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Some new classes of neutrosophic minimal open sets

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Abstract: This article focuses on N_m-β-open, β-interior and β-closure operators using neutrosophic minimal structures. We investigate properties of such concepts and we introduced the concepts of N_m-β-continuous, N_m-β-closed graph, N_m-β-compact and almost N_m-β-compact. Finally, we introduced the concepts of N_m-regular-open sets and N_m-π-open sets and investigate some properties.

Key words: N_m-β-continuous, N_m-β-closed graph, N_m-β-compact, almost N_m-β-compact, N_m-regular-open and N_m-π-open

1. Introduction

Zadeh's [17] Fuzzy set laid the foundation of many fields such as intuitionistic fuzzy, neutrosophic set, rough sets. Later, researchers developed K. T. Atanassov's [4] intuitionistic fuzzy set theory in many fields such as differential equations, topology, computerscience and so on. F. Smarandache [15, 16] found that some objects have indeterminacy or neutral other than membership and non-membership. So he coined the notion of neutrosophy. V. Popa & T. Noiri [12] introduced the notions of minimal structure which is a generalization of a topology on a given nonempty set. We introduced the concepts of $\mathcal{M}$-continuous maps. M. Karthika et al [11] studied neutrosophic minimal structure spaces. S. Ganesan and F. Smarandache [9] studied N_m-semi-open in neutrosophic minimal structure spaces. S. Ganesan et al [10] studied N_m-pre-continuous maps. This article focuses on N_m-β-open, β-interior and β-closure operators using neutrosophic minimal structures. We investigate properties of such concepts and we introduced the notions of N_m-β-continuous, N_m-β-closed graph, N_m-β-compact and almost N_m-β-compact and investigate some properties for such concepts. Finally, we introduced N_m-regular-open, N_m-π-open sets and investigate fundamental properties.

2. Preliminaries

Definition 2.1. [15, 16] Neutrosophic set (in short ns) K on a set $G \neq \emptyset$ is defined by $K = \{\prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in G\}$, where $P_K : G \to [0,1]$, $Q_K : G \to [0,1]$ and $R_K : G \to [0,1]$ denotes the membership of an object, indeterminacy and non-membership of an object, for each a on G to K, respectively and $0 \leq P_K(a) + Q_K(a) + R_K(a) \leq 3$ for each $a \in G$.

Proposition 2.1. *[13] For any ns S, then the following conditions are holds:*

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1. *$0_\sim \leq S$, $0_\sim \leq 0_\sim$.*
2. *$S \leq 1_\sim$, $1_\sim \leq 1_\sim$.*

Definition 2.2. [13] Let K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ G$\}$ be a ns.

1. A ns K is an empty set i.e., K = $0_\sim$ if 0 is membership of an object and 0 is an indeterminacy and 1 is an non-membership of an object respectively. i.e., $0_\sim = \{g, (0, 0, 1) : g \in G\}$
2. A ns K is a universal set i.e., K = $1_\sim$ if 1 is membership of an object and 1 is an indeterminacy and 0 is an non-membership of an object respectively. $1_\sim = \{g, (1, 1, 0) : g \in G\}$
3. $K_1 \cup K_2 = \{a, \max\{P_{K_1}(a), P_{K_2}(a)\}, \max\{Q_{K_1}(a), Q_{K_2}(a)\}, \min\{R_{K_1}(a), R_{K_2}(a)\} : a \in G\}$
4. $K_1 \cap K_2 = \{a, \min\{P_{K_1}(a), P_{K_2}(a)\}, \min\{Q_{K_1}(a), Q_{K_2}(a)\}, \max\{R_{K_1}(a), R_{K_2}(a)\} : a \in G\}$
5. $K_1^C = \{\prec a, R_K(a), 1 - Q_K(a), P{=}P_K(a) \succ : a \in G\}$

Definition 2.3. [13] Neutrosophic topology (nt) in Salama's sense on a nonempty set G is a family τ of ns in G satisfying three conditions:

1. Empty set ($0_\sim$) and universal set ($1_\sim$) are members of τ.
2. $K_1 \cap K_2 \in \tau$ where $K_1, K_2 \in \tau$.
3. $\cup K_\delta \in \tau$ for every $\{K_\delta : \delta \in \Delta\} \leq \tau$.

Definition 2.4. [11] The neutrosophic minimal structure space over a universal set G be denoted by N_m. N_m is said to be neutrosophic minimal structure space (in short, nms) over G if it satisfying following the axiom: $0_\sim, 1_\sim \in N_m$. A family of neutrosophic minimal structure space is denoted by (G, N_{mG}).
Note that neutrosophic empty set and neutrosophic universal set can form a topology and it is known as neutrosophic minimal structure space.

Remark 2.1. *[11] Each ns in nms is neutrosophic minimal open set (in short, nmo).*
Complement of nmo is neutrosophic minimal closed set (in short, nmc).

Definition 2.5. [11] A is N_m-closed if and only if $N_m$cl(A) = A. Similarly, A is a N_m-open if and only if $N_m$int(A) = A.

Definition 2.6. [11] Let N_m be any nms and A be any neutrosophic set. Then

1. Every A $\in N_m$ is open and its complement is N_m closed.
2. N_m-closure of A = min {F : F is a nmc and F $\geq$ A} and it is denoted by $N_m$cl(A).
3. N_m-interior of A = max {F : F is a nmo and F $\leq$ A} and it is denoted by $N_m$int(A).

In general $N_m$int(A) is subset of A and A is a subset of $N_m$cl(A).

Proposition 2.2. *[11] Let R and S are any ns of nms N_m over G. Then*

1. *$N_m^C = \{0, 1, R_i^C\}$ where R_i^C is a complement of ns R_i.*

2. *$G - N_m int(S) = N_m cl(G - S)$.*

3. *$G - N_m cl(S) = N_m int(G - S)$.*

4. *$N_m cl(R^C) = (N_m cl(R))^C = N_m int(R)$.*

5. *N_m closure of an empty set is an empty set and N_m closure of a universal set is a universal set. Similarly, N_m interior of an empty set and universal set respectively an empty and a universal set.*

6. *If S is a subset of R then $N_m cl(S) \leq N_m cl(R)$ and $N_m int(S) \leq N_m int(R)$.*

7. *$N_m cl(N_m cl(R)) = N_m cl(R)$ and $N_m int(N_m int(R)) = N_m int(R)$.*

8. *$N_m cl(R \vee S) = N_m cl(R) \vee N_m cl(S)$.*

9. *$N_m cl(R \wedge S) = N_m cl(R) \wedge N_m cl(S)$.*

Definition 2.7. Let (G, N_{mG}) be a nms and S $\leq$ G is said to be

1. N_m-semi-open set (in short, N_m-so) [9] if S $\leq$ $N_m$cl($N_m$int(S)).
2. N_m-pre-open set (in short, N_m-po) [10] if S $\leq$ $N_m$int($N_m$cl(S)).
   The complement of above N_m-open set is called an N_m-closed set.

Definition 2.8. [11] Let (G, N_{mG}) be nms.

1. Arbitrary union of nmo in (G, N_{mG}) is nmo. (Union Property).
2. Finite intersection of nmo in (G, N_{mG}) is nmo. (intersection Property).

Definition 2.9. [11] A function f: (G, N_{mG}) $\rightarrow$ (H, N_{mH}) is called neutrosophic minimal continuous map iff $f^{-1}(V) \in N_{mG}$ whenever V $\in$ N_{mH}.

Definition 2.10. [11] let A be a ns in nms (G, N_{mG}). Then Y is said to be neutrosophic minimal subspace if (H, N_{mH}) = {A $\cap$ U : U $\in$ N_{mH}}.

3. N_m-β-open sets

Definition 3.1. (G, N_{mG}) be a nms & S $\leq$ G is said to be N_m-β-open set (in short, N_m-βo) if S $\leq$ $N_m$cl($N_m$int($N_m$cl(S))).
The complement of an N_m-βo is called an N_m-β-closed set(in short, N_m-βc)

Remark 3.1. *(G, $\mathcal{T}$) be a nt & $S \leq G$ is said to be $\mathcal{N}$-β-open set [3] if $S \leq \mathcal{N} cl(\mathcal{N} int(\mathcal{N} cl(S)))$. If the nms N_{mG} is a topology, clearly an N_m-βo is $\mathcal{N}$-β-open.*

Above definition of 3.1, trivially the following statement are obtained.

Lemma 3.1. *Consider (G, N_{mG}) be a nms.*

1. *Every N_m-open is N_m-βo.*
2. *S is an N_m-βo iff $S \leq N_m cl(N_m int(N_m cl(S)))$.*

3. *Every N_m-closed set is N_m-β-closed.*

4. *S is an N_m-β-closed set iff N_m int(N_m cl(N_m int(S)) $\leq$ S.*

Theorem 3.1. *(G, N_{mG}) be a nms. Any union of N_m-β o is N_m-β o.*

Proof. Suppose A_δ be an N_m-βo for $\delta \in \Delta$. Above definition 3.1 and Proposition 2.2(6), $A_\delta \leq N_m cl(N_m int(N_m cl(A_\delta)))$ $\leq N_m cl(N_m int(N_m cl(\bigcup A_\delta)))$. This implies $\bigcup A_\delta \leq N_m cl(N_m int(N_m cl(\bigcup A_\delta)))$. Hence $\bigcup A_\delta$ is an N_m-βo. ☐

Remark 3.2. *Consider (G, N_{mG}) be a nms. Intersection of any 2 N_m-β o may not be N_m-β o.*

Example 3.1. *Consider G = {a} with $N_m = \{0_\sim, P, Q, R, S, 1_\sim\}$ and $N_m^C = \{1_\sim, I, J, K, L, 0_\sim\}$ where*
$P = \prec (0.5, 0.6, 0.6) \succ$; $Q = \prec (0.4, 0.6, 0.8) \succ$
$R = \prec (0.4, 0.7, 0.9) \succ$; $S = \prec (0.5, 0.7, 0.6) \succ$
$I = \prec (0.6, 0.4, 0.5) \succ$; $J = \prec (0.8, 0.4, 0.4) \succ$
$K = \prec (0.9, 0.3, 0.4) \succ$; $L = \prec (0.6, 0.3, 0.5) \succ$
We know that $0_\sim = \{\prec g, 0, 0, 1 \succ : g \in G\}$, $1_\sim = \{\prec g, 1, 1, 0 \succ : g \in G\}$ and $0_\sim^C = \{\prec g, 1, 1, 0 \succ : g \in G\}$, $1_\sim^C = \{\prec g, 0, 0, 1 \succ : g \in G\}$.
Now we define the two N_m-β os as follows:
$A = \prec (0.6, 0.7, 0.9) \succ$; $B = \prec (0.5, 0.8, 0.4) \succ$
Here $N_m cl(A) = 0_\sim^C$, $N_m int(N_m cl(A)) = 1_\sim$, $N_m cl(N_m int(N_m cl(A))) = 0_\sim^C$ and
$N_m cl(B) = 0_\sim^C$, $N_m int(N_m cl(B)) = 1_\sim$, $N_m cl(N_m int(N_m cl(A))) = 0_\sim^C$. But $A \wedge B = \prec (0.5, 0.7, 0.9) \succ$ is not a N_m-β o in G.

Proposition 3.1. *Let (G, N_{mG}) be a nms.*

1. *If S is a N_m so then it is a N_m-β o.*
2. *If S is a N_m-po then it is a N_m-β o.*

Proof. (1) The proof is straightforward from the definitions.
(2) The proof is straightforward from the definitions. ☐

Definition 3.2. Let (G, N_{mG}) be a nms.

1. N_m-β-closure of A = min {S : S is N_m-β-closed set and S $\geq$ A} and it is denoted by N_m-βcl (A).
2. N_m-β-interior of A = max {V : V is N_m-βo and V $\leq$ A} and it is denoted by N_m-βint(A).

Theorem 3.2. *Suppose (G, N_{mG}) be a nms and R, S $\leq$ G. Then*

1. *N_m-β int($0_\sim$) = $0_\sim$.*
2. *N_m-β int($1_\sim$) = $1_\sim$.*
3. *N_m-β int(R) $\leq$ R.*
4. *If R $\leq$ S, then N_m-β int(R) $\leq$ N_m-β int(S).*

5. *R is N_m-β o iff N_m-β int(R) = R.*

6. *N_m-β int(N_m-β int(R)) = N_m-β int(R).*

7. *N_m-β cl ($G - R$) = $G - N_m$-β int(R).*

Proof. (1), (2) are Obvious.
(3), (4) are Obvious.
(5) It follows from Theorem 3.1.
(6) It follows condition from (5).
(7) For $R \leq G$, $G - N_m$-βint(R) = $G -$ max $\{U : U \leq R, U$ is N_m-βo$\}$ = min $\{ G - U : U \leq R, U$ is N_m-βo$\}$ = min $\{G - U : G - R \leq G - U\}$, U is N_m-βo$\}$ = N_m-βcl ($G - R$). □

Theorem 3.3. *Let (G, N_{mG}) be a nms and R, $S \leq G$. Then*

1. *N_m-β cl ($0_\sim$) = $0_\sim$.*

2. *N_m-β cl ($1_\sim$) = $1_\sim$.*

3. *$R \leq N_m$-β cl (R).*

4. *If $R \leq S$, then N_m-β cl (R) $\leq N_m$-β cl (S).*

5. *R is N_m-β c iff N_m-β cl (R) = R.*

6. *N_m-β cl (N_m-β cl (R)) = N_m-β cl (R).*

7. *N_m-β int($G - R$) = $G - N_m$-β cl (R).*

Proof. It is similar to the proof of above Theorem 3.2. □

Theorem 3.4. *Let (G, N_{mG}) be a nms and $S \leq G$. Then*

1. *$g \in N_m$-β cl (S) iff $S \cap V \neq \emptyset$ for every N_m-β o V containing g.*

2. *$g \in N_m$-β int(S) iff there exists an N_m-β o U such that $U \leq S$.*

Proof. (1) Suppose there is an N_m- βo V containing g such that $S \cap V = \emptyset$. Then $G - V$ is an N_m-βc such that $S \leq G - V$, $g \notin G - V$. This implies $g \notin N_m$-βcl (S).

The reverse relation is obvious.
(2) Obvious. □

Lemma 3.2. *Let (G, N_{mG}) be a nms and $S \leq G$. Then*

1. *N_m int(N_m cl(N_m int(S))) $\leq N_m$ int(N_m cl(N_m int(N_m-β int(S))) $\leq N_m$-β int(S).*

2. *N_m-β cl (S) $\leq N_m$ cl(N_m int(N_m cl(N_m-β cl(S))) $\leq N_m$ cl(N_m int(N_m cl(S))).*

Proof. (1) For $S \leq G$, by Theorem 3.3, N_m-βcl (S) is an N_m-βc set. Hence from Lemma 3.1, we have $N_m$int($N_m$cl($N_m$int(S))) $\leq N_m$int($N_m$cl($N_m$int(N_m-βint(S)))) $\leq N_m$-βint(S).
(2) It is similar to the proof of (1). □

4. N_m-β-continuous map

Definition 4.1. Map f : (G, N_{mG}) → (H, N_{mH}) is said to be N_m-β-continuous if f^{-1}(O) is a N_m-βo in G, for each N_m-open O in H.

Theorem 4.1. *1. Every neutrosophic minimal continuous is N_m-β-continuous but not conversely.*

2. Every N_m-semi-continuous is N_m-β-continuous but not conversely.

3. Every N_m-pre-continuous is N_m-β-continuous but not conversely.

Proof. (1) The proof follows from [Lemma 3.1 (1)].
(2) The proof follows from [Proposition 3.1 (1)].
(3) The proof follows from [Proposition 3.1 (2)]. □

Theorem 4.2. *Map f : G → H be a function on 2 nms (G, N_{mG}) and (H, N_{mH}). Then the following statements are equivalent:*

1. *f is N_m-β-continuous.*
2. *f^{-1}(O) is an N_m-βo, for each N_m-open set O in H.*
3. *f^{-1}(S) is an N_m-βc set, for each N_m-closed S in H.*
4. *f(N_m-βcl (R)) ≤ $N_m$cl(f(R)), for R ≤ G.*
5. *N_m-βcl (f^{-1}(S)) ≤ f^{-1}($N_m$cl(S)), for S ≤ H.*
6. *f^{-1}($N_m$int(S)) ≤ N_m-βint(f^{-1}(S)), for S ≤ H.*

Proof. (1) ⇒ (2) Let O be an N_m-open in H and g ∈ f^{-1}(O). By hypothesis, there exists an N_m-βo U_g containing g such that f(U) ≤ O. This implies g ∈ U_g ≤ f^{-1}(O) for all g ∈ f^{-1}(O). Hence by Theorem 3.1, f^{-1}(O) is N_m-βo.
(2) ⇒ (3) Obvious.
(3) ⇒ (4) For R ≤ G, f^{-1}($N_m$cl(f(R))) = f^{-1}(min {F ≤ H : f(R) ≤ F and F is N_m-closed}) = min {f^{-1}(F) ≤ G : R ≤ f^{-1}(F) and F is N_m-βc} ≥ min {K ≤ G : R ≤ K and K is N_m-βc} = N_m-βcl (R). Hence f(N_m-βcl (R)) ≤ $N_m$cl(f(R)).
(4) ⇒ (5) For R ≤ G, from (4), it follows f(N_m-βcl (f^{-1}(R))) ≤ $N_m$cl(f(f^{-1}(R))) ≤ $N_m$cl(R). Hence we get (5).
(5) ⇒ (6) For S ≤ H, from $N_m$int(S) = Y − $N_m$cl(H − S) and (5), it follows: f^{-1}($N_m$int(S)) = f^{-1}(Y − $N_m$cl(H − S)) = G − f^{-1}($N_m$cl(H − S)) ≤ G − N_m-βcl (f^{-1}(H − S)) = N_m-βint(f^{-1}(S)). Hence (6) is obtained.
(6) ⇒ (1) Let g ∈ G and O an N_m-open set containing f(g). Then from (6) and Proposition 2.2, it follows g ∈ f^{-1}(O) = f^{-1}($N_m$int(O)) ≤ N_m-βint(f^{-1}(O)). So from Theorem 3.4, we can say that there exists an N_m-βo U containing g such that g ∈ U ≤ f^{-1}(O). Hence f is N_m-β-continuous. □

Theorem 4.3. *Map f : G → H be a function on 2 nms (G, N_{mG}) and (H, N_{mH}). Then the following statements are equivalent:*

1. *f is N_m-β-continuous.*
2. *$f^{-1}(O) \leq N_m cl(N_m int(f^{-1}(O)))$, for each N_m-open O in H.*
3. *$N_m int(N_m cl(f^{-1}(F))) \leq f^{-1}(F)$, for each N_m-closed set F in H.*
4. *$f(N_m int(N_m cl(R))) \leq N_m cl(f(R))$, for $R \leq G$.*
5. *$N_m int(N_m cl(f^{-1}(S))) \leq f^{-1}(N_m cl(S))$, for $S \leq H$.*
6. *$f^{-1}(N_m int(S)) \leq N_m cl(N_m int(f^{-1}(S)))$, for $S \leq H$.*

Proof. (1) ⇔ (2) It follows from Theorem 4.2 and Definition of N_m-βos.
(1) ⇔ (3) It follows from Theorem 4.2 and Lemma 3.1.
(3) ⇒ (4) Let R ≤ X. Then from Theorem 4.2(4) and Lemma 3.2, it follows $N_m int(N_m cl(R)) \leq N_m$-$\beta cl(R)$) $\leq f^{-1}(N_m cl(f(R)))$. Hence $f(N_m int(N_m cl(R))) \leq N_m cl(f(R))$.
(4) ⇒ (5) Obvious.
(5) ⇒ (6) From (5) and Proposition 2.2, it follows: $f^{-1}(N_m int(S)) = f^{-1}(H - N_m cl(H - S)) = G - f^{-1}(N_m cl(H - S)) \leq G - N_m int(N_m cl(f^{-1}(H - S)))$
$= N_m cl(N_m int(f^{-1}(S)))$. Hence, (6) is obtained.
(6) ⇒ (1) Let O be an N_m-open in H. Then by (6) and Proposition 2.2, we have $f^{-1}(O) = f^{-1}(N_m int(O)) \leq N_m cl(N_m int(f^{-1}(O)))$. This implies $f^{-1}(O)$ is an N_m-βo. Hence by (2), f is N_m-β-continuous. □

Definition 4.2. [10] (G, N_{mG}) be a nms. Then G is said to be N_m-T_2 if for each distinct points g and h of G, there exist two disjoint N_m-open U, V such that g ∈ U and h ∈ V.

Definition 4.3. (G, N_{mG}) be a nms. Then G is said to be N_m-β-T_2 if for any distinct points g and h of G, there exist disjoint N_m-βo C, D such that g ∈ C and h ∈ D.

Theorem 4.4. *Map $f : G \to H$ be a map on two nms (G, N_{mG}) and (H, N_{mH}). If f is an injective and N_m-β continuous map and if H is N_m-T_2, then G is N_m-β-T_2.*

Proof. Obvious. □

Theorem 4.5. *Map $f : G \to H$ be a map on two nms (G, N_{mG}) and (H, N_{mH}). If f is an injective and N_m-β continuous map with an N_m-β-closed graph, then G is N_m-β-T_2.*

Proof. Suppose g_1 and g_2 be any distinct points of G. Then $f(g_1) \neq f(g_2)$, so $(g_1, f(g_2)) \in (G \times H) - L(f)$. Since the graph L(f) is N_m-βc, there exist an N_m-βo containing g_1 and $D \in N_{mH}$ containing $f(g_2)$ such that $f(C) \cap D = \emptyset$. Since f is N_m-βcontinuous, $f^{-1}(D)$ is an N_m-βo containing g_2 such that $C \cap f^{-1}(D) = \emptyset$. Hence G is N_m-β-T_2. □

Definition 4.4. [10] (G, N_{mG}) be a nms and S ≤ G, S is called N_m-compact (respectively, almost N_m-compact) relative to S if every collection $\{U_i : i \in \Delta\}$ of N_m-open subsets of G such that $S \leq \max\{U_i : i \in \Delta\}$, there exists a finite subset Δ_0 of Δ such that $S \leq \max\{U_j : j \in \Delta_0\}$ (respectively, $S \leq \max\{N_m cl(U_j) : j \in \Delta_0\}$). (G, N_{mG}) be a nms and S ≤ G, S is said to be N_m-compact (respectively, almost N_m-compact) if S is N_m-compact (respectively, almost N_m-compact) as a neutrosophic minimal subspace of G.

Definition 4.5. (G, N_{mG}) be a nms and S $\leq$ G, S is called N$_m$-β-compact (respectively, almost N$_m$-β-compact) relative to S if every collection $\{U_\delta : \delta \in \Delta\}$ of N$_m$-β-open subsets of G such that S $\leq$ max $\{U_\delta : \delta \in \Delta\}$, there exists a finite subset Ω of Δ such that S $\leq$ max $\{U_\omega : \omega \in \Omega\}$ (respectively, S $\leq$ max $\{N_m \beta cl(U_\omega) : \omega \in \Omega\}$). (G, N_{mG}) be a nms and S $\leq$ G, S is said to be N$_m$-β-compact (respectively, almost N$_m$-β-compact) if S is N$_m$-β-compact (resp. almost N$_m$-β-compact) as a neutrosophic minimal subspace of G.

Theorem 4.6. *Map f : G $\rightarrow$ H be a map on 2 nms (G, N_{mG}) and (H, N_{mH}). If S is an N_m-β-compact set, then f(S) is N_m-compact.*

Proof. Obvious. ☐

5. N$_m$-regular open

We introduce following definitions

Definition 5.1. (G, N_{mG}) be a nms and A $\leq$ G, A is called N$_m$-regular open (in short, N$_m$-ro) if A = N$_m$int(N$_m$cl(A)).

Theorem 5.1. *Any N_m-ro is N_m-open.*

Proof. If A is N$_m$-ro in (G, N_{mG}), A = N$_m$int(N$_m$cl(A)). Then N$_m$int(A) = N$_m$int(N$_m$int(N$_m$cl(A))) = N$_m$int(N$_m$cl(A)) = A. That is, Ais N$_m$-open in (G, N_{mG}). ☐

Example 5.1. *G = {a} with $N_m = \{0_\sim, P, 1_\sim\}$ and $N_m^C = \{1_\sim, Q, 0_\sim\}$ where*
P = $\prec$ (0.5, 0.5, 0.5)$\succ$; Q = $\prec$ (0.5, 0.5, 0.5)$\succ$
Now we define the N_m-ro sets as follows:
A = $\prec$ (0.5, 0.5, 0.5)$\succ$
Here N_m cl(A) = Q, N_m int(N_m cl(A)) = P is a N_m-ro in G.

Definition 5.2. (G, N_{mG}) be a nms and S $\leq$ G, S is said to be N$_m$-π-open set if S is the finite union of N$_m$-ro.

Remark 5.1. *For a subset of A of an nms (G, N_{mG}), we have following implications:*

N_m-regular open $\Rightarrow$ N_m-π-open $\Rightarrow$ N_m-open

Diagram-I

Example 5.2. *G = {a} with $N_m = \{0_\sim, P, L, 1_\sim\}$ and $N_m^C = \{1_\sim, M, N, 0_\sim\}$ where*
P = $\prec$ (0.1, 0.5, 0.1)$\succ$; L = $\prec$ (0.5, 0.5, 0.5)$\succ$
M = $\prec$ (0.1, 0.5, 0.1)$\succ$; N = $\prec$ (0.5, 0.5, 0.5)$\succ$
Now we define the two N_m-ro sets as follows:
A = $\prec$ (0.1, 0.5, 0.1)$\succ$
B = $\prec$ (0.5, 0.5, 0.5)$\succ$
Here N_m cl(A) = M, N_m int(N_m cl(A)) = P ; N_m cl(B) = N, N_m int(N_m cl(B)) = L is a N_m-ro set in G. Here, A $\vee$ B = $\prec$ (0.5, 0.5, 0.1)$\succ$ is a N_m-π-open sets but it is not a N_m-ro.

Example 5.3. *$G = \{a\}$ with $N_m = \{0_\sim, A, 1_\sim\}$ and $N_m^C = \{1_\sim, B, 0_\sim\}$ where*
$A = \prec (0.6, 0.7, 0.3) \succ$; $B = \prec (0.3, 0.3, 0.6) \succ$
Now we define the N_m-ro sets as follows:
$R = \prec (0, 0, 1) \succ$; $S = \prec (1, 1, 0) \succ$
Here $R \vee S \prec (1, 1, 0) \succ$ is a N_m-π-open set in G. Here, $A = \prec (0.6, 0.7, 0.3) \succ$ is N_m-open but it is not a N_m-π-open.

Conclusion

We presented several definitions, properties, explanations and examples inspired from the concept of N_m-β-open, N_m-regular-open and N_m-π-open. The results of this study may be help in many reserches.

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