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Research Article

Some Results in Neutrosophic Soft Topology Concerning Neutrosophic Soft $*_b$ Open Sets

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In this article, new generalised neutrosophic soft open known as neutrosophic soft $*_b$ open set is introduced in neutrosophic soft topological spaces. Neutrosophic soft $*_b$open set is generated with the help of neutrosophic soft semiopen and neutrosophic soft preopen sets. Then, with the application of this new definition, some soft neutrosophical separation axioms, countability theorems, and countable space can be Hausdorff space under the subjection of neutrosophic soft sequence which is convergent, the cardinality of neutrosophic soft countable space, engagement of neutrosophic soft countable and uncountable spaces, neutrosophic soft topological features of the various spaces, soft neutrosophical continuity, the product of different soft neutrosophical spaces, and neutrosophic soft countably compact that has the characteristics of Bolzano Weierstrass Property (BVP) are studied. In addition to this, BVP shifting from one space to another through neutrosophic soft continuous functions, neutrosophic soft sequence convergence, and its marriage with neutrosophic soft compact space, sequentially compactness are addressed.

1. Introduction and Preliminaries

During the study towards possible applications in classical and nonclassical logic, fuzzy soft sets, vague soft set, and neutrosophic soft set are absolutely important. Nowadays, researchers daily deal with the complexities of modelling uncertain data in economics, engineering, environmental science, sociology, medical science, and many other fields. Classical methods are not always successful due to the reason that uncertainties appearing in these domains may be of various types. Zadeh [1] originated a new access of fuzzy set theory, which proved to be the most suitable agenda for dealing with uncertainties. While probability theory, rough sets [2], and other mathematical tools are considered as useful approaches to designate uncertainty. Each of these theories has its own inherent difficulties as pointed out by Molodtsov [3]. Molodtsov [4] suggested a completely new sophisticated approach of soft sets theory for modelling vagueness and uncertainty which is free from the complications affecting existing methods. In soft set theory the problem of setting the membership function, among other related problems, simply does not arise. Soft sets are considered as neighbourhood systems and are a special case of context-dependent fuzzy sets. Soft set theory has potential applications in many different fields, counting the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory. Maji et al. [5] functionalized soft sets in multicriteria decision-making problems by applying the technique of knowledge reduction to the information table induced by the soft set. In [6], they defined and studied several basic notions of soft set theory. In 2005, Pei and Miao [7] and Chen [8] improved the work of Maji et al. Cagman et al. [9] defined the concept of soft topology on a soft set and presented its related properties. The authors also discussed the foundations of the theory of soft topological spaces. Shabir and Naz [10] introduced soft topological spaces over an initial universe with a fixed set of parameters. The notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighbourhood of a point, soft separation axioms, and their basic properties were investigated. It is shown that a soft topological space gives a parametrized family of topological spaces. The authors discussed soft subspaces of a soft topological space and investigated characterization with respect to soft open and soft closed sets. Finally, soft Ti-spaces and notions of soft normal and soft regular spaces are addressed. Bayramov and Gunduz [11] investigated some basic notions of soft topological spaces by using the soft point notion. Later, the authors addressed T_i-soft spaces and the relationships between them. Lastly, the author defined soft compactness and explored some of its significant characteristics.

Khattak et al. [12] introduced the concept of soft (α, β) open sets and their characterization in soft single point topology. Zadeh [1] introduced the concept of a fuzzy set. The author described that a fuzzy set is a class of objects with a continuum of grades of membership. Further, the authors characterized the set by a membership function, assigning membership grade to each member of the set. The notions of inclusion, union, intersection, complement, relation, convexity, etc. were extended to such sets, and various properties of these notions in the context of fuzzy sets were established. In particular, a separation theorem for convex fuzzy sets was proved ignoring the prerequisites of mutually exclusive fuzzy sets.

Atanassov [13] introduced the concept of intuitionistic fuzzy set (IFS) which is an extension of the concept "fuzzy set." The authors discussed various properties including operations and relations over sets. Bayramov and Gunduz [14] introduced some important properties of intuitionistic fuzzy soft topological spaces and defined the intuitionistic fuzzy soft closure and interior of an intuitionistic fuzzy soft set. Furthermore, intuitionistic fuzzy soft continuous mapping and structural characteristics were discussed in their study. Deli and Broumi [15] defined for the first define a relation on neutrosophic soft sets. The new concept allows the composition of two neutrosophic soft sets. It is devised to derive useful information through the composition of two neutrosophic soft sets. Finally, a decision-making method on neutrosophic soft sets is presented.

Bera and Mahapatra [16] introduced the concept of cartesian product and the relationship on neutrosophic soft sets with a new attempt. Some properties of this concept have been discussed and verified with appropriate real-life examples. The neutrosophic soft composition has been defined and verified with the help of examples. Then, some basic properties neutrosophic set have been established. Further, the authors introduced neutrosophic soft function its properties have been introduced and some suitable examples. Injective, surjective, bijective, constant, and identity neutrosophic soft functions have been defined. Finally, properties of inverse neutrosophic soft function have been discussed with proper examples.

Maji [17] used the concept of the neutrosophic set in the soft set and introduced the neutrosophic soft set. Some definitions and related operations were introduced on the neutrosophic soft set.

Kamal et al. [18] introduced the concept of multivalued interval neutrosophic soft set which amalgamates multivalued interval neutrosophic set and soft set. According to the authors, the proposed set extended the notions of fuzzy set, intuitionistic fuzzy set, neutrosophic set, interval-valued neutrosophic set, multivalued neutrosophic set, soft set, and neutrosophic soft set. Further, the authors studied some basic operations such as complement, equality, inclusion, union, intersection, "AND", and "OR" for multivalued interval neutrosophic soft elements and discussed its associated properties. Moreover, the derivation of its properties, related examples, and some proofs on the propositions are included.

Karaaslan and Deli [19] introduced the concept of soft neutrosophic classical sets and its set theoretical operations such as union, intersection, complement, AND-product, and OR-product. In addition to these concepts and operations, the authors defined four basic types of sets of degenerate elements in a soft neutrosophic classical set. Then, the authors proposed a group decision-making method based on soft neutrosophic classical sets and gave the algorithm of the proposed method. The authors also made an application of the proposed method on a problem including soft neutrosophic classical data.

Karaaslan [20] studied the concept of single-valued neutrosophic refined soft set is defined as an extension of single-valued neutrosophic refined set. Also, set theoretical operations between two single-valued neutrosophic refined soft sets are defined, and some basic properties of these operations are investigated. Furthermore, two methods to calculate the correlation coefficient between two single-valued neutrosophic refined soft sets are proposed, and a clustering analysis application of one of the proposed methods is given.

Karaaslan [21] introduced the concept of possibility neutrosophic soft set and defined some related concepts such as possibility neutrosophic soft subset, possibility neutrosophic soft null set, and possibility neutrosophic soft universal set. Then, based on definitions of n-norm and n-conorm, we define set theoretical operations of possibility neutrosophic soft sets such as union, intersection and complement, and investigate some properties of these operations. The author also introduced AND-product and OR-product operations between two possibility neutrosophic soft sets. The author proposed a decision-making method called the possibility neutrosophic soft decision-making method (PNS decision-making method) which can be applied to the decision-making problems involving uncertainty based on AND-product operation. The author finally gave a numerical example to display the application of the method that can be successfully applied to the problems.

Karaaslan [22] introduced a similarity measure between possibility neutrosophic soft sets (PNS-set) is defined, and its properties are studied. A decision-making method is established based on the proposed similarity measure. Finally, an application of this similarity measure involving the real-life problem is given.

Karaaslan [23] addressed firstly Maji's definitions (Maji-2013) and verified that some propositions are incorrect by a counterexample. The author then redefined the concept of neutrosophic soft set and their operation based on Çağman (Çagman-2014) and investigated some properties of these operations. The author then constructed decision-making method and group decision-making which selects an optimum element from the alternatives by using weights of parameters. The author finally presented an example which shows that the method can be successfully applied to many problems that contain uncertainties.

Bera and Mahapatra [24] introduced the construction of topology on a neutrosophic soft set (NSS). The notion of the neutrosophic soft interior, neutrosophic soft closure, neutrosophic soft neighbourhood, neutrosophic soft boundary, regular NSS, and their basic properties are studied in this study. The base for neutrosophic soft topology and subspace topology on NSS are defined with suitable examples. Some related properties were also established. Moreover, the concept of separation axioms on neutrosophic soft topological space has been introduced along with the investigation of several structural characteristics.

Khattak et al. [25] for the first time bounced up the idea of the neutrosophic soft b open set, neutrosophic soft b closed sets and their properties. Also, the idea of neutrosophic soft b-neighbourhood and neutrosophic soft b-separation axioms in neutrosophic soft topological structures are reflected here. Later on, the important results are discussed related to these newly defined concepts with respect to soft points. The concept of neutrosophic soft b-separation axioms of neutrosophic soft topological spaces is diffused in different results with respect to soft points. Furthermore, properties of neutrosophic soft bT_i-space ($i = 0, 1, 2, 3, 4$) and some associations between them are discussed.

In this article, the concept of vague soft topology is initiated, and its structural characteristics are attempted with new definitions. Some fundamental definitions are given which are necessary for the upcoming study. Neutrosophic soft union, neutrosophic soft intersection, null neutrosophic soft set, absolute neutrosophic soft set, neutrosophic soft topology, and neutrosophic soft neighbourhood are discussed. Three new definitions are introduced with respect to soft points in neutrosophic soft topological spaces. These three definitions are neutrosophic soft semiopen, neutrosophic soft preopen, and neutrosophic soft $*_b$ open sets. Neutrosophic soft $*_b$ open set is generated with the help of neutrosophic soft semiopen and neutrosophic soft preopen sets. Then, with the application of this new definition, some neutrosophic soft separation axioms and neutrosophic soft other separation axioms are generated with respect to soft points of the spaces. The interplay among these neutrosophic soft separation axioms is also displayed with respect to soft points of the spaces. These results are verified with examples. The second criteria of $*_{b1}$ space are displayed. The engagement of neutrosophic soft $*_{b1}$ space with NS$*_b$ open set, neutrosophic soft $*_{b3}$ spaces relation with neutrosophic soft closer with respect to soft points of the spaces.

Neutrosophic soft countability, neutrosophic soft first countability and neutrosophic soft second countability, the relationship between these results, neutrosophic soft sequences and their convergence in NS $*_b$ Hausdorff space, and the cardinality results are discussed. In continuation, neutrosophic soft topological characteristics and inverse neutrosophic soft topological characteristics are addressed with respect to soft points. Neutrosophic soft product of neutrosophic soft structures is inaugurated. Characterization of Bolzano Weierstrass Property, neutrosophic soft compactness, related results, and neutrosophic sequentially compactness are discussed. In the end, some concluding remarks and future work are planted.

Definition 1 (see [25]). A neutrosophic set A on the father set $\langle \mathscr{X} \rangle$ is defined as follows:

$$A = \{\langle x, \mathbb{T}A(x), \mathbb{I}A(x), \mathbb{F}A(x) : x \in \langle \mathscr{X} \rangle \rangle\}, \tag{1}$$

where $\mathbb{T} : \langle \mathscr{X} \rangle \longrightarrow]0-,1+[, \mathbb{I} : \langle \mathscr{X} \rangle \longrightarrow]0-,1+[, \mathbb{F} : \langle \mathscr{X} \rangle \longrightarrow]0-,1+[$ and $0- \preccurlyeq \mathbb{T}A(x) + \mathbb{I}A(x) + \mathbb{F}A(x) \preccurlyeq 3+$.

Definition 2 (see [15]). Let $\langle \mathscr{X} \rangle$ be a father set, θ be a set of all parameters, and $\mathscr{L}(\langle \mathscr{X} \rangle)$ denotes the power set of $\langle \mathscr{X} \rangle$. A pair $(\mathfrak{f}, \theta)$ is referred to as a soft set over $\langle \mathscr{X} \rangle$, where $\mathfrak{f}$ is a map given by $\mathfrak{f} : \theta \longrightarrow \mathscr{L}(\langle \mathscr{X} \rangle)$. Then, a NS set $(\tilde{\mathfrak{f}}, \theta)$ over $\langle \mathscr{X} \rangle$ is a set defined by a set of valued functions signifying a mapping $\tilde{\mathfrak{f}} : \theta \longrightarrow \mathscr{L}(\langle \mathscr{X} \rangle)$ and is referred to as the approximate NS set function $(\tilde{\mathfrak{f}}, \theta)$. It can be written as a set of ordered pairs:

$$\left(\tilde{\mathfrak{f}}, \theta\right) = \left\{ \left(\left(n, [\![x, \mathbb{T}_{\tilde{\mathfrak{f}}(n)^{(x)}}, \mathbb{I}_{\tilde{\mathfrak{f}}(n)^{(x)}}, \mathbb{F}_{\tilde{\mathfrak{f}}(n)^{(x)}} : x \in \langle \mathscr{X} \rangle]\!] \right) : n \in \theta \right\}. \tag{2}$$

Definition 3 (see [24]). Let $(\tilde{\mathfrak{f}}, \theta)$ be a NSS over the father set $\langle \mathscr{X} \rangle$. The complement of $(\tilde{\mathfrak{f}}, \theta)$ is signified $(\tilde{\mathfrak{f}}, \theta)^c$ and is defined as follows:

$$\left(\tilde{\mathfrak{f}}, \theta\right)^c = \left\{ \left(\left(n, [\![x, \mathbb{T}_{\tilde{\mathfrak{f}}(n)^{(x)}}, 1 - \mathbb{I}_{\tilde{\mathfrak{f}}(n)^{(x)}}, \mathbb{F}_{\tilde{\mathfrak{f}}(n)^{(x)}} : x \in \langle \mathscr{X} \rangle]\!] \right) : n \in \theta \right\}. \tag{3}$$

It is clear that

$$\left(\left(\tilde{\mathfrak{f}}, \theta\right)^c\right)^c = \left(\tilde{\mathfrak{f}}, \theta\right). \tag{4}$$

Definition 4 (see [17]). Let $(\tilde{\mathfrak{f}}, \theta)$ and $(\tilde{\rho}, \theta)$ two NSS over the father $\langle \mathscr{X} \rangle$. $(\tilde{\mathfrak{f}}, \theta)$ is supposed to be NSSS of $(\tilde{\rho}, \theta)$ if $\mathbb{T}_{\tilde{\mathfrak{f}}(n)^{(x)}} \preccurlyeq \mathbb{T}_{\tilde{\rho}(n)^{(x)}}$, $\mathbb{I}_{\tilde{\mathfrak{f}}(n)^{(x)}} \preccurlyeq \mathbb{I}_{\tilde{\rho}(n)^{(x)}}$, $\mathbb{F}_{\tilde{\mathfrak{f}}(n)^{(x)}} \succcurlyeq \mathbb{F}_{\tilde{\rho}(n)^{(x)}}$, $\forall n \in \theta \& \forall x \in \langle \mathscr{X} \rangle$. It is signified as $(\tilde{\mathfrak{f}}, \theta) \sqsubseteq (\tilde{\rho}, \theta)$. $(\tilde{\mathfrak{f}}, \theta)$ is said to be NS equal to $(\tilde{\rho}, \theta)$ if $(\tilde{\mathfrak{f}}, \theta)$ is NSSS of $(\tilde{\rho}, \theta)$ and $(\tilde{\rho}, \theta)$ is NSSS of if $(\tilde{\mathfrak{f}}, \theta)$. It is symbolized as $(\tilde{\mathfrak{f}}, \theta) = (\tilde{\rho}, \theta)$.

Definition 5 (see [25]). Let $(\tilde{\mathfrak{f}}_1, \theta)\&(\tilde{\mathfrak{f}}_2, \theta)$ be two NSSS over father set$\langle \mathcal{X} \rangle$ such that $(\tilde{\mathfrak{f}}_1, \theta) \neq (\tilde{\mathfrak{f}}_2, \theta)$. Then, their union is signifies as $(\tilde{\mathfrak{f}}_1,\theta) \bigsqcup^{\sim} (\tilde{\mathfrak{f}}_2, \theta) = (\tilde{\mathfrak{f}}_3, \theta)$ and is defined as-
$(\tilde{\mathfrak{f}}_3, \theta) = \{ ((n, [\![x, \mathbb{T}_{\tilde{\mathfrak{f}}_3(n)^{(x)}}, \mathbb{I}_{\tilde{\mathfrak{f}}_3(n)^{(x)}}, \mathbb{F}_{\tilde{\mathfrak{f}}_3(n)^{(x)}} : x \in \langle \mathcal{X} \rangle]\!]) : n \in \theta\}$, where

$$\begin{cases} \mathbb{T}_{\tilde{\mathfrak{f}}_3(n)^{(x)}} = \max \left[\mathbb{T}_{\tilde{\mathfrak{f}}_1(n)^{(x)}}, \mathbb{T}_{\tilde{\mathfrak{f}}_2(n)^{(x)}}, \right. \\ \mathbb{I}_{\tilde{\mathfrak{f}}_3(n)^{(x)}} = \max \left[\mathbb{I}_{\tilde{\mathfrak{f}}_1(n)^{(x)}}, \mathbb{I}_{\tilde{\mathfrak{f}}_2(n)^{(x)}}, \right. \\ \mathbb{F}_{\tilde{\mathfrak{f}}_3(n)^{(x)}} = \min \left[\mathbb{F}_{\tilde{\mathfrak{f}}_1(n)^{(x)}}, \mathbb{F}_{\tilde{\mathfrak{f}}_2(n)^{(x)}}. \right. \end{cases} \tag{5}$$

Definition 6 (see [25]). Let $(\tilde{\mathfrak{f}}_1, \theta)$ and $(\tilde{\mathfrak{f}}_2, \theta)$ be two NSSS over the father set $\langle \mathcal{X} \rangle$ $s.t.(\tilde{\mathfrak{f}}_1, \theta) \neq (\tilde{\mathfrak{f}}_2, \theta)$. Then, their intersection is signified as $(\tilde{\mathfrak{f}}_1,\theta) \sqcap^{\sim} (\tilde{\mathfrak{f}}_2, \theta) = (\tilde{\mathfrak{f}}_3, \theta)$ and is defined as $(\tilde{\mathfrak{f}}_3, n) = \{ ((n, [\![x, \mathbb{T}_{\tilde{\mathfrak{f}}_3(n)^{(x)}}, \mathbb{I}_{\tilde{\mathfrak{f}}_3(n)^{(x)}}, \mathbb{F}_{\tilde{\mathfrak{f}}_3(n)^{(x)}} : \langle \mathcal{X} \rangle]\!]) : n \in \theta\}$,
where

$$\begin{cases} \mathbb{T}_{\tilde{\mathfrak{f}}_3(n)^{(x)}} = \min \left[\mathbb{T}_{\tilde{\mathfrak{f}}_1(n)^{(x)}}, \mathbb{T}_{\tilde{\mathfrak{f}}_2(n)^{(x)}}, \right. \\ \mathbb{I}_{\tilde{\mathfrak{f}}_3(n)^{(x)}} = \min \left[\mathbb{I}_{\tilde{\mathfrak{f}}_1(n)^{(x)}}, \mathbb{I}_{\tilde{\mathfrak{f}}_2(n)^{(x)}}, \right. \\ \mathbb{F}_{\tilde{\mathfrak{f}}_3(n)^{(x)}} = \max \left[\mathbb{F}_{\tilde{\mathfrak{f}}_1(n)^{(x)}}, \mathbb{F}_{\tilde{\mathfrak{f}}_2(n)^{(x)}}. \right. \end{cases} \tag{6}$$

Definition 7 (see [25]). Let $(\tilde{\mathfrak{f}}, \theta)$ be a NSS over the father set $\langle \mathcal{X} \rangle$. The complement of $(\tilde{\mathfrak{f}}, \theta)$ is signified $(\tilde{\mathfrak{f}}, \theta)^c$ and is defined as follows:

$$\left(\tilde{\mathfrak{f}}, \theta\right)^c = \left\{ \left(\left(n, [\![x, \mathbb{F}_{\tilde{\mathfrak{f}}(n)^{(x)}}, 1 - \mathbb{I}_{\tilde{\mathfrak{f}}(n)^{(x)}}, \mathbb{T}_{\tilde{\mathfrak{f}}(n)^{(x)}} : x \in \langle \mathcal{X} \rangle]\!] \right) : n \in \theta \right\}. \tag{7}$$

It is clear that

$$\left(\left(\tilde{\mathfrak{f}}, \theta\right)^c\right)^c = \left(\tilde{\mathfrak{f}}, \theta\right). \tag{8}$$

Definition 8 (see [25]). NSS $(\tilde{\mathfrak{f}}, \theta)$ be a NSS over the father set $\langle \mathcal{X} \rangle$ is said to be a null neutrosophic soft set. If $\mathbb{T}_{\tilde{\mathfrak{f}}(n)^{(x)}} = 0, \mathbb{I}_{\tilde{\mathfrak{f}}(n)^{(x)}} = 0, \mathbb{F}_{\tilde{\mathfrak{f}}(n)^{(x)}} = 1$; , $\forall n \in \theta \& \forall x \in \langle \mathcal{X} \rangle$.

It is signified as $0_{(\langle \mathcal{X} \rangle, \theta)}$.

Definition 9 (see [25]). NSS $(\tilde{\mathfrak{f}}, \theta)$ over the father set $\langle \mathcal{X} \rangle$ is an absolute NSS if $\mathbb{T}_{\tilde{\mathfrak{f}}(n)^{(x)}} = 1, \mathbb{I}_{\tilde{\mathfrak{f}}(n)^{(x)}} = 1, \mathbb{F}_{\tilde{\mathfrak{f}}(n)^{(x)}} = 0$; , $\forall n \in \theta \& \forall$ $x \in \langle \mathcal{X} \rangle$.

It is signified as $1_{(\langle \mathcal{X} \rangle, n)}$. Clearly, $0_{(\langle \mathcal{X} \rangle, n)}{}^c = 1_{(\langle \mathcal{X} \rangle, n)}$ and $1_{(\langle \mathcal{X} \rangle, n)}{}^c = 0_{(\langle \mathcal{X} \rangle, n)}$.

Definition 10 (see [25]). Let NSS $(\langle \tilde{\mathcal{X}} \rangle, \theta)$ be the family of all NS soft sets and $\tau \subset \text{NSS}(\langle \tilde{\mathcal{X}} \rangle, \theta)$. Then, τ is said to be a NS soft topology on $\langle \tilde{\mathcal{X}} \rangle$ if (1). $0_{(\langle \mathcal{X} \rangle, n)}, 1_{(\langle \mathcal{X} \rangle, n)} \in \tau$, (2). The union of any number of NS soft sets in $\tau \in \tau$, (3). The intersection of a finite number of NS soft sets in $\tau \in \tau$. Then, $(\langle \tilde{\mathcal{X}} \rangle, \tau, \theta)$ is said to be a NSTS over $\langle \tilde{\mathcal{X}} \rangle$.

Definition 11 (see [25]). Let NS be the family of all NS over $\langle \tilde{\mathcal{X}} \rangle$ and $x \in \widetilde{\langle \mathcal{X} \rangle}$, then NS $x_{(\mathfrak{a}, \mathfrak{b}, \mathfrak{c})}$ is supposed to be a N point, for $0 < \mathfrak{a}, \mathfrak{b}, \mathfrak{c} \leq 1$ and is defined as follows:

$$x_{(\mathfrak{a}, \mathfrak{b}, \mathfrak{c})^{(y)}} = \left\lfloor \begin{cases} (\mathfrak{a}, \mathfrak{b}, \mathfrak{c}) \text{ provided } y = x \} \\ (0, 0, 1) \text{provided } y \neq x \} \end{cases} \right\rfloor. \tag{9}$$

Definition 12 (see [25]). Let NSS $(\widetilde{\langle \mathcal{X} \rangle})$ be the family of all N soft sets over the father set $\widetilde{\langle \mathcal{X} \rangle}$. Then, NSS $(x_{(\mathfrak{a}, \mathfrak{b}, \mathfrak{c})})^e$ is called a NS point, for every $x \in \widetilde{\langle \mathcal{X} \rangle}, 0 \prec \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\} \preccurlyeq 1$, $e \propto \theta$, and is defined as follows:

$$x^e{}_{(\mathfrak{a}, \mathfrak{b}, \mathfrak{c})^{e'(y)}} = \begin{pmatrix} (\mathfrak{a}, \mathfrak{b}, \mathfrak{c}) \text{ provided } e' = e \wedge y = x \\ (0, 0, 1) \text{ provided } e' \neq e \wedge y \neq x \end{pmatrix}. \tag{10}$$

Definition 13 (see [25]). Let $(\tilde{\mathfrak{f}}, \theta)$ be a NSS over the father set $\langle \tilde{\mathcal{X}} \rangle$. We say that $x^e{}_{(\mathfrak{a}, \mathfrak{b}, \mathfrak{c})} \in (\tilde{\mathfrak{f}}, \theta)$ read as belonging to the NSS $(\tilde{\mathfrak{f}}, \theta)$ whenever

$$\mathfrak{a} \preccurlyeq \mathbb{T}_{\tilde{\mathfrak{f}}(x)^{(x)}}, \mathfrak{b} \preccurlyeq \mathbb{I}_{\tilde{\mathfrak{f}}(x)^{(x)}}, \mathfrak{c} \succcurlyeq \mathbb{F}_{\tilde{\mathfrak{f}}(x)^{(x)}}. \tag{11}$$

Definition 14 (see [25]). $(\langle \tilde{\mathcal{X}} \rangle, \tau, \theta)$ be a NSTS over $\langle \tilde{\mathcal{X}} \rangle$ and $(\tilde{\mathfrak{f}}, \theta)$ be a NSS over $\langle \tilde{\mathcal{X}} \rangle$. NSS $(\tilde{\mathfrak{f}}, \theta)$ in $(\langle \tilde{\mathcal{X}} \rangle, \tau, \theta)$ is called a NS nbhd of the NS point $x^e{}_{(\mathfrak{a}, \mathfrak{b}, \mathfrak{c})} \in (\tilde{\mathfrak{f}}, \theta)$, if there exists a NS open set $(\tilde{\mathcal{g}}, \theta)$ such that $x^e{}_{(\mathfrak{a}, \mathfrak{b}, \mathfrak{c})} \in (\tilde{\mathcal{g}}, \theta) \sqsubset (\tilde{\mathfrak{f}}, \theta)$.

2. Main Results

In this section, three new definitions are introduced with respect to soft points in neutrosophic soft topological spaces. These three definitions are neutrosophic soft semiopen, neutrosophic soft preopen, and neutrosophic soft $*_b$ open sets. Neutrosophic soft $*_b$ open set is generated with the help of neutrosophic soft semiopen and neutrosophic soft preopen sets. Then, with the help of this new definition, some important results are discussed. Examples are also given for a better understanding of some results. Neutrosophic soft countability that neutrosophic soft first countability and neutrosophic soft second countability, the relationship between these results, neutrosophic soft sequences and their convergence in NS $*_b$ Hausdorff space, and the cardinality results are discussed.

Definition 15. Let $(\langle \tilde{\mathcal{X}} \rangle, \tau, \theta)$ be a NSTS over $\langle \tilde{\mathcal{X}} \rangle \& (\tilde{\mathfrak{f}}, \theta)$ be a NS set over $\langle \tilde{\mathcal{X}} \rangle$. Then, $(\tilde{\mathfrak{f}}, \theta)$ is as follows:

(1) Neutrosophic soft semiopen if $(\tilde{\mathfrak{f}}, \theta) \subseteq \text{VScl}(\text{VSint}(\tilde{\mathfrak{f}}, \theta))$

(2) Neutrosophic soft preopen if $(\tilde{\mathfrak{f}}, \theta) \subseteq \text{VSint}(\text{VScl}(\tilde{\mathfrak{f}}, \theta))$

(3) Neutrosophic soft $*_b$open if $(\tilde{\mathfrak{f}}, \theta) \subseteq \text{VScl}(\text{VSint}(\tilde{\mathfrak{f}}, \theta))\sqcup\text{VSint}(\text{VScl}(\tilde{\mathfrak{f}}, \theta))$ and neutrosophic soft $*_b$ close if $(\tilde{\mathfrak{f}}, \theta) \supseteq \text{VScl}(\text{VSint}(\tilde{\mathfrak{f}}, \theta))\sqcap\text{VSint}(\text{VScl}(\tilde{\mathfrak{f}}, \theta))$

Definition 16. Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS over $\langle\tilde{\mathscr{X}}\rangle$, $x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})} \neq y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}$ are NS points. If there exist NS $*_b$ open sets $(\tilde{\mathfrak{f}}, \theta)$ and $(\tilde{\mathscr{g}}, \theta)$ such that $x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})} \in (\tilde{\mathfrak{f}}, \theta), x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}\sqcap(\tilde{\mathscr{g}}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$ or $y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')} \in (\tilde{\mathscr{g}}, \theta), y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}\sqcap(\tilde{\mathfrak{f}}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. Then, $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is called a NS $*_{b0}$space.

Definition 17. Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS over $\langle\tilde{\mathscr{X}}\rangle$, $x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})} \neq y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}$ are NS points. If there exist NS$*_b$ open sets $(\tilde{\mathfrak{f}}, \theta)$ and $(\tilde{\mathscr{g}}, \theta)$ such that $x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})} \in (\tilde{\mathfrak{f}}, \theta), x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}\sqcap(\tilde{\mathscr{g}}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$ and $y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')} \in (\tilde{\mathscr{g}}, \theta), y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}\sqcap(\tilde{\mathfrak{f}}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. Then, $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is called a NS$*_{b1}$space.

Definition 18. Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS over $\widetilde{\langle\mathscr{X}\rangle}$,$x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})} \neq y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}$ are NS points. If $\exists$NS $*_b$ open set $(\tilde{\mathfrak{f}}, \theta)$ and $(\tilde{\mathscr{g}}, \theta)$ such that $x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})} \in (\tilde{\mathfrak{f}}, \theta)$ and $y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')} \in (\tilde{\mathscr{g}}, \theta)$ and $(\tilde{\mathfrak{f}}, \theta)\sqcap(\tilde{\mathscr{g}}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. Then, $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is called a NS$**_{b2}$ space.

Definition 19. Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS. $(\tilde{\mathfrak{f}}, \theta)$ be a NS $*_b$ closed set and $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)\sqcap(\tilde{\mathfrak{f}}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. If there exists NS$*_b$ open sets $(\tilde{\mathscr{g}}_1, \theta)$ and $(\tilde{\mathscr{g}}_2, \theta)$ such that $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{\mathscr{g}}_1, \theta), (\tilde{\mathfrak{f}}, \theta) \subset (\tilde{\mathscr{g}}_2, \theta)$ and $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)\sqcap(\tilde{\mathscr{g}}_1, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$, then $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is called a NS $*_b$-regular space. $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is said to be NS$*_{b3}$ space, if is both a NS regular and NS $*_{b1}$space.

Definition 20. Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS. This space is a NS $**_b$normal space, if for every pair of disjoint NS $*_b$ close sets $(\tilde{\mathfrak{f}}_1, \theta)$ and $(\tilde{\mathfrak{f}}_2, \theta)$, $\exists$ disjoint NS $**_b$ open sets $(\tilde{\mathscr{g}}_1, \theta)$ and $(\tilde{\mathscr{g}}_2, \theta)$ $s.t.(\tilde{\mathfrak{f}}_1, \theta) \subset (\tilde{\mathscr{g}}_1, \theta)$ and $(\tilde{\mathfrak{f}}_2, \theta) \subset (\tilde{\mathscr{g}}_2, \theta)$.

$(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is said to be a NS $*_{b4}$space if it is both a NS $*_b$ normal and NS $*_{b1}$space.

Example 21. Suppose that the father set $\langle\tilde{\mathscr{X}}\rangle$ is assumed to be $\langle\tilde{\mathscr{X}}\rangle = \{x_1, x_2\}$ and the set of conditions by $\theta = \{e_1, e_2\}$. Let us consider NS set $(\tilde{\mathfrak{f}}, \theta)$ and $x^{e_1}{}_{1(0.1,0.4,0.7)}$, $x^{e_2}{}_{1(0.2,0.5,0.6)}$, $x^{e_1}{}_{2(0.3,0.3,0.5)}$, and $x^{e_1}{}_{2(0.4,0.4,0.4)}$ be NS points. Then, the family $\tau = \{0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}, 1_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}, (\tilde{\mathfrak{f}}_1, \theta), (\tilde{\mathfrak{f}}_2, \theta), (\tilde{\mathfrak{f}}_3, \theta), (\tilde{\mathfrak{f}}_4, \theta), (\tilde{\mathfrak{f}}_5, \theta), (\tilde{\mathfrak{f}}_6, \theta), (\tilde{\mathfrak{f}}_7, \theta), (\tilde{\mathfrak{f}}_8, \theta)\}$, where $(\tilde{\mathfrak{f}}_1, \theta) = x^{e_1}{}_{1(0.1,0.4,0.7)}$, $(\tilde{\mathfrak{f}}_2, \theta) = x^{e_2}{}_{1(0.2,0.5,0.6)}$, $(\tilde{\mathfrak{f}}_3, \theta) = x^{e_1}{}_{2(0.3,0.3,0.5)}$, $(\tilde{\mathfrak{f}}_4, \theta) = (\tilde{\mathfrak{f}}_1, \theta) \cup (\tilde{\mathfrak{f}}_2, \theta)$, $(\tilde{\mathfrak{f}}_5, \theta) = (\tilde{\mathfrak{f}}_1, \theta) \cup (\tilde{\mathfrak{f}}_3, \theta)$, $(\tilde{\mathfrak{f}}_6, \theta) = (\tilde{\mathfrak{f}}_2, \theta) \cup (\tilde{\mathfrak{f}}_3, \theta)$, $(\tilde{\mathfrak{f}}_7, \theta) = (\tilde{\mathfrak{f}}_1, \theta) \cup (\tilde{\mathfrak{f}}_2, \theta) \cup (\tilde{\mathfrak{f}}_3, \theta)$, $(\tilde{\mathfrak{f}}_8, \theta) = \{x^{e_1}{}_{1(0.1,0.4,0.7)}, x^{e_2}{}_{1(0.2,0.5,0.6)}, x^{e_1}{}_{2(0.3,0.3,0.5)}, x^{e_2}{}_{2(0.4,0.4,0.4)}\}$ is a NSTS. Thus, $(\langle\tilde{\mathscr{X}}\rangle, \tau, Ⅎ^{\text{parameter}})$ be a NSTS. Also, $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is NS$*_{b0}$ structure but it is not NS $**_{b1}$ because for NS points $x^{e_1}{}_{1(0.1,0.4,0.7)}, x^{e_2}{}_{2(0.4,0.4,0.4)}$,$(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ not NS$*_{b1}$.

Example 22. Suppose that the father set $\langle\tilde{\mathscr{X}}\rangle$ is assumed to be $\langle\tilde{\mathscr{X}}\rangle = \{x_1, x_2\}$ and the set of conditions by $Ⅎ^{\text{parameter}} = \{e_1, e_2\}$. Let us consider NS set $(\tilde{\mathfrak{f}}, Ⅎ^{\text{parameter}})$ and $x^{e_1}{}_{1(0.1,0.4,0.7)}$, $x^{e_2}{}_{1(0.2,0.5,0.6)}$, $x^{e_1}{}_{2(0.3,0.3,0.5)}$, and $x^{e_1}{}_{2(0.4,0.4,0.4)}$ be NS points. Then, the family $\tau = \{0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}, 1_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}, (\tilde{\mathfrak{f}}_1, \theta), (\tilde{\mathfrak{f}}_2, \theta), (\tilde{\mathfrak{f}}_3, \theta), (\tilde{\mathfrak{f}}_4, \theta), (\tilde{\mathfrak{f}}_5, \theta), (\tilde{\mathfrak{f}}_6, \theta), (\tilde{\mathfrak{f}}_7, \theta), (\tilde{\mathfrak{f}}_8, \theta), \cdots\cdots\ \cdots\ \cdots (\tilde{\mathfrak{f}}_{15}, \theta)\}$, where $(\tilde{\mathfrak{f}}_1, \theta) = x^{e_1}{}_{1(0.1,0.4,0.7)}$, $(\tilde{\mathfrak{f}}_2, \theta) = x^{e_2}{}_{1(0.2,0.5,0.6)}$, $(\tilde{\mathfrak{f}}_3, \theta) = x^{e_1}{}_{2(0.3,0.3,0.5)}$, $(\tilde{\mathfrak{f}}_4, \theta) = x^{e_2}{}_{2(0.4,0.4,0.4)}$ $(\tilde{\mathfrak{f}}_5, \theta) = (\tilde{\mathfrak{f}}_1, \theta) \cup (\tilde{\mathfrak{f}}_2, \theta)$, $(\tilde{\mathfrak{f}}_6, \theta) = (\tilde{\mathfrak{f}}_1, \theta) \cup (\tilde{\mathfrak{f}}_3, \theta)$, $(\tilde{\mathfrak{f}}_7, \theta) = (\tilde{\mathfrak{f}}_2, \theta) \cup (\tilde{\mathfrak{f}}_4, \theta)$, $(\tilde{\mathfrak{f}}_8, \theta) = (\tilde{\mathfrak{f}}_2, \theta) \cup (\tilde{\mathfrak{f}}_3, \theta)$, $(\tilde{\mathfrak{f}}_9, \theta) = (\tilde{\mathfrak{f}}_2, \theta) \cup (\tilde{\mathfrak{f}}_4, \theta)$, $(\tilde{\mathfrak{f}}_{10}, \theta) = (\tilde{\mathfrak{f}}_3, \theta) \cup (\tilde{\mathfrak{f}}_4, \theta)$, $(\tilde{\mathfrak{f}}_{11}, \theta) = (\tilde{\mathfrak{f}}_1, \theta) \cup (\tilde{\mathfrak{f}}_2, \theta) \cup (\tilde{\mathfrak{f}}_3, \theta)$, $(\tilde{\mathfrak{f}}_{12}, \theta) = (\tilde{\mathfrak{f}}_1, \theta) \cup (\tilde{\mathfrak{f}}_2, \theta) \cup (\tilde{\mathfrak{f}}_4, \theta)$, $(\tilde{\mathfrak{f}}_{13}, \theta) = (\tilde{\mathfrak{f}}_2, \theta) \cup (\tilde{\mathfrak{f}}_3, \theta) \cup (\tilde{\mathfrak{f}}_4, \theta)$, $(\tilde{\mathfrak{f}}_{14}, \theta) = (\tilde{\mathfrak{f}}_1, \theta) \cup (\tilde{\mathfrak{f}}_3, \theta) \cup (\tilde{\mathfrak{f}}_4, \theta)$, $(\tilde{\mathfrak{f}}_{15}, \theta) = \{x^{e_1}{}_{1(0.1,0.4,0.7)}, x^{e_2}{}_{1(0.2,0.5,0.6)}, x^{e_1}{}_{2(0.3,0.3,0.5)}, x^{e_2}{}_{2(0.4,0.4,0.4)}\}$ is a NSTS. Thus, $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS. Also, $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is NS$*_{b2}$ structure.

Theorem 23. *Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS. Then, $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NS$*_{b1}$structure if each NS point is a NS $*_b$-close.*

Proof. Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS over $\widetilde{\langle\mathscr{X}\rangle}$.$(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ be an arbitrary NS point. We establish $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ is a *N* soft $*_b$ open set. Let $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \in (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$. Then, either $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \succ (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, Ⅎ^{p\theta})or(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$ or$(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \succ\succ(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ or$(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)\prec\prec(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$. This means that $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$ and $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ are two are distinct NS points. Thus, $x \succ y$ or $x \prec y$ or $e' \succ e$ or $e' \prec e$ or $x \succ\succ y$ or $x \prec\prec y$ or $e' \succ\succ e$ or $e' \prec\prec e$. Since $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NS$*_{b1}$ structure, $\exists$a NS$*_b$ open set $(\tilde{\mathscr{g}}, \theta)$ so that $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \in (\tilde{\mathscr{g}}, \theta)$ and $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)\sqcap(\tilde{\mathscr{g}}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. Since, $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)\sqcap(\tilde{\mathscr{g}}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. So $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \in (\tilde{\mathscr{g}}, \theta) \subset (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$. Thus, $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ is a NS$*_b$ open set, i.e., $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ is a NS$*_b$ closed set. Suppose that each NS point $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ is a NS$**_b$ closed set. Then, $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)^c$ is a NS$*_b$ open set. Let $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)\sqcap(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. Thus, $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \in (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)^c$ and $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)\sqcap(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)^c = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. So $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NS-$*_{b1}$ space.□

Theorem 24. *Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS over the father set $\widetilde{\langle\mathscr{X}\rangle}$. Then, $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is NS-$*_{b2}$ space if and only if for distinct NS points $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ and $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$, there exists a NS$*_b$ open set $(\tilde{\mathfrak{f}}, \theta)$ containing there exists but not $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$ such that $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \notin \overline{(\tilde{\mathfrak{f}}, \theta)}$.*

Proof. Let $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$ be two NS points in NS$*_{b2}$ space. Then, there exist disjoint NS $*_b$ open sets $(\tilde{\mathfrak{f}}, \theta)$ and $(\tilde{g}, \theta)$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{\mathfrak{f}}, \theta)$ and $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \in (\tilde{g}, \theta)$. Since $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \sqcap (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$ and $(\tilde{\mathfrak{f}}, \theta) \sqcap (\tilde{g}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \notin (\tilde{\mathfrak{f}}, \theta) \Longrightarrow (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \notin \overline{(\tilde{\mathfrak{f}}, \theta)}$. Next, suppose that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$, there exists a NS$*_b$ open set $(\tilde{\mathfrak{f}}, \theta)$ containing $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ but not $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$ $s.t.(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \notin \overline{(\tilde{\mathfrak{f}}, \theta)}^c$ that is $(\tilde{\mathfrak{f}}, \theta)$ and $\overline{(\tilde{\mathfrak{f}}, \theta)}^c$ are mutually exclusive NS $*_b$ open sets supposing $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ and $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$ in turn.□

Theorem 25. *Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS. Then, $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is NS$*_{b1}$space if every NS point $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{\mathfrak{f}}, \theta) \in (\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$. If there exists a NS $*_b$ open set $(\tilde{g}, \theta)$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{g}, \theta) \subset \overline{(\tilde{g}, \theta)} \subset (\tilde{\mathfrak{f}}, \theta)$, then $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ a NS$*_{b2}$ space.*

Proof. Suppose $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \sqcap (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. Since $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is NS$*_{b1}$ space. $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ and $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$ are NS$*_b$ closed sets in $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$. Then, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in ((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta))^c \in (\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$. Thus, there exists a NS$*_b$ open set $(\tilde{g}, \theta) \in (\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{g}, \theta) \subset \overline{(\tilde{g}, \theta)} \subset ((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta))^c$. So we have $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \in (\tilde{g}, \theta)$ and $(\tilde{g}, \theta) \sqcap ((\tilde{g}, \theta))^c = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$, that is $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is a NS soft $*_{b2}$ space.□

Theorem 26. *Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS.$(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is soft $*_{b3}$ space if for every $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{\mathfrak{f}}, \theta)$ that is $(\tilde{g}, \theta) \in (\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{g}, \theta) \subset \overline{(\tilde{g}, \theta)} \subset (\tilde{\mathfrak{f}}, \theta)$.*

Proof. Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is NS$*_{b3}$ space and $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{\mathfrak{f}}, \theta) \in (\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$. Since $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is NS$*_{b3}$ space for the NS point $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ and NS$*_b$ closed set $(\tilde{\mathfrak{f}}, \theta)^c$, $\exists(\tilde{g}_1, \theta)$, and $(\tilde{g}_2, \theta)$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{g}_1, \theta)$, $(\tilde{\mathfrak{f}}, \theta)^c \subset (\tilde{g}_2, \theta)$ and $(\tilde{g}_1, \theta) \cap (\tilde{g}_2, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. Then, we have $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{g}_1, \theta) \subset (\tilde{g}_2, \theta)^c \subset (\tilde{\mathfrak{f}}, \theta)$. Since $(\tilde{g}_2, \theta)^c$NS $*_b$ closed set. $\overline{(\tilde{g}_1, \theta)} \subset (\tilde{g}_2, \theta)^c$. Conversely, let $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \cap (\tilde{\mathscr{H}}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$ and $(\tilde{\mathscr{H}}, \theta)$ be a NS$*_b$ closed set. $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \propto (\tilde{\mathscr{H}}, \theta)^c$ and from the condition of the theorem, we have $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{g}, \theta) \subset \overline{(\tilde{g}, \theta)} \subset (\tilde{\mathscr{H}}, \theta)^c$. Thus, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in (\tilde{g}, \theta)$, $(\tilde{\mathscr{H}}, \theta) \subset \overline{(\tilde{g}, \theta)}^c$&$(\tilde{g}, \theta) \cap \overline{(\tilde{g}, \theta)}^c = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. So $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ is NS$*_{b3}$ space.□

Theorem 27. *Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NSTS over the father set $\widetilde{\langle\mathscr{X}\rangle}$. This space is a NS$*_{b4}$ space, if and only if for each NS $*_b$ closed set $(\tilde{\mathfrak{f}}, \theta)$ and NS$*_b$ open set $(\tilde{g}, \theta)$ with $(\tilde{\mathfrak{f}}, \theta) \subset (\tilde{g}, \theta)$, there exists a NS$*_b$ open set $(\tilde{\mathscr{D}}, \theta) s.t. (\tilde{\mathfrak{f}}, \theta) \subset (\tilde{\mathscr{D}}, \theta) \subset \overline{(\tilde{\mathscr{D}}, \theta)} \subset (\tilde{g}, \theta)$.*

Proof. Let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NS$*_{b4}$ over the father set $\widetilde{\langle\mathscr{X}\rangle}$ and let $(\tilde{\mathfrak{f}}, \theta) \subset (\tilde{g}, \theta)$. Then, $(\tilde{g}, \theta)^c$ is a NS$*_b$ closed set and $(\tilde{\mathfrak{f}}, \theta) \cap (\tilde{g}, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. Since $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NS$*_{b4}$ space, $\exists$NS $*_b$ open sets $(\tilde{\mathscr{D}}_1, \theta)$ and $(\tilde{\mathscr{D}}_2, \theta) s.t. (\tilde{\mathfrak{f}}, \theta) \subset (\tilde{\mathscr{D}}_1, \theta)$, $(\tilde{g}, \theta)^c \subset (\tilde{\mathscr{D}}_2, \theta)$ and $(\tilde{\mathscr{D}}_1, \theta) \cap (\tilde{\mathscr{D}}_2, \theta) = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. Thus, $(\tilde{\mathfrak{f}}, \theta) \subset (\tilde{\mathscr{D}}_1, \theta) \subset (\tilde{\mathscr{D}}_2, \theta)^c \subset (\tilde{g}, \theta)$, $(\tilde{\mathscr{D}}_2, \theta)^c$ is a NS$*_b$ closed set and $\overline{(\tilde{\mathscr{D}}_1, \theta)} \subset (\tilde{\mathscr{D}}_2, \theta)^c$. So $(\tilde{\mathfrak{f}}, \theta) \subset (\tilde{\mathscr{D}}_1, \theta) \subset \overline{(\tilde{\mathscr{D}}_1, \theta)} \subset (\tilde{g}, \theta)$.

Conversely, $(\tilde{\mathfrak{f}}_1, \theta)$ and $(\tilde{\mathfrak{f}}_2, \theta)$ be two disjoint NS$*_b$ closed sets. Then, $(\tilde{\mathfrak{f}}_1, \theta) \subset \overline{(\tilde{\mathfrak{f}}_2, \theta)}^c$ implies $\exists$NS$*_b$ open set $(\tilde{\mathscr{D}}, \theta)$ $s.t.(\tilde{\mathfrak{f}}_1, \theta) \subset (\tilde{\mathscr{D}}, \theta) \subset \overline{(\tilde{\mathscr{D}}_1, \theta)} \subset (\tilde{\mathfrak{f}}_2, \theta)^c$. Thus, $(\tilde{\mathscr{D}}, \theta)$ and $\overline{(\tilde{\mathscr{D}}, \theta)}^c$ are NS$*_b$ open sets and $(\tilde{\mathfrak{f}}_1, \theta) \subset (\tilde{\mathfrak{f}}_2, \theta)^c$, $(\tilde{\mathfrak{f}}_2, \theta) \subset \overline{(\tilde{\mathscr{D}}, \theta)}^c$ and $(\tilde{\mathscr{D}}, \theta)$ and $\overline{(\tilde{\mathscr{D}}, \theta)}^c = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be a NS $*_{b4}$ space.□

Theorem 28. *Let $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ be a point in a NS first countable space$(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ and let $\{\langle\mathscr{W}, \theta\rangle_i : i = 1, 2, 3, \cdots\}$ generates a NS countable $*_b$open base about the point $\varkappa$; then, there exists an infinite soft subsequence $\{\langle\mathscr{V}, \theta\rangle_i : i = 1, 2, 3, \cdots\}$ of the NS sequence $\{\langle\mathscr{W}, \theta\rangle_i : i = 1, 2, 3, \cdots\}$, such that (i) for any NS$*_b$ open set $\langle\mathscr{U}, \partial\rangle$, containing $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$,$\exists$ a suffix m such that $\langle\mathscr{V}, \theta\rangle_i \Subset \langle\mathscr{U}, \theta\rangle$ for all $i \geqslant m$; and (ii) if $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be, in particular, a NS$*_{b1}$-space, then $\cap^{\sim}\{\langle\mathscr{V}, \theta\rangle_i : i = 1, 2, 3, \cdots\} = \{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)\}$.*

Proof. Given $\langle\mathscr{W},\theta\rangle_1 \cap^{\sim} \langle\mathscr{W},\theta\rangle_2 \cap^{\sim} \langle\mathscr{W},\theta\rangle_3 \cap^{\sim} \cdots .. \cap^{\sim} \langle\mathscr{W}, \theta\rangle_k$ is NS$*_b$ open sets, containing the point $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$. As the NS sets $\{\langle\mathscr{W}, \theta\rangle_i : i = 1, 2, 3, \cdots\}$ form NS$*_b$open base about $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$, there exists one among the NS sets $\{\langle\mathscr{W}, \partial\rangle_i : i = 1, 2, 3, \cdots\}$, which we shall denote by $\langle\mathscr{V}, \theta\rangle_k$, such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathscr{V}, \theta\rangle_k \Subset (\langle\mathscr{W},\theta\rangle_1 \cap^{\sim} \langle\mathscr{W},\theta\rangle_2 \cap^{\sim} \langle\mathscr{W},\theta\rangle_3 \cap^{\sim} \cdots .. \cap^{\sim} \langle\mathscr{W}, \theta\rangle_k)$, for $k = 1, 2, 3, \cdots$. The NS sequence $\{\langle\mathscr{V}, \theta\rangle_i : i = 1, 2, 3, \cdots\}$, thus obtained, has the required properties. In fact, if $\langle\mathscr{U}, \theta\rangle$ is any NS$**_b$ open set, containing $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$, then $\exists$a NS set $\langle\mathscr{W}, \theta\rangle_m$ say, belonging to the family $\{\langle\mathscr{W}, \theta\rangle_i : i = 1, 2, 3, \cdots\}$, such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathscr{W}, \theta\rangle_m \Subset \langle\mathscr{U}, \theta\rangle$. Also, since $\langle\mathscr{V}, \theta\rangle_i \Subset \langle\mathscr{W}, \theta\rangle_m$, for all $i \geqslant m$. Next, let $(\langle\tilde{\mathscr{X}}\rangle, \tau, \theta)$ be NS$*_{b1}$-space, and let $\cap^{\sim}\{\langle\mathscr{V}, \theta\rangle_i : i = 1, 2, 3, \cdots\} = \langle M, \theta\rangle$. As $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ is contained in each $\{\langle\mathscr{V}, \theta\rangle_i : i = 1, 2, 3, \cdots\}$ that is $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathscr{V}, \theta\rangle_1$, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathscr{V}, \theta\rangle_1$, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathscr{V}, \theta\rangle_2$, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathscr{V}, \partial\rangle_3$, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathscr{V}, \theta\rangle_4$, $\cdots \cdots ..(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}$

$,\theta) \in \langle \mathscr{V},\theta\rangle_n$, it follows that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle M,\theta\rangle$. Let $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$ be any point of $\langle\tilde{\mathscr{X}}\rangle$,from $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$ that is $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$ or $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$. By definition of NS$*_{b1}$-space, there exists NS$*_b$ open set $\langle\mathscr{U},\theta\rangle$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle\mathscr{U},\theta\rangle$ and $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \notin \langle\mathscr{U},\theta\rangle$. There exists a suffix, $s.t.\langle\mathscr{V},\theta\rangle_1 \Subset \langle\mathscr{U},\theta\rangle, \langle\mathscr{V},\theta\rangle_2 \Subset \langle\mathscr{U},\theta\rangle, \langle\mathscr{V},\theta\rangle_3 \Subset \langle\mathscr{U},\theta\rangle, \langle\mathscr{V},\theta\rangle_4 \Subset \Subset\langle\mathscr{U},\theta\rangle$ for all $i \succcurlyeq m$. Consequently, $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \notin \langle\mathscr{V},\theta\rangle_i$ for all $i \succcurlyeq m$; hence, $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \notin \langle M,\theta\rangle$. Thus, $\langle M,\theta\rangle$ consists of the point $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$ only.

Theorem 29. *NS second count-ability is NS first count-ability.*

Proof. Let $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ be a soft 2nd NS countable space. Then, this situation permits that there lives a NS countable base $\mathfrak{W}$ for $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$. In order to justify that $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ is NS, we proceed as, let $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle\tilde{\mathscr{X}}\rangle$ be an arbitrary point. Let us assemble those members of $\mathfrak{W}$ which absorbs $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$ and named as $\mathfrak{W}^{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$. If $\langle\aleph,\partial\rangle$ is soft n-hood of $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$, then this permit $\exists$NS$*_b$ open set $\langle\ell,\theta\rangle$ arresting $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$, in $\mathfrak{W}$ and so in $\mathfrak{W}^{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle\ell,\theta\rangle \Subset \langle\aleph,\theta\rangle$. This justifies that is a NS local base at $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$. One step more, $\mathfrak{W}^{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$ being a subfamily of a NS countable family $\mathfrak{W}$, it is therefore NS countable. Thus, every crisp point of $\langle\tilde{\mathscr{X}}\rangle$ supposes a countable NS local base. This leads us to say $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ is soft first N countable.

Theorem 30. *A NS countable space in which every NS convergent sequence has a unique soft limit is a NS $*_b$ Hausdorff space.*

Proof. Let $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ be NS Hausdorff space and let $\langle\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n\rangle$ be a soft convergent sequence in $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$. We prove that the limit of this sequence is unique. We prove this result by contradiction. Suppose $\langle\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n\rangle$ converges to two soft points $\tilde{l}$ and $\tilde{m}$ such that $\tilde{l} \neq \tilde{m}$. Then, by trichotomy law either $\tilde{l} < \tilde{m}$ or $\tilde{l} > \tilde{m}$. Since it possess the NS$*_b$ Hausdorff characteristics, there must happen two NS$*_b$ open sets $\langle\ell,\theta\rangle$ and $\langle\rho,\theta\rangle$ such that $\langle\ell,\theta\rangle \,\tilde{\cap}\, \langle\rho,\theta\rangle = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$.

Now, $\langle\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n\rangle$ converges to $\tilde{l}$ so there exists an integer n_1 such that $\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n \in \langle\ell,\partial\rangle \forall n \geq n_1$. Also, $\langle\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n\rangle$ converges to $\tilde{m}$ so there exists an integer n_2 such that $\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n \in \langle\rho,\theta\rangle \forall n \geq n_2$. We are interested to discuss the maximum possibility, for that we must suppose the maximum of both the integers which will enable us to discuss the soft sequence for a single soft number. Max $(n_1, n_2) = n_0$, which leads to the situation $\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n \in \langle\ell,\theta\rangle \forall n \geq n_0$ and $\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n \in \langle\rho,\theta\rangle \forall n \geq n_0$. This implies that $\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n \in \langle\ell,\theta\rangle$ and $\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n \in \langle\rho,\theta\rangle \forall n \geq n_0$. This guarantees that $\widetilde{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n \in (\langle\ell,\partial\rangle \,\tilde{\cap}\, \langle\rho,\theta\rangle) \forall n \geq n_0$, which beautifully contradict the fact that $\langle\ell,\theta\rangle \,\tilde{\cap}\, \langle\rho,\theta\rangle = 0_{\widetilde{(\langle\tilde{\mathscr{X}}\rangle,\theta)}}$. Hence, the limit of the NS sequence must be unique.

Theorem 31. *The cardinality of all NS open sets in a NS second countable space is at most equal to $\complement$ (the power of the continuum).*

Proof. Let $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ be NSTS such that is NS second countable space. Let $\langle\mathscr{G},\theta\rangle$ be any soft set of $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$, then $\langle\mathscr{G},\theta\rangle$ is the soft union of a certain soft subcollection of the NS countable collection $\{\langle\mathscr{W},\theta\rangle_i : i = 1, 2, 3, \cdots.\}$. Hence, the cardinality of the set of all soft $*_b$ open sets in $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ is not greater than the cardinality of the soft set of all soft subcollections of the NS countable collection $\{\langle\mathscr{W},\theta\rangle_i : i = 1, 2, 3, \cdots.\}$. Thus, the cardinality of $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta) \prec \complement$.

Theorem 32. *Any collection of mutually exclusive NS$*_b$ open sets in a NS$*_b$ second countable space is at most NS$*_b$ countable.*

Proof. Let $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ be NSTS such that it is NS second countable. Let $\langle\sqsupset,\theta\rangle$ signifies any collection of mutually exclusive NS$*_b$ open sets in $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$. Let $\langle\mathscr{U},\theta\rangle \in \langle\sqsupset,\theta\rangle$, then $\exists$at least one soft set $\langle\mathscr{W},\theta\rangle_m$, belonging to the collection $\{\langle\mathscr{W},\theta\rangle_i : i = 1, 2, 3, \cdots\}$, such that $\langle\mathscr{W},\theta\rangle_m \Subset \langle\mathscr{U},\theta\rangle$. Let n be the smallest suffix for which $\langle\mathscr{W},\theta\rangle_n \Subset \langle\mathscr{U},\theta\rangle$. Since the soft sets $\langle\mathscr{U},\theta\rangle$ in $\langle\sqsupset,\theta\rangle$ are mutually disjoint, it follows that, for different soft sets $\langle\mathscr{U},\theta\rangle \in \langle\sqsupset,\theta\rangle$, there correspond soft sets $\langle\mathscr{W},\theta\rangle_n$ with different suffices n. Hence, the soft sets in $\langle\sqsupset,\theta\rangle$ are in a $(1,1)$-correspondence with a soft subcollection of the NS countable collection $\{\langle\mathscr{W},\theta\rangle_i : i = 1, 2, 3, \cdots\}$; consequently, the cardinality of $\langle\sqsupset,\theta\rangle$ is less than or equal to d.

Theorem 33. *Let $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ be NSTS such that it is NS second countabley NS$*_b$ Hausdorff space. Then, set of all NS$*_b$ open sets has the cardinality C.*

Proof. Let $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ be second countable NS$*_b$ Hausdorff space. There exists in NSTS $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ an infinite soft sequence of NS$*_b$ open sets $\langle\mathscr{G},\theta\rangle_1, \langle\mathscr{G},\theta\rangle_2, \langle\mathscr{G},\theta\rangle_3, \langle\mathscr{G},\theta\rangle_4, \langle\mathscr{G},\theta\rangle_5, \cdots, \langle\mathscr{G},\theta\rangle_n, \cdots\cdots$ such that $\langle\mathscr{G},\theta\rangle_1 \neq 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}, \langle\mathscr{G},\theta\rangle_2 \neq 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}, \langle\mathscr{G},\theta\rangle_3 \neq 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}, \langle\mathscr{G},\theta\rangle_4 \neq 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}, \cdots.\langle\mathscr{G},\theta\rangle_n \neq 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)} \cdots\cdots$

$\langle\mathscr{G},\theta\rangle_1 \,\tilde{\cap}\, \langle\mathscr{G},\theta\rangle_2 \,\tilde{\cap}\, \langle\mathscr{G},\theta\rangle_3 \,\tilde{\cap}\, \langle\mathscr{G},\theta\rangle_4 \,\tilde{\cap}\, \cdots. \,\tilde{\cap}\, \langle\mathscr{G},\theta\rangle_n \,\tilde{\cap}\, \cdots = 0_{(\langle\tilde{\mathscr{X}}\rangle,\theta)}$. Different soft subsequences of the sequences $\langle\mathscr{G},\theta\rangle_1, \langle\mathscr{G},\theta\rangle_2, \langle\mathscr{G},\theta\rangle_3, \langle\mathscr{G},\theta\rangle_4, \langle\mathscr{G},\theta\rangle_5, \cdots, \langle\mathscr{G},\theta\rangle_n$ will determine, as their unions, different NS$*_b$ open sets. But since the soft set of all soft subsets of a countable soft set has the cardinality C, it follows that soft set of all the NS$*_b$ open sets in $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ has the cardinality $\succcurlyeq C$. Again, the cardinality of the soft set of all NS$*_b$ open sets in $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta) \preccurlyeq C$.

Consequently, the cardinality of all NS$*_b$ open sets in $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$ is exactly equal to C.

Theorem 34. *Let $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$ be NSST such that it is NS second countable $*_{b1}$ space; the soft set of all NS points in this space has the cardinality at most equal to C (i.e., possess the power of the continuum at most).*

Proof. Given that $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$ is NS second countable $*_{b1}$ space, for which the NS sets $\langle\mathcal{W}, \theta\rangle_1, \langle\mathcal{W}, \theta\rangle_2, \langle\mathcal{W}, \theta\rangle_3, \langle\mathcal{W}, \theta\rangle_4, \langle\mathcal{W}, \theta\rangle_5, \cdots$. Forms a soft countable NS$*_b$ open base of $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$. Then, for any given point $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\tilde{\mathcal{X}}\rangle$, $\langle\mathcal{W}, \theta\rangle_{\varsigma 1}, \langle\mathcal{W}, \theta\rangle_{\varsigma 2}, \langle\mathcal{W}, \theta\rangle_{\varsigma 3}, \langle\mathcal{W}, \theta\rangle_{\varsigma 4}, \langle\mathcal{W}, \theta\rangle_{\varsigma 5}, \cdots$. generate countable NS$*_b$ open base about the point ς in NSST$(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$,where ς $i, i = 1, 2, 3, \cdots$.., is a soft subsequence of the soft sequence $\langle\mathcal{W}, \theta\rangle_n : n = 1, 2, 3, \cdots$, consisting of all those $\langle\mathcal{W}, \theta\rangle_n$ which contain the point ς. Since a second NS countable space is necessary first NS countable. So corresponding to the point ς, there exists an infinite soft sequence of NS$*_b$ open sets $\{\langle\mathcal{O}, \partial\rangle_i : i = i, 2, 3, \cdots\}$, which is a soft subsequence of the soft sequence $\{\langle\mathcal{W}, \theta\rangle_{pn} : n = 1, 2, 3, ..\}$ and, therefore, also a soft subsequence of the soft sequence $\{\langle\mathcal{W}, \theta\rangle_n : n = 1, 2, 3, ..\}$, such that $\cap^\sim\{\langle\mathcal{O}, \theta\rangle_i : i = i, 2, 3, \cdots\} = \{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)\}$. Thus, to each point $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ in $\langle\tilde{\mathcal{X}}\rangle$, there corresponds a soft subsequence $\{\langle\mathcal{O}, \theta\rangle_i : i = i, 2, 3, \cdots\}$ of the soft sequence $\{\langle\mathcal{W}, \theta\rangle_n : n = 1, 2, 3, ..\}$, and two in different points, there correspond two such different soft subspaces $\{\langle\mathcal{O}, \theta\rangle_i : i = i, 2, 3, \cdots\}$. Hence, the soft set of all points in $\langle\tilde{\mathcal{X}}\rangle$, i.e., the crisp set $\langle\tilde{\mathcal{X}}\rangle$, has the same cardinality as that of a certain soft subcollection of the soft collection of all soft subspace of the sequence $\{\langle\mathcal{W}, \theta\rangle_n : n = 1, 2, 3, ..\}$. Thus, the cardinality of $\langle\tilde{\mathcal{X}}\rangle$ is less than or equal to $\mathfrak{C}$. In other words, the crisp set $\langle\tilde{\mathcal{X}}\rangle$ has the power of the continuum at most.

Theorem 35. *Let $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$ be NSTS such that it is NS second countable. Every soft uncountable soft subset contains a point of condensation.*

Proof. Since $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$ is NS second countable space and $\{\langle\mathcal{W}, \theta\rangle_i : i = 1, 2, 3, ..\}$ be a soft countable NS$*_b$ open base of $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$. Let $\langle\mathcal{f}, \theta\rangle$ be a NS subset of $\langle\tilde{\mathcal{X}}\rangle$ such that $\langle\mathcal{f}, \theta\rangle$ does not contain any point of condensation. For each point $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathcal{f}, \partial\rangle$, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ is not the point of condensation of $\langle\mathcal{f}, \theta\rangle$. Hence, there exists NS open $*_b$ set $\langle\mathcal{G}, \partial\rangle$, containing $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$ such that $\langle\mathcal{G},\partial\rangle \cap^\sim \langle\mathcal{f}, \theta\rangle$ is soft countable at most. There exists a suffix $n((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta))$, such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathcal{W}, \theta\rangle_{n(p)} \Subset \langle\mathcal{G}, \theta\rangle$, and then, $\langle\mathcal{f},\theta\rangle \cap^\sim \langle\mathcal{W}, \theta\rangle_{n(p)}$ is also NS countable at most. But we can express $\langle\mathcal{f}, \partial\rangle$ in the form $\langle\mathcal{f}, \theta\rangle = \cup^\sim \{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$: $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathcal{f},\theta\rangle\}\cup^\sim\{\langle\mathcal{f},\theta\rangle \cap^\sim \langle\mathcal{W}, \theta\rangle_{n((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))} : (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathcal{f}, \theta\rangle\}$, and there can be at most a NS countable number of different suffices. So, $\langle\mathcal{f}, \theta\rangle$ is at most a NS union of NS uncountable soft subset, that is, $\langle\mathcal{f}, \theta\rangle$ is at most a NS countable soft subset of $\langle\mathcal{X}\rangle$. Consequently, if $\langle\mathcal{f}, \theta\rangle$ is a soft uncountable soft subset, then it must possess a point of condensation.

Theorem 36. *$(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$ be NSTS such that it is NS second countable. If $\langle\psi, \theta\rangle$ is uncountable NS subset of $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$, then the soft subset $\langle\varpi, \theta\rangle$, consisting of all those N points of $\langle\psi, \theta\rangle$ which are not points of condensation of $\langle\psi, \theta\rangle$, is at most NS countable.*

Proof. Since $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$ is NS second countable space, $\{\langle\mathcal{W}, \theta\rangle_i : i = 1, 2, 3, ..\}$ be a countable NS$*_b$ open base of $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$. Let $\{\langle\mathcal{V}, \theta\rangle_i : i = 1, 2, 3, \cdots\}$ be a soft subsequence of the soft sequence $\{\langle\mathcal{W}, \theta\rangle_i : i = 1, 2, 3, ..\}$, consisting of all those soft sets $\langle\mathcal{W}, \theta\rangle_j$ for which $\langle\mathcal{W},\theta\rangle_j \cap^\sim \langle\psi, \theta\rangle$ is at most soft countable. Then, $\{\langle\mathcal{V},\theta\rangle_i\} \cap^\sim \langle\psi, \theta\rangle$: $i = 1, 2, 3, \cdots$ is at most NS countable. We shall establish that $\langle\varpi, \theta\rangle = \langle\psi,\theta\rangle \cap^\sim(\langle\mathcal{V},\theta\rangle_1 \cup^\sim \langle\mathcal{V},\theta\rangle_2 \cup^\sim, \cdots)$. $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\varpi, \theta\rangle$, there is not a point of condensation of $\langle\psi, \theta\rangle$; hence, there exists a soft nbhd $\langle\mathcal{U}, \partial\rangle$ of $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)$, such that $\langle\mathcal{U},\theta\rangle \cap^\sim \langle\psi, \theta\rangle$ is at most NS countable. Also, there exists a soft set $\langle\mathcal{W}, \theta\rangle_j$, belonging to the soft sequence $\{\langle\mathcal{W}, \theta\rangle_i : i = 1, 2, 3, ..\}$, satisfying $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathcal{W}, \theta\rangle_j \Subset \langle\mathcal{U}, \theta\rangle$. Then, $\langle\mathcal{W},\theta\rangle_j \cap^\sim \langle\psi, \theta\rangle$ is at most NS countable, and so $\langle\mathcal{W}, \theta\rangle_j$ must be one of the soft sets $\langle\mathcal{V}, \theta\rangle_i$ and therefore $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\mathcal{V}, \theta\rangle_i$. Again, since $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\psi, \partial\rangle$, it follows $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta) \in \langle\psi,\theta\rangle \cap^\sim(\langle\mathcal{V},\theta\rangle_1 \cup^\sim \langle\mathcal{V},\partial\rangle_2 \cup^\sim, \cdots)$. Next, let $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \in \langle\psi,\theta\rangle \cap^\sim(\langle\mathcal{V},\theta\rangle_1 \cup^\sim \langle\mathcal{V},\theta\rangle_2 \cup^\sim, \cdots)$, then $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$ belongs to some $\langle\mathcal{V}, \theta\rangle_i$. Now, as $\langle\mathcal{V}, \theta\rangle_i$ is a soft nbd of $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$, and $\langle\mathcal{V},\theta\rangle_i \cap^\sim \langle\psi, \theta\rangle$ is at most NS countable. $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta)$ cannot be a point of condensation of $\langle\psi, \partial\rangle$. Also, as $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \in \langle\psi, \theta\rangle$. It follows that $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}, \theta) \in \langle\varpi, \theta\rangle$. Thus, $\langle\varpi, \theta\rangle = \langle\psi,\theta\rangle \cap^\sim (\langle\mathcal{V},\theta\rangle_1 \cup^\sim \langle\mathcal{V},\theta\rangle_2 \cup^\sim, \cdots) = (\langle\psi,\theta\rangle \cap^\sim \langle\mathcal{V},\theta\rangle_1) \cup^\sim \langle\psi,\theta\rangle \cap^\sim \langle\mathcal{V},\theta\rangle_2 \cup^\sim, \cdots)$ and since each $\langle\psi,\theta\rangle \cup^\sim \langle\mathcal{V}, \theta\rangle_i$ is at most NS countable. It follows that $\langle\varpi, \partial\rangle$ is also at most NS countable.□

3. Main More Results

In this section, neutrosophic soft topological characteristics and inverse neutrosophic soft topological characteristics are addressed with respect to soft points. Neutrosophic soft product of neutrosophic soft structures is inaugurated. Characterization of Bolzano Weierstrass Property, neutrosophic soft compactness and related results, and neutrosophic sequentially compactness are discussed.

Theorem 37. *Let $(\langle\tilde{\mathcal{X}}\rangle, \tau, \theta)$ be NSTS such that it is NS$*_b$ Hausdorff space and $(\langle\tilde{Y}\rangle, \mathfrak{F}, \theta)$ be any NSST. Let $\langle\mathcal{f}, \partial\rangle : (\langle\tilde{\mathcal{X}}\rangle, \tau, \theta) \longrightarrow (\langle\tilde{Y}\rangle, \mathfrak{F}, \theta)$ be a soft function such that it is soft monotone and continuous. Then, $(\langle\tilde{Y}\rangle, \mathfrak{F}, \theta)$ is also of characteristics of NS$*_b$ Hausdorffness.*

Proof. Suppose $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1, (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \in \langle\tilde{\mathcal{X}}\rangle$ such that either $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \succ (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$ or $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \prec (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$. Since $\langle f,\theta\rangle$ is soft monotone. Let us suppose the monotonically increasing case. So, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \succ f_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2}$ or $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \prec f_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2}$ implies that $f_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1} \succ f(x_2)$ or $f_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1} \prec f_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2}$, respectively. Suppose $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1$, $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2 \in \widetilde{\langle Y\rangle}$ such that $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$ or $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$, so $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$ or $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$, respectively, such that $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) = f_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1}, (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2 = f_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2}$. Since, $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ is NS$*_b$ Hausdorff space so there exists mutually disjoint NS$*_b$ open sets $\langle k_1,\theta\rangle$ and $\langle k_2,\theta\rangle \in \langle\tilde{\mathcal{X}}\rangle,\tau,\theta) \Longrightarrow f(\langle k_1,\theta\rangle)$ and $f(\langle k_2,\theta\rangle) \in \langle\tilde{Y}\rangle,\mathfrak{F},\theta)$. We claim that $f(\langle k_1,\theta\rangle) \cap^{\sim} f(\langle k_2,\theta\rangle) = 0_{\widetilde{(\langle\tilde{\mathcal{X}}\rangle,\theta}}$. Otherwise, $f(\langle k_1,\theta\rangle) \cap^{\sim} f(\langle k_2,\theta\rangle) \neq 0_{\widetilde{(\langle\tilde{\mathcal{X}}\rangle,\theta)}}$. Suppose $\exists (t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_1 \in f(\langle k_1,\theta\rangle) \cap^{\sim} f(\langle k_2,\theta\rangle) \Longrightarrow (t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_1 \in f(\langle k_1,\theta\rangle)$& $(t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_1 \in f(\langle k_2,\theta\rangle), (t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_1 \in f(\langle k_1,\theta\rangle)$, f is soft one – one$\exists (t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_2 \in \langle k_1,\theta\rangle$ such that $(t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_1 = f((t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_2), (t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_1 \in f(\langle k_2,\partial\rangle) \Longrightarrow \exists (t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_3 \in \langle k_2,\partial\rangle$ such that $f((t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_3) \Longrightarrow f((t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_2) = f((t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_3)$. Since, f is soft one-one $\Longrightarrow (t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_2 = (t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_3$ implies that $(t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_2 \in f(\langle k_1,\partial\rangle)$, $(t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_2 \in f(\langle k_2,\partial\rangle)$ implies that $(t^{e''}{}_{(\mathfrak{a}'',\mathfrak{b}'',\mathfrak{c}'')},\theta)_2 \in f(\langle k_1,\partial\rangle) \cap^{\sim} f(\langle k_2,\partial\rangle)$. This is a contradiction because $\langle k_1,\partial\rangle \cap^{\sim} \langle k_2,\partial\rangle = 0_{(\langle\tilde{\mathcal{X}}\rangle,\theta)}$. Therefore, $f(\langle k_1,\partial\rangle) \cap^{\sim} f(\langle k_2,\partial\rangle) = 0_{(\langle\tilde{\mathcal{X}}\rangle,\theta)}$. Finally, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \succ (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$ or $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \prec (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2 \Longrightarrow (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \neq (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2 \Longrightarrow f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1) \succ f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2)$ or $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1) \prec f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2) \Longrightarrow f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1) \neq f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2)$. Given a pair of points $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1, (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2 \in \langle\tilde{Y}\rangle \ni (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \neq (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$. We are able to find out mutually exclusive NS$*_b$ open sets $f(\langle k_1,\theta\rangle), f(\langle k_2,\theta\rangle) \in (\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ $s.t.$ $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \in f(\langle k_1,\partial\rangle), (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2 \in f(\langle k_2,\theta\rangle)$. This proves that $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ is NS$*_b$ Husdorff space.□

Theorem 38. *Let $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ be NSTS and $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ be another NSTS which satisfies one more condition of NS$*_b$ Hausdorffness. Let $\langle f,\theta\rangle$: $(\langle\tilde{\mathcal{X}}\rangle,\tau,\theta) \longrightarrow (\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ a soft function s.t. it is soft monotone and continuous. Then, $(\widetilde{\langle\mathcal{X}\rangle},\tau,\theta)$ is also of characteristics of NS$*_b$ Hausdorfness.*

Proof. Suppose $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1, (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2 \in \langle\tilde{\mathcal{X}}\rangle$ such that either $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \succ (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$ or $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \prec (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$. Let us suppose the NS monotonically increasing case. So, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \succ (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$ or $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \prec (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$ implies that $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1) \succ f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2)$ or $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1) \prec f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2)$, respectively. Suppose $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1, (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2 \in \langle\tilde{Y}\rangle$ such that $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$ or $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$. So, $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$ or $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$, respectively, such that $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 = f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1), (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2 = f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2)$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 = f^{-1}(y_1)$ and $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2 = f^{-1}((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2)$. Since $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1, (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2 \in \langle\tilde{Y}\rangle$ but $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ is NS$*_b$ Hausdorff space. So according to definition $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$ or $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2$. There definitely exist NS$*_b$ open sets $\langle k_1,\theta\rangle$ and $\langle k_2,\theta\rangle \in (\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ such that $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \in \langle k_1,\theta\rangle$ and $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2 \in \langle k_2,\theta\rangle$and these two NS$*_b$ open sets are disjoint. Since $f^{-1}(\langle k_1,\theta\rangle)$ and $f^{-1}(\langle k_2,\partial\rangle)$ are NS$*_b$ open in $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$. Now, $f^{-1}(\langle k_1,\theta\rangle) \cap^{\sim} f^{-1}(\langle k_1,\theta\rangle) = f^{-1}(\langle k_1,\theta\rangle \cap^{\sim} \langle k_2,\theta\rangle) = f^{-1}(\varnothing^{\sim}) = 0_{\widetilde{(\langle\tilde{\mathcal{X}}\rangle,\partial)}}$ and $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1 \in \langle k_1,\theta\rangle \Rightarrow f^{-1}((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_1) \in f^{-1}(\langle k_1,\theta\rangle) \Rightarrow (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \in (\langle k_1,\theta\rangle)$, $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2 \in \langle k_2,\partial\rangle \Rightarrow f^{-1}((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_2) \in f^{-1}(\langle k_2,\theta\rangle)$ implies that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2 \in (\langle k_2,\theta\rangle)$. We see that it has been shown for every pair of distinct points of $\langle\mathcal{X}\rangle$, there suppose disjoint NS$*_b$ open sets, namely, $f^{-1}(\langle k_1,\theta\rangle)$ and $f^{-1}(\langle k_2,\theta\rangle)$ belong to $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \in f^{-1}(\langle k_1,\theta\rangle)$ and $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2 \in f^{-1}(\langle k_2,\theta\rangle)$. Accordingly, NSTS is NS$*_b$ Hausdorff space.

Theorem 39. *Let $(\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ be NSST and $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ be another NSTS. Let $\langle f,\partial\rangle$:$\langle\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)\rangle \longrightarrow (\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ be a soft mapping such that it is continuous mapping. Let $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ is NS$*_b$ Hausdorff space, then it is guaranteed that $\{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta), (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}$ is a NS$*_b$ closed subset of $(\langle\tilde{\mathcal{X}}\rangle,\tau,\theta) \times (\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$.*

Proof. Given that $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ be NSTS and $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ be another NSTS. Let $\langle f,\partial\rangle$:$\langle\tilde{\mathcal{X}}\rangle,\tau,\theta) \longrightarrow (\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ be a soft mapping such that it is continuous mapping. $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ is NS$*_b$ Hausdorff space. Then, we will prove that $\{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta'), (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}$ is a NS$*_b$ closed subset of $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta) \times (\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$. Equivalently, we will prove that $\{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta), (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)\}^c$ is NS$*_b$ open subset of $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta) \times (\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$. Let $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta), (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)) \in \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta), (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \succ f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}^c$or $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta), (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)) \in \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta), (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},$

$\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \prec f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}^c$. Then, $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \succ f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$ or $\prec f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \prec f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$ accordingly. Since, $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ is NS$*_b$ Hausdorff space. Certainly, $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)), f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$ are points of $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$; there exists NS$*_b$ open sets $\langle\mathcal{G},\partial\rangle,\langle k,\partial\rangle \in (\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ such that $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \in \langle\mathcal{G},\partial\rangle$ and $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \in \langle k,\partial\rangle$ provided $\langle\mathcal{G},\partial\rangle \cap^{\sim} \langle k,\partial\rangle = 0_{\widetilde{(\langle\tilde{\mathcal{X}}\rangle,\theta)_Y}}$. Since $\langle f,\theta\rangle$ is soft continuous, $f^{-1}(\langle\mathcal{G},\theta\rangle$ and $f^{-1}(\langle k,\theta\rangle$ are NS$*_b$ open sets in $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ supposing $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$ and $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$, respectively, and so $f^{-1}(\langle\mathcal{G},\partial\rangle \times f^{-1}(\langle\mathcal{H},\partial\rangle$ is basic NS$*_b$ open set in $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)\times(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ containing $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta), (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$. Since $\langle\mathcal{G},\theta\rangle \cap^{\sim} \langle k,\theta\rangle = \widetilde{0_{\theta Y}}$, it is clear by the definition of $\{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}$ that is $\{f^{-1}(\langle\mathcal{G},\theta\rangle \& f^{-1}(\langle k,\theta\rangle\} \cap^{\sim} \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$: $f(x) = f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\} = 0$ $(\langle\tilde{\mathcal{X}}\rangle,\theta)$, that is $f^{-1}(\langle\mathcal{G},\partial\rangle \times f^{-1}(\langle k,\theta\rangle \Subset \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}^c$. Hence, $\{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}^c$ implies that $\{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}$ is NS$*_b$ closed.

Theorem 40. *Let $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ be NSTS and $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ be another NSSTS. Let $\langle f,\partial\rangle:\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)\longrightarrow(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ be a NS $*_b$ open mapping such that it is onto. If the soft set $\{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}$ is NS$*_b$ closed in $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)\times(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$, then $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ will behave as NS$*_b$ Hausdorff space.*

Proof. Suppose $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)), f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$ be two points of $\langle\tilde{Y}\rangle$ such that either $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \succ f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$ or $\prec f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \prec f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$. Then, $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)) \notin \{(x,(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$ with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \succ f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)\}$or $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)) \notin \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$ with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \prec f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}$, that is $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)) \in \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \succ f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}^c$ or $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)) \in \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \prec f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}^c$. Since, $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)) \in \{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta, y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \succ f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}^c$ or $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)) \in \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \prec f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}^c$ is soft in $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)\times(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$, then there exists NS$**_b$ open sets $\langle\mathcal{G},\theta\rangle$ and $\langle k,\theta\rangle$ in $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ such that $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \in \langle\mathcal{G},\theta\rangle\times\langle k,\theta\rangle \Subset \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \succ f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}^c$or $((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)) \in \langle\mathcal{G},\partial\rangle\times\langle k,\theta\rangle \Subset \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \prec f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\}^c$. Then, since f is NS$*_b$ open, $f(\langle\mathcal{G},\theta\rangle)$ and $f(\langle k,\theta\rangle)$ are NS$*_b$ open sets in $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ containing $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))$ and $f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$, respectively, and $f(\langle\mathcal{G},\theta\rangle) \cap^{\sim} f(\langle k,\theta\rangle) = 0_{\widetilde{(\langle\tilde{\mathcal{X}}\rangle,\theta)}}$ otherwise $f(\langle\mathcal{G},\theta\rangle)\times f(\langle k,\theta\rangle)$ $\cap^{\sim}$ $\{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),)$ with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \succ (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \succ f(y)\}$ or $(((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))) \notin \{((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta'),(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$ with $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \prec (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$: $f((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \prec f((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))\} = 0_{\widetilde{(\langle\tilde{\mathcal{X}}\rangle,\theta)}}$. It follows that $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ is NS$*_b$ Hausdorff space.

Theorem 41. *Let $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ be a NS second countable space then it is guaranteed that every family of nonempty disjoint NS$*_b$ open subsets of a NS second countable space $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ is NS countable.*

Proof. Given that $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ be a NS second countable space.

Then, $\exists$ a NS countable base $\mathfrak{W} = \langle\mathcal{B}^1,\mathcal{B}^2,\mathcal{B}^3,\mathcal{B}^4,\cdots ..\mathcal{B}^n : n\in\mathbb{N}\rangle$for $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$. Let $\langle\mathcal{C},\theta\rangle$ be a family of nonvacuous mutually exclusive NS$*_b$ open subsets of $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$. Then, for each $\langle f,\theta\rangle$ of in $\langle\mathcal{C},\theta\rangle$, there exists a soft $\mathcal{B}^n \in \mathfrak{W}$ in such a way that $\mathcal{B}^n \Subset \langle f,\theta\rangle$. Let us attach with $\langle f,\theta\rangle$, the smallest positive inter n such that $\mathcal{B}^n \Subset \langle f,\theta\rangle$. Since the candidates of $\langle\mathcal{C},\theta\rangle$ are mutually exclusive because of this behaviour distinct candidates will be associated with distinct positive integers. Now, if we put the elements of $\langle\mathcal{C},\theta\rangle$ in order so that the order is increasing relative to the positive integers associated with them, we obtain a sequence of candidates of $\langle\mathcal{C},\theta\rangle$. This verifies that $\langle\mathcal{C},\theta\rangle$ is NS countable.□

Theorem 42. *Let $(\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$ be a NS second countable space and let $\langle f,\theta\rangle$ be NS uncountable subset of $(\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$. Then, at least one point of $\langle f,\theta\rangle$ is a soft limit point of $\langle f,\theta\rangle$.*

Proof.
Let $\mathfrak{W} = \langle\mathcal{B}_1,\mathcal{B}_2,\mathcal{B}_3,\mathcal{B}_4,\cdots..\mathcal{B}_n : n\in\mathbb{N}\rangle$ for $\langle\tilde{\mathcal{X}}\rangle,\tau,\theta)$.

Let, if possible, no point of $\langle f,\theta\rangle$ be a soft limit point of $\langle f,\theta\rangle$. Then, for each $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle f,\theta\rangle$, there exists NS $*_b$ open set $\langle\rho,\theta\rangle_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle\rho,\theta\rangle_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$ and $\langle\rho,\theta\rangle_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)} \cap^{\sim} \langle f,\theta\rangle = \{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}$. Since $\mathfrak{W}$ is a soft base $\exists\mathcal{B}_{n(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)} \in \mathfrak{W}$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \mathcal{B}_{n(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)} \Subset \langle\rho,\theta\rangle_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$. Therefore, $\mathcal{B}_{n(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)} \cap^{\sim} \langle f,\theta\rangle \Subset \langle\rho,\theta\rangle_{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)} \cap^{\sim} \langle f,\theta\rangle = \{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}$. Moreover, if $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1$ and $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$ be any two NS points such that $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \neq (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$ which means either $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \succ (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$ or $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \prec (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$, then there exists $\mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1}$ and $\mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2}$ in $\mathfrak{W}$ such that $\mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1} \cap^{\sim} \langle f,\theta\rangle = \{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1\}$ and $\mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2} \cap^{\sim} \langle f,\theta\rangle = \{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2\}$. Now, $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1 \neq (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2$ which guarantees that $\{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1\} \neq \{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2\}$ which implies that

$\mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1} \cap^{\sim} \langle f,\theta\rangle \neq \mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2} \cap^{\sim} \langle f,\theta\rangle$ which implies $\mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_1} \neq \mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_2}$. Thus, there exists a one to one soft correspondence of $\langle f,\theta\rangle$ on to $\{\mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)} : (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle f,\theta\rangle\}$. Now, $\langle f,\theta\rangle$ being NS uncountable, it follows that $\{\mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)} : (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle f,\theta\rangle\}$ is NS uncountable. But, this is purely a contradiction, since $\{\mathscr{B}_{n(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)} : (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle f,\theta\rangle\}$ being a NS subfamily of the NS countable collection $\mathfrak{W}$. This contradiction is taking birth that on point of $\langle f,\theta\rangle$ is a soft limit point of $\langle f,\theta\rangle$, so at least one point of $\langle f,\theta\rangle$ is a soft limit point of $\langle f,\theta\rangle$.□

Theorem 43. *Let $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ NSTS such that is is NS countably compact then this space has the characteristics of Bolzano Weierstrass Property(BWP).*

Proof. Let $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ be a NS countably compact space and suppose, if possible, that it negates the Bolzano Weierstrass Property (BWP). Then, there must exists an infinite NS set $\langle f,\partial\rangle$ supposing no soft limit point. Further, suppose $\langle\rho,\theta\rangle$ be soft countability infinite soft subset $\langle f,\theta\rangle$ that is $\langle\rho,\theta\rangle \Subset \langle f,\theta\rangle$. Then, this guarantees $\langle\rho,\theta\rangle$ has no soft limit point. It follows that $\langle\rho,\theta\rangle$ is $\mathrm{NS}*_b$ closed set. Also for each $(x^{e}\widetilde{{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}},\theta)_n \in \langle\rho,\theta\rangle$, $(x^{e}\widetilde{{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}},\theta)_n$ is not a soft limit point of $\langle\rho,\langle\mathfrak{k},\theta\rangle\rangle$. Hence, there exists $\mathrm{NS}**_b$ open set $\langle\mathscr{G}_n,\theta\rangle$, such that $(x^{e}\widetilde{{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}},\theta)_n \in \langle\mathscr{G}_n,\langle\mathfrak{k},\theta\rangle\rangle$ and $\langle\mathscr{G}_n,\theta\rangle \cap^{\sim} \langle\rho,\theta\rangle = \{(x^{e}\widetilde{{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}},\theta)_n\}$. Then, the collection $\{\langle\mathscr{G}_n,\langle\mathfrak{k},\theta\rangle : n\in N\} \cap^{\sim} \langle\rho,\langle\mathfrak{k},\theta\rangle\rangle^c$ is countable $\mathrm{NS}*_b$ open cover of $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$. This soft cover has no finite subcover. For this, we exhaust a single $\langle\mathscr{G}_n,\theta\rangle$, it would not be a soft cover of $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ since then $\langle(x^{e}\widetilde{{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}},\theta)_n\rangle$ would be covered. Result in $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ is not NS countably compact.

Theorem 44. *Let $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ and $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ be two NSTS and suppose $\langle\mathfrak{f},\theta\rangle$ be a NS continuous function such that $\langle\mathfrak{f},\partial\rangle$: $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta) \longrightarrow (\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ is NS continuous function and let $\langle\mathscr{L},\theta\rangle \Subset (\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ supposes the $B.V.P.$ then safely $\mathfrak{f}(\langle\mathscr{L},\theta\rangle)$ has the $B.V.P.$*

Proof. Suppose $\langle\mathscr{L},\theta\rangle$ be an infinite NS subset of $\langle\mathfrak{f},\theta\rangle$, so that $\langle\mathscr{L},\theta\rangle$ contains an enumerable NS set $\langle(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_n : n\in N\rangle$ then there exists enumerable NS set $\langle(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_n : n\in N\rangle \Subset \langle\mathscr{L},\theta\rangle$ $s.t.\mathfrak{f}((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_n) = (x^{e}(\mathfrak{a},\mathfrak{b},\mathfrak{c}),\theta)_n$. $\langle\mathscr{L},\theta\rangle$ has $B.V.P$ implies that every infinite soft subset of $\langle\mathscr{L},\theta\rangle$ supposes soft accumulation point belonging to $\langle\mathscr{L},\theta\rangle$ this implies that $\langle(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_n : n\in N\rangle$ has a soft neutrosophic limit point, say, $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_0 \in \langle\mathscr{L},\theta\rangle$ implies that the limit of the soft sequence $\langle(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_n : n\in N\rangle$ is $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_0 \in \langle\mathscr{L},\partial\rangle \Longrightarrow (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_n \longrightarrow (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_0 \in \langle\mathscr{L},\theta\rangle$. $\mathfrak{f}$ is soft continuous implies that it is soft continuous. Furthermore, $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_n \longrightarrow (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_0 \in \langle\mathscr{L},\theta\rangle \Longrightarrow \mathfrak{f}(((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_n)) \longrightarrow \mathfrak{f}(((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_0)) \in \mathfrak{f}(\langle\mathscr{L},\theta\rangle) \Longrightarrow (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_n \longrightarrow \mathfrak{f}((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_0) \in \mathfrak{f}(\langle\mathscr{L},\theta\rangle)$ implies that limit of a soft sequence $\langle(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_n : n\in N\rangle$ is $\mathfrak{f}((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_0) \in \mathfrak{f}(\langle\mathscr{L},\theta\rangle)$ implies that limit of a soft sequence $\langle(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_n : n\in N\rangle$ is $\mathfrak{f}((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_0) \in \langle\mathfrak{f},\theta\rangle(\langle\mathscr{L},\theta\rangle)$. Finally, we have shown that there exists an infinite soft subset $\langle(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)_n : n\in N\rangle$ of $\mathfrak{f}(\langle\mathscr{L},\theta\rangle)$ containing a limit point $\mathfrak{f}((y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)_0) \in \mathfrak{f}(\langle\mathscr{L},\theta\rangle)$. This guarantees that $\mathfrak{f}(\langle\mathscr{L},\theta\rangle)$ has $B.V.P.$□

Theorem 45. *Let $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ be a NSTS and $\langle\mathfrak{f},\mathfrak{T},\theta\rangle$ be NS subspace of $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$. The necessary and sufficient condition for $\langle\mathfrak{f},\theta\rangle$ to be $\mathrm{NS}*_b$ compact relative to $\langle\mathfrak{f},\mathfrak{T},\theta\rangle$ is that $\langle\mathfrak{f},\theta\rangle$ is $\mathrm{NS}*_b$ compact relative to $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$.*

Proof. First, we prove that $\langle\mathfrak{f},\theta\rangle$ relative to $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$.

Let $\{\langle\mathfrak{k},\theta\rangle_i : i\in I\}$ that is $\{\langle\mathfrak{k},\theta\rangle_1,\langle\mathfrak{k},\theta\rangle_2,\langle\mathfrak{k},\theta\rangle_3,\langle\mathfrak{k},\theta\rangle_4,\cdots\}$ be $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$–$\mathrm{NS}*_b$ open cover of $\langle\mathfrak{f},\theta\rangle$, then $\langle\mathfrak{f},\theta\rangle \Subset \tilde{\cup}_i \langle\mathfrak{k},\partial\rangle_i.\langle\mathfrak{k},\partial\rangle_i \in \langle\tilde{\mathscr{X}}\rangle,\tau,\theta) \Longrightarrow \exists\langle g,\partial\rangle_i \in \langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ such that $\langle\mathfrak{k},\theta\rangle_i = \langle g,\theta\rangle_i \cap^{\sim} \langle\mathfrak{f},\theta\rangle \Subset \langle g,\theta\rangle_i$ implies that there exists $\langle g,\theta\rangle_i \in \langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ such that $\langle\mathfrak{k},\theta\rangle_i \Subset \langle g,\theta\rangle_i \Longrightarrow \tilde{\cup}_i\langle\mathfrak{k},\theta\rangle_i \Subset \tilde{\cup}_i$ but $\langle\mathfrak{f},\theta\rangle \Subset \langle\mathfrak{k},\theta\rangle_i$. So that $\langle\mathfrak{f},\theta\rangle \Subset \tilde{\cup}_i\langle\mathfrak{k},\theta\rangle_i$. This guarantees that $\{\langle g,\theta\rangle_i : i\in I\}$ is a $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta) - \mathrm{NS}*_b$ open cover of $\langle\mathfrak{f},\theta\rangle$ which is known to be $\mathrm{NS}*_b$ compact relative $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$, and hence, the soft cover $\{\langle g,\theta\rangle_i : i(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}$ must be freezable to a finite soft cub cover, say, $\{\langle g,\theta\rangle_{ir} : r = 1,2,3,4,\cdots.,n\}$. Then, $\langle\mathfrak{f},\theta\rangle \Subset \bigcup_{r=1}^{n}\widetilde{\langle G,\theta\rangle}_{ir} \Longrightarrow \langle\mathfrak{f},\theta\rangle \cap^{\sim} \langle\mathfrak{f},\theta\rangle \Subset \langle\mathfrak{f}\ crip,\partial\rangle \cap^{\sim} [\widetilde{\bigcup_{r=1}^{n}} \langle G,\theta\rangle_{ir}] = \bigcup_{r=1}^{n}(\langle\mathfrak{f},\theta\rangle \cap^{\sim}\langle g,\theta\rangle_{ir} = \bigcup_{r=1}^{n}\widetilde{\langle\mathfrak{k},\theta\rangle}_{ir}$ or $\langle\mathfrak{f},\theta\rangle \Subset \bigcup_{r=1}^{n}\widetilde{\langle\mathfrak{k},\theta\rangle}_{ir} \Longrightarrow \{\langle\mathfrak{k},\theta\rangle_{ir} : 1\le r\le n\}$ is a $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta) - \mathrm{NS}*_b$ open cover of $\langle\mathfrak{f},\theta\rangle$. Stepping from an arbitrary $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta) - *_b$ open cover of $\langle\mathfrak{f},\theta\rangle$, we are able to show that the $\mathrm{NS}*_b$ cover is freezable to a finite soft subcover $\{\langle\mathfrak{f},\theta\rangle_{ir} : 1\le r\le n\}$ of $\langle\mathfrak{k},\theta\rangle$, meaning there by $\langle\mathfrak{f},\theta\rangle$ is $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta) - \mathrm{NS}*_b$ compact. The condition is sufficient: suppose $\langle\mathfrak{f},\mathfrak{T},\theta\rangle$ be soft subspace of $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ and also $\langle\mathfrak{f},\theta\rangle$ is $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$–NS compact. We have to prove that $\langle\mathfrak{f},\theta\rangle$ is $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta) - \mathrm{NS}*_b$ compact. Let $\{\langle\mathfrak{k},\theta\rangle_1,\langle\mathfrak{k},\theta\rangle_2,\langle\mathfrak{k},\theta\rangle_3,\langle\mathfrak{k},\theta\rangle_4,\cdots\}$ be soft $\langle\tilde{\mathscr{X}}\rangle,\tau,\theta) - \mathrm{NS}*_b$ open cover of $\langle\mathfrak{f},\theta\rangle$, so that $\langle\mathfrak{f},\theta\rangle \Subset \tilde{\cup}_i\langle g,\theta\rangle_i$ from which $\langle\mathfrak{f},\theta\rangle \cap^{\sim}\langle\mathfrak{f},\theta\rangle \Subset \langle\mathfrak{f},\theta\rangle \cap^{\sim} (\tilde{\cup}_i\langle g,\theta\rangle_i)$or, $\langle\mathfrak{f},\theta\rangle \Subset \tilde{\cup}_i((\langle\mathfrak{f},\theta\rangle \cap^{\sim}\langle g,\theta\rangle_i)$. On taking $\langle\mathfrak{k},\theta\rangle_i = \langle g,\theta\rangle_i \cap \sim\langle\mathfrak{f},\theta\rangle$, we get $\langle\mathfrak{f},\theta\rangle \Subset \cup^{\sim} \langle\mathfrak{k},\theta\rangle_i$, $\langle g,\theta\rangle_i \in \langle\tilde{\mathscr{X}}\rangle,\tau,\theta) \Longrightarrow \langle\mathfrak{k},\theta\rangle_i = \langle g,\theta\rangle_i \cap^{\sim}\langle\mathfrak{f},\theta\rangle \in \langle\mathfrak{k},\mathfrak{H},\theta\rangle \cdots .(1)$. Now from (1) it is clear that $\{\langle \mathfrak{k},\theta\rangle_1, \langle \mathfrak{k},\theta\rangle_2, \langle \mathfrak{k},\theta\rangle_3, \langle \mathfrak{k},\theta\rangle_4, \cdots\}$ is $\langle \mathfrak{f}, \mathfrak{T}, \theta\rangle - \mathrm{NS}{*_b}$ open soft cover of $\langle \mathfrak{f},\theta\rangle$ which is known to be $\langle \mathfrak{f}, \mathfrak{T}, \theta\rangle - \mathrm{NS}{*_b}$ compact; hence, this soft cover must be reducible to a finite soft subcover say, $\{\langle \mathfrak{k},\theta\rangle_{ir} : 1 \le r \le n\}$. This $\Longrightarrow \langle \mathfrak{f},\theta\rangle \Subset \bigcup_{r=1}^{n} \widetilde{\langle \mathfrak{k},\theta\rangle}_{ir} = \bigcup_{r=1}^{n}((\langle \mathscr{g},\theta\rangle_{ir}) \cap^{\sim} \langle \mathfrak{f},\theta\rangle \in \langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$, or

$$\langle \mathfrak{f},\theta\rangle \Subset \left(\bigcup_{r=1}^{n}\left(\left(\langle \mathscr{g},\theta\rangle_{ir}\right) \cap^{\sim} \langle \mathfrak{f}^{crip},\theta\rangle\right) \Subset \bigcup_{r=1}^{n} \widetilde{\langle \mathscr{g},\theta\rangle}_{ir}, \text{or} \langle \mathfrak{f},\theta\rangle \bigcup_{r=1}^{n} \langle \mathscr{g},\theta\rangle_{ir}.\right. \tag{12}$$

This proves that $\{\langle \mathscr{g},\theta\rangle_{ir} : 1 \le r \le n\}$ is a finite soft subcover to the soft cover $\langle \mathscr{g},\theta\rangle_i$. Starting from an arbitrary $\langle \tilde{\mathscr{X}}\rangle, \tau, \theta) - \mathrm{NS}{*_b}$ open soft cover of $\langle \mathfrak{f},\theta\rangle$, we are able to show that this soft neutrosophic open cover is freezable to a finite soft subcover, showing there by $\langle \mathfrak{f},\theta\rangle$ is $\langle \tilde{\mathscr{X}}\rangle, \tau, \theta) - N{*_b}S$ compact.

Theorem 46. *Let $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ NSTS and let $\langle (\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_n\rangle$ be a NS sequence in $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ such that it converges to a point $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_0$, then the soft set $\langle \mathscr{g},\theta\rangle$ consisting of the points $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_{n_0}$ and $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_n (n = 1, 2, 3, \cdots\cdots)$ is soft NS ${*_b}$ compact.*

Proof. Given $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ NSTS and let $\langle (\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_n\rangle$ be a NS sequence in $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ such that it converges to a point $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_{n_0}$ that is $(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_n \longrightarrow (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_{n_0} \in \langle \tilde{\mathscr{X}}\rangle$. Let $\langle \mathscr{g},\theta\rangle = \Big\langle (\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_1, (\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_2, (\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_3, (\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_4, (\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_5, (\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_7, \cdots\rangle$. Let $\{\langle \mathfrak{S},\theta\rangle_\alpha : \alpha \in \Delta\}$ be $\mathrm{NS}{*_b}$ open covering of $\langle \mathscr{g},\theta\rangle$ so that $\langle \mathscr{g},\theta\rangle \Subset \cup^{\sim}\{\langle \mathfrak{S},\theta\rangle_\alpha : \alpha \in \Delta\}$, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_{n_0} \in \langle \mathscr{g},\theta\rangle$ implies that $\exists \alpha_0 \in \Delta$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_{n_0} \in \langle \mathfrak{S},\theta\rangle_{\alpha_0}$. According to the definition of soft convergence, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_{n_0} \in \langle \mathfrak{S},\theta\rangle_{\alpha_0} \in (\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ implies that there exists $n_0 \in N\, s.t. n \ge n_0$ and $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_n \in \langle \mathfrak{S},\theta\rangle_{\alpha_0}$. Evidently, $\langle \mathfrak{S},\theta\rangle_{\alpha_0}$ contains the points $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_{n_0}$,

$$\begin{gathered}\left(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta\right)_{n_{0+1}}, \left(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta\right)_{n_{0+2}}, \\ \left(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta\right)_{n_{0+3}}, \left(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta\right), \cdots.\end{gathered} \tag{13}$$

$(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_{n_{0+n}}, \cdots\cdots$ Look carefully at the points and train them in a way as, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_1, (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_2, (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_3, (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_4, \cdots$.

$(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta), \cdots\cdots$ generating a finite soft set. Let $1 \le n_{0-1}$. Then, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_i \in \langle \mathscr{g},\theta\rangle$. For this i, $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_i \in \langle \mathscr{g},\theta\rangle$. Hence, there exists $\alpha_i \in \Delta$ such that $(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta)_i \in \langle \mathfrak{S},\theta\rangle_{\alpha_i}$. Evidently, $\langle \mathscr{g},\theta\rangle \Subset \bigcup_{r=0}^{n_{0-1}} \widetilde{\langle \mathfrak{S},\theta\rangle}_{\alpha_i}$. This shows that $\{\langle \mathfrak{S},\theta\rangle_{\alpha_i} : 0 \le n_{0-1}\}$ is $\mathrm{NS}{*_b}$ open cover of $\langle \mathscr{g},\theta\rangle$. Thus, an arbitrary $\mathrm{NS}{*_b}$ open cover $\{\langle \mathfrak{S},\theta\rangle_\alpha : \alpha \in \Delta\}$ of $\langle \mathscr{g},\theta\rangle$ is reducible to a finite NS cub-cover $\{\langle \mathfrak{S},\theta\rangle_{\alpha i} : i = 0, 1, 2, 3, \cdots n_{0-1}\}$, it follows that $\langle \mathscr{g},\theta\rangle$ is soft $\mathrm{NS}{*_b}$ compact.□

Theorem 47. *If $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ NSTS such that it has the characteristics of $\mathrm{NS}{*_b}$ sequentially compactness. Then, $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ is safely $\mathrm{NS}{*_b}$ countably compact.*

Proof. Let $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ NSTS and let $\langle \rho,\theta\rangle$ be finite soft subset of $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$. Let

$$\left\langle \begin{gathered}\left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_1, \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_2, \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_3, \\ \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_4, \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_5, \\ \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_6, \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_7, \cdots\end{gathered} \right\rangle \tag{14}$$

be a soft sequence of soft points of $\langle \rho,\theta\rangle$. Then, $\langle \rho,\theta\rangle$ being finite, at least one of the elements. In $\langle \rho,\partial\rangle$, say $(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_0$ must be duplicated an infinite number of times in the NS sequence. Hence,

$$\left\langle \begin{gathered}\left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_0, \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_0, \\ \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_0, \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_0, \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_0, \\ \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_0, \left(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta}\right)_0, \cdots\end{gathered} \right\rangle \tag{15}$$

is a soft subsequence of $\langle (\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_n\rangle$ such that it is soft constant sequence and repeatedly constructed by a single soft number $(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_0$ and we know that a soft constant sequence converges on its self. So it converges to $(\widetilde{x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}, \theta})_0$ which belongs to $\langle \rho,\theta\rangle$. Hence, $\langle \rho,\theta\rangle$ is soft sequentially $\mathrm{NS}{*_b}$ compact.

Theorem 48. *Let $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ NSTS and $(\langle \tilde{\mathscr{Y}}\rangle, \mathfrak{T}, \theta)$ be another NSTS. Let $\langle \mathscr{f},\theta\rangle$ be a soft continuous mapping of a soft neutrosophic sequentially compact $\mathrm{NS}{*_b}$ space $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ into $(\langle \tilde{\mathscr{Y}}\rangle, \mathfrak{T}, \theta)$. Then, $\langle \mathscr{f},\theta\rangle(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ is $\mathrm{NS}{*_b}$ sequentially compact.*

Proof. Given $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ NSTS and $(\langle \tilde{\mathscr{Y}}\rangle, \mathfrak{T}, \theta)$ be another NSTS. Let $\langle \mathscr{f},\theta\rangle$ be a soft continuous mapping of a NS sequentially compact space $(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ into $(\langle \tilde{\mathscr{Y}}\rangle, \mathfrak{T}, \theta)$. Then, we have to prove $\langle \mathscr{f},\theta\rangle(\langle \tilde{\mathscr{X}}\rangle, \tau, \theta)$ NS sequentially. For this, we proceed. Let

$$\left\langle \begin{array}{c} \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_1, \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_2, \\ ,\widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_5, \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_6, \\ \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_7, \cdots \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_n, \cdots. \end{array} \right\rangle \quad (16)$$

be a soft sequence of NS points in $\langle f,\theta\rangle(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$. Then, for each $n\in N$, there exists

$$\left\langle \begin{array}{c} \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_1, \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_2, \\ \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_4, \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_5, \\ \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_7, \cdots \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_n, \cdots \in \left(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta\right) \end{array} \right\rangle \quad (17)$$

such that

$$\langle f,\partial\rangle \left(\left\langle \begin{array}{c} \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_1, \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_2, \\ \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_3, \\ \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_7, \cdots \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_n, \cdots. \end{array} \right\rangle \right)$$
$$= \left\langle \begin{array}{c} \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_1, \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_2, \\ \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_3, \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_4, \\ \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_6, \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_7, \\ \cdots \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_n, \cdots. \end{array} \right\rangle. \quad (18)$$

Thus, we obtain a soft sequence

$$\left\langle \begin{array}{c} \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_1, \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_2, \\ \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_3, \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_4, \\ \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}, \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_6, \\ \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_7, \cdots \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_n, \cdots. \end{array} \right\rangle \quad (19)$$

in $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$. But $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ being soft sequentially NS $*_b$ compact, there is a NS subsequence $\langle\widetilde{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_{n_i}\rangle$ of $\langle\widetilde{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n\rangle$ such that $\langle\widetilde{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_{n_i}\rangle \longrightarrow \widetilde{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)} \in (\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$. So, by NS$*_b$ continuity of $\langle f,\theta\rangle$, $\langle\widetilde{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_{n_i}\rangle \longrightarrow \widetilde{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$ implies that $\langle f,\theta\rangle(\langle\widetilde{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_{n_i}\rangle) \longrightarrow \langle f,\theta\rangle(\langle\widetilde{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_n\rangle) \in \langle f,\theta\rangle(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$. Thus, $\langle f,\partial\rangle(\langle\widetilde{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}_{n_i}\rangle)$ is a soft subsequence of

$$\left\langle \begin{array}{c} \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_1, \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_2, \\ \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}, \widetilde{\left(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta\right)}_4, \\ \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_5, \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_6, \\ \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')}\theta\right)}_7, \cdots \widetilde{\left(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta\right)}_n, \cdots. \end{array} \right\rangle \quad (20)$$

converges to $(\langle f,\theta\rangle)(\tilde{\varkappa})$ in $\langle f,\theta\rangle(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$. Hence, $\langle f,\theta\rangle(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ is NS$*_b$ sequentially compact.□

Theorem 49. *Let $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ be a NS$*_{b1}$ space and $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$, $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \in \langle\tilde{\mathscr{X}}\rangle$ such that $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) > (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$ or $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) < (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$. If $\mathfrak{W}_{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$ is a NS local base at $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$, then there exists at least one member of $\mathfrak{W}_{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$ which does not suppose $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$.*

Proof. Since $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ be a NS$*_{b1}$ space and $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) > (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$ or $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) < (y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$, $\exists$NS$*_b$ open sets $\langle\mathscr{G},\theta\rangle$ and $\langle\mathscr{H},\theta\rangle$ such that $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle\mathscr{G},\theta\rangle$ but $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \notin \langle\mathscr{G},\partial\rangle$ and $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \in \langle\mathscr{H},\partial\rangle$ but $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \notin \langle\mathscr{H},\theta\rangle$. Since, $\mathfrak{W}_{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$ is NS local base at $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$; there exists $(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in B \Subset \langle\mathscr{G},\partial\rangle$. Since $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \notin \langle\mathscr{G},\theta\rangle$ and $B \Subset \langle\mathscr{G},\theta\rangle$, so $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \notin B$. Thus, $B \in \mathfrak{W}_{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)}$ such that $(y^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \notin B$.

Theorem 50. *Let $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ NSTS and suppose $\langle f,\theta\rangle$, $\langle g,\theta\rangle$ be two NS continuous functions on a NSTS$(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$ in to a NSTS $(\langle\tilde{Y}\rangle,\mathfrak{F},\theta)$ which is NS$*_b$ Hausdorff. Then, soft set $\{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle\tilde{\mathscr{X}}\rangle : (f)((x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (g)\}$ is NS$*_b$ closed of $(\langle\tilde{\mathscr{X}}\rangle,\tau,\theta)$.*

Proof. Let if $\{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle\tilde{\mathscr{X}}\rangle : (f)((x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (g)((x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))\}$ is a NS set of function. If $\{(x^{e}\ (\mathfrak{a},\mathfrak{b},\mathfrak{c}),\theta) \in \langle\tilde{\mathscr{X}}\rangle : (f)((x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (g)(\varkappa)\}^c = \varnothing^{\sim}$, it is clearly NS$*_b$ open, and therefore, $\{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle\tilde{\mathscr{X}}\rangle : (f)((x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (g)((x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))\}$ is NS$*_b$closed, which is nothing is proved in this case. Let us consider the case when $\{(x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle\tilde{\mathscr{X}}\rangle : (f)((x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (g)((x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))\}^c \neq (x^{e}{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)$. And let $\rho \in \{(x\ {}^{e}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \mathscr{X}^{crip} : (f)((

$x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (\mathscr{g})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))\}^c$. Then, ρ does not belong $\{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle \tilde{\mathscr{X}} \rangle : (\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (\mathscr{g})(\varkappa)\}$. Result in $(\mathscr{f})(\rho) \neq (\mathscr{g})(\rho)$. Now, $\langle Y^{crip}, \mathfrak{F}, \partial \rangle$ being NS$*_b$ Hausdorff space so there exists NS$*_b$ open sets $\langle \mathscr{g}, \theta \rangle$ and $\langle \mathfrak{H}, \theta \rangle$ of $(\mathscr{f})(\rho)$ and $(\mathscr{g})(\rho)$, respectively, such that $\langle \mathscr{g}, \theta \rangle$ and $\langle \mathfrak{H}, \theta \rangle$ such that these NS sets such that the possibility of one rules out the possibility of other. By soft continuity of $\langle \mathscr{f}, \theta \rangle, \langle \mathscr{g}, \theta \rangle$ $,\langle \mathscr{f}, \theta \rangle^{-1}$ as well as $\langle \mathscr{g}, \theta \rangle^{-1}$is NS $**_b$ open nhd of ρ, and therefore, so is $\langle \mathscr{f},\theta \rangle^{-1} \cap {}^{\sim} \langle \mathscr{g}, \theta \rangle^{-1}$ is contained in $\{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}$ $,\theta) \in \langle \tilde{\mathscr{X}} \rangle : (\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (\mathscr{g})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))\}$, for, $(x^e$ $_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in (\langle \mathscr{f},\partial \rangle^{-1} \cap {}^{\sim} \langle \mathscr{g},\theta \rangle^{-1}) \Longrightarrow (\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \in \langle \mathscr{g},$ $\theta \rangle$ and $(\mathscr{g})((\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) \neq (\mathscr{g})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))$ because $\langle \mathscr{g}$ $,\partial \rangle$ and $\langle \mathfrak{H}, \theta \rangle$ are mutually exclusive. This implies that x does not belong to $\{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle \tilde{\mathscr{X}} \rangle : (\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) =$ $(\mathscr{g})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))\}$. Therefore, $\rho \in (\mathscr{f})^{-1}(\langle \mathscr{g},\theta \rangle) \cap {}^{\sim}(\mathscr{g})^{-1}(\langle \mathscr{g},$ $\theta \rangle) \Subset \{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle \tilde{\mathscr{X}} \rangle : (\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (\mathscr{g})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},$ $\theta))\}^c$. This shows that $\{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle \tilde{\mathscr{X}} \rangle : (\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta$ $)) = (\mathscr{g})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))\}^c$is nhd of each of its points. So, $\{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle \tilde{\mathscr{X}} \rangle : (\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (\mathscr{g})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))\}^c$ NS$*_b$ open, and hence, $\{(x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta) \in \langle \tilde{\mathscr{X}} \rangle : (\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},$ $\theta)) = (\mathscr{g})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta))\}$ is NS$*_b$ closed.

Theorem 51. *Let $(\langle \tilde{\mathscr{X}} \rangle, \tau, \theta)$ NSTS such that it is NS$*_b$ Hausdorff space and let $(\mathscr{f})$ be a soft continuous function of $(\langle \tilde{\mathscr{X}} \rangle$ $,\tau, \theta)$ into itself. Then, the NS set of fixed points under $(\mathscr{f})$ is a NS$*_b$ closed set.*

Proof. Let $\delta = \{(\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}$. If $\delta^c = \widetilde{\varnothing}$, then is NS$*_b$ open, and therefore, $\{(\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = ($ $x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}NS*_b$closed. So, let $\{(\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = ($ $x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}^c \neq \varnothing^{\sim}$ and let $(\mathscr{y}^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \in \{(\mathscr{f})((x^e\ (\mathfrak{a},\mathfrak{b},\mathfrak{c}),$ $\theta)) = (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}^c$. Then, $(\mathscr{y}^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$ does not belong to $\{(\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}$, and therefore, $(\mathscr{f})(($ $\mathscr{y}^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)) \neq (\mathscr{y}^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$. Now, $(\mathscr{y}^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta)$ and $(\mathscr{f})($ $(\mathscr{y}^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta))$ being two distinct points of the NS$*_b$ Hausdorff space $(\langle \tilde{\mathscr{X}} \rangle, \tau, \theta)$, so there exists NS$*_b$ open sets $\langle \mathscr{g}, \partial \rangle$ and $\langle \mathfrak{H}, \theta \rangle$ such that $(\mathscr{y}^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \in \langle \mathscr{g},\theta \rangle, (\mathscr{f})((\mathscr{y}^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},$ $\theta)) \in \langle \mathfrak{H}, \partial \rangle$ and $\langle \mathscr{g},\theta \rangle, \langle \mathfrak{H},\theta \rangle$ are disjoint. Also, by the NS continuity of $(\mathscr{f})$, $(\mathscr{f})^{-1}(\langle H,\theta \rangle$ is NS$*_b$ open set containing $\mathscr{y}$. We pretend that $\langle \mathscr{g},\theta \rangle \cap {}^{\sim}(\mathscr{f})^{-1}(\langle \mathfrak{H},\theta \rangle) \Subset \{\ (\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})}$ $,\theta)) = (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}^c$. Since $\mu \in \langle \mathscr{g},\theta \rangle \cap {}^{\sim}(\mathscr{f})^{-1}(\langle \mathfrak{H},\theta \rangle) \Longrightarrow \mu$ $\in \langle \mathscr{g},\theta \rangle \& \mu \in (\mathscr{f})^{-1} \Longrightarrow \mu \in \langle \mathscr{g},\theta \rangle \& (\mathscr{f})(\mu) \in \langle \mathfrak{H},\theta \rangle \Longrightarrow \mu \neq$ $(\mathscr{f})(\mu)$. As $\langle \mathscr{g},\theta \rangle \cap {}^{\sim}\langle \mathfrak{H},\theta \rangle = \varnothing^{\sim}$ implies that μ does not belong to $\{(\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\} \Longrightarrow \mu \in \{(\mathscr{f})\ (($ $x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}^c$. Therefore, $(\mathscr{y}^{e'}{}_{(\mathfrak{a}',\mathfrak{b}',\mathfrak{c}')},\theta) \in \langle \mathscr{g},$ $\partial \rangle \cap {}^{\sim}(\mathscr{f})^{-1}(\langle \mathfrak{H},\theta \rangle) \Subset \{(\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}^c$. Thus, $\{(\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}^c$ is the NS nhd of each of its points. So, $\{(\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}^c$ is NS$*_b$ open, and hence, $\{(\mathscr{f})((x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)) = (x^e{}_{(\mathfrak{a},\mathfrak{b},\mathfrak{c})},\theta)\}$ is NS$*_b$ closed.□

4. Conclusion

Crisp topology is such an important branch of mathematics which is used as applied mathematics as well as pure mathematics. Soft topology is an extension of crisp topology. It actually discusses the behaviour of the subsets of the crisp set with the help of parameters. Fuzzy soft topology only discusses the membership value, and it has nothing to do with the nonmembership value. This is the drawback of fuzzy soft topology. The intuitionistic fuzzy soft topology is an extension of fuzzy soft topology. It addresses both the degree of membership as well as the degree of nonmembership. Intuitionistic fuzzy soft topology is failed to address the indeterminacy case. This deficiency is filled by neutrosophic soft topology. This neutrosophic soft topology addresses acceptance, rejection, and also indeterminacy case. Characterization of neutrosophic soft points, neutrosophic soft separation axioms, countability theorems, and countable space can be Hausdorff space under the restriction of neutrosophic soft sequence which is convergent, cardinality of neutrosophic soft countable space, engagement of neutrosophic soft countable and uncountable spaces, neutrosophic soft topological characteristics of different spaces, neutrosophic soft continuity, product of different neutrosophic soft spaces, and neutrosophic soft countably compact has the characteristics of Bolzano Weierstrass Property are studied. BVP shifting from one space to another through neutrosophic soft continuous functions, neutrosophic soft sequence convergence and its linking with neutrosophic soft compact space, sequentially compactness are addressed.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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