Computers & Industrial Engineering 116 (2018) 144–162

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Some new Hamacher aggregation operators under single-valued neutrosophic 2-tuple linguistic environment and their applications to multi-attribute group decision making

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ARTICLE INFO

Keywords:
Multiple attribute group decision making
Single valued neutrosophic 2-tuple linguistic set
Aggregation operator
Weights
Emergency management

ABSTRACT

This paper proposes an approach to linguistic multiple attribute group decision making (MAGDM) problem with single-valued neutrosophic 2-tuple linguistic (SVN2TL) assessment information by adding a subjective imprecise estimation of reliability of the 2-tuple linguistic terms (2TLTs). SVN2TL includes the truth-membership (*TM*), indeterminacy-membership (*IM*) and faulty-membership (*FM*), which can express the incomplete, indeterminate and inconsistent information perfectly and avoid information and precision losing in aggregation process ideally. We first propose the concept of SVN2TL set (SVN2TLS) and single valued neutrosophic 2-tuple linguistic element (SVN2TLE), basic operational rules on SVN2TLEs via Hamacher triangular norms, and ranking method for SVN2TLEs. Then, some SVN2TL aggregation operators including SVN2TL Hamacher weighted averaging (SVN2TLHWA) operator, SVN2TL Hamacher geometric weighted averaging (SVN2TLHGWA) operator, are developed, their some properties are investigated as well. Moreover, we apply new operators to develop approach to MAGDM problem with SVN2TL assessment information, where a model for optimal weighting vector is constructed. Finally, an numerical example related to evaluation of emergency response solutions for sustainable community development is provided to show the utility and effectiveness of the method described in this paper. A sensitivity and comparative analysis are also conducted to demonstrate the strength and practicality of the proposed method.

1. Introduction

Multiple attribute group decision making (MAGDM) is an important and hot topic in modern decision fields. Since Zadeh (1965) proposed the fuzzy set (FS) theory, researches on FS have made a large number of achievements in many fields, including intelligent fuzzy control (Filip & Szeidert, 2016; Mendel & Wu, 2010), decision support system (Gong, Xu, Li, & Xu, 2015; Gong, Zhang, Forrest, Li, & Xu, 2015; Merigó, Gil-Lafuente, & Martorell, 2012) and so on. However, one of the shortcomings of FS is that it only reflects the degree of membership, but dose not take the degree of non-membership into consideration. Therefore, as an extension of FS, Atanassov (1986) introduced the intuitionistic fuzzy set (IFS), which can effectively make up the shortcomings of FS by adding a non-membership. Although FS and IFS have been successfully applied and improved, they are not capable of dealing with each sort of special cases under fuzzy condition in practical matters. For instance, in daily life, people prefer to express their evaluation by linguistic term sets (LTSs), terms like 'very bad', 'somewhat bad', 'good' and so on instead of numeric values or FS and IFS, thus, LTSs are effective when dealing with complex or ill-defined situations.

When it comes to representation models of LTSs, the most frequently-used three main methods for computing with words (CWW) are as follows: (1) Models based on transformation functions, where there is a one-to-one correspondence between fuzzy numbers and LTSs, such as triangular fuzzy number, trapezoidal fuzzy number and so on (Delgado, Herrera, Herrera-Viedma, & Martinez, 1998; Jiang, Fan, & Ma, 2008; Zadeh, 1975); (2) Models based on the linguistic symbolic representation, which compute words with the index of linguistic labels directly (Merigó, Casanovas, & Palacios-Marqus, 2014; Xu, Merigó, & Wang, 2012); (3) Models based on 2-tuple symbolic representation (Merigó & Gil-Lafuente, 2013; Wu et al., 2015; Xu & Wang, 2011), which transform linguistic information into consecutive 2-tuple linguistic terms (2TLTs). For the former two methods, the evaluated LTSs are discrete, so the calculation results may not meet the initial LTSs, which could

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https://doi.org/10.1016/j.cie.2017.12.024
Received 7 June 2017; Received in revised form 20 November 2017; Accepted 25 December 2017
Available online 27 December 2017
0360-8352/

cause loss of details and information. While the 2-tuple linguistic representation proposed by Herrera and Martínez (2000a, 2000b), has exact characteristic in expressing any counting of information in the universe of discourse, information distortion and losing in the linguistic information processing can be solved satisfactorily, thus it has been widely used in many domains and gotten favorable performance in the past few years (Dong & Herrera-Viedma, 2015; Dong, Li, & Herrera, 2016; Gong, Forrest, & Yang, 2013; Martínez & Herrera, 2012; Tao, Chen, Zhou, & Liu, 2014; Wang, Wang, Zhang, & Chen, 2016; Zhang, Xu, & Wang, 2016).

Nevertheless, in fact, when we use LTs or 2TLTs to express our preference, it is impliedly consented that the truth-membership (*TM*) associated with the LTs or 2TLTs is 1, while the faulty-membership (*FM*) associated with it is unclear. In order to conquer this deficiency, more and more researchers turn their attention to the combination of linguistic representation models and some specific FSs. For instance, the intuitionistic linguistic sets (ILSs), proposed by Wang and Li (2009), can be seen as the combination of LTSs and IFS. Chen, Liu, and Pei (2015) proposed linguistic intuitionistic fuzzy numbers (LIFNs) by integrating linguistic models and IFSs. The linguistic truth-valued intuitionistic fuzzy sets (LTVIFSs) (Zou, Wen, & Wang, 2016), in which the LTVIFS is based on the point view of IFS and linguistic truth-valued lattice implication algebra.

For ILS, there still exists a defect that it can't settle some incomplete or inconsistent information in practical problems. For instance, in assessment of emergency response solutions for sustainable community development, suppose that one DM states his/her assessment on the attribute of rescuing capacity is 'good', moreover, he/she estimates that the possibility of that his/her judge is right is about 70%, the possibility of that his/her judge is false is about 40%, the possibility of that his/her judge is unsure is about 30%. This issue can't be directly disposed by means of FSs or IFSs. Therefore, new theory and method need to be presented. To deal with this particular type of problem, Smarandache (Smarandache, 1999) originally proposed the neutrosophic set (NS) by adding an independent indeterminacy-membership (*IM*) on the basis of IFS, which can be seen as a generalization of FS, IFS and so on. The NS consists of three parts of memberships including the *TM*, the *FM* and the *IM*, among which the three variables are completely independent. In fact, the NS comes from the point of philosophical view, thus it is difficult to put it into practical scientific and engineering operations. To overcome the complication mentioned above, several subclasses of NSs have been put forward over the years, and it has become an interesting and popular research subject. The single-valued neutrosophic set (SVNS), proposed by Wang, Smarandache, Zhang, and Sunderraman (2010), is a specific subclass of the NS which changes the condition to that $T(x),I(x),F(x) \in [0,1]$ and $0 \leqslant T(x) + I(x) + F(x) \leqslant 3$. What we can observe from the SVNS is that it gives us an additional possibility to represent imprecise, incomplete and inconsistent information which exists in real world. By this way, it would be more reasonable in handling with indeterminate and inconsistent information. Since then, other subclasses of NSs such as interval neutrosophic sets (INSs) (Wang, Smarandache, Zhang, & Sunderraman, 2005), neutrosophic soft set (Deli & Broumi, 2015), single-valued neutrosophic linguistic set (SVNLS) and other NSs (Broumi & Deli, 2016; Broumi & Smarandache, 2014; Deli, 2017; Ji, Wang, & Zhang, 2016; Liang, Wang, & Li, 2016; Liu & Wang, 2014; Li, Zhang, & Wang, 2017; Peng, Wang, Wang, Zhang, & Chen, 2016; Tian, Wang, Wang, & Zhang, 2017; Tian, Wang, Zhang, & Wang, 2016; Tian, Zhang, Wang, Wang, & Chen, 2016; Ye, 2013, 2014a, 2015a, 2015b, 2017a) have been investigated by many researchers.

The solution for linguistic decision problems usually includes three steps: (1) Selecting suitable LTS; (2) Choosing appropriate aggregation technique; (3) Obtaining the best alternative(s). With the fact that step (2) plays a key role in information fusion, therefore, researches on linguistic aggregation operators under NS environment have been receiving more attention. For example, Liu and Shi (2017) proposed the neutrosophic uncertain linguistic number improved generalized weighted Heronian mean (NULNIGWHM) operator. Wang, Yang, and Li (2016) extended a series of Maclaurin symmetric mean aggregation techniques under single-valued neutrosophic linguistic environment and applied them for solving MCDM problems. Tian, Wang, Zhang, Chen, and Wang (2015) proposed a MCDM approach based on the simplified neutrosophic linguistic normalized weighted Bonferroni mean (SNLNWBM) operator. Ma, Wang, Wang, and Wu (2017) developed some prioritized harmonic mean operators in an interval neutrosophic linguistic environment and applied them to a practical medical treatment selection problem. Ye (2014b) defined two aggregation operators for interval neutrosophic uncertain linguistic information and applied them to solve MAGDM problems.

All aforementioned aggregation operators rely on the simple handling of subscripts of LTs in the linguistic part, and the NSs part are mainly based on the Algebraic t-norm and t-conorm. It is noted that the operational laws defined in the linguistic part are not closed and fail to process original information. Drawbacks on granularity and logical problems for existing operation laws are verified as follows:

(1) **Granularity problem**: All operations are carried out directly on the basis of the subscripts of LTs. In fact, operational rules mentioned above are defined on a given LTS, while the calculated results would go out of the original LTS. For example, Let $H = \{h_0,h_1,...,h_6\}$ be a LTS, $a_1 = \langle h_4,(0.3,0.5,0.2)\rangle$ and $a_2 = \langle h_5,(0.6,0.2,0.2)\rangle$ be two single-valued neutrosophic linguistic numbers (SVNLNs). According to the operational rules defined in Ye (2014b), we can obtain $a_1 \oplus a_2 = \langle h_9,(0.72,0.1,0.04)\rangle$ and $a_1 \otimes a_2 = \langle h_{20},(0.18,0.6,0.36)\rangle$, it is obvious that the LTs h_9 and h_{20} exceed the range of H;
(2) **Logical problem**: On one hand, for a LTS $H = \{h_0,h_1,...,h_6\}$, the addition identifies a new LTS with $2g$ LTs, while the corresponding number of LTs on multiplication operation is g^2. In other words, we need to use different granularity standards to give the assessment of LTs; on the other hand, the linear operations cannot reflect the non-linearity of logical thinking.

To overcome these drawbacks, aggregation operators based on the Archimedean t-norms and t-conorms have been developed over the last decades. Xia, Xu, and Zhu (2012) extended some intuitionistic fuzzy aggregation operators based on Archimedean t-conorm and t-norm. Qin, Liu, and Pedrycz (2016) developed a family of hesitant fuzzy aggregation operators with the help of Frank t-conorm and t-norm. Garg (2016) constructed a number of generalized intuitionistic fuzzy interactive geometric interaction operators using Einstein t-norm and t-conorm. Peng, Wang, Wu, and Tian (2017) proposed the hesitant intuitionistic fuzzy weighted averaging operator and the hesitant intuitionistic fuzzy power weighted averaging operator based on the Archimedean t-norms and t-conorms. Tan, Yi, and Chen (2015) introduced some new operational rules of hesitant fuzzy sets based on the Hamacher t-norm and t-conorm, and then proposed a family of hesitant fuzzy Hamacher operators.

As an important style of the Archimedean t-norm and t-conorm, Hamacher t-norm and t-conorm (Hamacher, 1975) were used to construct a number of fuzzy aggregation operators. The Hamacher t-norm and t-conorm, which are interesting generalizations of Algebraic and Einstein t-norms and t-conorms, are general and flexible family of continuous triangular norms. Since the Hamacher t-norm and t-conorm involve a certain parameter, this makes them more flexible in the process of information fusion and more adequate to model practical decision making problems. This indicates why Hamacher aggregation operators are addressed in this paper.

There is also a certain point which needs to be highlighted, to the best of our knowledge, there exists few literature dealing with neutrosophic 2-tuple linguistic set and neutrosophic 2-tuple linguistic element, both of which are very important in practical decision applications. In order to express and deal with complex and imprecise linguistic assessment information, we define a new subclass of NSs, single-valued neutrosophic 2-tuple linguistic sets (SVN2TLSs), which can reflect uncertainty of DM and settle some incomplete or inconsistent information in practical problems perfectly. Meanwhile, we notice that the operational rules defined for some subclasses of NSs are not closed and may be irrational. Motivated by this fact, it becomes beneficial to study a family of aggregation operators based on Hamacher operations with regard to SVN2TL information and subsequently apply them to MAGDM problems. Those are the two strong motivating factors behind the study reported in this paper.

At the same time, in practical MAGDM problem, the attribute weights may be unknown. In this circumstance, we should determine the attribute weights firstly. There are many methods for obtaining attribute weights (Dong, Xiao, Zhang, & Wang, 2016; Jin, Ni, Chen, Li, & Zhou, 2016; Tian, Wang, Wang, & Zhang, 2017; Xu & Da, 2010; Yager, 1988; Zhou, Chen, & Liu, 2012), which can mainly be divided into two categories: objective weighting methods (Dong et al., 2016; Jin et al., 2016; Xu & Da, 2010; Yager, 1988) and subjective weighting methods (Tian et al., 2017; Zhou et al., 2012). Objective weighting methods are based on some mathematical models where DMs fail in determining the relative importance of attributes, while subjective weighting methods determine the weighting vector based on preferences of DMs. In this paper, an objective optimal weighting model based on the maximizing deviation method is proposed to obtain the effective and reliable weights of attributes. Finally, based on the SVN2TL Hamacher operators and new model for the optimal weights, we develop an approach for SVN2TL MAGDM problem.

To achieve above contents, the reminder of this paper is allocated as follows. In Section 2, we briefly review some basic definitions and notations including the 2-tuple linguistic representation model, SVNLSs and the Hamacher t-norm and t-conorm. Section 3 proposes the concept of SVN2TLS and Hamacher operational rules for SVN2TLEs, some properties of proposed operational rules, such as the closure, commutativity, associativity and so on, are also considered in this subsection. Besides, a ranking method for SVN2TLEs related to four indexes is proposed for comparison. The SVN2TLHWA and the SVN2TLHGWA aggregation operators are put forward in Section 4, we also prove some desirable properties and discuss some special cases. Section 5 proposes a MAGDM method based on the new aggregation operators with SVN2TLEs, where a new model for the optimal weights is proposed as well. In Section 6, an example is provided to illustrate the efficiency of the proposed method, and some previous methods are mentioned to make comparisons with it. Section 7 gives some concluding remarks.

2. Preliminaries

This section succinctly introduces some basic concepts associated with SVN2TLS, including the 2-tuple linguistic representation model, the SVNLS and the Hamacher t-norm and t-conorm.

2.1. The 2-tuple linguistic representation model

Suppose that $H=\{h_0,h_1,...,h_g\}$ is a pre-established finite and totally ordered discrete term set with odd cardinalities, where $h_p(p=0,1,...,g)$ denotes the pth LT of H and represents a possible value for a linguistic evaluation information, $g+1$ is the cardinality of H and $g+1\geqslant 0$. H satisfies the following characteristics:

(1) The set is ordered: $h_p \geqslant h_q$ if $p \geqslant q$;
(2) There is a negation operator: $Neg(h_p)=h_q$, such that $q=g-p$;
(3) Maximum operator: $\max\{h_p,h_q\}=h_p$, if $h_p \geqslant h_q$;
(4) Minimum operator: $\min\{h_p,h_q\}=h_p$, if $h_p \leqslant h_q$.

To minimize the loss of linguistic information, the discrete LTS H can be extended to a continuous linguistic label, such that $h_p \in H=\{h_p|p\in[0,g]\}$. Just consider the opposite result, assume that there exists an aggregation value $\beta\in[0,g]$, but $\beta \notin \{0,1,...,g\}$. To obtain a better approximation for expressing the aggregation result β, a linguistic representation model which represents the aggregation result β by means of 2-tuples (h_p,α) is defined as follows.

Definition 1 (Herrera and Martínez, 2000b). Suppose that $H=\{h_0,h_1,...,h_g\}$ is a pre-established finite and totally ordered discrete LTS, $\beta\in[0,g]$ is a number value representing the aggregation result of linguistic symbolic. Then the function Δ identified to obtain the 2-tuple linguistic representation that expresses information equivalent to β is defined as:

$$\Delta\colon [0,g] \to H\times[-0.5,0.5). \tag{1}$$

$$\Delta(\beta)=(h_p,\alpha) \text{ with } \begin{cases} h_p, & p=round(\beta) \\ \alpha=\beta-p, & \alpha\in[-0.5,0.5) \end{cases} \tag{2}$$

where $round(.)$ is the usual round operation. h_p is the closest index label to β and α is the value of the symbolic translation, (h_p,α) is called a 2-tuple linguistic term (2TLT)

Definition 2 (Herrera and Martínez, 2000b). Suppose that $H=\{h_0,h_1,...,h_g\}$ is a pre-established finite and totally ordered discrete LTS and (h_p,α) be a 2TLT. There always exists an inverse function Δ^{-1} to transform the 2TLT into its equivalent numerical value, $\beta\in[0,g]\in R$, where

$$\Delta^{-1}\colon H\times[-0.5,0.5)\to[0,g], \tag{3}$$

$$\Delta^{-1}(h_p,\alpha)=p+\alpha=\beta. \tag{4}$$

Correspondingly, the additional characteristics of the 2TLTs can be defined as follows:

(1) There is a negation operator: $Neg((h_p,\alpha))=\Delta(g-\Delta^{-1}(h_p,\alpha))$;
(2) Ranking method: Let (h_p,α_p) and (h_q,α_q) be two 2TLTs, then
    (1) $\Delta^{-1}(h_p,\alpha_p)>\Delta^{-1}(h_q,\alpha_q)$, then (h_q,α_q) is smaller than (h_p,α_p);
    (2) $\Delta^{-1}(h_p,\alpha_p)=\Delta^{-1}(h_q,\alpha_q)$, then (h_q,α_q) and (h_p,α_p) represent the same information;
    (3) $\Delta^{-1}(h_p,\alpha_p)<\Delta^{-1}(h_q,\alpha_q)$, then (h_p,α_p) is smaller than (h_q,α_q).

2.2. The single-valued neutrosophic linguistic set

Definition 3 (Ye, 2015a). Let X be a space of points with a generic element in X, denoted by x and H be a set of LTS. A SVNLS in X is defined as:

$$A=\{\langle x,[h_{\theta(x)},(T_A(x),I_A(x),F_A(x))]\rangle | x\in X\}, \tag{5}$$

where $h_{\theta(x)}\in H, T_A(x),I_A(x),F_A(x)\in[0,1]$, with the condition $0\leqslant T_A(x)+I_A(x)+F_A(x)\leqslant 3$ for any $x\in X$. $T_A(x),I_A(x)$ and $F_A(x)$ represent the *TM* degree, the *IM* degree and the *FM* degree of the element x in X, respectively.

For convenience, the quaternary $\langle h_{\theta(x)},(T_A(x),I_A(x),F_A(x))\rangle$ is called a single valued neutrosophic linguistic element and A can be viewed as the collection of SVNLEs. Hence, the SVNLS can also be represented as:

$$A=\{\langle h_{\theta(x)},(T_A(x),I_A(x),F_A(x))\rangle | x\in X\}. \tag{6}$$

Specially, if $T_A(x)=1, I_A(x)=0$ and $F_A(x)=0$, then the SVNLE A degenerates to a normal linguistic variable.

2.3. Hamacher t-norm and t-conorm

Since Zadeh first introduced the max and min operations, the triangular norms have been intensively studied (Garg, 2016; Peng et al., 2017; Qin et al., 2016; Tan et al., 2015; Xia et al., 2012). Various triangular norms and corresponding triangular conorms, such as product t-norm and probabilistic sum t-conorm (Xia et al., 2012), Einstein t-norm and t-conorm (Garg, 2016), Algebraic t-norm and t-conorm (Ye, 2017a), etc., are vehicles to operate on FSs.

Hamacher t-norm and t-conorm, as a generalized form of Algebraic and Einstein triangular norms, are more general and flexible in practical operational rules.

The family $(T_\lambda^H)_{\lambda\in[0,\infty]}$ of Hamacher t-norm is given by

$$T_\lambda^H(x,y)=\frac{xy}{\lambda+(1-\lambda)(x+y-xy)},\lambda\geqslant 0. \tag{7}$$

The family $(S_\lambda^H)_{\lambda\in[0,\infty]}$ of Hamacher t-conorm is given by

$$S_\lambda^H(x,y)=\frac{x+y-xy-(1-\lambda)xy}{1-(1-\lambda)xy},\lambda\geqslant 0. \tag{8}$$

Some special cases of $T_\lambda^H(x,y)$ and $S_\lambda^H(x,y)$ are listed as follows:

(1) When $\lambda=0$, the Hamacher t-norm and t-conorm reduce to the Hamacher product and Hamacher sum, where:

$$T_0^H(x,y)=\frac{xy}{x+y-xy}, \tag{9}$$

$$S_0^H(x,y)=\frac{x+y-2xy}{1-xy}. \tag{10}$$

(2) When $\lambda=1$, the Hamacher t-norm and t-conorm reduce to the Algebraic t-norm and t-conorm:

$$T_1^H(x,y)=T_A(x,y)=x\cdot y, \tag{11}$$

$$S_1^H(x,y)=S_A(x,y)=x+y-x\cdot y. \tag{12}$$

(3) When , the Hamacher t-norm and t-conorm reduce to the Einstein t-norm and t-conorm:

$$T_2^H(x,y)=T_E(x,y)=\frac{xy}{1+(1-x)(1-y)}, \tag{13}$$

$$S_2^H(x,y)=S_E(x,y)=\frac{x+y}{1+xy}. \tag{14}$$

3. The single-valued neutrosophic 2-tuple linguistic sets and Hamacher operational rules of SVN2TLEs

This section introduces the advantages and applications of SVN2TLS. Then, operational rules via Hamacher t-norm and t-conorm and comparison rule for SVN2TLEs are presented.

3.1. The single-valued neutrosophic 2-tuple linguistic set

Definition 4. Let X be a space of points with a generic element in X, denoted by x, and H be a set of LTS. A SVNLS in X is defined as:

$$A=\{\langle x,[(h_{\theta(x)},\alpha_{\theta(x)}),(T_A(x),I_A(x),F_A(x))]\rangle|x\in X\}, \tag{15}$$

where $h_{\theta(x)}\in H,\alpha_{\theta(x)}\in[-0.5,0.5),T_A(x),I_A(x),F_A(x)\in[0,1]$, with the condition $0\leqslant T_A(x)+I_A(x)+F_A(x)\leqslant 3$ for any $x\in X$. $T_A(x),I_A(x)$ and $F_A(x)$ represent the *TM* degree, the *IM* degree and the *FM* degree of the element x in X to the 2TLT $(h_{\theta(x)},\alpha_{\theta(x)})$, respectively.

For convenience, the quintuplet $\langle(h_{\theta(x)},\alpha_{\theta(x)}),(T_A(x),I_A(x),F_A(x))\rangle$ is called a SVN2TLE and A can be viewed as the set of all SVN2TLEs. Hence, the SVN2TLS can be represented as:

$$\tilde{a}=\langle\left(h_{\tilde{a}},\alpha_{\tilde{a}}\right),\left(t_{\tilde{a}},i_{\tilde{a}},f_{\tilde{a}}\right)\rangle. \tag{16}$$

Specially, if $t_{\tilde{a}}=1,i_{\tilde{a}}=0$ and $f_{\tilde{a}}=0$, then the SVN2TLE $\tilde{a}$ degenerates to a normal 2TLT.

A SVN2TLE is an extension of the 2TLT and a single-valued neutrosophic number (SVNN). It is an interesting and new concept combining many benefits and advantages of 2TLT and SVNN. On one hand, the SVN2TLE can embody the closeness degree of an evaluation object and a 2TLT, and it can depict uncertainty and fuzziness more accurately. On the other hand, the SVN2TLE assigns *TM*,*IM*,*FM* functions to a specific 2TLT. Thus, SVN2TLEs are effective tools in solving problems with quantitative and qualitative expressions that involve incomplete and inconsistent information.

Compared with some new linguistic variables based on FSs and LTSs model, such as ILSs (Wang & Li, 2009), simplified neutrosophic linguistic sets (SNLSs) (Tian et al., 2017), single-valued neutrosophic linguistic set (SVNLS) (Ye, 2015a), the proposed SVN2TLSs have a wider range of application. The SVN2TLSs can express and deal with more complex linguistic assessment, such as incomplete or inconsistent information of a LT. Compared with the SVNLS introduced by Ye (2015a), SVN2TLSs can ensure information integrity in aggregation process. For SVNLS, the calculated results maybe do not match the initial LTs, so an approximation procedure should be introduced to express the result in the initial expression domain. The approximation procedure can make information lost. While SVN2TLSs do not need the approximation procedure and can avoid information loss and distortion.

3.2. Hamacher operational rules of SVN2TLEs

Definition 5 (*Ye, 2015a*). Let $a_p=\langle h_{\theta(a_p)},(T(a_p),I(a_p),F(a_p))\rangle(p=1,2)$ be two SVNLNs and $\lambda\geqslant 0$, then the operations of SVNLNs are defined as follows:

(1) $a_1\oplus a_2=\langle h_{\theta(a_1)+\theta(a_2)},(T(a_1)+T(a_2)$;
$-T(a_1)T(a_2),I(a_1)I(a_2),F(a_1)F(a_2))\rangle$
(2) $a_1\otimes a_2=\langle h_{\theta(a_1)\times\theta(a_2)},(T(a_1)T(a_2),I(a_1)+I(a_2)-I(a_1)I(a_2),F(a_1);$
$+F(a_2)-F(a_1)F(a_2))\rangle$
(3) $\lambda a_1=\langle h_{\lambda\theta(a_i)},(1-(1-T(a_1))^\lambda,I^\lambda(a_1),F^\lambda(a_1))\rangle$;
(4) $a_1^\lambda=\langle h_{\theta^\lambda(a_i)},(T^\lambda(a_1),1-(1-I(a_1))^\lambda,1-(1-F(a_1))^\lambda)\rangle$.

As for the issue discussed in Introduction part, the operational rules presented above are not closed and illogical. Therefore, it is meaningful and necessary to do some improvements. Hamacher's family of t-norm and t-conorm supply a wide class of t-norms and t-conorms operators. According to the Hamacher t-norm and t-conorm, Tan et al. (2015) proposed a family of hesitant fuzzy Hamacher operators for aggregating hesitant fuzzy information. Since the operational rules produced by the Hamacher t-norm and t-conorm are closed, inspired by this idea, we will propose some closed operational rules as follows.

Definition 6. Let $\tilde{a}_1=\langle(h_{\tilde{a}_1},\alpha_{\tilde{a}_1}),(t_{\tilde{a}_1},i_{\tilde{a}_1},f_{\tilde{a}_1})\rangle$ and $\tilde{a}_2=\langle(h_{\tilde{a}_2},\alpha_{\tilde{a}_2}),(t_{\tilde{a}_2},i_{\tilde{a}_2},f_{\tilde{a}_2})\rangle$ be two SVN2TLEs, and $\lambda\geqslant 0$, then the Hamacher operational rules of SVN2TLEs are defined as

Example 1. Let $H=\{h_0,h_1,...,h_6\}$ be a LTS, $a_1=\langle h_4,(0.3,0.5,0.2)\rangle$ and $a_2=\langle h_5,(0.6,0.2,0.2)\rangle$ be two SVNLNs, $\eta=2$ and . According to the

(1) $\tilde{a}_1 \oplus_H \tilde{a}_2 = \langle (h_{\tilde{a}_1},\alpha_{\tilde{a}_1}) \oplus_H (h_{\tilde{a}_2},\alpha_{\tilde{a}_2}),(t_{\tilde{a}_1} \oplus_H t_{\tilde{a}_2}, i_{\tilde{a}_1} \oplus_H i_{\tilde{a}_2}, f_{\tilde{a}_1} \oplus_H f_{\tilde{a}_2})\rangle$

$$= \left\langle \Delta\left(g\cdot\frac{g\cdot\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1}) + g\cdot\Delta^{-1}(h_{\tilde{a}_2},\alpha_{\tilde{a}_2}) - \Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1})\cdot\Delta^{-1}(h_{\tilde{a}_2},\alpha_{\tilde{a}_2}) - (1-\lambda)(\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1})\cdot\Delta^{-1}(h_{\tilde{a}_2},\alpha_{\tilde{a}_2}))}{g^2-(1-\lambda)(\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1})\cdot\Delta^{-1}(h_{\tilde{a}_2},\alpha_{\tilde{a}_2}))}\right), \left(\frac{t_{\tilde{a}_1} + t_{\tilde{a}_2} - t_{\tilde{a}_1}t_{\tilde{a}_2} - (1-\lambda)t_{\tilde{a}_1}t_{\tilde{a}_2}}{1-(1-\lambda)t_{\tilde{a}_1}t_{\tilde{a}_2}}, \frac{i_{\tilde{a}_1}i_{\tilde{a}_2}}{\lambda + (1-\lambda)(i_{\tilde{a}_1} + i_{\tilde{a}_2} - i_{\tilde{a}_1}i_{\tilde{a}_2})}, \frac{f_{\tilde{a}_1}f_{\tilde{a}_2}}{\lambda + (1-\lambda)(f_{\tilde{a}_1} + f_{\tilde{a}_2} - f_{\tilde{a}_1}f_{\tilde{a}_2})}\right)\right\rangle$$

(2) $\tilde{a}_1 \otimes_H \tilde{a}_2 = \langle (h_{\tilde{a}_1},\alpha_{\tilde{a}_1}) \otimes_H (h_{\tilde{a}_2},\alpha_{\tilde{a}_2}),(t_{\tilde{a}_1} \otimes_H t_{\tilde{a}_2}, i_{\tilde{a}_1} \otimes_H i_{\tilde{a}_2}, f_{\tilde{a}_1} \otimes_H f_{\tilde{a}_2})\rangle$

$$= \left\langle \Delta\left(g\cdot\frac{\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1})\cdot\Delta^{-1}(h_{\tilde{a}_2},\alpha_{\tilde{a}_2})}{\lambda g^2 + (1-\lambda)(\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1})\cdot g + \Delta^{-1}(h_{\tilde{a}_2},\alpha_{\tilde{a}_2})\cdot g - \Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1})\cdot\Delta^{-1}(h_{\tilde{a}_2},\alpha_{\tilde{a}_2}))}\right), \left(\frac{t_{\tilde{a}_1}t_{\tilde{a}_2}}{\lambda + (1-\lambda)(t_{\tilde{a}_1} + t_{\tilde{a}_2} - t_{\tilde{a}_1}t_{\tilde{a}_2})}, \frac{i_{\tilde{a}_1} + i_{\tilde{a}_2} - i_{\tilde{a}_1}i_{\tilde{a}_2} - (1-\lambda)i_{\tilde{a}_1}i_{\tilde{a}_2}}{1-(1-\lambda)i_{\tilde{a}_1}i_{\tilde{a}_2}}, \frac{f_{\tilde{a}_1} + f_{\tilde{a}_2} - f_{\tilde{a}_1}f_{\tilde{a}_2} - (1-\lambda)f_{\tilde{a}_1}f_{\tilde{a}_2}}{1-(1-\lambda)f_{\tilde{a}_1}f_{\tilde{a}_2}}\right)\right\rangle$$

(3) $$\eta \odot_H \tilde{a}_1 = \langle \eta \odot_H (h_{\tilde{a}_1},\alpha_{\tilde{a}_1}),(\eta \odot_H t_{\tilde{a}_1}, \eta \odot_H i_{\tilde{a}_1}, \eta \odot_H f_{\tilde{a}_1})\rangle = \left\langle \Delta\left(g\cdot\frac{(g + (\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1}))^{\eta} - (g-\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1}))^{\eta}}{(g + (\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1}))^{\eta} + (\lambda-1)(g-\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1}))^{\eta}}\right), \left(\frac{(1 + (\lambda-1)\cdot t_{\tilde{a}_1})^{\eta} - (1-t_{\tilde{a}_1})^{\eta}}{(1 + (\lambda-1)\cdot t_{\tilde{a}_1})^{\eta} + (\lambda-1)(1-t_{\tilde{a}_1})^{\eta}}, \frac{\lambda\cdot(i_{\tilde{a}_1})^{\eta}}{(1 + (\lambda-1)\cdot(1-i_{\tilde{a}_1}))^{\eta} + (\lambda-1)(i_{\tilde{a}_1})^{\eta}}, \frac{\lambda\cdot(f_{\tilde{a}_1})^{\eta}}{(1 + (\lambda-1)\cdot(1-f_{\tilde{a}_1}))^{\eta} + (\lambda-1)(f_{\tilde{a}_1})^{\eta}}\right)\right\rangle$$

(4) $$(\tilde{a}_1)^{\eta} = \left\langle \Delta\left(g\cdot\frac{\lambda\cdot(\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1}))^{\eta}}{(g + (\lambda-1)\cdot(g-\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1})))^{\eta} + (\lambda-1)(\Delta^{-1}(h_{\tilde{a}_1},\alpha_{\tilde{a}_1}))^{\eta}}\right), \left(\frac{\lambda\cdot(t_{\tilde{a}_1})^{\eta}}{(1 + (\lambda-1)\cdot(1-t_{\tilde{a}_1}))^{\eta} + (\lambda-1)(t_{\tilde{a}_1})^{\eta}}, \frac{(1 + (\lambda-1)\cdot i_{\tilde{a}_1})^{\eta} - (1-i_{\tilde{a}_1})^{\eta}}{(1 + (\lambda-1)\cdot i_{\tilde{a}_1})^{\eta} + (\lambda-1)(1-i_{\tilde{a}_1})^{\eta}}, \frac{(1 + (\lambda-1)\cdot f_{\tilde{a}_1})^{\eta} - (1-f_{\tilde{a}_1})^{\eta}}{(1 + (\lambda-1)\cdot f_{\tilde{a}_1})^{\eta} + (\lambda-1)(1-f_{\tilde{a}_1})^{\eta}}\right)\right\rangle$$

operational rules defined above, then:

(1) $a_1 \oplus a_2 = \langle (h_6,-0.2143),(0.7627,0.0714,0.0244)\rangle$;
(2) $a_1 \otimes a_2 = \langle (h_3,0.1579),(0.1406,0.6364,0.3846)\rangle$;
(3) $2a_1 = \langle (h_6,-0.4615),(0.5505,0.2,0.0604)\rangle$;
(4) $(a_1)^2 = \langle (h_2,0.4),(0.0604,0.8,0.3846)\rangle$.

Example 1 shows that the new defined operational laws can commendably overcome the granularity and logical problems for existing operation laws.

Remark 1. Some special and reasonable properties about these operational laws of SVN2TLEs are shown as follows:

(1) The linguistic part in SVNLNs must be h_g no matter what LT plus the maximum h_g; Similarly, the linguistic part in SVNLNs must be itself no matter what LT plus the minimum h_0, the memberships part in SVNLNs satisfies this similar property as well.
(2) The linguistic part in SVNLNs must be itself no matter what LT multiplies the maximum h_g, and the linguistic part in SVNLNs must be h_0 no matter what LT multiplies the minimum h_0, the memberships part in SVNLNs satisfies this similar property as well.
(3) The linguistic part in SVNLNs must be h_g no matter what positive real number multiplies the maximum h_g, and the linguistic part in SVNLNs must be h_0 no matter what positive real number multiplies the minimum h_0, the memberships part in SVNLNs satisfies this similar property as well.
(4) The exponentiation operation of the maximum h_g must be h_g no matter what the positive real number be, the memberships part in SVNLNs satisfies this similar property as well.

All of these properties conform to the common sense of people, and these operational laws of SVN2TLEs are reasonable and effective when computing with SVN2TLEs.

Theorem 1. *Suppose that $H = \{h_0,h_1,...,h_g\}$ is a pre-established finite and totally ordered discrete term set. Let $\Omega = \{\langle (h_\Lambda,\alpha_\Lambda),(t_\Lambda,i_\Lambda,f_\Lambda)\rangle | h_\Lambda \in H, \alpha_\Lambda \in [-0.5,0.5), t_\Lambda \in [0,1], i_\Lambda \in [0,1], f_\Lambda \in [0,1]\}$ be the set of all SVN2TLEs generated based on H, $\tilde{a}_1,\tilde{a}_2 \in \Omega$ and $\eta \geqslant 0$, then operations rules on SVN2TLEs defined by the Hamacher t-norm and t-conorm are closed, i.e.,*

(1) $\tilde{a}_1 \oplus_H \tilde{a}_2 \in \Omega$;
(2) $\tilde{a}_1 \otimes_H \tilde{a}_2 \in \Omega$;
(3) $\eta \odot_H \tilde{a}_1 \in \Omega$;
(4) $(\tilde{a}_1)^{\eta} \in \Omega$.

Theorem 2. *Let $\tilde{a}_p = \langle (h_{\tilde{a}_p},\alpha_{\tilde{a}_p}),(t_{\tilde{a}_p},i_{\tilde{a}_p},f_{\tilde{a}_p})\rangle (p = 1,2,3)$ be three SVN2TLEs, $\eta,\eta_1,\eta_2 \geqslant 0$ are three scalars, then the following Hamacher operational rules of SVN2TLEs are true:*

(1) $\tilde{a}_1 \oplus_H \tilde{a}_2 = \tilde{a}_2 \oplus_H \tilde{a}_1$;
(2) $(\tilde{a}_1 \oplus_H \tilde{a}_2) \oplus_H \tilde{a}_3 = \tilde{a}_1 \oplus_H (\tilde{a}_2 \oplus_H \tilde{a}_3)$;
(3) $\tilde{a}_1 \otimes_H \tilde{a}_2 = \tilde{a}_2 \otimes_H \tilde{a}_1$;
(4) $(\tilde{a}_1 \otimes_H \tilde{a}_2) \otimes_H \tilde{a}_3 = \tilde{a}_1 \otimes_H (\tilde{a}_2 \otimes_H \tilde{a}_3)$;
(5) $\eta \odot_H (\tilde{a}_1 \oplus_H \tilde{a}_2) = (\eta \odot_H \tilde{a}_1) \oplus_H (\eta \odot_H \tilde{a}_2)$;
(6) $(\eta_1 + \eta_2) \odot_H \tilde{a}_1 = (\eta_1 \odot_H \tilde{a}_1) \oplus_H (\eta_2 \odot_H \tilde{a}_1)$;
(7) $(\tilde{a}_1)^{\eta_1+\eta_2} = (\tilde{a}_1)^{\eta_1} \otimes_H (\tilde{a}_2)^{\eta_2}$;
(8) $(\tilde{a}_1 \otimes_H \tilde{a}_2)^{\eta} = (\tilde{a}_1)^{\eta} \otimes_H (\tilde{a}_2)^{\eta}$.

3.3. Ranking method for SVN2TLEs

For any two SVN2TLEs $\tilde{a}_p = \langle (h_{\tilde{a}_p},\alpha_{\tilde{a}_p}),(t_{\tilde{a}_p},i_{\tilde{a}_p},f_{\tilde{a}_p})\rangle (p = 1,2)$, how to provide a rational comparison method is significant for ranking alternatives. We say that $\tilde{a}_1 \leqslant_A \tilde{a}_2$ if $(h_{\tilde{a}_1},\alpha_{\tilde{a}_1}) \leqslant (h_{\tilde{a}_2},\alpha_{\tilde{a}_2}), t_{\tilde{a}_1} \leqslant t_{\tilde{a}_2}, i_{\tilde{a}_1} \geqslant i_{\tilde{a}_2}$ and $f_{\tilde{a}_1} \geqslant f_{\tilde{a}_2}$. Unfortunately, $\leqslant_A$ is just a partial order. Up to now, there have been several total orders of SVNLNs available in Wang et al. (2016), Tian et al. (2015), Ma et al. (2017), Ye (2014b). In these literatures, some meaningful indexes are proposed as follows:

Definition 7 (*Ye, 2014b*). For any SVNLN $a = \langle h_{\theta(a)},(T(a),I(a),F(a))\rangle$, the score function, accuracy function and certainty function for a can be defined, respectively, as follows:

(1) $S(a) = (2 + T(a) - I(a) - F(a)) \cdot h_{\theta(a)}$;
(2) $A(a) = (T(a) - F(a)) \cdot h_{\theta(a)}$;
(3) $C(a) = T(a) \cdot h_{\theta(a)}$.

These three index functions are operated upon according to the subscripts of the LTs and degrees of memberships. As we know, the smallest LT h_0 is regarded as 0 in their operations. Thus, if h_0 is involved in multiplicative operations in these three indexes, inaccurate results could be obtained. These limitations yield unreliable and inaccurate results.

Example 2. Let $a_1 = \langle h_0,(0.6,0.2,0.1)\rangle$ and $a_2 = \langle h_0,(0.5,0.3,0.2)\rangle$ be two SVNLNs, Then according to the score, accuracy and certainty functions introduced above, we have

$$S(a_1) = S(a_2) = A(a_1) = A(a_2) = C(a_1) = C(a_2) = 0.$$

Example 2 means that a_1 and a_2 cannot be compared using the above functions. However, a_1 is known to be superior to a_2. In order to overcome these limitations and improve their applicability, the linguistic scale functions (Ma et al., 2017; Tian et al., 2015; Wang et al., 2016) were modified. These modifications are efficient and flexible by converting various LTs into real numbers.

Definition 8 (*Tian et al., 2015*). For any SVNLN $a = \langle h_{\theta(a)},(T(a),I(a),F(a))\rangle$, the score function, accuracy function and certainty function for a can be defined, respectively, as follows:

(1) $S(a) = f^*(h_{\theta(a)})(T(a) + 1 - I(a) + 1 - F(a))$;
(2) $A(a) = f^*(h_{\theta(a)})(T(a) - F(a))$;
(3) $C(a) = f^*(h_{\theta(a)})T(a)$,

where f^* is the linguistic scale function (LSF).

The LSFs are preferable in practice because they can yield more deterministic results when faced with differences in semantics. However, these functions do not equate the *TM* to the *FM*. This may lead to unreliable and inaccurate results as well.

Example 3. Let $a_1 = \langle h_2,(0.5,0.3,0.1)\rangle$ and $a_2 = \langle h_2,(0.6,0.3,0.2)\rangle$ be two SVNLNs, Then by Definition 8, we have

$$S(a_1) = S(a_2), A(a_1) = A(a_2), C(a_2) > C(a_1).$$

Then $a_2 > a_1$. However, the memberships in a SVNLN should be treated equally, and we cannot conclude that $a_2 > a_1$ under the condition of $C(a_2) > C(a_1)$. Because if we redefine an index called uncertainty function (UF) for a, that is, $UC(a) = f^*(h_{\theta(a)})F(a)$, then we have $a_1 > a_2$ in the case of $UC(a_1) < UC(a_2)$, which bears an uncanny resemblance to the method in Tian et al. (2015), but these two indexes get the opposite results. In order to overcome these limitations and improve their applicability, we provide four indexes to rank SVN2TLEs including the score function, the score knowledge measure, the certain knowledge measure and the hesitant-related knowledge measure. They are represented as follows:

(1) Score function: $S(\widetilde{a}_p) = \Delta\left(\frac{\Delta^{-1}(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p})}{3}(t_{\widetilde{a}_p} + 2 - i_{\widetilde{a}_p} - f_{\widetilde{a}_p})\right)$;
(2) Score knowledge measure: $SK(\widetilde{a}_p) = \frac{(t_{\widetilde{a}_p} + 2 - i_{\widetilde{a}_p} - f_{\widetilde{a}_p})}{3}$;
(3) Certainty knowledge measure: $CK(\widetilde{a}_p) = \frac{2(t_{\widetilde{a}_p} + f_{\widetilde{a}_p} - i_{\widetilde{a}_p})}{\sqrt{t_{\widetilde{a}_p}^2 + f_{\widetilde{a}_p}^2 + i_{\widetilde{a}_p}^2} + 1}$;
(4) Hesitant related knowledge measure: $HRK(\widetilde{a}_p) = \frac{(1 - t_{\widetilde{a}_p} + i_{\widetilde{a}_p} + f_{\widetilde{a}_p})\cdot(1 + i_{\widetilde{a}_p})}{6}$.

The knowledge measures of a SVN2TLE is to evaluate the score, certainty and hesitation knowledge of a SVN2TLE separately. Here, some specific SVNSs and existing measures are proposed as evaluation criterions, which provide a framework to describe and analysis data in a flexible way. Some specific SVNSs such as the smallest SVNSs $A_S = \langle 0,1,1\rangle$, the fuzziest SVNSs $A_F = \langle 0,1,0\rangle$ and so on. It follows that three new knowledge measures of a SVN2TLE on the basic of distance, projection, distance-based measure to the smallest, fuzziest, largest SVNSs are proposed. They are explained below:

(1) The score knowledge measure: The new knowledge measure is defined as a normalized Hamming distance (Ye, 2017b) from SVNS $A_{\widetilde{a}_p} = \langle t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p}\rangle$ for the smallest SVNS $A_S = \langle 0,1,1\rangle$ and can be expressed as:

$$SK(\widetilde{a}_p) = d(\langle t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p}\rangle,\langle 0,1,1\rangle) = \frac{1}{3}(|t_{\widetilde{a}_p} - 0| + |i_{\widetilde{a}_p} - 1| + |f_{\widetilde{a}_p} - 1|) = (t_{\widetilde{a}_p} + 2 - i_{\widetilde{a}_p} - f_{\widetilde{a}_p})/3.$$

(2) The certain knowledge measure: The fuzziest and most certain SVNSs are $A_F = \langle 0,1,0\rangle, A_{C_1} = \langle 1,0,0\rangle$ and $A_{C_2} = \langle 0,0,1\rangle$, respectively. The new knowledge measure is defined by calculating the combine harmonic averaging projection measure (Ouyang & Pedrycz, 2016) between SVNS $A_{\widetilde{a}_p} = \langle t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p}\rangle$ and above three specific SVNSs, which is expressed as

$$CK(\widetilde{a}_p) = P(A_{\widetilde{a}_p},A_{C_1}) + P(A_{\widetilde{a}_p},A_{C_2}) - P(A_{\widetilde{a}_p},A_F) = \frac{2t_{\widetilde{a}_p}}{\sqrt{t_{\widetilde{a}_p}^2 + f_{\widetilde{a}_p}^2 + i_{\widetilde{a}_p}^2} + 1} + \frac{2f_{\widetilde{a}_p}}{\sqrt{t_{\widetilde{a}_p}^2 + f_{\widetilde{a}_p}^2 + i_{\widetilde{a}_p}^2} + 1} - \frac{2i_{\widetilde{a}_p}}{\sqrt{t_{\widetilde{a}_p}^2 + f_{\widetilde{a}_p}^2 + i_{\widetilde{a}_p}^2} + 1} = \frac{2(t_{\widetilde{a}_p} + f_{\widetilde{a}_p} - i_{\widetilde{a}_p})}{\sqrt{t_{\widetilde{a}_p}^2 + f_{\widetilde{a}_p}^2 + i_{\widetilde{a}_p}^2} + 1}.$$

The projection $P(A_1,A_2)$ is a measure that considers not only both the distance and the included angle but also bidirectional projection magnitudes between A_1 and A_2. In general, the larger the value of $P(A_1,A_2)$ is, the closer A_1 is to A_2. Clearly, the larger the value of $CK(\widetilde{a}_p)$ is, the closer the SVN2TLE $\widetilde{a}_p$ and its corresponding most certain SVN2TLE are. Therefore, the defined index is effective to measure the certain knowledge measure of a SVN2TLE.

(3) The hesitant related knowledge measure: The new knowledge measure proposed is determined by the *IM* degree and the normalized Hamming distance between SVNS $A_{\widetilde{a}_p} = \langle t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p}\rangle$ and the largest SVNS $A_L = \langle 1,0,0\rangle$, which can be presented as:

$$HRK(\widetilde{a}_p) = 0.5\cdot d(A_L,A_{\widetilde{a}_p})\cdot(1 + i_{\widetilde{a}_p}) = \frac{1}{6}(|t_{\widetilde{a}_p} - 1| + |i_{\widetilde{a}_p} - 0| + |f_{\widetilde{a}_p} - 0|)\cdot(1 + i_{\widetilde{a}_p}) = \frac{(1 - t_{\widetilde{a}_p} + i_{\widetilde{a}_p} + f_{\widetilde{a}_p})\cdot(1 + i_{\widetilde{a}_p})}{6}.$$

This index has the similar form as the ranking method for IFVs in (Ju, Wang, & Liu, 2012). The hesitant related knowledge measure $HRK(\widetilde{a}_p)$ is constructed by strongly taking not only the amount of information related to a SVN2TLE but also the reliability of information represented by a SVN2TLE into account. This knowledge measure tells us about the quality of a SVN2TLE, that the lower the value of $HRK(\widetilde{a}_p)$, the better the SVN2TLE in the sense of the amount of positive information included, and reliability of information.

The proposed four indexes above satisfy the following properties:

Remark 2. For any SVN2TLE $\widetilde{a}_p$, if its linguistic part is h_0, then it need to be denoted as h_ε in the calculation process.

Remark 3. For a SVN2TLE $\widetilde{a}_p$, in its SVNS part, if the distance from SVNS $A_{\widetilde{a}_p} = \langle t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p}\rangle$ for the smallest SVNS $A_S = \langle 0,1,1\rangle$ is farther, which means that $t_{\widetilde{a}_p}$ with respect to the 2TLT $(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p})$ is bigger and $i_{\widetilde{a}_p} + f_{\widetilde{a}_p}$ corresponding to $(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p})$ is smaller, then the larger the score function $S(\widetilde{a}_p)$.

Remark 4. For two SVN2TLEs $\widetilde{a}_p(p=1,2)$, in the condition that their score functions are the same, if the difference between $t_{\widetilde{a}_p}$ and $i_{\widetilde{a}_p}+f_{\widetilde{a}_p}$ with respect to $(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p})$ is bigger, then the SVN2TLE $S(\widetilde{a}_p)$ is more affirmative, i.e., the score knowledge measure of $S(\widetilde{a}_p)$ is higher.

Remark 5. For two SVN2TLEs $\widetilde{a}_p(p=1,2)$, if the projection value between SVNS $A_{\widetilde{a}_p}$ and the fuzziest SVNS A_F is smaller, the projection values between SVNS $A_{\widetilde{a}_p}$ and the most certain SVNS A_{C_1} and A_{C_2} are larger, then the SVN2TLE $\widetilde{a}_p$ is more determinate, i.g., the certain knowledge measure of $\widetilde{a}_p$ is larger.

Remark 6. For a SVN2TLEs $\widetilde{a}_p$, the best SV2TLE is $A_L=\langle 1,0,0\rangle$ for which $HRK(A_L)=0$. The maximal value of HRK, i.e. 1, we obtain for $A_S=\langle 0,1,1\rangle$ for which both the distance from A_L and *IM* are equal 1.

From the above analysis of the four new defined indexes that used for making comparison among the SVN2TLEs, the ranking method for SVN2TLEs can be obtained as follows:

Definition 9. Let $\widetilde{a}_p=\langle(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}),(t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p})\rangle(p=1,2)$ be two SVN2TLEs.

(1) If $S(\widetilde{a}_1)>S(\widetilde{a}_2)$, then $\widetilde{a}_1>\widetilde{a}_2$;
(2) If $S(\widetilde{a}_1)<S(\widetilde{a}_2)$, then $\widetilde{a}_1<\widetilde{a}_2$;
(3) If $S(\widetilde{a}_1)=S(\widetilde{a}_2)$, then
  (1) If $SK(\widetilde{a}_1)>SK(\widetilde{a}_2)$, then $\widetilde{a}_1>\widetilde{a}_2$;
  (2) If $SK(\widetilde{a}_1)<SK(\widetilde{a}_2)$, then $\widetilde{a}_1<\widetilde{a}_2$;
  (3) If $SK(\widetilde{a}_1)=SK(\widetilde{a}_2)$, then
    (i) If $CK(\widetilde{a}_1)>CK(\widetilde{a}_2)$, then $\widetilde{a}_1>\widetilde{a}_2$;
    (ii) If $CK(\widetilde{a}_1)<CK(\widetilde{a}_2)$, then $\widetilde{a}_1<\widetilde{a}_2$;
    (iii) If $CK(\widetilde{a}_1)=CK(\widetilde{a}_2)$, then
      ① If $HRK(\widetilde{a}_1)<HRK(\widetilde{a}_2)$, then $\widetilde{a}_1>\widetilde{a}_2$;
      ② If $HRK(\widetilde{a}_1)>HRK(\widetilde{a}_2)$, then $\widetilde{a}_1<\widetilde{a}_2$.

Example 4. Let $a_1=\langle h_2,(0.5,0.3,0.1)\rangle$ and $a_2=\langle h_2,(0.6,0.3,0.2)\rangle$ be two SVNLNs, then by Definition 9, we have $S(a_1)=S(a_2),SK(a_1)=SK(a_2),CK(a_2)>CK(a_1)$, in this case, we can say that SVNLNs a_2 is better than a_1.

The advantages and differences of the new defined ranking method for SVN2TLEs can be concluded as follows:

(1) For the order based on the score and accuracy often only score is taken into account, which may produce counter intuitive results, such as SVN2TLEs with lower LTs and higher *IM* are pointed out as the better ones. This paper proposes a ranking method with multi-indexes, which provides a full and complete comparison.
(2) The existing accuracy function and certainty function for SVNLN are related to the LTS, while the LTs are only used to show their linguistic evaluation in fact, but cannot reflect the specific measures of a SVNLNS, the proposed new ranking method here are mainly on the basis of its SVN part, which is far easier and more rational.
(3) Projection measure is a very suitable tool when dealing with the certain knowledge measure for that it can consider not only the distance but also the included angle between objects evaluated, while the existing cosine similarity measure of SNSs (Ye, 2015b) introduced in vector space may show some unreasonable result in some cases.
(4) The method takes into account the amount of the information (both positive and negative) associated with an alternative (measured by a distance to the positive ideal alternative), and how reliable the information is (which is measured by the alternative's *IM*).

4. Some new aggregation operators based on SVN2TLEs and Hamacher t-norm and t-conorm

In course of practical application, there are more than two SVN2TLEs need to be fused. Therefore, it's necessary to apply such operations to aggregate *n* SVN2TLEs. In the following, we propose the SVN2TLHWA and SVN2TLHGWA operators.

4.1. Single-valued neutrosophic 2-tuple linguistic Hamacher weighted averaging operator

The SVN2TLHWA operator is defined as follows:

Definition 10. Let $\widetilde{a}_p=\langle(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}),(t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p})\rangle(p=1,2,\ldots,n)$ be n SVN2TLEs, the Hamacher weighted averaging operator is mapping $SVN2TLHWA:\Omega^n\rightarrow\Omega$, with associated weighting vector $\omega=(\omega_1,\omega_2,\ldots,\omega_n)$ which satisfies that $\omega_p\in[0,1]$ and $\sum_{p=1}^{n}\omega_p=1$, such that

$$SVN2TLHWA(\widetilde{a}_1,\widetilde{a}_2,\ldots,\widetilde{a}_n)=\bigoplus_{H\,p=1}^{n}(\omega_p\odot_H\widetilde{a}_p) \tag{17}$$

where Ω is the set of all SVN2TLEs, then the mapping *SVN2TLHWA* is called the single-valued neutrosophic 2-tuple linguistic Hamacher weighted averaging operator.

On the basic of the operational rules on SVN2TLEs via the Hamacher t-norm and t-conorm described in Section 3.2, we can educe the result shown as Theorem 3.

Theorem 3. *Let $\widetilde{a}_p=\langle(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}),(t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p})\rangle(p=1,2,\ldots,n)$ be n SVN2TLEs, then the Hamacher weighted averaging of n SVN2TLEs by using Eq. (17) is still a SVN2TLE, and*

$$\begin{aligned}&SVN2TLHWA(\widetilde{a}_1,\widetilde{a}_2,\ldots,\widetilde{a}_n)\\&=\left\langle\Delta\left(g\cdot\frac{\prod_{p=1}^{n}(g+(\lambda-1)\cdot\Delta^{-1}(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}))^{\omega_p}-\prod_{p=1}^{n}(g-\Delta^{-1}(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}))^{\omega_p}}{\prod_{p=1}^{n}(g+(\lambda-1)\cdot\Delta^{-1}(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(g-\Delta^{-1}(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}))^{\omega_p}}\right),\right.\\&\left.\left(\frac{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{a}_p})^{\omega_p}-\prod_{p=1}^{n}(1-t_{\widetilde{a}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{a}_p})^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(1-t_{\widetilde{a}_p})^{\omega_p}},\frac{\lambda\prod_{p=1}^{n}(i_{\widetilde{a}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-i_{\widetilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(i_{\widetilde{a}_p})^{\omega_p}},\frac{\lambda\prod_{p=1}^{n}(f_{\widetilde{a}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-f_{\widetilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(f_{\widetilde{a}_p})^{\omega_p}}\right)\right\rangle\end{aligned} \tag{18}$$

Proof.

(1) When $n=1,\omega_1=1$, it follows that

$$\begin{aligned}&SVN2TLHWA(\widetilde{a}_1,\widetilde{a}_2,\ldots,\widetilde{a}_n)=SVN2TLHWA(\widetilde{a}_1)\\&=\left\langle\Delta\left(g\cdot\frac{g+(\lambda-1)\cdot\Delta^{-1}(h_{\widetilde{a}_1},\alpha_{\widetilde{a}_1})-(g-\Delta^{-1}(h_{\widetilde{a}_1},\alpha_{\widetilde{a}_2}))}{(g+(\lambda-1)\cdot\Delta^{-1}(h_{\widetilde{a}_1},\alpha_{\widetilde{a}_2}))+(\lambda-1)(g-\Delta^{-1}(h_{\widetilde{a}_1},\alpha_{\widetilde{a}_2}))}\right),\left(\frac{(1+(\lambda-1)\cdot t_{\widetilde{a}_1})-(1-t_{\widetilde{a}_1})}{(1+(\lambda-1)\cdot t_{\widetilde{a}_1})+(\lambda-1)(1-t_{\widetilde{a}_1})},\right.\right.\\&\left.\left.\frac{\lambda\cdot i_{\widetilde{a}_1}}{1+(\lambda-1)\cdot(1-i_{\widetilde{a}_1})+(\lambda-1)i_{\widetilde{a}_1}},\frac{\lambda\cdot f_{\widetilde{a}_1}}{1+(\lambda-1)\cdot(1-f_{\widetilde{a}_1})+(\lambda-1)f_{\widetilde{a}_1}}\right)\right\rangle\\&=\langle(t_{\widetilde{a}_1},\alpha_{\widetilde{a}_1}),(t_{\widetilde{a}_1},i_{\widetilde{a}_1},f_{\widetilde{a}_1})\rangle\end{aligned}$$

(2) When $n=2$, it follows that

$$SVN2TLHWA(\widetilde{a}_1,\widetilde{a}_2,\ldots,\widetilde{a}_n)=(\omega_1\odot_H\widetilde{a}_1)\oplus_H(\omega_2\odot_H\widetilde{a}_2).$$

Here we only need to prove that

$$(\omega_1 \odot_H (h_{\tilde{a}_1},\alpha_{\tilde{a}_1}))\oplus_H(\omega_2 \odot_H (h_{\tilde{a}_2},\alpha_{\tilde{a}_2}))=\Delta\left(g\cdot\frac{\prod_{p=1}^2(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}-\prod_{p=1}^2(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}}{\prod_{p=1}^2(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^2(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}}\right)$$

Let $x_p=(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}$ and $y_p=(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}$, then

$$\begin{aligned}&(\omega_1 \odot_H (h_1,\alpha_1))\oplus_H(\omega_2 \odot_H (h_2,\alpha_2))\\&=\Delta\left(g\cdot\frac{x_1-y_1}{x_1+(\lambda-1)y_1}\right)\oplus_H\Delta\left(g\cdot\frac{x_2-y_2}{x_2+(\lambda-1)y_2}\right)\\&=\Delta\left(g\cdot\frac{\lambda\cdot x_1x_2-\lambda\cdot y_1y_2}{\lambda\cdot x_1x_2+\lambda(\lambda-1)\cdot y_1y_2}\right)\\&=\Delta\left(g\cdot\frac{\prod_{p=1}^2(g+(\lambda-1)\cdot\Delta^{-1}(h_p,\alpha_p))^{\omega_p}-\prod_{p=1}^2(g-\Delta^{-1}(h_p,\alpha_p))^{\omega_p}}{\prod_{p=1}^2(g+(\lambda-1)\cdot\Delta^{-1}(h_p,\alpha_p))^{\omega_p}+(\lambda-1)\prod_{p=1}^2(g-\Delta^{-1}(h_p,\alpha_p))^{\omega_p}}\right).\end{aligned}$$

(3) Assume that Eq. (18) hold true for $n=k$, we have

$$\begin{aligned}&\overset{k}{\underset{p=1}{\oplus_H}}(\omega_p \odot_H \tilde{a}_p)\\&=\left\langle\Delta\left(g\cdot\frac{\prod_{p=1}^k(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}-\prod_{p=1}^k(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}}{\prod_{p=1}^k(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^k(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}}\right),\right.\\&\left(\frac{\prod_{p=1}^k(1+(\lambda-1)\cdot t_{\tilde{a}_p})^{\omega_p}-\prod_{p=1}^k(1-t_{\tilde{a}_p})^{\omega_p}}{\prod_{p=1}^k(1+(\lambda-1)\cdot t_{\tilde{a}_p})^{\omega_p}+(\lambda-1)\prod_{p=1}^k(1-t_{\tilde{a}_p})^{\omega_p}}\right.\\&,\frac{\lambda\prod_{p=1}^k(i_{\tilde{a}_p})^{\omega_p}}{\prod_{p=1}^k(1+(\lambda-1)\cdot(1-i_{\tilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^k(i_{\tilde{a}_p})^{\omega_p}},\\&\left.\left.\frac{\lambda\prod_{p=1}^k(f_{\tilde{a}_p})^{\omega_p}}{\prod_{p=1}^k(1+(\lambda-1)\cdot(1-f_{\tilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^k(f_{\tilde{a}_p})^{\omega_p}}\right)\right\rangle\end{aligned}$$

then for $n=k+1$, we have $SVN2TLHWA(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_k,\tilde{a}_{k+1})=SVN2TLHWA(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_k)\oplus_H(\omega_{k+1}\odot_H \tilde{a}_{k+1})$

Here we only need to prove

$$\left(\overset{k}{\underset{p=1}{\oplus_H}}(\omega_p \odot_H (h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))\right)\oplus_H(\omega_{k+1}\odot_H (h_{\tilde{a}_{k+1}},\alpha_{\tilde{a}_{k+1}}))=\Delta\left(g\cdot\frac{\prod_{p=1}^{k+1}(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}-\prod_{p=1}^{k+1}(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}}{\prod_{p=1}^{k+1}(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{k+1}(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}}\right)$$

Let $x_p=(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}$ and $y_p=(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}$, then

$$\begin{aligned}&\left(\overset{k}{\underset{p=1}{\oplus_H}}(\omega_p \odot_H (h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))\right)\oplus_H(\omega_{k+1}\odot_H (h_{\tilde{a}_{k+1}},\alpha_{\tilde{a}_{k+1}}))\\&=\Delta\left(g\cdot\frac{\prod_{p=1}^k x_p-\prod_{p=1}^k y_p}{\prod_{p=1}^k x_p+(\lambda-1)\prod_{p=1}^k y_p}\right)\oplus_H\Delta\left(g\cdot\frac{x_{k+1}-y_{k+1}}{x_{k+1}+(\lambda-1)y_{k+1}}\right)\\&=\Delta\left(g\cdot\frac{\lambda\cdot\prod_{p=1}^{k+1}x_p-\lambda\cdot\prod_{p=1}^{k+1}y_p}{\lambda\cdot\prod_{p=1}^{k+1}x_p+\lambda(\lambda-1)\cdot\prod_{p=1}^{k+1}y_p}\right)\\&=\Delta\left(g\cdot\frac{\prod_{p=1}^{k+1}(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}-\prod_{p=1}^{k+1}(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}}{\prod_{p=1}^{k+1}(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{k+1}(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}}\right)\end{aligned}$$

Therefore, when $n=k+1$, Eq. (18) holds.

According to mathematical induction, we can get Eq. (18) holds true for any n. □

In the following, we can investigate some desirable properties of the SVN2TLHWA operator.

Theorem 4 (*Idempotency*). *Let $\tilde{a}_p=\langle(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}),(t_{\tilde{a}_p},i_{\tilde{a}_p},f_{\tilde{a}_p})\rangle(p=1,2,\ldots,n)$ be n SVN2TLEs, if $\tilde{a}_p=\tilde{a}=\langle(h_{\tilde{a}},\alpha_{\tilde{a}}),(t_{\tilde{a}},i_{\tilde{a}},f_{\tilde{a}})\rangle$ for all p, then*

$$SVN2TLHWA(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\tilde{a}=\langle(h_{\tilde{a}},\alpha_{\tilde{a}}),(t_{\tilde{a}},i_{\tilde{a}},f_{\tilde{a}})\rangle. \tag{19}$$

Theorem 5 (*Boundedness*). *Let $\tilde{a}_p=\langle(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}),(t_{\tilde{a}_p},i_{\tilde{a}_p},f_{\tilde{a}_p})\rangle(p=1,2,\ldots,n)$ be n SVN2TLEs, $\tilde{a}=\langle(h_{\tilde{a}},\alpha_{\tilde{a}}),(t_{\tilde{a}},i_{\tilde{a}},f_{\tilde{a}})\rangle$ be the aggregated result by SVN2TLHWA operator and*

$$\tilde{a}^-=\langle\min_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}),(\min_p\{t_{\tilde{a}_p}\},\max_p\{i_{\tilde{a}_p}\},\max_p\{f_{\tilde{a}_p}\})\rangle, \tag{20}$$

$$\tilde{a}^+=\langle\max_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}),(\max_p\{t_{\tilde{a}_p}\},\min_p\{i_{\tilde{a}_p}\},\min_p\{f_{\tilde{a}_p}\})\rangle. \tag{21}$$

then

$$\tilde{a}^-\leqslant SVN2TLHWA(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\leqslant\tilde{a}^+. \tag{22}$$

Proof. For convenience and simplification of calculation, let

$$X=\prod_{p=1}^n(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p},Y=\prod_{p=1}^n(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p},$$

then

$$\begin{aligned}(h_{\tilde{a}},\alpha_{\tilde{a}})&=\Delta\left(g\cdot\frac{\prod_{p=1}^n(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}-\prod_{p=1}^n(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}}{\prod_{p=1}^n(g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^n(g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}))^{\omega_p}}\right)\\&=\Delta\left(g\cdot\frac{X-Y}{X+(\lambda-1)Y}\right)\\&=\Delta\left(g\cdot\left(1-\frac{\lambda}{X/Y+(\lambda-1)}\right)\right)\end{aligned}$$

$$\begin{aligned}X/Y&=\prod_{p=1}^n\left(\frac{g+(\lambda-1)\cdot\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})}{g-\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})}\right)^{\omega_p}\\&=\prod_{p=1}^n\left(1+\frac{\lambda}{g/\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})-1}\right)^{\omega_p}\end{aligned}$$

Obviously, X/Y is a monotonous increasing mapping of $(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})$, and $(h_{\tilde{a}},\alpha_{\tilde{a}})$ is a monotonous increasing mapping of X/Y, therefore, $(h_{\tilde{a}},\alpha_{\tilde{a}})$ is a monotonous increasing mapping of $(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})$, thus

$$\begin{aligned}&(h_{\tilde{a}},\alpha_{\tilde{a}})\\&\geqslant\Delta\left(g\cdot\frac{\prod_{p=1}^n(g+(\lambda-1)\cdot\Delta^{-1}(\min_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})))^{\omega_p}-\prod_{p=1}^n(g-\Delta^{-1}(\min_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})))^{\omega_p}}{\prod_{p=1}^n(g+(\lambda-1)\cdot\Delta^{-1}(\min_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})))^{\omega_p}+(\lambda-1)\prod_{p=1}^n(g-\Delta^{-1}(\min_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})))^{\omega_p}}\right)\\&=\min_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}),\end{aligned}$$

and

$$\begin{aligned}&(h_{\tilde{a}},\alpha_{\tilde{a}})\\&\leqslant\Delta\left(g\cdot\frac{\prod_{p=1}^n(g+(\lambda-1)\cdot\Delta^{-1}(\max_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})))^{\omega_p}-\prod_{p=1}^n(g-\Delta^{-1}(\max_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})))^{\omega_p}}{\prod_{p=1}^n(g+(\lambda-1)\cdot\Delta^{-1}(\max_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})))^{\omega_p}+(\lambda-1)\prod_{p=1}^n(g-\Delta^{-1}(\max_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})))^{\omega_p}}\right)\\&=\max_p(h_{\tilde{a}_p},\alpha_{\tilde{a}_p}).\end{aligned}$$

Similarly, for the *TM* part, *IM* part and *FM* part, the following inequalities hold, where $\min_p\{t_{\tilde{a}_p}\}\leqslant t_{\tilde{a}}\leqslant\max_p\{t_{\tilde{a}_p}\},\min_p\{i_{\tilde{a}_p}\}\leqslant i_{\tilde{a}}\leqslant\max_p\{i_{\tilde{a}_p}\},\min_p\{f_{\tilde{a}_p}\}\leqslant f_{\tilde{a}}\leqslant\max_p\{f_{\tilde{a}_p}\}$. For the score function

$$S(\tilde{a}_p)=\Delta\left(\frac{\Delta^{-1}(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})}{3}(2+t_{\tilde{a}_p}-i_{\tilde{a}_p}-f_{\tilde{a}_p})\right),$$

it is monotonous increasing with $(h_{\tilde{a}_p},\alpha_{\tilde{a}_p})$ and $t_{\tilde{a}_p}$, but monotonous decreasing with $i_{\tilde{a}_p}$ and $f_{\tilde{a}_p}$, that is

$$2\min_p\{t_{\tilde{a}_p}\}-\max_p\{i_{\tilde{a}_p}\}-2\max_p\{f_{\tilde{a}_p}\}\leqslant 2t_{\tilde{a}}-i_{\tilde{a}}-2f_{\tilde{a}}\leqslant 2\max_p\{t_{\tilde{a}_p}\}-\min_p\{i_{\tilde{a}_p}\}-2\min_p\{f_{\tilde{a}_p}\},$$

thus

$$\tilde{a}^-\leqslant SVN2TLHWA(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\leqslant\tilde{a}^+.$$

which completes the proof. □

In the following we can get some special cases of the SVN2TLHWA operator with different parameters λ.

(1) If $\lambda \to 0$, then we obtain the single-valued neutrosophic 2-tuple linguistic maximum (SVN2TLMA) operator;
(2) If $\lambda = 1$, it follows that:

$$SVN2TLHWA_{\lambda=1}(\widetilde{a}_1,\widetilde{a}_2,...,\widetilde{a}_n) = \left\langle \Delta\left(g-\prod_{p=1}^{n}(g-\Delta^{-1}(h_p,\alpha_p))^{\omega_p}\right), \left(1-\prod_{p=1}^{n}(1-t_{\widetilde{a}_p})^{\omega_p},\prod_{p=1}^{n}(i_{\widetilde{a}_p})^{\omega_p},\prod_{p=1}^{n}(f_{\widetilde{a}_p})^{\omega_p}\right)\right\rangle,$$

which is the single-valued neutrosophic 2-tuple linguistic weighted averaging (SVN2TLWA) operator;
(3) If , it follows that:

$$SVN2TLHWA_{\lambda=2}(\widetilde{a}_1,\widetilde{a}_2,...,\widetilde{a}_n) = \left\langle \Delta\left(g\cdot\frac{\prod_{p=1}^{n}(g+\Delta^{-1}(h_p,\alpha_p))^{\omega_p}-\prod_{i=1}^{n}(g-\Delta^{-1}(h_p,\alpha_p))^{\omega_p}}{\prod_{p=1}^{n}(g+\Delta^{-1}(h_p,\alpha_p))^{\omega_p}+\prod_{i=1}^{n}(g-\Delta^{-1}(h_p,\alpha_p))^{\omega_p}}\right), \left(\frac{\prod_{p=1}^{n}(1+t_{\widetilde{a}_p})^{\omega_p}-\prod_{p=1}^{n}(1-t_{\widetilde{a}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+t_{\widetilde{a}_p})^{\omega_p}+\prod_{p=1}^{n}(1-t_{\widetilde{a}_p})^{\omega_p}},\frac{2\prod_{p=1}^{n}(i_{\widetilde{a}_p})^{\omega_p}}{\prod_{p=1}^{n}(2-i_{\widetilde{a}_p}))^{\omega_p}+\prod_{p=1}^{n}(i_{\widetilde{a}_p})^{\omega_p}},\frac{2\prod_{p=1}^{n}(f_{\widetilde{a}_p})^{\omega_p}}{\prod_{p=1}^{n}(2-f_{\widetilde{a}_p}))^{\omega_p}+\prod_{p=1}^{n}(f_{\widetilde{a}_p})^{\omega_p}}\right)\right\rangle,$$

which is the single-valued neutrosophic 2-tuple linguistic Einstein weighted averaging (SVN2TLEWA) operator;
(4) If $\lambda \to +\infty$, then we can obtain the single-valued neutrosophic 2-tuple linguistic minimum (SVN2TLMI) operator.

4.2. *Single-valued neutrosophic 2-tuple linguistic Hamacher geometric weighted averaging operator*

The SVN2TLHGWA operator is defined as follows:

Definition 11. Let $\widetilde{a}_p = \langle(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}),(t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p})\rangle (p=1,2,...,n)$ be n SVN2TLEs, the Hamacher geometric weighted averaging operator is mapping $SVN2TLHGWA$: $\Omega^n \to \Omega$, with associated weighting vector $\omega = (\omega_1,\omega_2,...,\omega_n)$ which satisfies that $\omega_p \in [0,1]$ and $\sum_{p=1}^{n}\omega_p = 1$, such that

$$SVN2TLHGWA(\widetilde{a}_1,\widetilde{a}_2,...,\widetilde{a}_n) = \bigotimes_{H\,p=1}^{n}(\widetilde{a}_p^{\omega_p}), \tag{23}$$

where Ω is the set of all SVN2TLEs, then the mapping $SVN2TLHGWA$ is called the single-valued neutrosophic 2-tuple linguistic Hamacher geometric weighted averaging operator.

Similar to the SVN2TLHWA operator, based on the Hamacher operational rules of the SVN2TLEs, we can derive the following theorems.

Theorem 6. *Let $\widetilde{a}_p = \langle(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}),(t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p})\rangle (p=1,2,...,n)$ be n SVN2TLEs, then the Hamacher geometric weighted averaging of n SVN2TLEs is still a SVN2TLE, and*

$$SVN2TLHGWA(\widetilde{a}_1,\widetilde{a}_2,...,\widetilde{a}_n) = \left\langle \Delta\left(g\cdot\frac{\lambda\prod_{p=1}^{n}(\Delta^{-1}(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}))^{\omega_p}}{\prod_{p=1}^{n}(g+(\lambda-1)\cdot(g-\Delta^{-1}(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p})))^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(\Delta^{-1}(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}))^{\omega_p}}\right), \left(\frac{\lambda\prod_{p=1}^{n}(t_{\widetilde{a}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-t_{\widetilde{a}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(t_{\widetilde{a}_p})^{\omega_p}},\frac{\prod_{p=1}^{n}(1+(\lambda-1)i_{\widetilde{a}_p})^{\omega_p}-\prod_{p=1}^{n}(1-i_{\widetilde{a}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)i_{\widetilde{a}_p})^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(1-i_{\widetilde{a}_p})^{\omega_p}},\frac{\prod_{p=1}^{n}(1+(\lambda-1)f_{\widetilde{a}_p})^{\omega_p}-\prod_{p=1}^{n}(1-f_{\widetilde{a}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)f_{\widetilde{a}_p})^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(1-f_{\widetilde{a}_p})^{\omega_p}}\right)\right\rangle. \tag{24}$$

Theorem 7 (*Idempotency*). *Let $\widetilde{a}_p = \langle(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}),(t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p})\rangle (p=1,2,...,n)$ be n SVN2TLEs, if $\widetilde{a}_p = \widetilde{a} = \langle(h_{\widetilde{a}},\alpha_{\widetilde{a}}),(t_{\widetilde{a}},i_{\widetilde{a}},f_{\widetilde{a}})\rangle$ for all p, then*

$$SVN2TLHGWA(\widetilde{a}_1,\widetilde{a}_2,...,\widetilde{a}_n) = \widetilde{a} = \langle(h_{\widetilde{a}},\alpha_{\widetilde{a}}),(t_{\widetilde{a}},i_{\widetilde{a}},f_{\widetilde{a}})\rangle. \tag{25}$$

Theorem 8 (*Boundedness*). *Let $\widetilde{a}_p = \langle(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}),(t_{\widetilde{a}_p},i_{\widetilde{a}_p},f_{\widetilde{a}_p})\rangle (p=1,2,...,n)$ be n SVN2TLEs, $\widetilde{a} = \langle(h_{\widetilde{a}},\alpha_{\widetilde{a}}),(t_{\widetilde{a}},i_{\widetilde{a}},f_{\widetilde{a}})\rangle$ be the aggregated result by SVN2TLHGWA operator and*

$$\widetilde{a}^- = \langle\min_p(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}),(\min_p\{t_{\widetilde{a}_p}\},\max_p\{i_{\widetilde{a}_p}\},\max_p\{f_{\widetilde{a}_p}\})\rangle, \tag{26}$$

$$\widetilde{a}^+ = \langle\max_p(h_{\widetilde{a}_p},\alpha_{\widetilde{a}_p}),(\max_p\{t_{\widetilde{a}_p}\},\min_p\{i_{\widetilde{a}_p}\},\min_p\{f_{\widetilde{a}_p}\})\rangle. \tag{27}$$

then

$$\widetilde{a}^- \leqslant SVN2TLHGWA(\widetilde{a}_1,\widetilde{a}_2,...,\widetilde{a}_n) \leqslant \widetilde{a}^+. \tag{28}$$

4.3. *Analysis of Hamacher aggregation operators in MAGDM with SVN2TLS assessments*

To guarantee the rationality of decisions made with consideration of the parameter λ, it is necessary and important to analyze the relationships between the Hamacher weighted and geometric averaging operators in SVN2TLS contexts from a theoretical point of view. Here, two common rules can be extracted from the representative analysis of the movement of score values with SVN2TLS assessments with variation in the parameter λ in Hamacher weighted and geometric aggregation operators. They are formally presented below.

Theorem 9. *Score values decrease and increase with the increase of the parameter λ in Hamacher weighted and geometric aggregation operators respectively.*

Proof. The definitions of two functions $T_\lambda^H(x,y)$ and $S_\lambda^H(x,y)$ in Eqs. (7) and (8) indicate that the two parts in $S^{WA}(\widetilde{a}_p)$ (or $S^{GWA}(\widetilde{a}_p)$) are the functions with respect to the parameter λ. In this context the verification of Theorem 9 is equivalently transformed into the discussion of the monotonicity of the functions $T_\lambda^H(x,y)$ and $S_\lambda^H(x,y)$ with respect to λ. By Eq. (7), we have

$$T_\lambda^H(x,y) = \frac{xy}{\lambda+(1-\lambda)(x+y-xy)} = \frac{xy}{\lambda(1-x-y+xy)+(x+y-xy)} = \frac{xy}{\lambda(x-1)(y-1)+(x+y-xy)}.$$

With the condition that $(x-1)(y-1) \geqslant 0$ and $x+y-xy \geqslant 0$, it is easy to know that the family of Hamacher t-norm is strictly decreasing and the family of Hamacher t-conorm is strictly increasing as parameter λ increases. Therefore, score values decrease and increase with the increase of the parameter λ in Hamacher weighted and geometric aggregation operators respectively. □

Based on this monotonicity, the relationship between $S^{WA}(\widetilde{a}_p)$ and $S^{GWA}(\widetilde{a}_p)$ can be obtained.

Theorem 10. *Score values generated by Hamacher weighted operator $S^{WA}(\widetilde{a}_p)$ are always larger than those generated by Hamacher geometric operator $S^{GWA}(\widetilde{a}_p)$ given the same λ when the two operators are applied in*

MAGDM with SVN2TLS assessments, that is

$$S^{WA}(\widetilde{\alpha}_p) > S^{GWA}(\widetilde{\alpha}_p), \lambda \in (0,+\infty). \quad (29)$$

Proof. Theorems 9 indicates $S^{WA}(\widetilde{\alpha}_p)$ and $S^{GWA}(\widetilde{\alpha}_p)$ are monotonously decreasing and increasing with respect to λ, respectively. Thus, the conclusion in Eq. (29) can be transformed into $\lim_{\lambda\to+\infty} S^{WA}(\widetilde{\alpha}_p) > \lim_{\lambda\to+\infty} S^{WGA}(\widetilde{\alpha}_p)$. It can be further converted into

$$\lim_{\lambda\to+\infty} \frac{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}-\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}}$$

$$\geqslant \lim_{r\to+\infty} \frac{\lambda\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-t_{\widetilde{\alpha}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}$$

We can reason from L'Hospital's rule that

$$\lim_{\lambda\to+\infty} \frac{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}-\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}}$$

$$= \lim_{\lambda\to+\infty} \frac{\partial(\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}-\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p})/\partial\lambda}{\partial(\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p})/\partial\lambda}$$

$$= \lim_{\lambda\to+\infty} \frac{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}\cdot\sum_{j=1}^{n}\frac{\omega_p t_{\widetilde{\alpha}_p}}{(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}\cdot\sum_{j=1}^{n}\frac{\omega_p t_{\widetilde{\alpha}_p}}{(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})}+\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}}$$

and

$$\lim_{\lambda\to+\infty} \frac{\lambda\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-t_{\widetilde{\alpha}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}$$

$$= \lim_{\lambda\to+\infty} \frac{\partial(\lambda\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p})/\partial\lambda}{\partial(\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-t_{\widetilde{\alpha}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p})/\partial\lambda}$$

$$= \lim_{\lambda\to+\infty} \frac{\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-t_{\widetilde{\alpha}_p}))^{\omega_p}\cdot\sum_{p=1}^{n}\frac{\omega_p(1-t_{\widetilde{\alpha}_p})}{1+(\lambda-1)\cdot(1-t_{\widetilde{\alpha}_p})}+\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}$$

The function $f(x) = x/(x+b)$ is monotonously increasing with respect to the parameter x with $x > 0$ and $b \geqslant 0$, therefore,

$$\lim_{\lambda\to+\infty} \frac{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}\cdot\sum_{p=1}^{n}\frac{\omega_p t_{\widetilde{\alpha}_p}}{(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}\cdot\sum_{p=1}^{n}\frac{\omega_p t_{\widetilde{\alpha}_p}}{(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})}+\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}}$$

$$\geqslant \lim_{\lambda\to+\infty} \frac{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}\cdot\prod_{p=1}^{n}\left(\frac{t_{\widetilde{\alpha}_p}}{(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})}\right)^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}\cdot\prod_{p=1}^{n}\left(\frac{t_{\widetilde{\alpha}_p}}{(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})}\right)^{\omega_p}+\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}}$$

$$= \frac{\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}+\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}}$$

and

$$\lim_{\lambda\to+\infty} \frac{\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-t_{\widetilde{\alpha}_p}))^{\omega_p}\cdot\sum_{p=1}^{n}\frac{\omega_p(1-t_{\widetilde{\alpha}_p})}{1+(\lambda-1)\cdot(1-t_{\widetilde{\alpha}_p})}+\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}$$

$$\leqslant \lim_{\lambda\to+\infty} \frac{\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-t_{\widetilde{\alpha}_p}))^{\omega_p}\cdot\prod_{p=1}^{n}\left(\frac{(1-t_{\widetilde{\alpha}_p})}{1+(\lambda-1)\cdot(1-t_{\widetilde{\alpha}_p})}\right)^{\omega_p}+\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}$$

$$= \frac{\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}+\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}$$

As a result,

$$\lim_{\lambda\to+\infty} \frac{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}-\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot t_{\widetilde{\alpha}_p})^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(1-t_{\widetilde{\alpha}_p})^{\omega_p}}$$

$$\geqslant \lim_{\lambda\to+\infty} \frac{\lambda\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-t_{\widetilde{\alpha}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(t_{\widetilde{\alpha}_p})^{\omega_p}}$$

holds, from which we deduce that

$$\lim_{\lambda\to+\infty} \frac{\lambda\prod_{p=1}^{n}(i_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot(1-i_{\widetilde{\alpha}_p}))^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(i_{\widetilde{\alpha}_p})^{\omega_p}}$$

$$\leqslant \lim_{\lambda\to+\infty} \frac{\prod_{p=1}^{n}(1+(\lambda-1)\cdot i_{\widetilde{\alpha}_p})^{\omega_p}-\prod_{p=1}^{n}(1-i_{\widetilde{\alpha}_p})^{\omega_p}}{\prod_{p=1}^{n}(1+(\lambda-1)\cdot i_{\widetilde{\alpha}_p})^{\omega_p}+(\lambda-1)\prod_{p=1}^{n}(1-i_{\widetilde{\alpha}_p})^{\omega_p}}$$

Similarly, the remaining two inequalities could be proved.

With the score function for SVN2TLE, the conclusion in Eq. (29) is verified for any $\lambda \in (0,+\infty)$. □

5. The process of MAGDM based on SVN2TL Hamacher aggregation operators

In this section, we will present the process of solving MAGDM problem by using the SVN2TL Hamacher aggregation operators, where the weights of DMs are given but the weights of attributes are completely unknown, the preference values take the form of SVNLNs.

5.1. Problem description

A MAGDM problem can be defined as a quadruple $<D,X,C,A>$, where

$D = \{d_1,d_2,...,d_l\}$ is the finite set of DMs and is indexed by k and $k \geqslant 2$;
$X = \{x_1,x_2,...,x_m\}$ is the discrete set of alternatives for DMs and is indexed by p and $m \geqslant 2$;
$C = \{c_1,c_2,...,c_n\}$ is the set of attributes for each alternative, and the attributes are assumed to be confluent and independent in this paper for simplicity;
$A^{(k)} = (a_{pq}^{(k)})_{m\times n}$ is the decision matrix provided by DM $d_k(k = 1,2,...,l)$, and $a_{pq}^{(k)}$ represents the preference value of alternative x_p with respect to attribute c_q.
$a_{pq}^{(k)}$ is in the form of SVNLNs $a_{pq}^{(k)} = \langle h_{a_{pq}^{(k)}},(t_{\widetilde{a}_{pq}^{(k)}},i_{\widetilde{a}_{pq}^{(k)}},f_{\widetilde{a}_{pq}^{(k)}})\rangle, h_{a_{pq}^{(k)}}$ is derived from a given LTS based on the subjective evaluation of all DMs. $\mu = (\mu_1,\mu_2,...,\mu_l)$ is the weight of DMs $d_k(k = 1,2,...,l), \mu_k \in [0,1]$ and $\sum_{k=1}^{l}\mu_k = 1$, the attribute weights are completely unknown.

5.2. Generation of attributes' weights by constructing optimization model based on the maximizing deviation method

Due to complexity and uncertainty in many MAGDM problems, with human thinking is inherently subjective, the information about attribute weights maybe unknown, we must determine the attribute weights in advance. Based on the maximizing deviation method (Xu & Da, 2010), here we extend it to the SVN2TL environment. Firstly, we define the deviation degree between any two SVN2TLEs, which is on the basis of Hamming distance measure, is defined as follows:

Definition 12. Let $\widetilde{\alpha}_p = \langle(h_{\widetilde{\alpha}_p},\alpha_{\widetilde{\alpha}_p}),(t_{\widetilde{\alpha}_p},i_{\widetilde{\alpha}_p},f_{\widetilde{\alpha}_p})\rangle(p = 1,2)$ be two SVN2TLEs, then the Hamming distance measure between any two SVN2TLEs is defined as

$$d(\widetilde{\alpha}_1,\widetilde{\alpha}_2) = |\Delta^{-1}(h_{\widetilde{\alpha}_1},\alpha_{\widetilde{\alpha}_1})t_{\widetilde{\alpha}_1}-\Delta^{-1}(h_{\widetilde{\alpha}_2},\alpha_{\widetilde{\alpha}_2})t_{\widetilde{\alpha}_2}| + |\Delta^{-1}(h_{\widetilde{\alpha}_1},\alpha_{\widetilde{\alpha}_1})(1-i_{\widetilde{\alpha}_1}) -\Delta^{-1}(h_{\widetilde{\alpha}_2},\alpha_{\widetilde{\alpha}_2})(1-i_{\widetilde{\alpha}_2})| + |\Delta^{-1}(h_{\widetilde{\alpha}_1},\alpha_{\widetilde{\alpha}_1})(1-f_{\widetilde{\alpha}_1}) -\Delta^{-1}(h_{\widetilde{\alpha}_2},\alpha_{\widetilde{\alpha}_2})(1-f_{\widetilde{\alpha}_2})|. \quad (30)$$

To determine the differences among the performance values of all alternatives, we adopt the deviation method. For the DM d_k and the attribute c_q, the deviation of alternative x_p to all the other alternatives can be expressed as:

$$d_{pq}^{(k)}(\omega) = \sum_{s=1}^{m}\omega_q d(\widetilde{a}_{pq}^{(k)},\widetilde{a}_{sq}^{(k)}), \quad p = 1,...,m, q = 1,...,n, \quad (31)$$

Then, the collective deviation degree (CDD) between the alternative x_p and all the other alternatives with respect to the attribute c_q can be given as follows:

$$d_{pq}(\omega) = \sum_{k=1}^{l}\sum_{s=1}^{m}\omega_q\mu_k d(\widetilde{a}_{pq}^{(k)},\widetilde{a}_{sq}^{(k)}), \quad p = 1,...,m, q = 1,...,n, \quad (32)$$

where μ_k is the weight of DM d_k.

The idea of maximizing deviation method is that if the CDD among alternatives is smaller for an attribute, then the attribute should be assigned a smaller weight, otherwise, it should be assigned a larger weight. Let

$$d_q(\omega) = \sum_{p=1}^{m} d_{pq}(\omega) = \sum_{k=1}^{l}\sum_{p=1}^{m}\sum_{s=1}^{m} \omega_q \mu_k d(\widetilde{\alpha}_{pq}^{(k)},\widetilde{\alpha}_{sq}^{(k)}),\ p = 1,...,m, q = 1,...,n, \tag{33}$$

where $d_q(\omega)$ denotes the CDD of one alternative and others with respect to the attribute c_q, and then let

$$d(\omega) = \sum_{q=1}^{n} d_q(\omega) = \sum_{q=1}^{n}\sum_{k=1}^{l}\sum_{p=1}^{m}\sum_{s=1}^{m} \omega_q \mu_k d(\widetilde{\alpha}_{pq}^{(k)},\widetilde{\alpha}_{sq}^{(k)}), \tag{34}$$

which expresses the sum of the CDDs among all attributes.

Then we can construct the following single-objective optimization model to determine the attribute weights $\omega = (\omega_1,\omega_2,...,\omega_n)$ so as to make the UDDs $d(\omega)$ as large as possible. To do so, we can construct the model as follows:

$$\text{(M}-1)\ \max\ d(\omega) = \sum_{q=1}^{n}\sum_{k=1}^{l}\sum_{p=1}^{m}\sum_{s=1}^{m} \omega_q \mu_k d(\widetilde{\alpha}_{pq}^{(k)},\widetilde{\alpha}_{sq}^{(k)})$$
$$s.\ t.\quad \omega_q \in [0,1], q = 1,...,n, \sum_{q=1}^{n} \omega_q^2 = 1. \tag{35}$$

To solve the above model, we construct the Lagrange function:

$$L(\omega,\eta) = \sum_{q=1}^{n}\sum_{k=1}^{l}\sum_{p=1}^{m}\sum_{s=1}^{m} \omega_q \mu_k d(\widetilde{\alpha}_{pq}^{(k)},\widetilde{\alpha}_{sq}^{(k)}) + \frac{\eta}{2}\left(\sum_{q=1}^{n} \omega_q^2 - 1\right), \tag{36}$$

where η is the Lagrange parameter. Since both functions $d(\omega)$ and $L(\omega,\eta)$ are differentiable for $\omega_q (q = 1,...,n)$; differentiating Eq. (36) with respect to $\omega_q (q = 1,...,n)$ and setting the partial derivatives equal to zero, we get the following set of equations:

$$\begin{cases} \frac{\partial L(\omega,\eta)}{\partial \omega_q} = \sum_{k=1}^{l}\sum_{p=1}^{m}\sum_{s=1}^{m} \mu_k d(\widetilde{\alpha}_{pq}^{(k)},\widetilde{\alpha}_{sq}^{(k)}) + \eta\omega_q = 0, q = 1,...,n \\ \frac{\partial L(\omega,\eta)}{\partial \eta} = \sum_{q=1}^{n} \omega_q^2 - 1 = 0 \end{cases}. \tag{37}$$

From Eq. (37), we get a simple and exact formula for determining the attribute weights as follows:

$$\omega_q^* = \frac{\sum_{k=1}^{l}\sum_{p=1}^{m}\sum_{s=1}^{m} \mu_k d(\widetilde{\alpha}_{pq}^{(k)},\widetilde{\alpha}_{sq}^{(k)})}{\sqrt{\sum_{q=1}^{n}\left(\sum_{k=1}^{l}\sum_{p=1}^{m}\sum_{s=1}^{m} \mu_k d(\widetilde{\alpha}_{pq}^{(k)},\widetilde{\alpha}_{sq}^{(k)})\right)^2}}. \tag{38}$$

From Eq. (38), it can be verified easily that $\omega_q^* (q = 1,...,n)$ are positive that they do satisfy the constrained conditions in model (M-1) and the solution is unique. Furthermore, the normalized optimal weighting vector $\omega = (\omega_1,\omega_2,...,\omega_n)$ can be obtained as follows:

$$\omega_q = \frac{\sum_{k=1}^{l}\sum_{p=1}^{m}\sum_{s=1}^{m} \mu_k d(\widetilde{\alpha}_{pq}^{(k)},\widetilde{\alpha}_{sq}^{(k)})}{\sum_{q=1}^{n}\sum_{k=1}^{l}\sum_{p=1}^{m}\sum_{s=1}^{m} \mu_k d(\widetilde{\alpha}_{pq}^{(k)},\widetilde{\alpha}_{sq}^{(k)})}. \tag{39}$$

Let

$$D_q = \sum_{k=1}^{l}\sum_{p=1}^{m}\sum_{s=1}^{m} \mu_k d(\widetilde{\alpha}_{pq}^{(k)},\widetilde{\alpha}_{sq}^{(k)}), q = 1,2,...,n, \tag{40}$$

then

$$\omega_q = \frac{D_q}{\sum_{q=1}^{n} D_q}, q = 1,2,...n. \tag{41}$$

As a mater of fact, D_q represents the CDD of all alternatives for the attribute c_q and for all the DMs. Because the larger D_q, the more important the attribute c_q is, Eq. (41) is obtained directly by using each D_q divide the sum of D_q. The theoretic foundation of this method is based on information theory, that is, the attribute providing more information should be assigned a bigger weight.

5.3. The decision making procedure

To obtain the best option(s), the process with the SVN2TL Hamacher aggregation operators in MAGDM involves the following steps:

Step 1. Transform each SVNLNs decision matrix $A^{(k)}$ into the SVN2TLEs decision matrix $\widetilde{A}^{(k)} = (\widetilde{\alpha}_{pq}^{(k)})_{m\times n}$, where $\widetilde{\alpha}_{pq}^{(k)}$ are in the form of SVN2TLEs.

Step 2. Normalization of the decision matrix.
Generally speaking, with respect to attributes of alternatives, there are two main types of attributes including benefit attributes (J_b) and cost attributes (J_c). To eliminate the effect of the final decision result caused by different types of attribute values, a normalization of the decision matrices needs to be developed. Thus, the cost attributes can be transformed into the benefit attributes by using the negation operator in 2-tuple linguistic environment. Of course, if all attributes are in the same type, there is no doubt the normalized process can be omitted.

Step 3. Generation of attributes' weights by constructing optimization model from an objective point of view based on the maximizing deviation method.

Step 4. Input of decision information fusion
Aggregate all the individual decision matrix $\widetilde{A}^{(k)} = (\widetilde{\alpha}_{pq}^{(k)})_{m\times n}$ into a synthesize decision matrix $\widetilde{A} = (\widetilde{\alpha}_{pq})_{m\times n}$ by using the SVN2TLHWA operator or SVN2TLHGWA operator, where

$$\widetilde{\alpha}_{pq} = SVN2TLHWA(\widetilde{\alpha}_{pq}^{(1)},\widetilde{\alpha}_{pq}^{(2)},...,\widetilde{\alpha}_{pq}^{(l)}) = \bigoplus_{H\,k=1}^{l}(\mu_k \odot_H \widetilde{\alpha}_{pq}^{(k)}), \tag{42}$$

$$\widetilde{\alpha}_{pq} = SVN2TLHGWA(\widetilde{\alpha}_{pq}^{(1)},\widetilde{\alpha}_{pq}^{(2)},...,\widetilde{\alpha}_{pq}^{(l)}) = \bigotimes_{H\,k=1}^{l}(\widetilde{\alpha}_{pq}^{(k)})^{\mu_k}. \tag{43}$$

Step 5. Output of comprehensive evaluation values for each alternative
Based on the attribute weighting vector obtained in Step 3, utilize the SVN2TLHWA operator or SVN2TLHGWA operator again to derive the overall collective preference values $\widetilde{r}_p$ in terms of SVN2TLEs for each alternative $x_p (p = 1,2,...,m)$, where

$$\widetilde{r}_p = SVN2TLHWA(\widetilde{\alpha}_{p1},\widetilde{\alpha}_{p2},...,\widetilde{\alpha}_{pn}) = \bigoplus_{H\,q=1}^{n}(\omega_q \odot_H \widetilde{\alpha}_{pq}), \tag{44}$$

$$\widetilde{r}_p = SVN2TLHGWA(\widetilde{\alpha}_{p1},\widetilde{\alpha}_{p2},...,\widetilde{\alpha}_{pn}) = \bigotimes_{H\,q=1}^{n}(\widetilde{\alpha}_{pq}^{\omega_q}). \tag{45}$$

Step 6. Calculation of score function, the score knowledge measures, the certain knowledge measures and the hesitant-related knowledge measures of $\widetilde{r}_p (p = 1,2,...,m)$.
Use the equations in Section 3.3 to calculate the Calculation of score function, the score knowledge measures, the certain knowledge measures and the hesitant-related knowledge measures of $\widetilde{r}_p (p = 1,2,...,m)$, denoted by $S(\widetilde{r}_p), SK(\widetilde{r}_p), CK(\widetilde{r}_p)$ and

$HRK(\tilde{r}_p)$ of $\tilde{r}_p(p = 1,2,...,m)$, respectively.

Step 7. Ranking of all alternatives
Use the comparison method described in Definition 3.6 to rank all the alternatives and select the best one(s) according to $S(\tilde{r}_p), SK(\tilde{r}_p), CK(\tilde{r}_p)$ and $HRK(\tilde{r}_p)$ of $\tilde{r}_p(p = 1,2,...,m)$.

Step 8. End.

6. Illustrative example

In this section, we employ an evaluation of emergency response solutions for sustainable community development by applying the proposed MAGDM approach, and give an example to demonstrate its validity and effectiveness.

6.1. Background

Over the past several decades, economic development has been recognized as the only approach to improve quality of life and social status in communities and cities of different areas, especially developing countries. However, along with rapid economic development, recently, increasing natural and man-made disasters (such as earthquakes, floods, air pollution, and urban fire disaster) have urged governments to reconsider community development planning by encouraging using local resources in a sustainable way that enhances economic opportunities while improving social and environmental conditions. During the process of planning and implementing sustainable community development, one of the major components is emergency management that is designated to minimize the huge impacts by potentially catastrophic events on every socioeconomic aspect in local community. Emergency management is vital in implementing sustainable community development, for which community planning must include emergency response solutions to potential natural and man-made hazards.

6.2. Case study

In order to mitigate the damage of natural or man-made disaster in highly populated areas, more and more municipal governments in China have established emergency departments to provide rescue capacity. Considering one of the emergency management problems (adapted from (Ju et al., 2012)), the community development department of a major city that holds a state-level special economic zone needs to regularly evaluate a set of alternative response solutions against urban fire hazards.

Suppose there are four alternative rescue plans $\{x_1,x_2,x_3,x_4\}$ for evaluation against an urban fire disaster. Three DM teams $d_k(k = 1,2,3)$, i.e., employees team (d_1), external experts team (d_2) and senior management team (d_3), have been organized to evaluate the alternatives under three attributes: (c_1) Accident identifying capacity, (c_2) Rescuing capacity, (c_3) Emergency response resources supplying capacity. Due to the highly-unstructured characteristics of this management activity, assessment values are hardly to be assigned with crisp numbers and DMs are often inclined to be hesitant or irresolute in assigning those assessments. Therefore, in this case study, DMs are empowered to provide their preferences in terms of SVNLNs on the response solutions $x_p(p = 1,2,3,4)$ under the three attributes $c_q(q = 1,2,3)$. Assume that the four alternative rescue plans are to be evaluated using the following LTS $H = \{h_0$: Very bad, h_1: Bad, h_2: Somewhat bad, h_3: Fair, h_4: Somewhat good, h_5: Good, h_6: Very good$\}$.

To help maintain such solution repository, we investigate effective MAGDM approach for the complex problems of evaluating alternative emergency response solutions, where the weighting vector of DMs is $\mu = (0.37,0.33,0.3)^T$, the attributes' weights are completely unknown. Then, three SVNLNs matrices are collected and listed in following Tables 1–3.

Table 1
Decision matrix $A^{(1)}$.

	c_1	c_2	c_3
x_1	$\langle h_4,(0.4,0.2,0.3)\rangle$	$\langle h_5,(0.4,0.2,0.3)\rangle$	$\langle h_5,(0.3,0.2,0.5)\rangle$
x_2	$\langle h_3,(0.6,0.1,0.2)\rangle$	$\langle h_5,(0.6,0.1,0.2)\rangle$	$\langle h_4,(0.5,0.2,0.2)\rangle$
x_3	$\langle h_4,(0.3,0.2,0.3)\rangle$	$\langle h_4,(0.5,0.2,0.3)\rangle$	$\langle h_3,(0.5,0.3,0.1)\rangle$
x_4	$\langle h_4,(0.7,0.1,0.1)\rangle$	$\langle h_3,(0.6,0.1,0.2)\rangle$	$\langle h_2,(0.3,0.1,0.2)\rangle$

Table 2
Decision matrix $A^{(2)}$.

	c_1	c_2	c_3
x_1	$\langle h_5,(0.4,0.3,0.4)\rangle$	$\langle h_5,(0.5,0.3,0.2)\rangle$	$\langle h_2,(0.3,0.1,0.6)\rangle$
x_2	$\langle h_3,(0.4,0.2,0.3)\rangle$	$\langle h_5,(0.3,0.2,0.3)\rangle$	$\langle h_3,(0.6,0.2,0.2)\rangle$
x_3	$\langle h_3,(0.4,0.2,0.4)\rangle$	$\langle h_4,(0.6,0.3,0.4)\rangle$	$\langle h_3,(0.6,0.1,0.3)\rangle$
x_4	$\langle h_4,(0.8,0.1,0.2)\rangle$	$\langle h_4,(0.5,0.2,0.3)\rangle$	$\langle h_3,(0.4,0.2,0.2)\rangle$

Table 3
Decision matrix $A^{(3)}$.

	c_1	c_2	c_3
x_1	$\langle h_5,(0.5,0.2,0.3)\rangle$	$\langle h_4,(0.6,0.2,0.4)\rangle$	$\langle h_2,(0.2,0.1,0.6)\rangle$
x_2	$\langle h_4,(0.5,0.2,0.3)\rangle$	$\langle h_4,(0.7,0.2,0.2)\rangle$	$\langle h_2,(0.7,0.2,0.1)\rangle$
x_3	$\langle h_5,(0.5,0.1,0.3)\rangle$	$\langle h_5,(0.6,0.1,0.3)\rangle$	$\langle h_3,(0.6,0.2,0.1)\rangle$
x_4	$\langle h_4,(0.6,0.1,0.2)\rangle$	$\langle h_4,(0.5,0.2,0.2)\rangle$	$\langle h_4,(0.4,0.1,0.1)\rangle$

6.2.1. Procedure of MAGDM problem based on SVN2TL Hamacher aggregation operators

We adopt the proposed method to rank the alternatives in the example and select the best one(s). The decision steps are as follows:

Step 1. Transform each SVNLNs decision matrix $A^{(k)}$ into the SVN2TLEs decision matrix $\tilde{A}^{(k)} = (\tilde{a}_{pq}^{(k)})_{4\times 3}$, where $\tilde{a}_{pq}^{(k)}$ are in the form of SVN2TLEs, and they are shown as:

$$\tilde{A}^{(1)} = \begin{pmatrix} \langle(s_4,0),(0.4,0.2,0.3)\rangle & \langle(s_5,0),(0.4,0.2,0.3)\rangle & \langle(s_5,0),(0.3,0.2,0.5)\rangle \\ \langle(s_3,0),(0.6,0.1,0.2)\rangle & \langle(s_5,0),(0.6,0.1,0.2)\rangle & \langle(s_4,0),(0.5,0.2,0.2)\rangle \\ \langle(s_4,0),(0.3,0.2,0.3)\rangle & \langle(s_4,0),(0.5,0.2,0.3)\rangle & \langle(s_3,0),(0.5,0.3,0.1)\rangle \\ \langle(s_4,0),(0.7,0.1,0.1)\rangle & \langle(s_3,0),(0.6,0.1,0.2)\rangle & \langle(s_2,0),(0.3,0.1,0.2)\rangle \end{pmatrix},$$

$$\tilde{A}^{(2)} = \begin{pmatrix} \langle(s_5,0),(0.4,0.3,0.4)\rangle & \langle(s_5,0),(0.5,0.3,0.2)\rangle & \langle(s_2,0),(0.3,0.1,0.6)\rangle \\ \langle(s_3,0),(0.4,0.2,0.3)\rangle & \langle(s_5,0),(0.3,0.2,0.3)\rangle & \langle(s_3,0),(0.6,0.2,0.2)\rangle \\ \langle(s_3,0),(0.4,0.2,0.4)\rangle & \langle(s_4,0),(0.6,0.3,0.4)\rangle & \langle(s_3,0),(0.6,0.1,0.3)\rangle \\ \langle(s_4,0),(0.8,0.1,0.2)\rangle & \langle(s_4,0),(0.5,0.2,0.3)\rangle & \langle(s_3,0),(0.4,0.2,0.2)\rangle \end{pmatrix},$$

$$\tilde{A}^{(3)} = \begin{pmatrix} \langle(s_5,0),(0.5,0.2,0.3)\rangle & \langle(s_4,0),(0.6,0.2,0.4)\rangle & \langle(s_2,0),(0.2,0.1,0.6)\rangle \\ \langle(s_4,0),(0.5,0.2,0.3)\rangle & \langle(s_4,0),(0.7,0.2,0.2)\rangle & \langle(s_2,0),(0.7,0.2,0.1)\rangle \\ \langle(s_5,0),(0.5,0.1,0.3)\rangle & \langle(s_5,0),(0.6,0.1,0.3)\rangle & \langle(s_3,0),(0.6,0.2,0.1)\rangle \\ \langle(s_4,0),(0.6,0.1,0.2)\rangle & \langle(s_4,0),(0.5,0.2,0.2)\rangle & \langle(s_4,0),(0.4,0.1,0.1)\rangle \end{pmatrix}.$$

Step 2. Normalization of the decision matrix.

For all the measured attributes in this paper, we find that they are all benefit attributes, thus, they do not need normalization.

Step 3. Generation of attribute weights by constructing optimization model.

Utilize the objective optimal model (35), the attribute weights can be derived as follows:

$$\omega_1 = \frac{\sum_{k=1}^{3}\sum_{p=1}^{4}\sum_{s=1}^{4}\mu_k d(\tilde{a}_{p1}^{(k)},\tilde{a}_{s1}^{(k)})}{\sum_{q=1}^{3}\sum_{k=1}^{3}\sum_{p=1}^{4}\sum_{s=1}^{4}\mu_k d(\tilde{a}_{pq}^{(k)},\tilde{a}_{sq}^{(k)})} = 0.2926,$$

$$\omega_2 = \frac{\sum_{k=1}^{3}\sum_{p=1}^{4}\sum_{s=1}^{4}\mu_k d(\tilde{a}_{p2}^{(k)},\tilde{a}_{s2}^{(k)})}{\sum_{q=1}^{3}\sum_{k=1}^{3}\sum_{p=1}^{4}\sum_{s=1}^{4}\mu_k d(\tilde{a}_{pq}^{(k)},\tilde{a}_{sq}^{(k)})} = 0.3147,$$

$$\omega_3 = \frac{\sum_{k=1}^{3}\sum_{p=1}^{4}\sum_{s=1}^{4}\mu_k d(\widetilde{a}_{p3}^{(k)},\widetilde{a}_{s3}^{(k)})}{\sum_{q=1}^{3}\sum_{k=1}^{3}\sum_{p=1}^{4}\sum_{s=1}^{4}\mu_k d(\widetilde{a}_{pq}^{(k)},\widetilde{a}_{sq}^{(k)})} = 0.3927.$$

Step 4. Input of decision information fusion.

Aggregate all the individual decision matrix $\widetilde{A}^{(k)} = (\widetilde{a}_{pq}^{(k)})_{4\times3}$ into a synthesize decision matrix $\widetilde{A} = (\widetilde{a}_{pq})_{4\times3}$ by using the SVN2TLHWA operator, where

$$\widetilde{a}_{pq} = SVN2TLHWA(\widetilde{a}_{pq}^{(1)},\widetilde{a}_{pq}^{(2)},\widetilde{a}_{pq}^{(3)}) = \bigoplus_{k=1}^{3}{}_H(\mu_k \odot_H \widetilde{a}_{pq}^{(k)}),$$

then we can obtain the synthesize decision matrix with different parameters λ:

(1) $\lambda \to 0$ (the SVN2TLMA operator)

$$\widetilde{A} = \begin{pmatrix} \langle(s_5,-0.2273),(0.4340,0.2247,0.3270)\rangle & \langle(s_5,-0.1767),(0.5066,0.2247,0.2753)\rangle & \langle(s_4,0.1029),(0.2727,0.1227,0.5587)\rangle \\ \langle(s_3,0.3912),(0.5181,0.1460,0.2532)\rangle & \langle(s_5,-0.1767),(0.5827,0.1460,0.2247)\rangle & \langle(s_3,0.2971),(0.6101,0.2000,0.1539)\rangle \\ \langle(s_4,0.3187),(0.4042,0.1539,0.3270)\rangle & \langle(s_4,0.4612),(0.5680,0.1682,0.3270)\rangle & \langle(s_3,0.000),(0.5680,0.1659,0.1283)\rangle \\ \langle(s_4,0.0000),(0.7247,0.1000,0.1460)\rangle & \langle(s_4,-0.2815),(0.5423,0.1460,0.2247)\rangle & \langle(s_3,0.1629),(0.3665,0.1198,0.1539)\rangle \end{pmatrix}$$

(2) $\lambda = 1$ (the SVN2TLWA operator)

$$\widetilde{A} = \begin{pmatrix} \langle(s_5,-0.3020),(0.4312,0.2292,0.3305)\rangle & \langle(s_5,-0.2393),(0.4975,0.2292,0.2879)\rangle & \langle(s_3,0.4780),(0.2706,0.1298,0.5617)\rangle \\ \langle(s_3,0.3315),(0.5088,0.1554,0.2589)\rangle & \langle(s_5,-0.2393),(0.5503,0.1554,0.2292)\rangle & \langle(s_3,0.1490),(0.5994,0.2000,0.1631)\rangle \\ \langle(s_4,0.1107),(0.3962,0.1631,0.3305)\rangle & \langle(s_4,0.3638),(0.5648,0.1875,0.3305)\rangle & \langle(s_3,0.0000),(0.5648,0.1868,0.1455)\rangle \\ \langle(s_4,0.0000),(0.7121,0.1000,0.1554)\rangle & \langle(s_4,-0.3351),(0.5388,0.1554,0.2292)\rangle & \langle(s_3,0.0073),(0.3639,0.1262,0.1631)\rangle \end{pmatrix}$$

(3) (the SVN2TLEWA operator)

$$\widetilde{A} = \begin{pmatrix} \langle(s_5,-0.3151),(0.4295,0.2300,0.3315)\rangle & \langle(s_5,-0.2504),(0.4929,0.2300,0.2906)\rangle & \langle(s_3,0.2377),(0.2672,0.1304,0.5635)\rangle \\ \langle(s_3,0.3108),(0.5041,0.1563,0.2599)\rangle & \langle(s_5,-0.2504),(0.5311,0.1563,0.2300)\rangle & \langle(s_3,0.0728),(0.5958,0.2000,0.1639)\rangle \\ \langle(s_4,0.0611),(0.3900,0.1639,0.3315)\rangle & \langle(s_4,0.3484),(0.5635,0.1897,0.3315)\rangle & \langle(s_3,0.0000),(0.5635,0.1893,0.1478)\rangle \\ \langle(s_4,0.0000),(0.7094,0.1000,0.1563)\rangle & \langle(s_4,-0.3552),(0.5374,0.1563,0.2300)\rangle & \langle(s_3,-0.0728),(0.3615,0.1268,0.1639)\rangle \end{pmatrix}$$

(4) $\lambda \to +\infty$(the SVN2TLMI operator)

$$\widetilde{A} = \begin{pmatrix} \langle(s_5,-0.2924),(0.4319,0.2286,0.3299)\rangle & \langle(s_5,-0.2311),(0.4997,0.2286,0.2861)\rangle & \langle(s_4,-0.3950),(0.2714,0.1292,0.5609)\rangle \\ \langle(s_3,0.3436),(0.5111,0.1548,0.2582)\rangle & \langle(s_5,-0.2311),(0.5587,0.1548,0.2286)\rangle & \langle(s_3,0.1852),(0.6015,0.2000,0.1625)\rangle \\ \langle(s_4,0.1429),(0.3986,0.1625,0.3299)\rangle & \langle(s_4,0.3755),(0.5656,0.1857,0.3299)\rangle & \langle(s_3,0.0000),(0.5656,0.1849,0.1437)\rangle \\ \langle(s_4,0.0000),(0.7139,0.1000,0.1548)\rangle & \langle(s_4,-0.3237),(0.5396,0.1548,0.2286)\rangle & \langle(s_3,0.0452),(0.3648,0.1257,0.1625)\rangle \end{pmatrix}$$

Step 5. Output of comprehensive evaluation values for each alternative.

Based on the synthesis weighting vector of attributes obtained in Step 3, utilize the SVN2TLHWA operator again to derive the overall collective preference values $\widetilde{r}_p$ in terms of SVN2LEs for each alternative $x_p(p = 1,2,3,4)$, where

$$\widetilde{r}_p = SVN2TLHWA(\widetilde{a}_{p1},\widetilde{a}_{p2},\widetilde{a}_{p3}) = \bigoplus_{q=1}^{3}{}_H(\omega_q \odot_H \widetilde{a}_{pq})$$

with different parameters λ, we have:

(1) $\lambda \to 0$ (the SVN2TLMA operator)

$\widetilde{r}_1 = \langle(s_5,-0.4029),(0.4099,0.1695,0.3649)\rangle,\widetilde{r}_2,$

$= \langle(s_4,0.0943),(0.5778,0.1634,0.1958)\rangle$

$\widetilde{r}_3 = \langle(s_4,0.0368),(0.5302,0.1629,0.2034)\rangle,\widetilde{r}_4.$

$= \langle(s_4,-0.3661),(0.5781,0.1196,0.1679)\rangle$

(2) $\lambda = 1$ (the SVN2TLWA operator)

$\widetilde{r}_1 = \langle(s_4,0.3783),(0.3982,0.1827,0.3885)\rangle,\widetilde{r}_2,$

$= \langle(s_4,-0.1334),(0.5631,0.1712,0.2072)\rangle$

$\widetilde{r}_3 = \langle(s_4,-0.1495),(0.5222,0.1783,0.2380)\rangle,\widetilde{r}_4.$

$= \langle(s_4,-0.4440),(0.5455,0.1255,0.1783)\rangle$

(3) (the SVN2TLEWA operator)

$\widetilde{r}_1 = \langle(s_4,0.3215),(0.3934,0.1840,0.3943)\rangle,\widetilde{r}_2,$

$= \langle(s_4,-0.1813),(0.5585,0.1717,0.2083)\rangle$

$\widetilde{r}_3 = \langle(s_4,-0.1835),(0.5193,0.1798,0.2420)\rangle,\widetilde{r}_4.$

$= \langle(s_4,-0.4692),(0.5372,0.1260,0.1793)\rangle$

(4) $\lambda \to +\infty$(the SVN2TLMI operator)

$\widetilde{r}_1 = \langle(s_4,0.2085),(0.3803,0.1856,0.4047)\rangle,\widetilde{r}_2$

$= \langle(s_4,-0.2654),(0.5489,0.1724,0.2097)\rangle$

$\widetilde{r}_3 = \langle(s_4,-0.2358),(0.5125,0.1817,0.2471)\rangle,\widetilde{r}_4.$

$= \langle(s_3,0.4777),(0.5212,0.1266,0.1804)\rangle$

Step 6. Use the comparison method described in Definition 8 to rank all the alternatives and select the most appropriate one(s) in accordance with the value of $S(\widetilde{r}_p)(p = 1,2,3,4)$. The results are shown in Table 4. If the SVN2TLHWA operator is replaced by the SVN2TLHGWA operator in the above step 4, Table 5 lists the score values and rankings of the alternatives.

As we can see from Tables 4 and 5, depending on different aggregation operators, different orderings can we get. But these results may lead to the same decision that is, for all four rescue plans, response solution x_2 is the most appropriate one.

6.3. Sensitivity analysis of attribute weights and the parameter λ in Hamacher aggregation operators

To explore the effect of attribute weights and parameter λ in Hamacher aggregation operators on the ranking results, different weighting vectors and parameter λ are assigned for analysis.

6.3.1. Sensitivity analysis of attribute weights

Firstly, different attribute weights are assigned, and their corresponding score values and orderings are shown in Table 6. The final ranking order is sensitive to the attribute weights. As Fig. 1 shows, no matter how much the attribute weights change, the ranking values x_2 fluctuates little, in most cases, response solution x_2 is always the most appropriate one except some limiting cases, such as the attribute c_1 is assigned a higher weight, while the ranking value of x_1 is the most sensitive to the attribute weights. When the attributes c_1 and c_2 are only a small proportion of the all attribute weights, response solution x_1 has the lowest ranking value. In summary, the attribute weights have an important impact on selection of rescue plans, thus, determining the attribute weights is critical to the decision-making process. As the maximizing deviation method is an objective weight determination method, it can effectively avoid a subjective preference that may mislead the results. Further, the proposed weighting method has a relatively simple computation.

6.3.2. Meaning of the parameter λ in Hamacher aggregation operators

The analysis in Section 4.3 indicates that the parameter λ in Hamacher aggregation operators has a significant effect on the aggregated results of alternatives, and further on the solution to a MAGDM problem. It is possible to analyze how the different parameters λ affect the aggregation results. For $\lambda \in (0,+\infty)$, in this case, we take different values of λ into consideration: 0.01,1,2,…,50, which are provided by DMs. The results of collective overall score values $S(\widetilde{r}_p)(p = 1,2,3,4)$ obtained by the SVN2TLHWA operator and SVN2TLHGWA operator are shown in Figs. 2 and 3, respectively.

It is observed from Fig. 2 that all the score values obtained by the SVN2TLHWA operator decrease accordantly with the parameter λ increases, while score values obtained by the SVN2TLHGWA operator increase accordantly with the parameter λ increases. Cause for this is twofold. One is the Hamacher triangular norms themselves, that is the family of Hamacher t-norm is strictly decreasing with parameter λ increases and the family of Hamacher t-conorm is strictly increasing with parameter λ increases. Another is that the value of score function increases as $t_{\widetilde{r}_p}$, while decreases as $i_{\widetilde{r}_p}$ and $f_{\widetilde{r}_p}$. Meanwhile, we can observe that the score value obtained by the SVN2TLHWA operator is always smaller than the score value obtained by the SVN2TLHGWA operator for the same parameter value λ and the same aggregation argument values. The above results are consistent with theoretical analysis in Theorem 9 and 10.

With the use of Theorem 9 and 10, λ can be reasonably associated with the risk attitudes, in terms of the optimism and pessimism of DMs. To elaborate, a DM is risk-seeking when he/she prefers small λ, while the DM is risk-averse if he/she prefers large λ when the Hamacher weighted averaging aggregation operator is applied. The opposite conclusion can be drawn when the Hamacher geometric aggregation operator is applied. In the former situation, small λ indicates a large alternative score, while it indicates a small alternative score in the latter. DMs can choose the values of parameter λ in accordance with their preferences. From Fig. 3, for the SVN2TLHGWA operator, there is no influence on the final rankings of the alternatives for parameter, while in Fig. 2, for the SVN2TLHWA operator, the choice of parameter value λ has a great impact on the score values of the alternatives, and the ranking of the alternatives is affected, the rankings of alternatives with different parameter λ are given as follows:

(1) If $\lambda \in (0,1.33)$, then $x_2 \succ x_3 \succ x_1 \succ x_4$;
(2) If $\lambda = 1.33$, then $x_2 \succ x_3 \succ x_4 \sim x_1$;
(3) If $\lambda \in (1.33,+\infty)$, then $x_2 \succ x_3 \succ x_4 \succ x_1$.

From Figs. 2 and 3, we can see that the ordering of the alternatives is different, thus leading to different decisions. However, it seems that response solution x_2 is always the most appropriate one.

6.4. Comparative analysis and discussion

In order to demonstrate the feasibility and applicability of the proposed neutrosophic linguistic MAGDM method in this paper, a set of comparative study was conducted with the relevant frequently-used aggregation approach and classical decision making method (Tian et al., 2015; Wang et al., 2016; Ye, 2014b, 2015a), and the analysis is based on the same illustrative example described above.

6.4.1. Comparison with existing neutrosophic linguistic aggregation operators

Firstly, we compare our methods with previous neutrosophic linguistic aggregation operators including the weighted single-valued neutrosophic linguistic Maclaurin symmetric mean (WSVNLMSM) operator (or WSVNLGeoMSM operator) (Wang et al., 2016), SNLNWBM operator (Tian et al., 2015), interval neutrosophic linguistic weighted arithmetic average (INLWAA) operator (or INLWGA operator) (Ye, 2014b), and interval neutrosophic uncertain linguistic weighted arithmetic averaging (INULWAA) operator (or INULWGA operator) (Ye, 2017a). Here, we focus on the calculation process of INLWAA and INLWGA operators.

In Ye (2014b), Ye proposed the concepts of an interval neutrosophic linguistic set (INLS) and an interval neutrosophic linguistic number (INLN) as a further generalization of the concepts of an ILS and an intuitionistic linguistic fuzzy number MADM problems with interval neutrosophic linguistic information. In order to use the INLWAA operator and INLWGA operator, the evaluation values of this paper need to be transformed into interval 2-tuple neutrosophic linguistic information firstly. That is, $\widetilde{\alpha} = \langle (h_{\widetilde{\alpha}},\alpha_{\widetilde{\alpha}}),(t_{\widetilde{\alpha}},i_{\widetilde{\alpha}},f_{\widetilde{\alpha}})\rangle = \langle (h_{\widetilde{\alpha}},\alpha_{\widetilde{\alpha}}),([t_{\widetilde{\alpha}},t_{\widetilde{\alpha}}],[i_{\widetilde{\alpha}},i_{\widetilde{\alpha}}],[f_{\widetilde{\alpha}},f_{\widetilde{\alpha}}])\rangle$, where the lower and upper bounds are equal. In his method, the operational laws of INLNs have the similar form as Definition 3.2. The steps involved by using the INLWAA and INLWGA operators are as follows:

Step 1. According to the SVNLNs matrices $A^{(k)}(k = 1,2,3)$ provided by DMs, utilize the INLWAA or INLWGA operator to aggregate all the individual SVNLNs matrices $A^{(k)}(k = 1,2,3)$ into the collective SVNLNs matrix A, where

$$a_{pq} = INLWAA(a_{pq}^{(1)},a_{pq}^{(2)},a_{pq}^{(3)}) = \bigoplus_{k=1}^{3}(\lambda_k \odot a_{pq}^{(k)})$$

Thus, we can obtain the collective SVNLNs matrix A as

Table 4
Orderings with different cases of SVN2TLHWA operators.

Aggregation operators	Score values	Score value rankings	Final orderings
SVN2TLMI	$S(\tilde{r}_1) = (s_2,-0.0014)$ $S(\tilde{r}_2) = (s_2,0.3763)$ $S(\tilde{r}_3) = (s_2,0.3747)$ $S(\tilde{r}_4) = (s_2,0.3313)$	$S(\tilde{r}_2) > S(\tilde{r}_3) > S(\tilde{r}_4) > S(\tilde{r}_1)$	$x_2 > x_3 > x_4 > x_1$
SVN2TLGWA	$S(\tilde{r}_1) = (s_2,0.2130)$ $S(\tilde{r}_2) = (s_3,-0.4987)$ $S(\tilde{r}_3) = (s_2,0.4489)$ $S(\tilde{r}_4) = (s_3,0.4385)$	$S(\tilde{r}_2) > S(\tilde{r}_3) > S(\tilde{r}_4) > S(\tilde{r}_1)$	$x_2 > x_3 > x_4 > x_1$
SVN2TLEGWA	$S(\tilde{r}_1) = (s_2,0.2930)$ $S(\tilde{r}_2) = (s_3,-0.4496)$ $S(\tilde{r}_3) = (s_2,0.4843)$ $S(\tilde{r}_4) = (s_2,0.4752)$	$S(\tilde{r}_2) > S(\tilde{r}_3) > S(\tilde{r}_4) > S(\tilde{r}_1)$	$x_2 > x_3 > x_4 > x_1$
SVN2TLMA	$S(\tilde{r}_1) = (s_3,-0.4889)$ $S(\tilde{r}_2) = (s_3,-0.3027)$ $S(\tilde{r}_3) = (s_3,-0.3856)$ $S(\tilde{r}_4) = (s_3,-0.4332)$	$S(\tilde{r}_2) > S(\tilde{r}_3) > S(\tilde{r}_4) > S(\tilde{r}_1)$	$x_2 > x_3 > x_4 > x_1$

Table 5
Orderings with different cases of SVN2TLHGWA operators.

Aggregation operators	Score values	Score value rankings	Final orderings
SVN2TLMI	$S(\tilde{r}_1) = (s_2,-0.0014)$ $S(\tilde{r}_2) = (s_2,0.3763)$ $S(\tilde{r}_3) = (s_2,0.3747)$ $S(\tilde{r}_4) = (s_2,0.3313)$	$S(\tilde{r}_2) > S(\tilde{r}_3) > S(\tilde{r}_4) > S(\tilde{r}_1)$	$x_2 > x_3 > x_4 > x_1$
SVN2TLGWA	$S(\tilde{r}_1) = (s_2,0.2130)$ $S(\tilde{r}_2) = (s_3,-0.4987)$ $S(\tilde{r}_3) = (s_2,0.4489)$ $S(\tilde{r}_4) = (s_3,0.4385)$	$S(\tilde{r}_2) > S(\tilde{r}_3) > S(\tilde{r}_4) > S(\tilde{r}_1)$	$x_2 > x_3 > x_4 > x_1$
SVN2TLEGWA	$S(\tilde{r}_1) = (s_2,0.2930)$ $S(\tilde{r}_2) = (s_3,-0.4496)$ $S(\tilde{r}_3) = (s_2,0.4843)$ $S(\tilde{r}_4) = (s_2,0.4752)$	$S(\tilde{r}_2) > S(\tilde{r}_3) > S(\tilde{r}_4) > S(\tilde{r}_1)$	$x_2 > x_3 > x_4 > x_1$
SVN2TLMA	$S(\tilde{r}_1) = (s_3,-0.4889)$ $S(\tilde{r}_2) = (s_3,-0.3027)$ $S(\tilde{r}_3) = (s_3,-0.3856)$ $S(\tilde{r}_4) = (s_3,-0.4332)$	$S(\tilde{r}_2) > S(\tilde{r}_3) > S(\tilde{r}_4) > S(\tilde{r}_1)$	$x_2 > x_3 > x_4 > x_1$

$$A = \begin{pmatrix} \langle s_{4.63},(0.4093,0.2286,0.3299)\rangle & \langle s_{4.70},(0.4997,0.2286,0.2861)\rangle & \langle s_{3.11},(0.2714,0.1292,0.5609)\rangle \\ \langle s_{3.30},(0.5111,0.1548,0.2582)\rangle & \langle s_{4.70},(0.5587,0.1548,0.2286)\rangle & \langle s_{3.07},(0.6015,0.2000,0.1625)\rangle \\ \langle s_{3.97},(0.3986,0.1625,0.3299)\rangle & \langle s_{4.30},(0.5656,0.1857,0.3299)\rangle & \langle s_{3.00},(0.5656,0.1849,0.1437)\rangle \\ \langle s_{4.00},(0.7139,0.1000,0.1548)\rangle & \langle s_{3.63},(0.5396,0.1548,0.2286)\rangle & \langle s_{2.93},(0.3648,0.1257,0.1625)\rangle \end{pmatrix}$$

Step 2. Utilize the INLWAA operator again to aggregate the neutrosophic linguistic argument collections to determine the collective evaluation value of each alternative $x_p (p = 1,2,3,4)$ as follows:

$$r_p = INLWAA(a_{p1},a_{p2},...,a_{pn}) = \bigoplus_{q=1}^{n} (w_q \odot a_{pq})$$

Analogously, the overall assessment of each alternative can be obtained

$$r_1 = \langle s_{4.0551},(0.3982,0.1827,0.3885)\rangle, r_2,$$
$$= \langle s_{3.6503},(0.5631,0.1712,0.2072)\rangle$$
$$r_3 = \langle s_{3.6929},(0.5222,0.1783,0.2380)\rangle, r_4.$$
$$= \langle s_{3.4634},(0.5455,0.1255,0.1783)\rangle$$

Step 3. Rank the overall evaluation value $r_p (p = 1,2,3,4)$ by using the score function, where

$S(r_1) = s_{2.4695}, S(r_2) = s_{2.6583}, S(r_3) = s_{2.5923}, S(r_4) = s_{2.5878}$.

Then, the ranking order among the alternative is $x_2 > x_3 > x_4 > x_1$, therefore, response solution x_2 is the most appropriate one. The comparisons are shown in Table 7. Compared with the existing neutrosophic linguistic aggregation operators, our proposed approaches have the following advantages:

(1) Compared with INLWAA (or INLWGA operator) proposed by Ye (2014b), the proposed operators based on Hamacher t-norms are more robust and can capture the relationship between the arguments. The SVN2TL Hamacher aggregation operators can contain almost all of the arithmetic aggregation operators and geometric aggregation operators for SVN2TLEs according to different values of parameter λ.

Table 6
Orderings with different groups of attribute weights.

Attribute weights	Score values	Final orderings
$\omega_1 = (0.2926,0.3147,0.3927)^T$	$S(\tilde{r}_1) = (s_3,-0.3337), S(\tilde{r}_2) = (s_2,-0.1843)$ $S(\tilde{r}_3) = (s_3,-0.2971), S(\tilde{r}_4) = (s_3,-0.3429)$	$x_2 > x_3 > x_1 > x_4$
$\omega_2 = (0.0653,0.1390,0.7957)^T$	$S(\tilde{r}_1) = (s_2,0.1870), S(\tilde{r}_2) = (s_2,-0.4040)$ $S(\tilde{r}_3) = (s_2,0.4449), S(\tilde{r}_4) = (s_2,0.2779)$	$x_2 > x_3 > x_4 > x_1$
$\omega_3 = (0.0959,0.2851,0.6190)^T$	$S(\tilde{r}_1) = (s_2,0.4250), S(\tilde{r}_2) = (s_3,-0.2052)$ $S(\tilde{r}_3) = (s_3,-0.3978), S(\tilde{r}_4) = (s_2,0.4002)$	$x_2 > x_3 > x_1 > x_4$
$\omega_4 = (0.3291,0.3986,0.2724)^T$	$S(\tilde{r}_1) = (s_3,-0.2004), S(\tilde{r}_2) = (s_3,-0.0923)$ $S(\tilde{r}_3) = (s_3,-0.2364), S(\tilde{r}_4) = (s_3,-0.2584)$	$x_2 > x_1 > x_3 > x_4$
$\omega_5 = (0.0196,0.3476,0.3409)^T$	$S(\tilde{r}_1) = (s_3,-0.4182), S(\tilde{r}_2) = (s_3,0.0142)$ $S(\tilde{r}_3) = (s_3,-0.2835), S(\tilde{r}_4) = (s_2,0.4031)$	$x_2 > x_3 > x_1 > x_4$
$\omega_6 = (0.3114,0.3476,0.3409)^T$	$S(\tilde{r}_1) = (s_3,-0.2760), S(\tilde{r}_2) = (s_3,-0.1473)$ $S(\tilde{r}_3) = (s_3,-0.2705), S(\tilde{r}_4) = (s_3,-0.3041)$	$x_2 > x_3 > x_1 > x_4$
$\omega_7 = (0.3218,0.5378,0.1404)^T$	$S(\tilde{r}_1) = (s_3,-0.0536), S(\tilde{r}_2) = (s_3,0.0495)$ $S(\tilde{r}_3) = (s_3,-0.1683), S(\tilde{r}_4) = (s_3,-0.2004)$	$x_2 > x_1 > x_3 > x_4$
$\omega_8 = (0.6957,0.0314,0.2729)^T$	$S(\tilde{r}_1) = (s_3,-0.2876), S(\tilde{r}_2) = (s_2,0.4080)$ $S(\tilde{r}_3) = (s_3,-0.3750), S(\tilde{r}_4) = (s_3,-0.0229)$	$x_4 > x_1 > x_3 > x_2$
$\omega_9 = (0.0478,0.1005,0.8518)^T$	$S(\tilde{r}_1) = (s_2,0.1101), S(\tilde{r}_2) = (s_3,-0.4613)$ $S(\tilde{r}_3) = (s_2,0.3918), S(\tilde{r}_4) = (s_2,0.2323)$	$x_2 > x_3 > x_4 > x_1$
$\omega_{10} = (0.7653,0.1390,0.0957)^T$	$S(\tilde{r}_1) = (s_3,-0.1062), S(\tilde{r}_2) = (s_3,-0.4492)$ $S(\tilde{r}_3) = (s_3,-0.3124), S(\tilde{r}_4) = (s_3,0.1007)$	$x_4 > x_1 > x_3 > x_2$

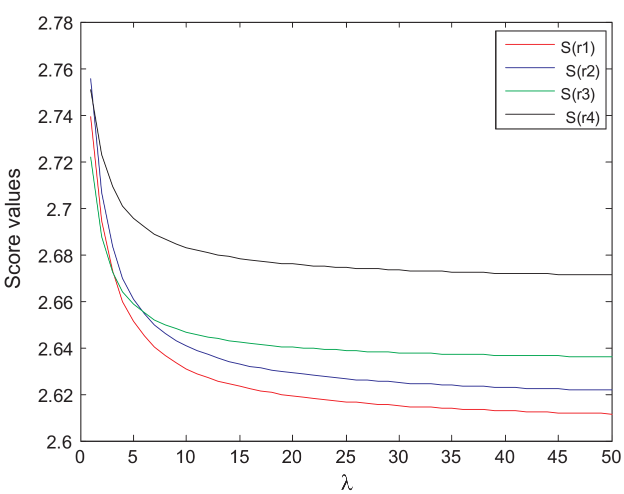

Fig. 1. Variation of the score values obtained by the SVN2TLHWA operator.

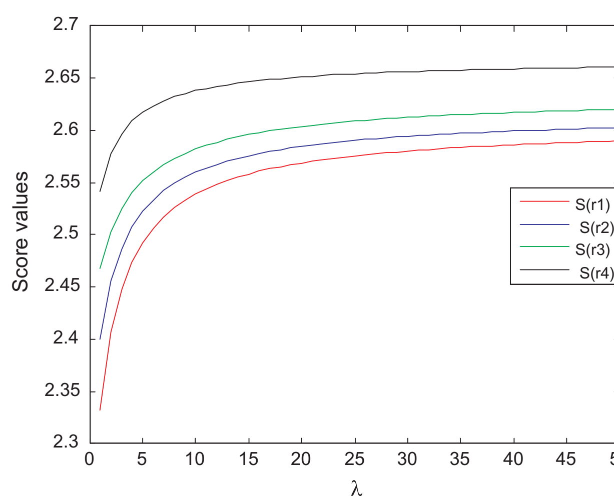

Fig. 2. Variation of the score values obtained by the SVN2TLHGWA operator.

By means of different parameter values, the dynamic variation trend of rankings of alternatives can be shown clearly. Relative to a static fixed evaluated result obtained by the existing neutrosophic linguistic aggregation operators, the dynamic evaluated result can better reflect the inherent variety law. So, our methods are more general.

(2) Compared with SNLNWBM operator proposed by Tian et al. (2015) and the WSVNLMSM (or WSVNLGeoMSM) operator proposed by Wang et al. (2016), the computational complexity of our methods are more simple. Meanwhile, the proposed methods include only one parameter, which can adjust the aggregate values based on the real decision needs, and capture many existing SVNSs aggregation operators, while the SNLNWBM operator includes two parameters, which makes it hard to determine two appropriate parameter values. Therefore, the benefit is that the proposed operators come with their higher conciseness and flexibility.

(3) Aggregation results obtained by comparative operators may not

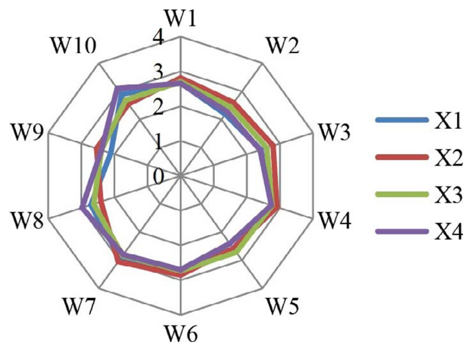

Fig. 3. The ranking of each response solution under ten different groups of attribute weights.

match any of the LTSs. For example, $r_1 = \langle s_{4.0551}, (0.3982, 0.1827, 0.3885)\rangle$. In such a case, as Tao et al. (2014) have pointed out in their introduction, there is an awareness that $s_{4.172}$ does not have any syntax or semantics assigned, because such a virtual LT makes sense only in comparison and operation. Moreover, in the calculation process, the product between the numerical value and the LT is usually employed to calculate the alternative collective evaluation value. For example, $0.28 \times s_2$, under the meaning of linguistic label, means "0.28 × *Very low*". However, what does "0.28 × *Very low*" mean in the actual decision problem? The 2-tuple linguistic representation model can make LTs continuous and prevent information from losing in aggregation process. So SVN2TL aggregation operators are more efficient and can avoid information loss and the lack of precision.

6.4.2. The TOPSIS method for neutrosophic linguistic MAGDM

In the following, the classical TOPSIS method is taken into consideration. The basic principle of the TOPSIS method is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The steps are involved by using the TOPSIS method based on Ye (2015a).

Step 1. Same as Step 1 in Section 6.3.1, we can obtain the collective SVNLNs matrix A as

Step 2. Define the neutrosophic linguistic positive-ideal solution (NLPIS) and neutrosophic linguistic negative-ideal solution (NLNIS). Since $h_{\theta(a_p)} \in H$, the smallest LT is h_0, and the largest LT is h_6. Thus, the NLPIS and NLNIS can be expressed as $v_s^+ = \langle h_6, (1,0,0)\rangle$ and $v_s^- = \langle h_0, (1,0,0)\rangle$, respectively.

Step 3. Calculate the distance between each alternative from NLPIS and NLNIS using the following equation, respectively:

$$d_p^+ = \sum_{q=1}^{n} \omega_p d(a_{pq}, v_s^+), d_p^- = \sum_{q=1}^{n} \omega_p d(a_{pq}, v_s^-). \tag{46}$$

The separation between alternatives can be measured by the Hamming distance or Euclidean distance. In order to measure the distances between SVN2TLEs, we adopt the SVNLNs Hammming distance proposed by Ye (2015a). Then, we can get d_p^+ and d_p^-, respectively. Obviously, for the attribute weights given, the smaller d_p^+ and the larger d_p^-, the better alternative.

Step 4. Calculate the closeness coefficient to ideal solution as

$$CC_p = \frac{d_p^-}{d_p^+ + d_p^-}, p = 1,2,3,4. \tag{47}$$

It follows that we can get the CC_p for alternative x_p as:

$$CC_1 = \frac{d_1^-}{d_1^+ + d_1^-} = 0.4114, CC_2 = \frac{d_2^-}{d_2^+ + d_2^-} = 0.4413,$$

$$CC_3 = \frac{d_3^-}{d_3^+ + d_3^-} = 0.4235, CC_4 = \frac{d_4^-}{d_4^+ + d_4^-} = 0.4295.$$

According to the closeness coefficient, we can determine the ranking of all alternatives as

$x_2 \succ x_4 \succ x_3 \succ x_1$.

It is obvious that the ranking of alternatives obtained by the single valued neutrosophic linguistic TOPSIS method is the same as that by the SVN2TL aggregation operators, which reflects the validity of the proposed method in this paper. Thus, response solution x_2 is the most appropriate one.

According to the comparison that focus on different angles, we find the result based on the SVN2TLHWA operator is the same as NLNWAA operator and TOPSIS methods. In fact, these methods have their own advantages and disadvantages correspondingly. In summary, the SVN2TLEs model proposed in this paper have the following characteristics:

(1) The SVN2TLEs consist of the 2TLVs and the subjective evaluation value on the reliability of the given 2TLVs, they not only reflect the principal assessment information for alternatives but also show the reliability of evaluation and the attitudes of the DMs in the process of MAGDM, the representation of SVN2TLEs is more reasonable than that of unique real numbers or 2TLVs. Thus, the decision-making method based on the SVN2TLEs is useful in handling complex decision-making problems.
(2) The primary advantage of using the SVN2TLHWA operator is that the aggregated results belong to the initial LTs, which is more appropriate and more easily comprehended. While in most of traditional linguistic MADM methods, the LTs may lead to information distortion and losing that occur in the process of information fusion.
(3) Compared with most aggregation operators based on Algebraic t-conorm and t-norm, new operational laws for SVN2TLEs based on Hamacher t-norm and t-conorm are closed and can overcome granularity and logical problems. The prominent characteristics of the SVN2TLHWA operator are not only for its effectiveness dealing with the preference information expressed by SVNLNs, but also for that it provides a very general formula including a wide range of aggregation operators, which can avoid losing and distorting the given preference information so as to make the final results accord with the real decision making problems, therefore, the aggregation operators based on Hamacher t-norm and t-conorm can provide another choice for DMs.
(4) Finally, we propose a model to deal with the situation where the weights information is unknown. The proposed model for optimal

Table 7
Comparison with existing neutrosophic linguistic aggregation operators.

Aggregation operators	Parameter number	Computation	Order of alternatives
INLWAA (or INLWGA operator) Ye (2014b)	None	Low	$x_2 > x_3 > x_4 > x_1$
INULWAA (or INULWGA operator) Ye (2017a)	None	Low	$x_2 > x_3 > x_4 > x_1$
WSVNLMSM (or WSVNLGeoMSM) operator Wang et al. (2016)	None	Median	$x_2 > x_3 > x_4 > x_1$
SNLNWBM operator Tian et al. (2015)	Two	High	$x_2 > x_3 > x_4 > x_1$
The proposed operators	One	Low	$x_2 > x_3 > x_4 > x_1$

weight vector is advantaged and effective, which takes objective weights information into consideration.

In summary, the developed method would be more suitable to handle indeterminate information and inconsistent information in complex decision-making problems. Therefore, it is more reasonable than existing methods.

7. Conclusions

This paper proposes a new class of FSs, which can be see as an extension of SVNLs, named SVN2TLs, they can satisfactorily reflect imprecise, incomplete, and inconsistent information in order to address decision-making situations that involve qualitative information rather than numerical information. Based on related research achievements in the literature, we propose some basic operational rules on SVN2TLEs via Hamacher triangular norms, which overcome the drawbacks of traditional operational rules of LTs. We also propose the Hamacher weighted averaging and Hamacher geometric weighted averaging of n SVN2TLEs based on the proposed operational rules. Finally, we give an numerical example to show the steps of the proposed method and discuss the influence of different parameters λ on the ranking results. In the future research, we can extend the application scopes of the proposed operators to other fields such as consensus models (Dong, Ding, Martínez, & Herrera, 2017; Gong, Forrest, Zhao, & Yang, 2012; Xu, Javier Cabrerizo, & Herrera-Viedma, 2017), preference relations (Chu, Liu, Wang, & Chin, 2016; Nie, Wang, & Li, 2017; Xu, Rui, & Wang, 2017; Zhou, Merigó, Chen, & Liu, 2016), and so on, or we can extend Hamacher triangular norms to several others decision environment (Broumi, Ye, & Smarandache, 2015; Fang & Ye, 2017; Zhou & Chen, 2013).

Acknowledgements

The work was supported by National Natural Science Foundation of China (Nos. 71771001, 71701001, 71301001, 71371011, 71501002, 71272047), Project of Anhui Province for Excellent Young Talents, Provincial Natural Science Research Project of Anhui Colleges (No. KJ2015A379), Anhui Provincial Philosophy and Social Science Program (No. AHSKQ2016D13), Scientific Research and Training Program of Anhui University (No. KYXL2016006), Innovation and Training Program of Anhui University (Nos. 201610357119, 201610357347, 201610357348, 201610357349, 201610357083), and the Doctoral Scientific Research Foundation of Anhui University.

References

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20*, 87–96.

Broumi, S., & Deli, I. (2016). Correlation measure for neutrosophic refined sets and its application in medical diagnosis. *Palestine Journal of Mathematics, 5*, 135–143.

Broumi, S., & Smarandache, F. (2014). Correlation coefficient of interval neutrosophic set. *Applied Mechanics and Materials, 436*, 511–517.

Broumi, S., Ye, J., & Smarandache, F. (2015). An extended TOPSIS method for multiple attribute decision making based on interval neutrosophic uncertain linguistic variables. *Neutrosophic Sets & Systems, 8*, 23–32.

Chen, Z. C., Liu, P. H., & Pei, Z. (2015). An approach to multiple attribute group decision making based on linguistic intuitionistic fuzzy numbers. *International Journal of Computational Intelligence Systems, 8*, 747–760.

Chu, J. F., Liu, X. W., Wang, Y. M., & Chin, K. S. (2016). A group decision making model considering both the additive consistency and group consensus of intuitionistic fuzzy preference relations. *Computers & Industrial Engineering, 101*, 227–242.

Delgado, M., Herrera, F., Herrera-Viedma, E., & Martinez, L. (1998). Combining numerical and linguistic information in group decision making. *Information Sciences, 107*, 177–194.

Deli, I. (2017). Interval-valued neutrosophic soft sets and its decision making. *International Journal of Machine Learning and Cybernetics, 8*, 665–676.

Deli, I., & Broumi, S. (2015). Neutrosophic soft matrices and NSM-decision making. *Journal of Intelligent & Fuzzy Systems, 28*, 2233–2241.

Dong, Y. C., Ding, Z. G., Martínez, L., & Herrera, F. (2017). Managing consensus based on leadership in opinion dynamics. *Information Sciences, 397*, 187–205.

Dong, Y. C., & Herrera-Viedma, E. (2015). Consistency-driven automatic methodology to set interval numerical scales of 2-tuple linguistic term sets and its use in the linguistic GDM with preference relation. *IEEE Transactions on Cybernetics, 45*, 780–792.

Dong, Y. C., Li, C. C., & Herrera, F. (2016). Connecting the linguistic hierarchy and the numerical scale for the 2-tuple linguistic model and its use to deal with hesitant unbalanced linguistic information. *Information Sciences, 376–368*, 259–278.

Dong, Y. C., Xiao, J., Zhang, H. J., & Wang, T. (2016). Managing consensus and weights in iterative multiple-attribute group decision making. *Applied Soft Computing, 48*, 80–90.

Fang, Z., & Ye, J. (2017). Multiple attribute group decision-making method based on linguistic neutrosophic numbers. *Symmetry, 9, 111*, 1–12.

Filip, I., & Szeidert, I. (2016). Adaptive fuzzy PI controller with shifted control singletons. *Expert Systems with Applications, 54*, 1–12.

Garg, H. (2016). Generalized intuitionistic fuzzy interactive geometric interaction operators using Einstein t-norm and t-conorm and their application to decision making. *Computers & Industrial Engineering, 101*, 53–69.

Gong, Z. W., Forrest, J., & Yang, Y. J. (2013). The optimal group consensus for 2-tuple linguistic preference relations. *Knowledge-Based Systems, 37*, 427–437.

Gong, Z. W., Forrest, J., Zhao, Y., & Yang, Y. J. (2012). The optimal group consensus deviation measure for multiplicative preference relations. *Expert Systems with Applications, 39*, 11548–11555.

Gong, Z. W., Xu, X. X., Li, L. S., & Xu, C. (2015). Consensus modeling with nonlinear utility and cost constraints: A case study. *Knowledge-Based Systems, 88*, 210–222.

Gong, Z. W., Zhang, H. H., Forrest, J., Li, L. S., & Xu, X. X. (2015). Two consensus models based on the minimum cost and maximum return regarding either all individuals or one individual. *European Journal of Operational Research, 240*, 183–192.

Hamacher, H. (1975). Uber logische Verknupfungen Aussagen und deren zugehorige Bewertungs-funktionen. Working Paper, 75/14. Aachen: RWTH.

Herrera, F., & Martínez, L. (2000a). A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems, 8*, 746–752.

Herrera, F., & Martínez, L. (2000b). An approach for combining linguistic and numerical information based on the 2-tuple fuzzy linguistic representation model in decision-making. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 8*, 539–562.

Jiang, Y. P., Fan, Z. P., & Ma, J. (2008). A method for group decision making with multi-granularity linguistic assessment information. *Information Sciences, 178*, 1098–1109.

Jin, F. F., Ni, Z. W., Chen, H. Y., Li, Y. P., & Zhou, L. G. (2016). Multiple attribute group decision making based on interval-valued hesitant fuzzy information measures. *Computers & Industrial Engineering, 101*, 103–115.

Ji, P., Wang, J. Q., & Zhang, H. Y. (2016). Frank prioritized Bonferroni mean operator with single-valued neutrosophic sets and its application in selecting third-party logistics providers. *Neural Computing and Applications*. http://dx.doi.org/10.1007/s00521-016-2660-6.

Ju, Y. B., Wang, A. H., & Liu, X. Y. (2012). Evaluating emergency response capacity by fuzzy AHP and 2-tuple fuzzy linguistic approach. *Expert Systems with Applications, 39*, 6972–6981.

Liang, R. X., Wang, J. Q., & Li, L. (2016). Multi-criteria group decision-making method based on interdependent inputs of single-valued trapezoidal neutrosophic information. *Neural Computing and Applications*. http://dx.doi.org/10.1007/s00521-016-2672-2.

Liu, P. D., & Shi, L. L. (2017). Some neutrosophic uncertain linguistic number Heronian mean operators and their application to multi-attribute group decision making. *Neural Computing and Applications, 28*, 1079–1093.

Liu, P. D., & Wang, Y. M. (2014). Multiple attribute decision making method based on single valued neutrosophic normalized weighted Bonferroni mean. *Neural Computing and Applications, 25*, 2001–2010.

Li, Y. Y., Zhang, H. Y., & Wang, J. Q. (2017). Linguistic neutrosophic sets and their application in multicriteria decision-making problems. *International Journal for Uncertainty Quantification, 7*, 135–154.

Martínez, L., & Herrera, F. (2012). An overview on the 2-tuple linguistic model for computing with words in decision making: Extensions, applications and challenges. *Information Sciences, 207*, 1–18.

Ma, Y. X., Wang, J. Q., Wang, J., & Wu, X. H. (2017). An interval neutrosophic linguistic multi-criteria group decision-making method and its application in selecting medical treatment options. *Neural Computing and Applications, 28*, 2745–2765.

Mendel, J. M., & Wu, D. (2010). *Perceptual computing: Aiding people in making subjective judgments.* Piscataway, New Jersey: IEEE – Wiley Press.

Merigó, J. M., Casanovas, M., & Palacios-Marqus, D. (2014). Linguistic group decision making with induced aggregation operators and probabilistic information. *Applied Soft Computing, 24*, 669–678.

Merigó, J. M., & Gil-Lafuente, A. M. (2013). Induced 2-tuple linguistic generalized aggregation operators and their application in decision-making. *Information Sciences, 236*, 1–16.

Merigó, J. M., Gil-Lafuente, A. M., & Martorell, O. (2012). Uncertain induced aggregation operators and its application to tourism management. *Expert Systems with Applications, 39*, 869–880.

Nie, R. X., Wang, J. Q., & Li, L. (2017). A shareholder voting method for proxy advisory firm selection based on 2-tuple linguistic picture preference relation. *Applied Soft Computing, 60*, 520–539.

Ouyang, Y., & Pedrycz, W. (2016). A new model for intuitionistic fuzzy multi-attributes decision making. *European Journal of Operational Research, 249*, 677–682.

Peng, J. J., Wang, J. Q., Wu, X. H., & Tian, C. (2017). Hesitant intuitionistic fuzzy aggregation operators based on the Archimedean t-norms and t-conorms. *International Journal of Fuzzy Systems, 19*, 702–714.

Peng, J. J., Wang, J. Q., Wang, J., Zhang, H. Y., & Chen, X. H. (2016). Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *International Journal of Systems Science, 47*, 2342–2358.

Qin, J. D., Liu, X. W., & Pedrycz, W. (2016). Frank aggregation operators and their application to hesitant fuzzymultiple attribute decision making. *Applied Soft Computing, 41*, 428–452.

Smarandache, F. (1999). *A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic.* Rehoboth: American Research Press.

Tan, C. Q., Yi, W. T., & Chen, X. H. (2015). Hesitant fuzzy Hamacher aggregation

operators for multicriteria decision making. *Applied Soft Computing, 26*, 325–349.
Tao, Z. F., Chen, H. Y., Zhou, L. G., & Liu, J. P. (2014). On new operational laws of 2-tuple linguistic information using Archimedean t-norm and s-norm. *Knowledge-Based Systems, 66*, 156–165.
Tian, Z. P., Wang, J., Wang, J. Q., & Zhang, H. Y. (2017). An improved MULTIMOORA approach for multi-criteria decision-making based on interdependent inputs of simplified neutrosophic linguistic information. *Neural Computing and Applications*. http://dx.doi.org/10.1007/s00521-016-2378-5.
Tian, Z. P., Wang, J., Wang, J. Q., & Zhang, H. Y. (2017). Simplified neutrosophic linguistic multi-criteria group decision-making approach to green product development. *Group Decision and Negotiation, 26*, 597–627.
Tian, Z. P., Wang, J., Zhang, H. Y., Chen, X. H., & Wang, J. Q. (2015). Simplified neutrosophic linguistic normalized weighted Bonferroni mean operator and its application to multi-criteria decision-making problems. *Filomat, 30*, 3339–3360.
Tian, Z. P., Wang, J., Zhang, H. Y., & Wang, J. Q. (2016). Multi-criteria decision-making based on generalized prioritized aggregation operators under simplified neutrosophic uncertain linguistic environment. *International Journal of Machine Learning and Cybernetics*. http://dx.doi.org/10.1007/s13042-016-0552-9.
Tian, Z. P., Zhang, H. Y., Wang, J., Wang, J. Q., & Chen, X. H. (2016). Multi-criteria decision-making method based on a cross-entropy with interval neutrosophic sets. *International Journal of Systems Science, 47*, 3598–3608.
Wang, J. Q., & Li, J. J. (2009). The multi-criteria group decision making method based on multi-granularity intuitionistic two semantics. *Science & Technology Information, 33*, 8–9.
Wang, H. B., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2005). *Interval neutrosophic sets and logic: Theory and applications in computing.* Phoenix: Hexis.
Wang, H. B., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure, 4*, 410–413.
Wang, J., Wang, J. Q., Zhang, H. Y., & Chen, X. H. (2016). Multi-criteria group decision-making approach based on 2-tuple linguistic aggregation operators with multi-hesitant fuzzy linguistic information. *International Journal of Fuzzy Systems, 18*, 81–97.
Wang, J. Q., Yang, Y., & Li, L. (2016). Multi-criteria decision-making method based on single-valued neutrosophic linguistic Maclaurin symmetric mean operators. *Neural Computing and Applications*. http://dx.doi.org/10.1007/s00521-016-2747-0.
Wu, Q., Wu, P., Zhou, Y. Y., Zhou, L., Chen, H., & Ma, X. (2015). Some 2-tuple linguistic generalized power aggregation operators and their applications to multiple attribute group decision making. *Journal of Intelligent & Fuzzy Systems, 29*, 423–436.
Xia, M. M., Xu, Z. S., & Zhu, B. (2012). Some issues on intuitionistic fuzzy aggregation operators based on Archimedean t-conorm and t-norm. *Knowledge-Based Systems, 31*, 78–88.
Xu, Y. J., & Da, Q. L. (2010). Standard and mean deviation methods for linguistic group decision making and their applications. *Expert Systems with Applications, 37*, 5905–5912.
Xu, Y. J., Javier Cabrerizo, F., & Herrera-Viedma, E. (2017). A consensus model for hesitant fuzzy preference relations and its application in water allocation management. *Applied Soft Computing, 58*, 265–284.
Xu, Y. J., Merigó, J. M., & Wang, H. M. (2012). Linguistic power aggregation operators and their application to multiple attribute group decision making. *Applied Mathematical Modelling, 36*, 5427–5444.
Xu, Y. J., Rui, D., & Wang, H. M. (2017). A dynamically weight adjustment in the consensus reaching process for group decision-making with hesitant fuzzy preference relations. *International Journal of Intelligent Systems, 48*, 1311–1321.
Xu, Y. J., & Wang, H. M. (2011). Approaches based on 2-tuple linguistic power aggregation operators for multiple attribute group decision making under linguistic environment. *Applied Soft Computing, 11*, 3988–3997.
Yager, R. R. (1988). On ordered weighted averaging aggregation in multicriteria decision making. *IEEE Transactions on systems, Man, and Cybernetics, 18*, 183–190.
Ye, J. (2013). Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *International Journal of General Systems, 42*, 386–394.
Ye, J. (2014a). Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. *Journal of Intelligent & Fuzzy Systems, 26*, 165–172.
Ye, J. (2014b). Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making. *Journal of Intelligent & Fuzzy Systems, 27*, 2231–2241.
Ye, J. (2015a). An extended TOPSIS method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers. *Journal of Intelligent & Fuzzy Systems, 28*, 247–255.
Ye, J. (2015b). Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. *Artificial Intelligence in Medicine, 63*, 171–179.
Ye, J. (2017a). Multiple attribute group decision making based on interval neutrosophic uncertain linguistic variables. *International Journal of Machine Learning and Cybernetics, 8*, 837–848.
Ye, J. (2017b). Simplified neutrosophic harmonic averaging projection-based method for multiple attribute decision-making problems. *International Journal of Machine Learning and Cybernetics, 8*, 981–987.
Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*, 338–356.
Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-I. *Information Sciences, 8*, 199–249.
Zhang, W. C., Xu, Y. J., & Wang, H. M. (2016). A consensus reaching model for 2-tuple linguistic multiple attribute group decision making with incomplete weight information. *International Journal of Systems Science, 47*, 389–405.
Zhou, L. G., & Chen, H. Y. (2013). The induced linguistic continuous ordered weighted geometric operator and its application to group decision making. *Computers & Industrial Engineering, 66*, 222–232.
Zhou, L. G., Chen, H. Y., & Liu, J. P. (2012). Generalized logarithmic proportional averaging operators and their applications to group decision making. *Knowledge-Based Systems, 36*, 268–279.
Zhou, L. G., Merigó, J. M., Chen, H. Y., & Liu, J. P. (2016). The optimal group continuous logarithm compatibility measure for interval multiplicative preference relations based on the COWGA operator. *Information Sciences, 328*, 250–269.
Zou, L., Wen, X., & Wang, Y. X. (2016). Linguistic truth-valued intuitionistic fuzzy reasoning with applications in human factors engineering. *Information Sciences, 327*, 201–216.