m int(O)) \leq N_m \beta int(f^{-1}(O))$. So from Theorem 3.4, we can say that there exists an $N_m$-$\beta$o $U$ containing $g$ such that $g \in U \leq f^{-1}(O)$. Hence $f$ is $N_m$-$\beta$-continuous.

**Theorem 4.3.** Map $f : G \to H$ be a function on 2 nms $(G, N_mG)$ and $(H, N_mH)$. Then the following statements are equivalent:
1. $f$ is $N_m$-$\beta$-continuous.

2. $f^{-1}(O) \leq N_m \text{cl}(N_m \text{int}(f^{-1}(O)))$, for each $N_m$-open $O$ in $H$.

3. $N_m \text{int}(N_m \text{cl}(f^{-1}(F))) \leq f^{-1}(F)$, for each $N_m$-closed set $F$ in $H$.

4. $f(N_m \text{int}(N_m \text{cl}(R))) \leq N_m \text{cl}(f(R))$, for $R \leq G$.

5. $N_m \text{int}(N_m \text{cl}(f^{-1}(S))) \leq f^{-1}(N_m \text{cl}(S))$, for $S \leq H$.

6. $f^{-1}(N_m \text{cl}(S)) \leq N_m \text{cl}(N_m \text{int}(f^{-1}(S)))$, for $S \leq H$.

**Proof.** (1) $\iff$ (2) It follows from Theorem 4.2 and Definition of $N_m$-$\beta$os.

(1) $\iff$ (3) It follows from Theorem 4.2 and Lemma 3.1.

(3) $\implies$ (4) Let $R \leq X$. Then from Theorem 4.2(4) and Lemma 3.2, it follows $N_m \text{int}(N_m \text{cl}(R)) \leq N_m \text{cl}(f(R)))$. Hence $f(N_m \text{int}(N_m \text{cl}(R))) \leq N_m \text{cl}(f(R))$.

(4) $\implies$ (5) Obvious.

(5) $\implies$ (6) From (5) and Proposition 2.2, it follows: $f^{-1}(N_m \text{int}(S)) = f^{-1}(H - N_m \text{cl}(H - S)) = G - f^{-1}(N_m \text{cl}(H - S)) \leq G - N_m \text{int}(N_m \text{cl}(f^{-1}(H - S)))$

$= N_m \text{cl}(N_m \text{int}(f^{-1}(S)))$. Hence, (6) is obtained.

(6) $\Rightarrow$ (1) Let $O$ be an $N_m$-open in $H$. Then by (6) and Proposition 2.2, we have $f^{-1}(O) = f^{-1}(N_m \text{int}(O)) \leq N_m \text{cl}(N_m \text{int}(f^{-1}(O)))$. This implies $f^{-1}(O)$ is an $N_m$-$\beta$-o. Hence by (2), $f$ is $N_m$-$\beta$-continuous. $\square$

**Definition 4.2.** [10] $(G, N_mG)$ be a nms. Then $G$ is said to be $N_m$-$T_2$ if for each distinct points $g$ and $h$ of $G$, there exist two disjoint $N_m$-open $U, V$ such that $g \in U$ and $h \in V$.

**Definition 4.3.** $(G, N_mG)$ be a nms. Then $G$ is said to be $N_m$-$\beta$-$T_2$ if for any distinct points $g$ and $h$ of $G$, there exist disjoint $N_m$-$\beta$-open $C, D$ such that $g \in C$ and $h \in D$.

**Theorem 4.4.** Map $f : G \to H$ be a map on two nms $(G, N_mG)$ and $(H, N_mH)$. If $f$ is an injective and $N_m$-$\beta$ continuous map and if $H$ is $N_m$-$T_2$, then $G$ is $N_m$-$\beta$-$T_2$.

**Proof.** Obvious. $\square$

**Theorem 4.5.** Map $f : G \to H$ be a map on two nms $(G, N_mG)$ and $(H, N_mH)$. If $f$ is an injective and $N_m$-$\beta$ continuous map with an $N_m$-$\beta$-closed graph, then $G$ is $N_m$-$\beta$-$T_2$.

**Proof.** Suppose $g_1$ and $g_2$ be any distinct points of $G$. Then $f(g_1) \neq f(g_2)$, so $(g_1, f(g_2)) \in (G \times H) - L(f)$. Since the graph $L(f)$ is $N_m$-$\beta$-c, there exist an $N_m$-$\beta$-o containing $g_1$ and $D \in N_mH$ containing $f(g_2)$ such that $f(C) \cap D = \emptyset$. Since $f$ is $N_m$-$\beta$ continuous, $f^{-1}(D)$ is an $N_m$-$\beta$-o containing $g_2$ such that $C \cap f^{-1}(D) = \emptyset$. Hence $G$ is $N_m$-$\beta$-$T_2$. $\square$

**Definition 4.4.** [10] $(G, N_mG)$ be a nms and $S \leq G$, $S$ is called $N_m$-compact (respectively, almost $N_m$-compact) relative to $S$ if every collection $\{U_i : i \in \Delta\}$ of $N_m$-open subsets of $G$ such that $S \leq \max \{U_i : i \in \Delta\}$, there exists a finite subset $\Delta_0$ of $\Delta$ such that $S \leq \max \{U_j : j \in \Delta_0\}$ (respectively, $S \leq \max \{N_m \text{cl}(U_j) : j \in \Delta_0\}$). $(G, N_mG)$ be a nms and $S \leq G$, $S$ is said to be $N_m$-compact (respectively, almost $N_m$-compact) if $S$ is $N_m$-compact (respectively, almost $N_m$-compact) as a neutrosophic minimal subspace of $G$. 109
Definition 4.5. \((G, N_mG)\) be a nms and \(S \leq G\), \(S\) is called \(N_m\)-\(\beta\)-compact (respectively, almost \(N_m\)-\(\beta\)-compact) relative to \(S\) if every collection \(\{U_\delta : \delta \in \Delta\}\) of \(N_m\)-\(\beta\)-open subsets of \(G\) such that \(S \leq \max \{U_\delta : \delta \in \Delta\}\) (respectively, \(S \leq \max \{N_m \beta cl (U_\omega) : \omega \in \Omega\}\)). \((G, N_mG)\) be a nms and \(S \leq G\), \(S\) is said to be \(N_m\)-\(\beta\)-compact (resp. almost \(N_m\)-\(\beta\)-compact) if \(S\) is \(N_m\)-\(\beta\)-compact (resp. almost \(N_m\)-\(\beta\)-compact) as a neutrosophic minimal subspace of \(G\).

Theorem 4.6. Map \(f : G \to H\) be a map on 2 nms \((G, N_mG)\) and \((H, N_mH)\). If \(S\) is an \(N_m\)-\(\beta\)-compact set, then \(f(S)\) is \(N_m\)-compact.

Proof. Obvious.

5. \(N_m\)-regular open
We introduce following definitions

Definition 5.1. \((G, N_mG)\) be a nms and \(A \leq G\), \(A\) is called \(N_m\)-regular open (in short, \(N_m\)-ro) if \(A = N_m int(N_m cl(A))\).

Theorem 5.1. Any \(N_m\)-ro is \(N_m\)-open.

Proof. If \(A\) is \(N_m\)-ro in \((G, N_mG)\), \(A = N_m int(N_m cl(A))\). Then \(N_m int(A) = N_m int(N_m int(N_m cl(A))) = N_m int(N_m cl(A)) = A\). That is, \(A\) is \(N_m\)-open in \((G, N_mG)\).

Example 5.1. \(G = \{a\}\) with \(N_m = \{0_~, P, 1_~\}\) and \(N_mC = \{1_~, Q, 0_~\}\) where \(P = \prec (0.1, 0.5, 0.1)\succ\); \(Q = \prec (0.5, 0.5, 0.5)\succ\)

Now we define the \(N_m\)-ro sets as follows:
\(A = \prec (0.5, 0.5, 0.5)\succ\)
Here \(N_m cl(A) = Q\), \(N_m int(N_m cl(A)) = P\) is a \(N_m\)-ro set in \(G\).

Definition 5.2. \((G, N_mG)\) be a nms and \(S \leq G\), \(S\) is said to be \(N_m\)-\(\pi\)-open set if \(S\) is the finite union of \(N_m\)-ro.

Remark 5.1. For a subset of \(A\) of an nms \((G, N_mG)\), we have following implications:

\[\text{\(N_m\)-regular open} \Rightarrow \text{\(N_m\)-\(\pi\)-open} \Rightarrow \text{\(N_m\)-open}\]

Diagram-I

Example 5.2. \(G = \{a\}\) with \(N_m = \{0_~, P, L, 1_~\}\) and \(N_mC = \{1_~, M, N, 0_~\}\) where \(P = \prec (0.1, 0.5, 0.1)\succ\); \(L = \prec (0.5, 0.5, 0.5)\succ\)

Now we define the two \(N_m\)-ro sets as follows:
\(A = \prec (0.1, 0.5, 0.1)\succ\)
\(B = \prec (0.5, 0.5, 0.5)\succ\)
Here \(N_m cl(A) = M\), \(N_m int(N_m cl(A)) = P\); \(N_m cl(B) = N\), \(N_m int(N_m cl(B)) = L\) is a \(N_m\)-ro set in \(G\). Here, \(A \lor B = \prec (0.5, 0.5, 0.1)\succ\) is a \(N_m\)-\(\pi\)-open sets but it is not a \(N_m\)-ro.
Example 5.3. \( G = \{ a \} \) with \( N_m = \{ 0_\sim, A, 1_\sim \} \) and \( N_m^C = \{ 1_\sim, B, 0_\sim \} \) where
\[
A = \prec (0.6, 0.7, 0.3) \succ ; B = \prec (0.3, 0.3, 0.6) \succ
\]
Now we define the \( N_m \)-ro sets as follows:
\[
R = \prec (0, 0, 1) \succ ; S = \prec (1, 1, 0) \succ
\]
Here \( R \lor S \prec (1, 1, 0) \succ \) is a \( N_m \)-\( \pi \)-open set in \( G \). Here, \( A = \prec (0.6, 0.7, 0.3) \succ \) is \( N_m \)-open but it is not a \( N_m \)-\( \pi \)-open.

Conclusion

We presented several definitions, properties, explanations and examples inspired from the concept of \( N_m \)-\( \beta \)-open, \( N_m \)-regular-open and \( N_m \)-\( \pi \)-open. The results of this study may be help in many reserches.

Acknowledgment

We thank to referees for giving their useful suggestions and help to improve this article.

References

[9] S Ganesan and F. Smarandache, On \( N_m \)-semi-open sets in neutrosophic minimal structure spaces (communicated)
