SPECIAL TYPES OF INTERVAL VALUED NEUTROSOPHIC GRAPHS

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Abstract. Neutrosophic theory has many applications in graph theory, interval valued neutrosophic graph (IVNG) is the generalization of fuzzy graph, intuitionistic fuzzy graph and single valued neutrosophic graph. In this paper, we introduced some types of IVNGs, which are subdivision IVNGs, middle IVNGs, total IVNGs and interval valued neutrosophic line graphs (IVNLGs), also discussed the isomorphism, co weak isomorphism and weak isomorphism properties of subdivision IVNGs, middle IVNGs, total IVNGs and IVNLGs.

Keywords: Interval valued neutrosophic line graph, Subdivision IVNG, middle IVNG, total IVNG.

1. Introduction

Neutrosopic sets were introduced by Smarandache in [1], which are the generalization of fuzzy sets and intuitionistic fuzzy sets. The single valued neutrosophic graphs were introduced by Broumi, Talea, Bakali and Smarandache in [3] and recently in [8, 9, 10] proposed some algorithms. A graph is a way to represent information between objects. The objects are represented by vertices and the relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a fuzzy graph Model. The perception of fuzzy graph was introduced by Rosenfeld in [6] and the some remarks on fuzzy graphs were explained by Bhattacharya in [5]. The special types and its truncations of fuzzy graphs were paid the way by Gani in [7]. The IVNGs have many applications in path problems, networks and computer science. The strong IVNG and complete IVNG are the special types of IVNG. In this paper, we introduce the another types of IVNGs, which are subdivision IVNGs, middle IVNGs, total IVNGs and IVNLGs. These are all the strong IVNGs, also we discuss their relations based on isomorphism, co weak isomorphism and weak isomorphism.

2. Preliminaries

Let G denotes IVNG and $G^* = (V, E)$ denotes its underlying crisp graph.

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Definition 2.1. [1, 2] *Let X be a crisp set, the single (interval) valued neutrosophic set (SVNS) A is characterized by three membership functions $T_A(x), I_A(x)$ and $F_A(x)$, such that for every $x \in X$, the membership values $T_A(x), I_A(x), F_A(x) \in [0,1]$ $(T_A(x), I_A(x), F_A(x) \subseteq [0,1]$.)*

Definition 2.2. [3] *Let C and D be a SVNSs of V and E, respectively. Then D is said to be single valued neutrosophic relation (SVNR) on C, whenever*

$$T_D(xy) \leq \min(T_C(x), T_C(y))$$
$$I_D(xy) \geq \max(I_C(x), I_C(y))$$
$$F_D(xy) \geq \max(F_C(x), F_C(y))$$

$\forall x, y \in V$.

Definition 2.3. [4] *Let C and D be IVNSs of a V and E, respectively. Then D is said to be interval valued neutrosophic relation (IVNR) on C, whenever*

$$T_{DL}(xy) \leq \min(T_{CL}(x), T_{CL}(y)),\ I_{DL}(xy) \geq \max(I_{CL}(x), I_{CL}(y))$$
$$F_{DL}(xy) \geq \max(F_{CL}(x), F_{CL}(y)),\ T_{DU}(xy) \leq \min(T_{CU}(x), T_{CU}(y))$$
$$I_{DU}(xy) \geq \max(I_{CU}(x), I_{CU}(y)),\ F_{DU}(xy) \geq \max(F_{CU}(x), F_{CU}(y))$$

$\forall x, y \in V$.

Definition 2.4. [4] *The interval valued neutrosophic graph (IVNG) is a pair $G = (C, D)$ of $G^* = (V, E)$, where C is IVNS on V and D is IVNS on E, such that*

$$T_{DL}(\alpha\beta) \leq \min(T_{CL}(\alpha), T_{CL}(\beta)),\ I_{DL}(\alpha\beta) \geq \max(I_{CL}(\alpha), I_{CL}(\beta))$$
$$F_{DL}(\alpha\beta) \geq \max(F_{CL}(\alpha), F_{CL}(\beta)),\ T_{DU}(\alpha\beta) \leq \min(T_{CU}(\alpha), T_{CU}(\beta))$$
$$I_{DU}(\alpha\beta) \geq \max(I_{CU}(\alpha), I_{CU}(\beta)),\ F_{DU}(\alpha\beta) \geq \max(F_{CU}(\alpha), F_{CU}(\beta))$$

whenever

$$0 \leq T_{DL}(\alpha\beta) + I_{DL}(\alpha\beta) + F_{DL}(\alpha\beta) \leq 3$$
$$0 \leq T_{DU}(\alpha\beta) + I_{DU}(\alpha\beta) + F_{DU}(\alpha\beta) \leq 3$$

$\forall\ \alpha, \beta \in V$. The IVNG G is said to be complete (strong) IVNG, if

$$T_{DL}(xy) = \min(T_{CL}(x), T_{CL}(y)),\ I_{DL}(xy) = \max(I_{CL}(x), I_{CL}(y))$$
$$F_{DL}(xy) = \max(F_{CL}(x), F_{CL}(y)),\ T_{DU}(xy) = \min(T_{CU}(x), T_{CU}(y))$$
$$I_{DU}(xy) = \max(I_{CU}(x), I_{CU}(y)),\ F_{DU}(xy) = \max(F_{CU}(x), F_{CU}(y))$$

$\forall\ x, y \in V (\forall xy \in E)$. The order and size of G and also degree of vertex defined below

$$O(G) = ([O_{TL}(G), O_{TU}(G)], [O_{IL}(G), O_{IU}(G)], [O_{FL}(G), O_{FU}(G)])$$

where

$$O_{TL}(G) = \sum_{\alpha \in V} T_{CL}(\alpha),\ O_{IL}(G) = \sum_{\alpha \in V} I_{CL}(\alpha),\ O_{FL}(G) = \sum_{\alpha \in V} F_{CL}(\alpha),$$
$$O_{TU}(G) = \sum_{\alpha \in V} T_{CU}(\alpha),\ O_{IU}(G) = \sum_{\alpha \in V} I_{CU}(\alpha),\ O_{FU}(G) = \sum_{\alpha \in V} F_{CU}(\alpha).$$
$$S(G) = ([S_{TL}(G), S_{TU}(G)], [S_{IL}(G), S_{IU}(G)], [S_{FL}(G), S_{FU}(G)])$$

where

$$S_{TL}(G) = \sum_{\alpha\beta \in E} T_{DL}(\alpha\beta),\ S_{IL}(G) = \sum_{\alpha\beta \in E} I_{DL}(\alpha\beta),\ S_{FL}(G) = \sum_{\alpha\beta \in E} F_{DL}(\alpha\beta),$$
$$S_{TU}(G) = \sum_{\alpha\beta \in E} T_{DU}(\alpha\beta),\ S_{IU}(G) = \sum_{\alpha\beta \in E} I_{DU}(\alpha\beta),\ S_{FU}(G) = \sum_{\alpha\beta \in E} F_{DU}(\alpha\beta).$$

$d_G(\alpha)$, *is defined by*

$$d_G(\alpha) = ([d_{TL}(\alpha), d_{TU}(\alpha)], [d_{IL}(\alpha), d_{IU}(\alpha)], [d_{FL}(\alpha), d_{FU}(\alpha)]),$$

where

$$d_{TL}(\alpha) = \sum_{\alpha\beta\in E} T_{DL}(\alpha\beta),\ d_{IL}(\alpha) = \sum_{\alpha\beta\in E} I_{DL}(\alpha\beta),\ d_{FL}(\alpha) = \sum_{\alpha\beta\in E} F_{DL}(\alpha\beta),$$

$$d_{TU}(\alpha) = \sum_{\alpha\beta\in E} T_{DU}(\alpha\beta),\ d_{IU}(\alpha) = \sum_{\alpha\beta\in E} I_{DU}(\alpha\beta),\ d_{FU}(\alpha) = \sum_{\alpha\beta\in E} F_{DU}(\alpha\beta).$$

3. Special Types of IVNGs

Definition 3.1. *Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two IVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then the homomorphism $\chi : V_1 \to V_2$ is a mapping from V_1 into V_2 satisfying following conditions*

$$T_{C_1L}(p) \leq T_{C_2L}(\chi(p)),\ I_{C_1L}(p) \geq I_{C_2L}(\chi(p)),\ F_{C_1L}(p) \geq F_{C_2L}(\chi(p))$$

$$T_{C_1U}(p) \leq T_{C_2U}(\chi(p)),\ I_{C_1U}(p) \geq I_{C_2U}(\chi(p)),\ F_{C_1U}(p) \geq F_{C_2U}(\chi(p))$$

$\forall p \in V_1$.

$$T_{D_1L}(pq) \leq T_{D_2L}(\chi(p)\chi(q)),\ I_{D_1L}(pq) \geq I_{D_2L}(\chi(p)\chi(q)),\ F_{D_1L}(pq) \geq F_{D_2L}(\chi(p)\chi(q))$$

$$T_{D_1U}(pq) \leq T_{D_2U}(\chi(p)\chi(q)),\ I_{D_1U}(pq) \geq I_{D_2U}(\chi(p)\chi(q)),\ F_{D_1U}(pq) \geq F_{D_2U}(\chi(p)\chi(q))$$

$\forall pq \in E_1$. *The weak isomorphism $\upsilon : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions*

$$T_{C_1L}(p) = T_{C_2L}(\upsilon(p)),\ I_{C_1L}(p) = I_{C_2L}(\upsilon(p)),\ F_{C_1L}(p) = F_{C_2L}(\upsilon(p))$$

$$T_{C_1U}(p) = T_{C_2U}(\upsilon(p)),\ I_{C_1U}(p) = I_{C_2U}(\upsilon(p)),\ F_{C_1U}(p) = F_{C_2U}(\upsilon(p))$$

$\forall p \in V_1$. *The co-weak isomorphism $\kappa : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions*

$$T_{D_1L}(pq) = T_{D_2L}(\kappa(p)\kappa(q)),\ I_{D_1L}(pq) = I_{D_2L}(\kappa(p)\kappa(q)),\ F_{D_1L}(pq) = F_{D_2L}(\kappa(p)\kappa(q))$$

$$T_{D_1U}(pq) = T_{D_2U}(\kappa(p)\kappa(q)),\ I_{D_1U}(pq) = I_{D_2U}(\kappa(p)\kappa(q)),\ F_{D_1U}(pq) = F_{D_2U}(\kappa(p)\kappa(q))$$

$\forall pq \in E_1$. *An isomorphism $\psi : V_1 \to V_2$ is a bijective homomorphism from V_1 into V_2 satisfying following conditions*

$$T_{C_1L}(p) = T_{C_2L}(\psi(p)), I_{C_1L}(p) = I_{C_2L}(v(p)), F_{C_1L}(p) = F_{C_2L}(\psi(p))$$

$$T_{C_1U}(p) = T_{C_2U}(\psi(p)), I_{C_1U}(p) = I_{C_2U}(v(p)), F_{C_1U}(p) = F_{C_2U}(\psi(p))$$

$\forall p \in V_1$.

$$T_{D_1L}(pq) = T_{D_2L}(\psi(p)\psi(q)),\ I_{D_1L}(pq) = I_{D_2L}(\psi(p)\psi(q)),\ F_{D_1L}(pq) = F_{D_2L}(\psi(p)\psi(q))$$

$$T_{D_1U}(pq) = T_{D_2U}(\psi(p)\psi(q)),\ I_{D_1U}(pq) = I_{D_2U}(\psi(p)\psi(q)),\ F_{D_1U}(pq) = F_{D_2U}(\psi(p)\psi(q))$$

$\forall pq \in E_1$.

Remark 3.1. *The weak isomorphism between two IVNGs preserves the orders.*

Remark 3.2. *The weak isomorphism between IVNGs is a partial order relation.*

Remark 3.3. *The co-weak isomorphism between two IVNGs preserves the sizes.*

Remark 3.4. *The co-weak isomorphism between IVNGs is a partial order relation.*

Remark 3.5. *The isomorphism between two IVNGs is an equivalence relation.*

Remark 3.6. *The isomorphism between two IVNGs preserves the orders and sizes.*

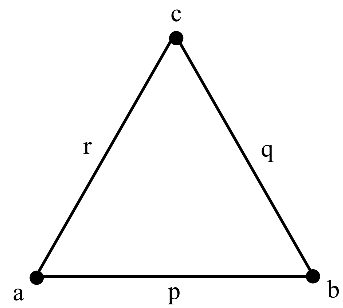

Figure 1: Crisp Graph of IVNG.

Remark 3.7. *The isomorphism between two IVNGs preserves the degrees of their vertices.*

Definition 3.2. *The subdivision IVNG $sd(G) = (C, D)$ of IVNG $G = (A, B)$, where C is a IVNS on $V \cup E$ and D is a IVNR on C, such that*
(**1**) $C = A$ *on* V *and* $C = B$ *on* E.
(**2**) *If* $v \in V$ *lie on edge* $e \in E$, *then*

$$T_{DL}(ve) = \min(T_{AL}(v), T_{BL}(e)),\ \ I_{DL}(ve) = \max(I_{AL}(v), I_{BL}(e))$$

$$F_{DL}(ve) = \max(F_{AL}(v), F_{BL}(e)),\ \ T_{DU}(ve) = \min(T_{AU}(v), T_{BU}(e))$$

$$I_{DU}(ve) = \max(I_{AU}(v), I_{BU}(e)),\ \ F_{DU}(ve) = \max(F_{AU}(v), F_{BU}(e))$$

else

$$D(ve) = O = ([0,0],[0,0],[0,0]).$$

Proposition 3.1. *Let G be a IVNG and $sd(G)$ be the subdivision IVNG of a IVNG G, then*
(**1**) $O(sd(G)) = O(G) + S(G)$.
(**2**) $S(sd(G)) = 2S(G)$.

Proposition 3.2. *If G is complete IVNG, then $sd(G)$ need not to be complete IVNG.*

Example 3.1. *Consider the crisp graph $G^* = (V, E)$ of IVNG $G = (A, B)$, which is shown in Figure 1. The IVNSs A and B over $V = \{a, b, c\}$ and $E = \{p = ab, q = bc, r = ac\}$, which are defined in Table 1.*

A	T_A	I_A	F_A	B	T_B	I_B	F_B
a	[0.2,0.3]	[0.1,0.2]	[0.4,0.5]	p	[0.2,0.3]	[0.4,0.5]	[0.5,0.6]
b	[0.3,0.4]	[0.2,0.3]	[0.5,0.6]	q	[0.3,0.4]	[0.8,0.9]	[0.6,0.7]
c	[0.4,0.5]	[0.7,0.8]	[0.6,0.7]	r	[0.1,0.2]	[0.7,0.8]	[0.9,1.0]

Table 1: IVNSs of IVNG.

The crisp graph of SDIVNG $sd(G) = (C, D)$ of a IVNG G, which is shown in Figure 2. By calculations the IVNSs C and D, which are defined in Table 2.

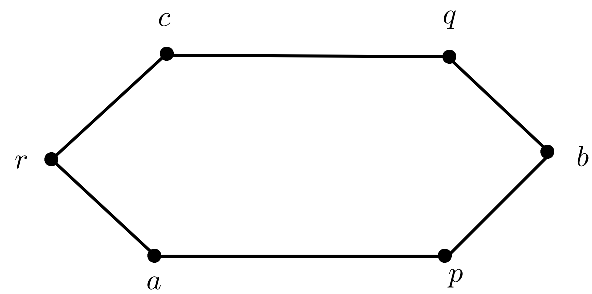

Figure 2: Crisp Graph of SDIVNG.

C	T_C	I_C	F_C	D	T_D	I_D	F_D
a	[0.2,0.3]	[0.1,0.2]	[0.4,0.5]	ap	[0.2,0.3]	[0.4,0.5]	[0.5,0.6]
p	[0.2,0.3]	[0.4,0.5]	[0.5,0.6]	pb	[0.2,0.3]	[0.4,0.5]	[0.5,0.6]
b	[0.3,0.4]	[0.2,0.3]	[0.5,0.6]	bq	[0.3,0.4]	[0.8,0.9]	[0.6,0.7]
q	[0.3,0.4]	[0.8,0.9]	[0.6,0.7]	qc	[0.3,0.4]	[0.8,0.9]	[0.6,0.7]
c	[0.4,0.5]	[0.7,0.8]	[0.6,0.7]	cr	[0.1,0.2]	[0.7,0.8]	[0.9,1.0]
r	[0.1,0.2]	[0.7,0.8]	[0.9,1.0]	ra	[0.1,0.2]	[0.7,0.8]	[0.9,1.0]

Table 2: IVNSs of SDIVNG.

Definition 3.3. *The total interval valued neutrosophic graph (TIVNG) $T(G) = (C, D)$ of $G = (A, B)$, where C is a IVNS on $V \cup E$ and D is a IVNR on C, such that*
(**1**) *$C = A$ on V and $C = B$ on E.*
(**2**) *If $v \in V$ lie on edge $e \in E$, then*

$$T_{DL}(ve) = \min(T_{AL}(v), T_{BL}(e)),\ I_{DL}(ve) = \max(I_{AL}(v), I_{BL}(e))$$

$$F_{DL}(ve) = \max(F_{AL}(v), F_{BL}(e)),\ T_{DU}(ve) = \min(T_{AU}(v), T_{BU}(e))$$

$$I_{DU}(ve) = \max(I_{AU}(v), I_{BU}(e)),\ F_{DU}(ve) = \max(F_{AU}(v), F_{BU}(e))$$

else

$$D(ve) = O = ([0, 0], [0, 0], [0, 0]).$$

(**3**) *If $\alpha, \beta \in E$, then*

$$T_{DL}(\alpha\beta) = T_{BL}(\alpha\beta),\ I_{DL}(\alpha\beta) = I_{BL}(\alpha\beta),\ F_{DL}(\alpha\beta) = F_{BL}(\alpha\beta),$$

$$T_{DU}(\alpha\beta) = T_{BU}(\alpha\beta),\ I_{DU}(\alpha\beta) = I_{BU}(\alpha\beta),\ F_{DU}(\alpha\beta) = F_{BU}(\alpha\beta).$$

(**4**) *If $e, f \in E$ have a common vertex, then*

$$T_{DL}(ef) = \min(T_{BL}(e), T_{BL}(f)),\ I_{DL}(ef) = \max(I_{BL}(e), I_{BL}(f))$$

$$F_{DL}(ef) = \max(F_{BL}(e), F_{BL}(f)),\ T_{DU}(ef) = \min(T_{BU}(e), T_{BU}(f))$$

$$I_{DU}(ef) = \max(I_{BU}(e), I_{BU}(f)),\ F_{DU}(ef) = \max(F_{BU}(e), F_{BU}(f))$$

else

$$D(ef) = O = ([0, 0], [0, 0], [0, 0]).$$

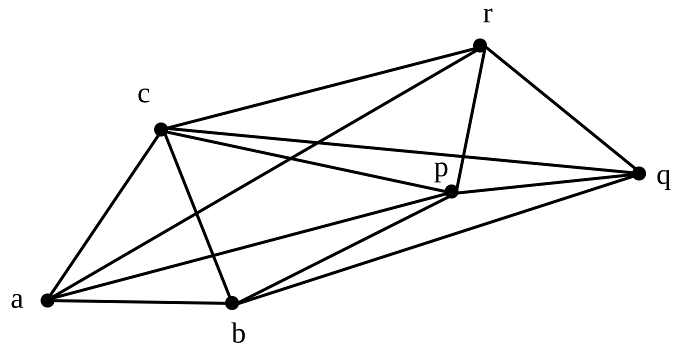

Figure 3: Crisp Graph of TIVNG.

D	T_D	I_D	F_D	D	T_D	I_D	F_D
ab	[0.2,0.3]	[0.4,0.5]	[0.5,0.6]	ap	[0.2,0.3]	[0.4,0.5]	[0.5,0.6]
bc	[0.3,0.4]	[0.8,0.9]	[0.6,0.7]	pb	[0.2,0.3]	[0.4,0.5]	[0.5,0.6]
ca	[0.1,0.2]	[0.7,0.8]	[0.9,1.0]	bq	[0.3,0.4]	[0.8,0.9]	[0.6,0.7]
pq	[0.2,0.3]	[0.8,0.9]	[0.6,0.7]	qc	[0.3,0.4]	[0.8,0.9]	[0.6,0.7]
qr	[0.1,0.2]	[0.8,0.9]	[0.9,1.0]	cr	[0.1,0.2]	[0.7,0.8]	[0.9,1.0]
rp	[0.1,0.2]	[0.7,0.8]	[0.9,1.0]	ra	[0.1,0.2]	[0.7,0.8]	[0.9,1.0]

Table 3: IVNS of TIVNG.

Example 3.2. *In Example 3.1, the crisp graph for TIVNG $T(G) = (C, D)$, which is shown in Figure 3. Here C is defined in Example 3.1. By calculations the IVNS D, which is defined in Table 3.*

Proposition 3.3. *Let G be a $IVNG$ and $T(G)$ be the TIVNG of G, then*
(**1**) $O(T(G)) = O(G) + S(G) = O(sd(G))$.
(**2**) $S(sd(G)) = 2S(G)$.

Proposition 3.4. *If G is a IVNG, then $sd(G)$ is weak isomorphic to $T(G)$.*

Definition 3.4. *The middle interval valued neutrosophic graph (MIVNG) $M(G) = (C, D)$ of $G = (A, B)$, where C is a IVNS on $V \cup E$ and D is a IVNR on C, such that*
(**1**) $C = A$ *on* V *and* $C = B$ *on* E, *else* $C = O = ([0, 0], [0, 0], [0, 0])$.
(**2**) *If* $v \in V$ *lie on edge* $e \in E$, *then*

$$T_{DL}(ve) = T_{BL}(e),\ I_{DL}(ve) = I_{BL}(e),\ F_{DL}(ve) = F_{BL}(e)$$

$$T_{DU}(ve) = T_{BU}(e),\ I_{DU}(ve) = I_{BU}(e),\ F_{DU}(ve) = F_{BU}(e)$$

else

$$D(ve) = O = ([0, 0], [0, 0], [0, 0]).$$

(**3**) *If* $u, v \in V$, *then*

$$D(uv) = O = ([0, 0], [0, 0], [0, 0]).$$

(**4**) *If* $e, f \in E$ *such that* e *and* f *are adjacent in* G, *then*

$$T_{DL}(ef) = T_{BL}(uv),\ I_{DL}(ef) = I_{BL}(uv),\ F_{DL}(ef) = F_{BL}(uv),$$

$$T_{DU}(ef) = T_{BU}(uv),\ I_{DU}(ef) = I_{BU}(uv),\ F_{DU}(ef) = F_{BU}(uv).$$

Remark 3.8. *If G is a IVNG and $M(G)$ is a MIVNG of G, then*
$O(M(G)) = O(G) + S(G)$.

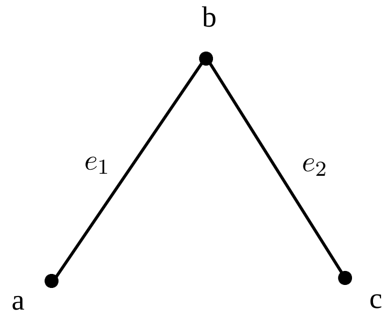

Figure 4: Crisp Graph of IVNG.

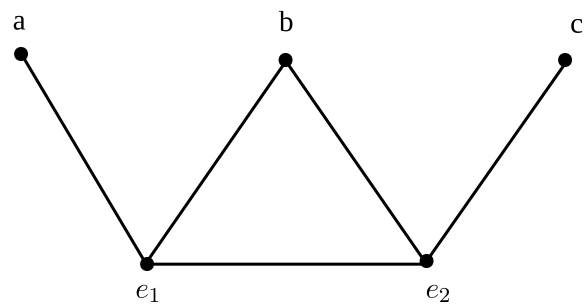

Figure 5: Crisp Graph of MIVNG.

Remark 3.9. *If G is a IVNG, then $M(G)$ is a strong IVNG.*

Remark 3.10. *If G is complete IVNG, then $M(G)$ need not to be complete IVNG.*

Example 3.3. *Consider the IVNG $G = (A, B)$ of a G^*, which is shown in Figure 4. The IVNSs A and B are defined in Table 4. The crisp graph of MIVNG $M(G) = (C, D)$, which is shown in Figure 5. By calculations, the IVNSs C and D are defined in Table 5.*

A	T_A	I_A	F_A
a	[0.3,0.4]	[0.4,0.5]	[0.4,0.5]
b	[0.7,0.8]	[0.6,0.7]	[0.3,0.4]
c	[0.9,1.0]	[0.7,0.8]	[0.2,0.3]
B	T_B	I_B	F_B
e_1	[0.2,0.3]	[0.6,0.7]	[0.6,0.7]
e_2	[0.4,0.5]	[0.8,0.9]	[0.7,0.8]

Table 4: IVNSs of IVNG.

C	T_C	I_C	F_C	D	T_D	I_D	F_D
a	[0.3,0.4]	[0.4,0.5]	[0.5,0.6]	e_1e_2	[0.2,0.3]	[0.8,0.9]	[0.7,0.8]
b	[0.7,0.8]	[0.6,0.7]	[0.3,0.4]	ae_1	[0.2,0.3]	[0.6,0.7]	[0.6,0.7]
c	[0.9,1.0]	[0.7,0.8]	[0.2,0.3]	be_1	[0.2,0.3]	[0.6,0.7]	[0.6,0.7]
e_1	[0.2,0.3]	[0.6,0.7]	[0.6,0.7]	be_2	[0.2,0.3]	[0.6,0.7]	[0.6,0.7]
e_2	[0.4,0.5]	[0.8,0.9]	[0.7,0.8]	ce_2	[0.4,0.5]	[0.8,0.9]	[0.7,0.8]

Table 5: IVNSs of MIVNG.

Proposition 3.5. *If G is a IVNG, then $sd(G)$ is weak isomorphic with $M(G)$.*

Proposition 3.6. *If G is a IVNG, then $M(G)$ is weak isomorphic with $T(G)$.*

Proposition 3.7. *If G is a IVNG, then $T(G)$ is isomorphic with $G \cup M(G)$.*

Definition 3.5. *Let the intersection graph be $P(X) = (X, Y)$ of a G^*, let C_1 and D_1 be IVNSs over V and E. Also let C_2 and D_2 be IVNSs over X and Y. Then the interval valued neutrosophic intersection graph (IVNIG) of a IVNG $G = (C_1, D_1)$ is a IVNG $P(G) = (C_2, D_2)$, such that*

$$T_{C_2L}(X_i) = T_{C_1L}(v_i), I_{C_2L}(X_i) = I_{C_1L}(v_i), F_{C_2L}(X_i) = F_{C_1L}(v_i)$$
$$T_{C_2U}(X_i) = T_{C_1U}(v_i), I_{C_2U}(X_i) = I_{C_1U}(v_i), F_{C_2U}(X_i) = F_{C_1U}(v_i)$$
$$T_{D_2L}(X_iX_j) = T_{D_1L}(v_iv_j), I_{D_2L}(X_iX_j) = I_{D_1L}(v_iv_j), F_{D_2L}(X_iX_j) = F_{D_1L}(v_iv_j)$$
$$T_{D_2U}(X_iX_j) = T_{D_1U}(v_iv_j), I_{D_2U}(X_iX_j) = I_{D_1U}(v_iv_j), F_{D_2U}(X_iX_j) = F_{D_1U}(v_iv_j)$$

$\forall\ X_i, X_j \in X$ *and* $X_iX_j \in Y$.

Proposition 3.8. *Let $G = (A_1, B_1)$ be a IVNG of $G^* = (V, E)$ and let $P(G) = (A_2, B_2)$ be a IVNIG, then*
(**1**) *The IVNIG is a IVNG.*
(**2**) *The IVNG is isomorphic to IVNIG.*

Proof. (**1**) By the definition of IVNIG

$$\begin{aligned}
T_{B_2L}(S_iS_j) &= T_{B_1L}(v_iv_j) \leq \min(T_{A_1L}(v_i), T_{A_1L}(v_j)) = \min(T_{A_2L}(S_i), T_{A_2L}(S_j)) \\
I_{B_2L}(S_iS_j) &= I_{B_1L}(v_iv_j) \geq \max(I_{A_1L}(v_i), I_{A_1L}(v_j)) = \max(I_{A_2L}(S_i), I_{A_2L}(S_j)) \\
F_{B_2L}(S_iS_j) &= F_{B_1L}(v_iv_j) \geq \max(F_{A_1L}(v_i), F_{A_1L}(v_j)) = \max(F_{A_2L}(S_i), F_{A_2L}(S_j)) \\
T_{B_2U}(S_iS_j) &= T_{B_1U}(v_iv_j) \leq \min(T_{A_1U}(v_i), T_{A_1U}(v_j)) = \min(T_{A_2U}(S_i), T_{A_2U}(S_j)) \\
I_{B_2U}(S_iS_j) &= I_{B_1U}(v_iv_j) \geq \max(I_{A_1U}(v_i), I_{A_1U}(v_j)) = \max(I_{A_2U}(S_i), I_{A_2U}(S_j)) \\
F_{B_2U}(S_iS_j) &= F_{B_1U}(v_iv_j) \geq \max(F_{A_1U}(v_i), F_{A_1U}(v_j)) = \max(F_{A_2U}(S_i), F_{A_2U}(S_j))
\end{aligned}$$

this shows that IVNIG is a IVNG.
(**2**) Define $f : V \to X$ by $f(v_i) = S_i$ for $i = 1, 2, 3, \ldots, n$ clearly f is bijective. Next $v_iv_j \in E$ if and only if $S_iS_j \in T$ and $T = \{f(v_i)f(v_j) : v_iv_j \in E\}$, also

$$T_{A_2L}(f(v_i)) = T_{A_2L}(S_i) = T_{A_1L}(v_i),\ I_{A_2L}(f(v_i)) = I_{A_2L}(S_i) = I_{A_1L}(v_i)$$
$$F_{A_2L}(f(v_i)) = F_{A_2L}(S_i) = F_{A_1L}(v_i),\ T_{A_2U}(f(v_i)) = T_{A_2U}(S_i) = T_{A_1U}(v_i)$$
$$I_{A_2U}(f(v_i)) = I_{A_2U}(S_i) = I_{A_1U}(v_i),\ F_{A_2U}(f(v_i)) = F_{A_2U}(S_i) = F_{A_1U}(v_i)$$

$\forall\ v_i \in V$.

$$T_{B_2L}(f(v_i)f(v_j)) = T_{B_2L}(S_iS_j) = T_{B_1L}(v_iv_j)$$
$$I_{B_2L}(f(v_i)f(v_j)) = I_{B_2L}(S_iS_j) = I_{B_1L}(v_iv_j)$$
$$F_{B_2L}(f(v_i)f(v_j)) = F_{B_2L}(S_iS_j) = F_{B_1L}(v_iv_j)$$
$$T_{B_2U}(f(v_i)f(v_j)) = T_{B_2U}(S_iS_j) = T_{B_1U}(v_iv_j)$$
$$I_{B_2U}(f(v_i)f(v_j)) = I_{B_2U}(S_iS_j) = I_{B_1U}(v_iv_j)$$
$$F_{B_2U}(f(v_i)f(v_j)) = F_{B_2U}(S_iS_j) = F_{B_1U}(v_iv_j)$$

$\forall\ v_iv_j \in E$. □

Definition 3.6. *Let $G^* = (V, E)$ and $L(G^*) = (X, Y)$ be its line graph, where A_1 and B_1 be IVNSs over V and E. Let A_2 and B_2 be IVNSs over X and Y. The interval valued neutrosophic line graph (IVNLG) of IVNG $G = (A_1, B_1)$ is IVNG $L(G) = (A_2, B_2)$, such that*

$$T_{A_2L}(S_x) = T_{B_1L}(x) = T_{B_1L}(u_xv_x),\ I_{A_2L}(S_x) = I_{B_1L}(x) = I_{B_1L}(u_xv_x)$$
$$F_{A_2L}(S_x) = F_{B_1L}(x) = F_{B_1L}(u_xv_x),\ T_{A_2U}(S_x) = T_{B_1U}(x) = T_{B_1U}(u_xv_x)$$
$$I_{A_2U}(S_x) = I_{B_1U}(x) = I_{B_1U}(u_xv_x),\ F_{A_2U}(S_x) = F_{B_1U}(x) = F_{B_1U}(u_xv_x)$$

$\forall\ S_x, S_y \in X$ *and*

$$T_{B_2L}(S_xS_y) = \min(T_{B_1L}(x), T_{B_1L}(y)),\ I_{B_2L}(S_xS_y) = \max(I_{B_1L}(x), I_{B_1L}(y))$$
$$F_{B_2L}(S_xS_y) = \max(F_{B_1L}(x), F_{B_1L}(y)),\ T_{B_2U}(S_xS_y) = \min(T_{B_1U}(x), T_{B_1U}(y))$$
$$I_{B_2U}(S_xS_y) = \max(I_{B_1U}(x), I_{B_1U}(y)),\ F_{B_2U}(S_xS_y) = \max(F_{B_1U}(x), F_{B_1U}(y))$$

$\forall\ S_xS_y \in Y$.

Example 3.4. *Consider the $G^* = (V, E)$, where $V = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ and $E = \{x_1 = \alpha_1\alpha_2, x_2 = \alpha_2\alpha_3, x_3 = \alpha_3\alpha_4, x_4 = \alpha_4\alpha_1\}$ and $G = (A_1, B_1)$ be a IVNG of $G^* = (V, E)$, which is defined in in Table 6. Consider the $L(G^*) = (X, Y)$, such that $X = \{\Gamma_{x_1}, \Gamma_{x_2}, \Gamma_{x_3}, \Gamma_{x_4}\}$ and $Y = \{\Gamma_{x_1}\Gamma_{x_2}, \Gamma_{x_2}\Gamma_{x_3}, \Gamma_{x_3}\Gamma_{x_4}, \Gamma_{x_4}\Gamma_{x_1}\}$. Let A_2 and B_2 be IVNSs over X and Y. Then by calculations, IVNLG $L(G)$ is defined in Table 7.*

A_1	T_{A_1}	I_{A_1}	F_{A_1}	B_1	T_{B_1}	I_{B_1}	F_{B_1}
α_1	[0.2,0.3]	[0.5,0.6]	[0.5,0.6]	x_1	[0.1,0.2]	[0.6,0.7]	[0.7,0.8]
α_2	[0.4,0.5]	[0.3,0.4]	[0.3,0.4]	x_2	[0.3,0.4]	[0.6,0.7]	[0.7,0.8]
α_3	[0.4,0.5]	[0.5,0.6]	[0.5,0.6]	x_3	[0.2,0.3]	[0.7,0.8]	[0.8,0.9]
α_4	[0.3,0.4]	[0.2,0.3]	[0.2,0.3]	x_4	[0.1,0.2]	[0.7,0.8]	[0.8,0.9]

Table 6: IVNSs of IVNG.

A_2	T_{A_2}	I_{A_2}	F_{A_2}	B_2	T_{B_2}	I_{B_2}	F_{B_2}
Γ_{x_1}	[0.1,0.2]	[0.6,0.7]	[0.7,0.8]	$\Gamma_{x_1}\Gamma_{x_2}$	[0.1,0.2]	[0.6,0.7]	[0.7,0.8]
Γ_{x_2}	[0.3,0.4]	[0.6,0.7]	[0.7,0.8]	$\Gamma_{x_2}\Gamma_{x_3}$	[0.2,0.3]	[0.7,0.8]	[0.8,0.9]
Γ_{x_3}	[0.2,0.3]	[0.7,0.8]	[0.8,0.9]	$\Gamma_{x_3}\Gamma_{x_4}$	[0.1,0.2]	[0.7,0.8]	[0.8,0.9]
Γ_{x_4}	[0.1,0.2]	[0.7,0.8]	[0.8,0.9]	$\Gamma_{x_4}\Gamma_{x_1}$	[0.1,0.2]	[0.7,0.8]	[0.8,0.9]

Table 7: IVNSs of IVNLG.

Proposition 3.9. *Every IVNLG is a strong IVNG.*

Proposition 3.10. *The $L(G) = (A_2, B_2)$ is a IVNLG corresponding to IVNG $G = (A_1, B_1)$.*

Proposition 3.11. *The $L(G) = (A_2, B_2)$ is a IVNLG of some IVNG $G = (A_1, B_1)$ if and only if*

$$T_{B_2L}(S_xS_y) = \min(T_{A_2L}(S_x), T_{A_2L}(S_y)),\ I_{B_2L}(S_xS_y) = \max(I_{A_2L}(S_x), I_{A_2L}(S_y))$$
$$F_{B_2L}(S_xS_y) = \max(F_{A_2L}(S_x), F_{A_2L}(S_y)),\ T_{B_2U}(S_xS_y) = \min(T_{A_2U}(S_x), T_{A_2U}(S_y))$$
$$I_{B_2U}(S_xS_y) = \max(I_{A_2U}(S_x), I_{A_2U}(S_y)),\ F_{B_2U}(S_xS_y) = \max(F_{A_2U}(S_x), F_{A_2U}(S_y))$$

$\forall\ S_xS_y \in Y$.

Proof. Assume that

$$T_{B_2L}(S_xS_y) = \min(T_{A_2L}(S_x), T_{A_2L}(S_y)),\ I_{B_2L}(S_xS_y) = \max(I_{A_2L}(S_x), I_{A_2L}(S_y))$$
$$F_{B_2L}(S_xS_y) = \max(F_{A_2L}(S_x), F_{A_2L}(S_y)),\ T_{B_2U}(S_xS_y) = \min(T_{A_2U}(S_x), T_{A_2U}(S_y))$$
$$I_{B_2U}(S_xS_y) = \max(I_{A_2U}(S_x), I_{A_2U}(S_y)),\ F_{B_2U}(S_xS_y) = \max(F_{A_2U}(S_x), F_{A_2U}(S_y))$$

$\forall\ S_xS_y \in Y$. Next define

$$T_{A_1L}(x) = T_{A_2L}(S_x),\ I_{A_1L}(x) = I_{A_2L}(S_x),\ F_{A_1L}(x) = F_{A_2L}(S_x)$$
$$T_{A_1U}(x) = T_{A_2U}(S_x),\ I_{A_1U}(x) = I_{A_2U}(S_x),\ F_{A_1U}(x) = F_{A_2U}(S_x)$$

$\forall\ x \in E$, then

$$T_{B_2L}(S_xS_y) = \min(T_{A_2L}(S_x), T_{A_2L}(S_y)) = \min(T_{A_2L}(x), T_{A_2L}(y))$$
$$I_{B_2L}(S_xS_y) = \max(I_{A_2L}(S_x), I_{A_2L}(S_y)) = \max(I_{A_2L}(x), I_{A_2L}(y))$$
$$F_{B_2L}(S_xS_y) = \max(F_{A_2L}(S_x), F_{A_2L}(S_y)) = \max(F_{A_2L}(x), F_{A_2L}(y))$$
$$T_{B_2U}(S_xS_y) = \min(T_{A_2U}(S_x), T_{A_2U}(S_y)) = \min(T_{A_2U}(x), T_{A_2U}(y))$$
$$I_{B_2U}(S_xS_y) = \max(I_{A_2U}(S_x), I_{A_2U}(S_y)) = \max(I_{A_2U}(x), I_{A_2U}(y))$$
$$F_{B_2U}(S_xS_y) = \max(F_{A_2U}(S_x), F_{A_2U}(S_y)) = \max(F_{A_2U}(x), F_{A_2U}(y))$$

The IVNS A_1 that yields the property

$$T_{B_1L}(xy) \leq \min(T_{A_1L}(x), T_{A_1L}(y)),\ I_{B_1L}(xy) \geq \max(I_{A_1L}(x), I_{A_1L}(y))$$
$$F_{B_1L}(xy) \geq \max(F_{A_1L}(x), F_{A_1L}(y)),\ T_{B_1U}(xy) \leq \min(T_{A_1U}(x), T_{A_1U}(y))$$
$$I_{B_1U}(xy) \geq \max(I_{A_1U}(x), I_{A_1U}(y)),\ F_{B_1U}(xy) \geq \max(F_{A_1U}(x), F_{A_1U}(y))$$

will suffice. Converse is straight forward. □

Proposition 3.12. *If $L(G) = (A_2, B_2)$ is IVNLG of IVNG $G = (A_1, B_1)$, then $L(G^*)$ is the crisp line graph of G^*.*

Proof. Since $L(G)$ be a IVNLG,

$$T_{A_2L}(S_x) = T_{B_1L}(x),\ I_{A_2L}(S_x) = I_{B_1L}(x),\ F_{A_2L}(S_x) = F_{B_1L}(x)$$

$\forall\ x \in E$ and so $S_x \in X$ if and only if $x \in E$, also

$$T_{B_2L}(S_xS_y) = \min(T_{B_1L}(x), T_{B_1L}(y)),\ I_{B_2L}(S_xS_y) = \max(I_{B_1L}(x), I_{B_1L}(y))$$
$$F_{B_2L}(S_xS_y) = \max(F_{B_1L}(x), F_{B_1L}(y)),\ T_{B_2U}(S_xS_y) = \min(T_{B_1U}(x), T_{B_1U}(y))$$
$$I_{B_2U}(S_xS_y) = \max(I_{B_1U}(x), I_{B_1U}(y)),\ F_{B_2U}(S_xS_y) = \max(F_{B_1U}(x), F_{B_1U}(y))$$

$\forall\ S_xS_y \in Y$ and so, $Y = \{S_xS_y : S_x \cap S_y \neq \phi, x, y \in E, x \neq y\}$. □

Proposition 3.13. *If $L(G) = (A_2, B_2)$ is IVNLG of IVNG $G = (A_1, B_1)$ if and only if $L(G^*) = (X, Y)$ is the line graph and*

$$T_{B_2L}(xy) = \min(T_{A_2L}(x), T_{A_2L}(y)),\ I_{B_2L}(xy) = \max(I_{A_2L}(x), I_{A_2L}(y))$$
$$F_{B_2L}(xy) = \max(F_{A_2L}(x), F_{A_2L}(y)),\ T_{B_2U}(xy) = \min(T_{A_2U}(x), T_{A_2U}(y))$$
$$I_{B_2U}(xy) = \max(I_{A_2U}(x), I_{A_2U}(y)),\ F_{B_2U}(xy) = \max(F_{A_2U}(x), F_{A_2U}(y))$$

$\forall\ xy \in Y$.

Proof. It follows from Propositions 3.11 and 3.12. □

Proposition 3.14. *Let G be a IVNG, then $M(G)$ is isomorphic with $sd(G) \cup L(G)$.*

Theorem 3.1. *Let $L(G) = (A_2, B_2)$ be IVNLG corresponding to IVNG $G = (A_1, B_1)$.*
(**a**) *If G is weak isomorphic onto $L(G)$ if and only if $\forall\ v \in V$, $x \in E$ and G^* to be a cycle, such that*

$$T_{A_1L}(v) = T_{B_1L}(x),\ I_{A_1L}(v) = I_{B_1L}(x),\ F_{A_1L}(v) = T_{B_1L}(x),$$
$$T_{A_1U}(v) = T_{B_1U}(x),\ I_{A_1U}(v) = I_{B_1U}(x),\ F_{A_1U}(v) = T_{B_1U}(x).$$

(**b**) *If G is weak isomorphic onto $L(G)$, then G and $L(G)$ are isomorphic.*

Proof. By hypothesis G^* is a cycle. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E = \{x_1 = v_1v_2, x_2 = v_2v_3, \ldots, x_n = v_nv_1\}$ where $P : v_1v_2v_3 \ldots v_n$ is a cycle, characterize a IVNS A_1 by $A_1(v_i) = ([p_i, p_i^{'}], [q_i, q_i^{'}], [r_i, r_i^{'}])$ and B_1 by $B_1(x_i) = ([a_i, a_i^{'}], [b_i, b_i^{'}], [c_i, c_i^{'}])$ for $i = 1, 2, 3, \ldots, n$ and $v_{n+1} = v_1$, if $p_{n+1} = p_1$, $q_{n+1} = q_1$, $r_{n+1} = r_1$, $p_{n+1}^{'} = p_1^{'}$, $q_{n+1}^{'} = q_1^{'}$, $r_{n+1}^{'} = r_1^{'}$. Thus

$$a_i \leq \min(p_i, p_{i+1}),\ b_i \geq \max(q_i, q_{i+1}),\ c_i \geq \max(r_i, r_{i+1})$$
$$a_i^{'} \leq \min(p_i^{'}, p_{i+1}^{'}),\ b_i^{'} \geq \max(q_i^{'}, q_{i+1}^{'}),\ c_i^{'} \geq \max(r_i^{'}, r_{i+1}^{'})$$

for $i = 1, 2, 3, \ldots, n$. Next $X = \{\Gamma_{x_1}, \Gamma_{x_2}, \ldots, \Gamma_{x_n}\}$ and $Y = \{\Gamma_{x_1}\Gamma_{x_2}, \Gamma_{x_2}\Gamma_{x_3}, \ldots, \Gamma_{x_n}\Gamma_{x_1}\}$, thus for $a_{n+1} = a_1, a_{n+1}^{'} = a_1^{'}, b_{n+1} = b_1, b_{n+1}^{'} = b_1^{'}, c_{n+1} = c_1, c_{n+1}^{'} = c_1^{'}$ to obtain

$$A_2(\Gamma_{x_i}) = B_1(x_i) = ([a_i, a_i^{'}], [b_i, b_i^{'}], [c_i, c_i^{'}])$$

and $B_2(\Gamma_{x_i}\Gamma_{x_{i+1}}) = ([\min(a_i, a_{i+1}), \min(a_i^{'}, a_{i+1}^{'})], [\max(b_i, b_{i+1}), \max(b_i^{'}, b_{i+1}^{'})], [\max(c_i, c_{i+1}), \max(c_i^{'}, c_{i+1}^{'})])$ for $i = 1, 2, 3, \ldots, n$ and $v_{n+1} = v_1$. Since f preserves adjacency, hence it induce permutation π of $\{1, 2, 3, \ldots, n\}$, $f(v_i) = \Gamma_{v_{\pi(i)}v_{\pi(i)+1}}$ and

$$v_iv_{i+1} \rightarrow f(v_i)f(v_{i+1}) = \Gamma_{v_{\pi(i)}v_{\pi(i)+1}}\Gamma_{v_{\pi(i+1)}v_{\pi(i+1)+1}}$$

for $i = 1, 2, 3, \ldots, n-1$. Therefore

$$p_i = T_{A_1L}(v_i) \leq T_{A_2L}(f(v_i)) = T_{A_2L}(\Gamma_{v_{\pi(i)}v_{\pi(i)+1}}) = T_{B_1L}(v_{\pi(i)}v_{\pi(i)+1}) = a_{\pi(i)}$$

Similarly, $p_i \leq a_{\pi(i)}$, $q_i \geq b_{\pi(i)}$, $r_i \geq c_{\pi(i)}$, $q_i^{'} \geq b_{\pi(i)}^{'}$, $r_i^{'} \geq c_{\pi(i)}^{'}$ and

$$\begin{aligned} a_i &= T_{B_1L}(v_iv_{i+1}) \leq T_{B_2L}(f(v_i)f(v_{i+1})) \\ &= T_{B_2L}(\Gamma_{v_{\pi(i)}v_{\pi(i)+1}}\Gamma_{v_{\pi(i+1)}v_{\pi(i+1)+1}}) \\ &= \min(T_{B_1L}(v_{\pi(i)}v_{\pi(i)+1}), T_{B_1L}(v_{\pi(i+1)}v_{\pi(i+1)+1})) \\ &= \min(a_{\pi(i)}, a_{\pi(i)+1}) \end{aligned}$$

similarly $a_i^{'} \leq \min(a_{\pi(i)}^{'}, a_{\pi(i)+1}^{'})$, $b_i \geq \max(b_{\pi(i)}, b_{\pi(i)+1})$, $b_i^{'} \geq \max(b_{\pi(i)}^{'}, b_{\pi(i)+1}^{'})$ and $c_i \geq \max(c_{\pi(i)}, c_{\pi(i)+1})$, $c_i^{'} \geq \max(c_{\pi(i)}^{'}, c_{\pi(i)+1}^{'})$ for $i = 1, 2, 3, \ldots, n$. Therefore

$$p_i \leq a_{\pi(i)}, q_i \geq b_{\pi(i)}, r_i \geq c_{\pi(i)},\ p_i^{'} \leq a_{\pi(i)}^{'}, q_i^{'} \geq b_{\pi(i)}^{'}, r_i^{'} \geq c_{\pi(i)}^{'}$$

and

$$a_i \leq \min(a_{\pi(i)}, a_{\pi(i)+1}),\ b_i \geq \max(b_{\pi(i)}, b_{\pi(i)+1}),\ c_i \geq \max(c_{\pi(i)}, c_{\pi(i)+1})$$
$$a_i^{'} \leq \min(a_{\pi(i)}^{'}, a_{\pi(i)+1}^{'}),\ b_i^{'} \geq \max(b_{\pi(i)}^{'}, b_{\pi(i)+1}^{'}),\ c_i^{'} \geq \max(c_{\pi(i)}^{'}, c_{\pi(i)+1}^{'})$$

thus

$$a_i \leq a_{\pi(i)}, b_i \geq b_{\pi(i)}, c_i \geq c_{\pi(i)},\ a_i^{'} \leq a_{\pi(i)}, b_i^{'} \geq b_{\pi(i)}^{'}, c_i^{'} \geq c_{\pi(i)}^{'}$$

and so

$$a_{\pi(i)} \leq a_{\pi(\pi(i))}, b_{\pi(i)} \geq b_{\pi(\pi(i))}, c_{\pi(i)} \geq c_{\pi(\pi(i))}$$
$$a_{\pi(i)}^{'} \leq a_{\pi(\pi(i))}^{'}, b_{\pi(i)}^{'} \geq b_{\pi(\pi(i))}^{'}, c_{\pi(i)}^{'} \geq c_{\pi(\pi(i))}^{'}$$

$\forall\ i = 1, 2, 3, \ldots, n$. Next to extend

$$a_i \leq a_{\pi(i)} \leq \ldots \leq a_{\pi^j(i)} \leq a_i,\ b_i \geq b_{\pi(i)} \geq \ldots \geq b_{\pi^j(i)} \geq b_i$$

$$c_i \geq c_{\pi(i)} \geq \ldots \geq c_{\pi^j(i)} \geq c_i,\ a_i^{'} \leq a_{\pi(i)}^{'} \leq \ldots \leq a_{\pi^j(i)}^{'} \leq a_i^{'}$$
$$b_i^{'} \geq b_{\pi(i)}^{'} \geq \ldots \geq b_{\pi^j(i)}^{'} \geq b_i^{'},\ c_i^{'} \geq c_{\pi(i)}^{'} \geq \ldots \geq c_{\pi^j(i)}^{'} \geq c_i^{'}$$

where π^{j+1} identity. Hence

$$a_i = a_{\pi(i)}, b_i = b_{\pi(i)}, c_i = c_{\pi(i)},\ a_i^{'} = a_{\pi(i)}^{'}, b_i^{'} = b_{\pi(i)}^{'}, c_i^{'} = c_{\pi(i)}^{'}$$

$\forall\ i = 1, 2, 3, \ldots, n$. Therefore

$$a_i \leq a_{\pi(i+1)} = a_{i+1}, b_i \geq b_{\pi(i+1)} = b_{i+1}, c_i \geq c_{\pi(i+1)} = c_{i+1}$$
$$a_i^{'} \leq a_{\pi(i+1)}^{'} = a_{i+1}^{'}, b_i^{'} \geq b_{\pi(i+1)}^{'} = b_{i+1}^{'}, c_i^{'} \geq c_{\pi(i+1)}^{'} = c_{i+1}^{'}$$

which together with

$$a_{n+1} = a_1, b_{n+1} = b_1, c_{n+1} = c_1,\ a_{n+1}^{'} = a_1^{'}, b_{n+1}^{'} = b_1^{'}, c_{n+1}^{'} = c_1^{'}$$

which implies that

$$a_i = a_1, b_i = b_1, c_i = c_1,\ a_i^{'} = a_1^{'}, b_i^{'} = b_1^{'}, c_i^{'} = c_1^{'}$$

$\forall\ i = 1, 2, 3, \ldots, n$. Thus

$$a_1 = a_2 = \ldots = a_n = p_1 = p_2 = \ldots = p_n$$
$$a_1^{'} = a_2^{'} = \ldots = a_n^{'} = p_1^{'} = p_2^{'} = \ldots = p_n^{'}$$
$$b_1 = b_2 = \ldots = b_n = q_1 = q_2 = \ldots = q_n$$
$$b_1^{'} = b_2^{'} = \ldots = b_n^{'} = q_1^{'} = q_2^{'} = \ldots = q_n^{'}$$
$$c_1 = c_2 = \ldots = c_n = r_1 = r_2 = \ldots = r_n$$
$$c_1^{'} = c_2^{'} = \ldots = c_n^{'} = r_1^{'} = r_2^{'} = \ldots = r_n^{'}$$

Therefore (a) and (b) holds, since converse of result (a) is straight forward. □

4. Conclusion

The neutrosophic graphs have many applications in path problems, networks and computer science. Strong IVNG and complete IVNG are the types of IVNG. In this paper, we discussed the special types of IVNGs, subdivision IVNGs, middle IVNGs, total IVNGs and IVNLGs of the given IVNGs. We investigated isomorphism properties of subdivision IVNGs, middle IVNGs, total IVNGs and IVNLGs.

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