Some New Kinds of Continuous Functions Via Fuzzy Neutrosophic Topological Spaces
Fatimah M. Mohammed, Shaymaa F. Motar
Department of Mathematics, College of Education for Pure Sciences, Tikrit University, Tikrit, Iraq

ABSTRACT
In this paper, we defined fuzzy neutrosophic-\(\tau_{0,2}\) continuous, fuzzy neutrosophic-\(\tau_{0,5}\) continuous, fuzzy neutrosophic-\(\tau_{0,1}\) contra continuous and fuzzy neutrosophic-\(\tau_{0,3}\) contra continuous functions. Then, we define the relationship between the define functions and studied functions with their comparative.

2. Some Basic of Topological Concepts
Definition 2.1 [8, 10]: Let X be a non-empty fixed set. Fuzzy neutrosophic set (FNS, for short), \(\lambda_N\) is an object having the form \(\lambda_N = \{< x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) > : x \in X \} \) where the functions \(\mu_{\lambda_N}, \sigma_{\lambda_N}, \nu_{\lambda_N} : X \rightarrow [0, 1] \) denote the degree of membership function (namely \(\mu_{\lambda_N}(x)\)), the degree of indeterminacy function (namely \(\sigma_{\lambda_N}(x)\)) and the degree of non-membership function (namely \(\nu_{\lambda_N}(x)\)) respectively, of each set \(\lambda_N\) we have, \(0 \leq \mu_{\lambda_N}(x) + \sigma_{\lambda_N}(x) + \nu_{\lambda_N}(x) \leq 3\), for each \(x \in X\).

Remark 2.2 [10]: FNS \(\lambda_N = \{< x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) > : x \in X \} \) can be identified to an ordered triple \(<x, \mu_{\lambda_N}, \sigma_{\lambda_N}, \nu_{\lambda_N} > \in [0, 1] \times X\).

Definition 2.3 [10]: Let X be a non-empty set and the FNSs \(\lambda_N\) and \(\beta_N\) be in the form \(\lambda_N = \{< x, \mu_{\lambda_N}(x), \sigma_{\lambda_N}(x), \nu_{\lambda_N}(x) > : x \in X \} \) and \(\beta_N = \{< x, \mu_{\beta_N}(x), \sigma_{\beta_N}(x), \nu_{\beta_N}(x) > : x \in X \} \). Then:

i. \(\lambda_N \subseteq \beta_N\) if \(\mu_{\lambda_N}(x) \leq \mu_{\beta_N}(x), \sigma_{\lambda_N}(x) \leq \sigma_{\beta_N}(x)\) and \(\nu_{\lambda_N}(x) \geq \nu_{\beta_N}(x)\). for all \(x \in X\),

ii. \(\lambda_N \equiv \beta_N\) if \(\lambda_N \subseteq \beta_N\) and \(\beta_N \subseteq \lambda_N\),

iii. \(1_N \lambda_N = \{< x, \nu_{\lambda_N}(x), 1 - \sigma_{\lambda_N}(x), \mu_{\lambda_N}(x) > : x \in X\},\)

iv. \(\lambda_N \cup \beta_N = \{< x, \max(\mu_{\lambda_N}(x), \mu_{\beta_N}(x)), \max(\sigma_{\lambda_N}(x), \sigma_{\beta_N}(x)), \min(\nu_{\lambda_N}(x), \nu_{\beta_N}(x)) > : x \in X\},\)

v. \(\lambda_N \cap \beta_N = \{< x, \min(\mu_{\lambda_N}(x), \mu_{\beta_N}(x)), \min(\sigma_{\lambda_N}(x), \sigma_{\beta_N}(x)), \max(\nu_{\lambda_N}(x), \nu_{\beta_N}(x)) > : x \in X\},\)

vi. \(\lambda_N - \beta_N = \{< x, \max(\mu_{\lambda_N}(x), \mu_{\beta_N}(x)) - \mu_{\beta_N}(x), \min(\sigma_{\lambda_N}(x), \sigma_{\beta_N}(x)) + \sigma_{\beta_N}(x), \min(\nu_{\lambda_N}(x), 1 - \nu_{\beta_N}(x)) > : x \in X\},\)

vii. \(\beta_N - \lambda_N = \{< x, \max(\mu_{\beta_N}(x), 1 - \mu_{\lambda_N}(x)), \min(\sigma_{\beta_N}(x), \sigma_{\lambda_N}(x)) + \sigma_{\lambda_N}(x), \max(\nu_{\beta_N}(x), \nu_{\lambda_N}(x)) > : x \in X\},\)

viii. \(\lambda_N \Delta \beta_N = \{< x, \max(\mu_{\lambda_N}(x), 1 - \mu_{\beta_N}(x)) - \mu_{\beta_N}(x), \min(\sigma_{\lambda_N}(x), \sigma_{\beta_N}(x)) + \sigma_{\beta_N}(x), \max(\nu_{\lambda_N}(x), 1 - \nu_{\beta_N}(x)) > : x \in X\},\)
\( \delta_n = \{ x, \min(\mu_n(x), \nu_n(x)), \min(\sigma_n(x), \sigma_n(x)), \max(\nu_n(x), \nu_n(x)) : x \in X \} \),

\( \lambda_n = \{ x, \mu_n(x), \sigma_n(x), \nu_n(x) : x \in X \} \),

\( \nu_n = \{ x, 0, 0, 1 \} \) and \( \delta_n = \{ x, 1, 1, 0 \} \).

**Definition 2.4** [10]: Fuzzy neutrosophic topology (FNT, for short) on a non-empty set \( X \) is a family \( \tau \) of fuzzy neutrosophic subsets in \( X \) satisfying the following axioms.

i. \( 0_n, 1_n \in \tau \),

ii. \( \lambda_{n1} \cap \lambda_{n2} \in \tau \) for any \( \lambda_{n1}, \lambda_{n2} \in \tau \),

iii. \( \lambda_{n1} \cup \lambda_{n2} \in \tau \) if \( \lambda_{n1} \in \tau \) and \( \lambda_{n2} \in \tau \),

iv. \( \lambda_{n1} \in \tau \) if \( \lambda_{n1} \in \tau \) and \( \lambda_{n2} \in \tau \).

Example 3.2: Let \( X = \{ a, b \} \) be FNTs \( \lambda_n \) in \( X \) and \( \beta_n \) in \( Y \) as follows:

\( \lambda_n = \{ x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.3}) \} > \) is FNT.

Then, \( f(x) \) is FNT-continuous (FN-\( \text{con.} \)) for \( f \) as follows:

\( f(a) = \text{FNT} \).

And, from \( f \) we get:

The family, \( \text{FNFr}_{0.1} = \{ 0_n, 1_n, \eta_n < x, (\frac{a}{0.4}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}) \} > \) is FNT.

**Definition 2.5** [9]: Let \( (X, \tau) \) be FNTs and \( \lambda_n = \{ x, \mu_n(x), \sigma_n(x), \nu_n(x) \} \) is FNS in \( X \). Then, the fuzzy neutrosophic-closure (FNC, for short) and fuzzy neutrosophic-Interior of \( \lambda_n \) (FNInt, for short) are defined by:

\( \text{FNC}(\lambda_n) = \{ y, \mu_n(y), \sigma_n(y), \nu_n(y) : y \in Y \} \) is FNS in \( Y \). Then, the inverses image of every FNTS (FN-\( \text{con.} \)) is called fuzzy neutrosophic-\( \tau \_0 \) continuous (FN-\( \text{con.} \) for short) if the inverse image of every FNS (FN-\( \text{closed} \)) set \( (Y, \text{FNFr}_{0.1}) \) is FNS (FN-\( \text{closed} \)) set \( (X, \text{FNFr}_{0.1}) \).

Some New Kinds of Continuous Functions Via Fuzzy Neutrosophic Topological Spaces

Now, we introduced a new concept in fuzzy neutrosophic topological spaces and called it fuzzy neutrosophic-\( \tau \_0 \) continuous, fuzzy neutrosophic-\( \tau \_0 \) continuous, fuzzy neutrosophic-\( \tau \_0 \) contra continuous and fuzzy neutrosophic-\( \tau \_0 \) contra continuous functions.
Then, \( f^{-1}(\Psi_{\eta}) = \{ x, (a, b) \} \in \text{FN}_{\text{r}0,2} \).

So, \( f^{-1}(\Psi_{\eta}) \) is FN-open set in \( \text{FN}_{\text{r}0,2} \). Hence, \( f \) is (FN-\( \text{r}_{0,\text{con}} \)) function.

**Theorem 3.3:**

Let \( (X, \tau_x), (Y, \tau_y) \) two FNTs and \( f: (X, \tau_x) \to (Y, \tau_y) \) is a function.

i. If, \( f \) is (FN-con.) function. Then, \( f \) is (FN-\( \text{r}_{0} \)) function.

ii. If, \( f \) is (FN-con.) function. Then, \( f \) is (FN-\( \text{r}_{0,\text{con}} \)) function.

**Proof:**

i. Let \( f \) be (FN-con.) function. Then, \( \beta_{\eta} = \{ y, \mu_{\eta}(y), \sigma_{\eta}(y), \nu_{\eta}(y) : y \in Y \} \) is FN-open set in \( \tau_x \), so

\[ f^{-1}(\beta_{\eta}) = \{ x, f^{-1}(\mu_{\eta}(x)), f^{-1}(\sigma_{\eta}(x)), f^{-1}(\nu_{\eta}(x)) : x \in X \} \]

isFN-open set in \( \tau_x \). And, by **Definition 2.8** we get:

\[ f^{-1}(\beta_{\eta}) = \{ x, f^{-1}(\mu_{\eta}(x)), f^{-1}(\sigma_{\eta}(x)), f^{-1}(\nu_{\eta}(x)) : x \in X \} \]

is FN-open set in \( \tau_x \). By **Definition 3.1** (i). Hence, \( f \) is (FN-\( \text{r}_{0,\text{con}} \)) function.

ii. Let \( f \) be (FN-con.) function. Then, \( \beta_{\eta} = \{ y, \mu_{\eta}(y), \sigma_{\eta}(y), \nu_{\eta}(y) : y \in Y \} \) is FN-open set in \( \tau_x \), so

\[ f^{-1}(\beta_{\eta}) = \{ x, f^{-1}(\mu_{\eta}(x)), f^{-1}(\sigma_{\eta}(x)), f^{-1}(\nu_{\eta}(x)) : x \in X \} \]

is FN-open set in \( \tau_x \). And, by **Definition 2.8** we get:

\[ f^{-1}(\Psi_{\eta}) = \{ x, f^{-1}(\psi_{\eta}(x)) : x \in X \} \]

is FN-open set in \( \tau_x \). By **Definition 3.1** (ii). Hence, \( f \) is (FN-\( \text{r}_{0,\text{con}} \)) function.

**Remark 3.4:**

The convers of **Theorem 3.3** is not true in general and we can show it by the following example.

**Example 3.5:** i. Let \( X = Y = \{ a, b \} \) define FNSs \( \lambda_{\eta} \) in \( X \) and \( \beta_{\eta} \) in \( Y \) as follows:

\[ \lambda_{\eta} = \{ \frac{a}{0.1}, \frac{b}{0.6} \} > \text{The family, } \tau_x = \{ 0_N, 1_N, \lambda_{\eta} \} \text{ is FNT.} \]

And, \( \beta_{\eta} = \{ \frac{a}{0.5}, \frac{b}{0.4} \} > \text{The family, } \tau_y = \{ 0_N, 1_N, \beta_{\eta} \} \text{ is FNT.} \)

Define \( f: (X, \tau_x) \to (Y, \tau_y) \) as follows: \( f(a) = b \) and \( f(b) = a \). Then, \( f^{-1}(\beta_{\eta}) > \{ x, \frac{a}{0.5}, \frac{b}{0.4} \} \in \text{FN}_{\text{r}0,2} \).

Hence, \( f \) is not (FN-con.) function.

But, from \( \tau_x \) we get:

The family, \( \text{FN}_{\text{r}0,1} = \{ 0_N, 1_N, \{ \frac{a}{0.4}, \frac{a}{0.5}, \frac{b}{0.5} \} \} \text{ is FNT.} \)

And, from \( \tau_y \) we get:

The family, \( \text{FN}_{\text{r}0,1} = \{ 0_N, 1_N, \{ \frac{a}{0.5}, \frac{b}{0.6} \} \} \text{ is FNT.} \)

Define \( f: (X, \text{FN}_{\text{r}0,1}) \to (Y, \text{FN}_{\text{r}0,1}) \) as follows: \( f(a) = b \) and \( f(b) = a \).

Then, \( f^{-1}(\beta_{\eta}) > \{ x, \frac{a}{0.5}, \frac{b}{0.6} \} \in \text{FN}_{\text{r}0,2} \).

Hence, \( f \) is not (FN-con.) function.

**Example 3.7:**

1. Take, **Example 3.5** (i). Then, \( f \) is (FN-\( \text{r}_{0,\text{con}} \)) function.
But, if \( f \) is not (FN-\( \tau_{0,2}\text{con.} \)) function. Since, from \( \tau_x \) we get:

The family, \( \text{FN}_{\tau_{0,2}}\{0_N, 1_N, < x, (a_{0.1}, b_{0.5}, \lambda_{0.5}), (a_{0.5}, b_{0.5}, 0) \} \) is FNT.

And, from \( \tau_y \) we get:

The family, \( \text{FN}_{\tau_{0,2}}\{0_N, 1_N, < y, (a_{0.5}, b_{0.3}, \lambda_{0.5}), (a_{0.3}, b_{0.5}, 0) \} \) is FNT.

Define \( f' : (X, \text{FN}_{\tau_{0,2}}) \to (Y, \text{FN}_{\tau_{0,2}}) \) as follows:

\[
\text{f(a)} = b \text{ and f(b)} = a.
\]

If, \( \Psi_N = < y, (a_{0.6}, b_{0.3}, \lambda_{0.5}, 0, 0) \rightarrow \text{FNT} \).

Then, \( f^{-1}(\Psi_N) = < x, (a_{0.6}, b_{0.3}, \lambda_{0.5}, 0, 0) \rightarrow \text{FNT} \).

And, from \( \tau_y \) we get:

The family, \( \text{FN}_{\tau_{0,1}}\{0_N, 1_N, < x, (a_{0.1}, b_{0.5}, \lambda_{0.5}), (a_{0.5}, b_{0.5}, 0) \} \) is FNT.

Then, \( f^{-1}(\Psi_N) = < x, (a_{0.6}, b_{0.3}, \lambda_{0.5}, 0, 0) \rightarrow \text{FNT} \).

Definition 3.8:

Let \( (X, \text{FN}_{\tau_{0,1}}) \) and \( (Y, \text{FN}_{\tau_{0,1}}) \) are two FNTSs. Then:

i. A function \( f : (X, \text{FN}_{\tau_{0,1}}) \to (Y, \text{FN}_{\tau_{0,1}}) \) is called fuzzy neutrosophic- \( \tau_{0,1}\text{contra continuous} \) (FN-\( \tau_{0,1}\text{con.} \), for short) if the inverse image of every FN-open (FN-closed) set in \( Y \) (FN-\( \tau_{0,1}\text{FNSs} \)) is FN-open (FN-closed) set in \( X \).

ii. A function \( f : (X, \text{FN}_{\tau_{0,2}}) \to (Y, \text{FN}_{\tau_{0,2}}) \) is called fuzzy neutrosophic- \( \tau_{0,2}\text{contra continuous} \) (FN-\( \tau_{0,2}\text{con.} \), for short) if the inverse image of every FN-open (FN-closed) set in \( Y \) (FN-\( \tau_{0,2}\text{FNSs} \)) is FN-closed (FN-open) set in \( X \).

Example 3.9: 1. Let \( X = Y = \{a, b\} \) define FNSs \( \lambda_N \) in \( X \) and \( \beta_N \) in \( Y \) as follows:

\[
\lambda_N = < x, (a_{0.9}, b_{0.6}, \lambda_{0.6}), (a_{0.6}, b_{0.5}, \lambda_{0.5}), (a_{0.5}, b_{0.4}, \lambda_{0.4}), (a_{0.4}, b_{0.3}, \lambda_{0.3}), (a_{0.3}, b_{0.2}, \lambda_{0.2}), (a_{0.2}, b_{0.1}, \lambda_{0.1}), (a_{0.1}, b_{0.0}, \lambda_{0.0}) \rightarrow \text{FNT}.
\]

The family, \( \tau_N = \{0_N, 1_N, \lambda_N, \beta_N \} \) is FNT.

Such that, \( 1_N \cap \tau_N = \{0_N, 0_N, \lambda_N \} \) is FNT.

Define \( f : (X, \tau_x) \to (Y, \tau_y) \) as follows:

\[
f(a) = b \text{ and f(b)} = a.
\]

And, if \( \beta_N = < y, (a_{0.6}, b_{0.3}, \lambda_{0.3}), (a_{0.3}, b_{0.5}, \lambda_{0.5}), (a_{0.5}, b_{0.4}, \lambda_{0.4}), (a_{0.4}, b_{0.2}, \lambda_{0.2}), (a_{0.2}, b_{0.1}, \lambda_{0.1}), (a_{0.1}, b_{0.0}, \lambda_{0.0}) \rightarrow \text{FNT}.
\]

The family, \( \tau_N = \{0_N, 1_N, \beta_N \} \) is FNT.

Then, \( f^{-1}(\beta_N) = < x, (a_{0.6}, b_{0.3}, \lambda_{0.3}), (a_{0.3}, b_{0.5}, \lambda_{0.5}), (a_{0.5}, b_{0.4}, \lambda_{0.4}), (a_{0.4}, b_{0.2}, \lambda_{0.2}), (a_{0.2}, b_{0.1}, \lambda_{0.1}), (a_{0.1}, b_{0.0}, \lambda_{0.0}) \rightarrow \text{FNT}.
\]

If, \( \beta_N = < y, (a_{0.6}, b_{0.3}, \lambda_{0.3}), (a_{0.3}, b_{0.5}, \lambda_{0.5}), (a_{0.5}, b_{0.4}, \lambda_{0.4}), (a_{0.4}, b_{0.2}, \lambda_{0.2}), (a_{0.2}, b_{0.1}, \lambda_{0.1}), (a_{0.1}, b_{0.0}, \lambda_{0.0}) \rightarrow \text{FNT}.
\]

Then, \( f^{-1}(\beta_N) = < y, (a_{0.6}, b_{0.3}, \lambda_{0.3}), (a_{0.3}, b_{0.5}, \lambda_{0.5}), (a_{0.5}, b_{0.4}, \lambda_{0.4}), (a_{0.4}, b_{0.2}, \lambda_{0.2}), (a_{0.2}, b_{0.1}, \lambda_{0.1}), (a_{0.1}, b_{0.0}, \lambda_{0.0}) \rightarrow \text{FNT}.
\]

TJPS
So, \( f^{-1}(\Psi_N) \) is FN-closed set in \( \text{FN} \tau_{0,2} \). Hence, \( f \) is (FN-\( \tau_{0,2} \)-ccon.) function.

**Remark 3.10:** i. The relation between (FN-ccon.) and (FN-\( \tau_{0,1} \)-ccon.) functions are independent.

ii. The relation between (FN-ccon.) and (FN-\( \tau_{0,2} \)-ccon.) functions are independent.

iii. The relation between (FN-\( \tau_{0,1} \)-ccon.) and (FN-\( \tau_{0,2} \)-ccon.) functions are independent.

And we can show it by the following example.

**Example 3.11:**

i. Take, **Example 3.9 (1)**. Then, \( f \) is (FN-ccon.) function.

But, \( f \) is not (FN-\( \tau_{0,1} \)-ccon.) function. Since, from \( \tau_x \) we get:

The family, \( \text{FN} \tau_{0,1} = \{0_N, 1_N, < x, (a_{0.9} b_{0.5}, (a_{0.5} b_{0.5}) \} \) is FNT.

Since, \( 1_N \text{FN} \tau_{0,1} = \{1_N, 0_N, < x, (a_{0.9} b_{0.5}, (a_{0.5} b_{0.5}) \} \)

And, from \( \tau_y \) we get:

The family, \( \text{FN} \tau_{0,1} = \{0_N, 1_N, < y, (a_{0.5} b_{0.4}, (a_{0.5} b_{0.5}) \}

(\( \frac{b}{0.5} \), (\( \frac{a}{0.5} \) b_{0.6}) \}) is FNT.

Define \( f : (X, \text{FN} \tau_{0,1}) \rightarrow (Y, \text{FN} \tau_{0,1}) \) as follows:

\( f(a) = b \) and \( f(b) = a \).

If, \( \eta_N < y, (\frac{b}{0.4} b_{0.4}, (\frac{a}{0.5} b_{0.5}) \) is FN-open set in \( \text{FN} \tau_{0,1} \).

Then, \( f^{-1}(\eta_N) = < x, (\frac{a}{0.4} b_{0.3}, (\frac{a}{0.5} b_{0.5}) \) \} is \( \text{FN} \tau_{0,1} \)

2- Take, **Example 3.9 (2)**. Then, \( f \) is (FN-\( \tau_{0,2} \)-ccon.) function.

But, \( f \) is not (FN-ccon.) function. Since, from \( \tau_x \) we get:

The family, \( \text{FN} \tau_{0,2} = \{0_N, 1_N, < x, (a_{0.5} b_{0.3}, (a_{0.5} b_{0.3}) \}

(\( \frac{b}{0.5} \), (\( \frac{a}{0.5} \) b_{0.5}) \} \)

Define \( f : (X, \text{FN} \tau_{0,2}) \rightarrow (Y, \text{FN} \tau_{0,2}) \) as follows:

\( f(a) = b \) and \( f(b) = a \).

If, \( \eta_N = < y, (\frac{b}{0.4} b_{0.7}, (\frac{a}{0.5} b_{0.5}) \) is FN-open set in \( \tau_y \).

Then, \( f^{-1}(\eta_N) = < x, (\frac{a}{0.4} b_{0.4}, (\frac{a}{0.5} b_{0.5}) \) \} is \( \text{FN} \tau_{0,2} \)

3- Take, **Example 3.9 (3)**. Then, \( f \) is (FN-\( \tau_{0,2} \)-ccon.) function.

But, \( f \) is not (FN-\( \tau_{0,2} \)-ccon.) function. Since, from \( \tau_x \) we get:

The family, \( \text{FN} \tau_{0,2} = \{0_N, 1_N, < x, (a_{0.5} b_{0.3}, (a_{0.5} b_{0.3}) \}

(\( \frac{b}{0.5} \), (\( \frac{a}{0.5} \) b_{0.5}) \} \)

Define \( f : (X, \text{FN} \tau_{0,2}) \rightarrow (Y, \text{FN} \tau_{0,2}) \) as follows:

\( f(a) = b \) and \( f(b) = a \).

If, \( \eta_N = < y, (\frac{b}{0.4} b_{0.7}, (\frac{a}{0.5} b_{0.5}) \) is FN-open set in \( \tau_y \).

Then, \( f^{-1}(\eta_N) = < x, (\frac{a}{0.4} b_{0.4}, (\frac{a}{0.5} b_{0.5}) \) \} is \( \text{FN} \tau_{0,2} \)
The relation between (FN-con.) and (FN-con.) are independent.

The relation between (FN-$\tau_{0,1}$con.) and (FN-$\tau_{0,1}$con.) are independent.

The relation between (FN-$\tau_{0,2}$con.) and (FN-$\tau_{0,2}$con.) are independent. And we can show it by the following example.

**Example 3.13:**

i. 1- Take, Example 3.9 (1). Then, $f$ is (FN-con.) function.

But, $f$ is not (FN-con.) function. Since, $f^{-1}(\beta_0) \notin \tau_x$.

2- Take, Example 3.2 (1). Then, $f$ is (FN-con.) function.

But, $f$ is not (FN-con.) function. Since, $f^{-1}(\beta_0) \notin 1_N^-$.

ii. 1- Take, Example 3.9 (2). Then, $f$ is (FN-$\tau_{0,1}$con.) function.

But, $f$ is not (FN-$\tau_{0,1}$con.) function. Since, $f^{-1}(\eta_0) \notin FN_{\tau_{0,1}}$.

2- Take, Example 3.2 (2). Then, $f$ is (FN-$\tau_{0,1}$con.) function.

But, $f$ is not (FN-$\tau_{0,1}$con.) function. Since, $f^{-1}(\eta_0) \notin 1_{N^-} FN_{\tau_{0,1}}$.

iii. 1- Take, Example 3.9 (3). Then, $f$ is (FN-$\tau_{0,2}$con.) function.

But, $f$ is not (FN-$\tau_{0,2}$con.) function. Since, $f^{-1}(\Psi_0) \notin FN_{\tau_{0,2}}$.

2- Take, Example 3.2 (3). Then, $f$ is (FN-$\tau_{0,2}$con.) function.

But, $f$ is not (FN-$\tau_{0,2}$con.) function. Since, $f^{-1}(\Psi_0) \notin 1_{N^-} FN_{\tau_{0,2}}$.

**Definition 3.14:**

Fuzzy neutrosophic subset $\lambda_0$ of FNTS $(X, \tau)$ is called fuzzy neutrosophic-clopen set (FN-clopen, for short) set if $\lambda_0$ is FN-closed set and FN-open set in same time.

**Theorem 3.15:**

i. Let $(X, \tau_x)$ and $(Y, \tau_y)$ are two FNTSs and $f: (X, \tau_x) \rightarrow (Y, \tau_y)$ is a function. $f$ is (FN-con.) iff $f$ is (FN-con.) whenever, every the invers image of any FNS in $\tau_y$ is FN-clopen set in $\tau_x$.

ii. Let $(X, FN_{\tau_{0,1}})$ and $(Y, FN_{\tau_{0,1}})$ are two FNTSs and $f: (X, FN_{\tau_{0,1}}) \rightarrow (Y, FN_{\tau_{0,1}})$ is a function. $f$ is (FN-$\tau_{0,1}$con.) iff $f$ is (FN-$\tau_{0,1}$con.) whenever, every the invers image of any FNS in $FN_{\tau_{0,1}}$ is FN-clopen set in $FN_{\tau_{0,1}}$.

iii. Let $(X, FN_{\tau_{0,2}})$ and $(Y, FN_{\tau_{0,2}})$ are two FNTSs and $f: (X, FN_{\tau_{0,2}}) \rightarrow (Y, FN_{\tau_{0,2}})$ is a function. $f$ is (FN-$\tau_{0,2}$con.) iff $f$ is (FN-$\tau_{0,2}$con.) whenever, every the invers image of any FNS in $FN_{\tau_{0,2}}$ is FN-clopen set in $FN_{\tau_{0,2}}$.

**Proof:**

i. Let $f$ be (FN-con.) function. If, $\beta_0$ be FN-open set in $\tau_y$.

Then, by **Definition 2.8** $f^{-1}(\beta_0) = \omega_N \in \tau_x$.

But, $\omega_N$ be FN-clopen set in $\tau_x$. Therefore, $f^{-1}(\beta_0) = \omega_N \in 1_{N^-}$.

Hence, by **Definition 2.9** $f$ is (FN-con.) function.

Conversely; the proof is direct.

ii. Let $f$ be (FN-$\tau_{0,1}$con.) function. If, $\eta_0$ be FN-open set in $FN_{\tau_{0,1}}$.

Then, by **Definition 3.1(i)** $f^{-1}(\eta_0) = \omega_N \in FN_{\tau_{0,1}}$.

But, $\omega_N$ be FN-clopen set in $FN_{\tau_{0,1}}$. So, $f^{-1}(\eta_0) = \omega_N \in 1_{N^-} FN_{\tau_{0,1}}$.

Hence, by **Definition 3.8 (i)** $f$ is (FN-$\tau_{0,1}$con.) function.

Conversely; the proof is direct.

iii. Let $f$ be (FN-$\tau_{0,2}$con.) function. If, $\Psi_0$ be FN-open set in $FN_{\tau_{0,2}}$.

Then, by **Definition 3.1(ii)** $f^{-1}(\Psi_0) = \omega_N \in FN_{\tau_{0,2}}$.

But, $\omega_N$ is FN-clopen set in $FN_{\tau_{0,2}}$. So, $f^{-1}(\Psi_0) = \omega_N \in 1_{N^-} FN_{\tau_{0,2}}$.

Hence, by **Definition 3.8 (ii)** $f$ is (FN-$\tau_{0,2}$con.) function.

Conversely; the proof is direct.

**Remark 3.16:** The next diagram showing the relationship between different functions. But the convers is not true in general.
References

بعض انواع جديدة من الدوال المستمرة من خلال فضاء تبولوجي نيوتروسوفك المضبب

فاطمة محمود محمد ، شيماء فائق مطر
قسم الرياضيات ، كلية التربية للعلوم الصرفة ، جامعة تكريت ، تكريت ، العراق

الملخص
في هذا البحث ، عرفنا كل من لدوال المستمرة $fuzzy\ neutrosophic-\tau_{0,1}$، $fuzzy\ neutrosophic-\tau_{0,1}\,contra$، $fuzzy\ neutrosophic-\tau_{0,2}$، $fuzzy\ neutrosophic-\tau_{0,2}\,contra$، ثم وجدنا العلاقات بين الدوال المذكورة والمدرسة مع بعض المقارنات.