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STRONG SUBSYSTEMS OF INTERVAL NEUTROSOPHIC AUTOMATA

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ABSTRACT. In this paper, we introduce the strong subsystems in interval neutrosophic automaton. Further, we show that every strong subsystem of interval neutrosophic automaton is subsystem but the converse need not be true.

1. INTRODUCTION

The theory of neutrosophy and neutrosophic set was introduced by Florentin Smarandache in 1999 [2]. A neutrosophic set N is classified by a Truth membership function T_N, Indeterminacy membership function I_N, and Falsity membership function F_N, where $T_N, I_N,$ andF_N are real standard and non-standard subsets of $]0^-, 1^+[$. Wang et al., [3] introduced the notion of interval-valued neutrosophic sets. The concept of interval neutrosophic finite state machine was introduced by Tahir Mahmood [1]. In this paper, we introduced the concept of strong subsystem of interval neutrosophic automaton.

We establish necessary and sufficient condition for N_Q to be strong subsystem of an interval neutrosophic automaton. Further, we have shown that every strong subsystem of interval neutrosophic automaton is subsystem but the converse need not be true.

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2. PRELIMINARIES

Definition 2.1. *[2] Let U be the universe of discourse. A neutrosophic set (NS) N in U is $N = \{\langle x, (T_N(x), I_N(x), F_N(x))\rangle, x \in U,\ T_A, I_A, F_A \in]0^-, 1^+[\ \}$ and with the condition $0^- \leq sup\, T_N(x) + sup\, I_N(x) + sup\, F_N(x) \leq 3^+$. We need to take the interval $[0,1]$ for technical applications instead of $]0^-, 1^+[$.*

Definition 2.2. *[3] Let U be a universal set. An interval neutrosophic set (INS for short) is of the form*

$$N = \{\langle \alpha_N(x), \beta_N(x), \gamma_N(x)\rangle \,|x \in U\} = \{\langle x, [\inf\ \alpha_N(x), \sup\ \alpha_N(x)], [\inf\ \beta_N(x), \sup\ \beta_N(x)], [\inf\ \gamma_N(x), \sup \gamma_N(x)]\rangle \,|x \in U\},$$

where $\alpha_N(x),\ \beta_N(x)$, and $\gamma_N(x) \subseteq [0,1]$ and the condition that

$$0 \leq \sup\ \alpha_N(x) +\ \sup\ \beta_N(x) +\ \sup\ \gamma_N(x) \leq\ 3.$$

3. INTERVAL NEUTROSOPHIC AUTOMATA

Definition 3.1. *[1] $M = (Q,\ \Sigma,\ N)$ is called interval neutrosophic automaton $(INAforshort)$, where Q and Σ are non-empty finite sets called the set of states and input symbols respectively, and $N = \{\langle \alpha_N(x), \beta_N(x), \gamma_N(x)\rangle\}$ is an INS in $Q \times \Sigma \times Q$. The set of all words of finite length of Σ is denoted by Σ^*. The empty word is denoted by ϵ, and the length of each $x \in \Sigma^*$ is denoted by $|x|$.*

Definition 3.2. *[1] $M = (Q,\ \Sigma,\ N)$ be an INA. Define an INS $N^* = \{\langle \alpha_{N^*}(x), \beta_{N^*}(x), \gamma_{N^*}(x)\rangle\}$ in $Q \times \Sigma^* \times Q$ by*

$$\alpha_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [1,1] & \textit{if } q_i = q_j \\ [0,0] & \textit{if } q_i \neq q_j \end{cases}$$

$$\beta_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [0,0] & \textit{if } q_i = q_j \\ [1,1] & \textit{if } q_i \neq q_j \end{cases}$$

$$\gamma_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [0,0] & \textit{if } q_i = q_j \\ [1,1] & \textit{if } q_i \neq q_j \end{cases}$$

$$\alpha_{N^*}(q_i, w, q_j) = \alpha_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\alpha_{N^*}(q_i, x, q_r) \cup \alpha_{N^*}(q_r, y, q_j)],$$

$$\beta_{N^*}(q_i, w, q_j) = \beta_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\beta_{N^*}(q_i, x, q_r) \cup \beta_{N^*}(q_r, y, q_j)],$$

$$\gamma_{N^*}(q_i, w, q_j) = \gamma_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\gamma_{N^*}(q_i, x, q_r) \cup \gamma_{N^*}(q_r, y, q_j)],$$

$\forall q_i, q_j \in Q$,$w = xy, x \in \Sigma^*$ *and* $y \in \Sigma$.

4. Strong Subsystems of Interval Neutrosophic Automata

Definition 4.1. *Let* $M = (Q, \Sigma, N)$ *be an interval neutrosophic automaton. Let*

$$N_Q = \{\langle \alpha_{N_Q}(q_i),\ \beta_{N_Q}(q_i),\ \gamma_{N_Q}(q_i)\rangle\} = \{\langle q_i, [\inf \alpha_{N_Q}(q_i), \sup \alpha_{N_Q}(q_i)], [\inf \beta_{N_Q}(q_i), \sup \beta_{N_Q}(q_i)], [\inf \gamma_{N_Q}(q_i), \sup \gamma_{N_Q}(q_i)]\rangle\}$$

$q_i \in Q$. *Then* (Q, N_Q, Σ, N) *is called a subsystem of* M *if* $\forall q_i, q_j \in Q$ *and* $x \in \Sigma$ *such that* $\alpha_{N_Q}(q_j) \geq \vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$, $\beta_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \beta_N(q_i, x, q_j)\}$ *and* $\gamma_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \gamma_N(q_i, x, q_j)\}$.

(Q, N_Q, Σ, N) *is a subsystem of* M, *then we write* N_Q *for* (Q, N_Q, Σ, N).

Definition 4.2. *Let* $M = (Q, \Sigma, N)$ *be an interval neutrosophic automaton. Let* $N_Q = \{\langle \alpha_{N_Q}(q_i),\ \beta_{N_Q}(q_i),\ \gamma_{N_Q}(q_i)\rangle\}$. *Then* N_Q *is called a strong subsystem of* M *if and only if* $\forall q_i, q_j \in Q$, *if* $\exists\ x \in \Sigma$ *such that* $\alpha_{N_Q}(q_j) \geq \vee_{q_i \in Q}\{\alpha_{N_Q}(q_i)\}$, $\beta_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\beta_{N_Q}(q_i)\}$, *and* $\gamma_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i)\}$.

Theorem 4.1. *Let* $M = (Q,\ \Sigma,\ N)$ *be an interval neutrosophic automaton. Let* $N_Q = \{\langle \alpha_{N_Q}(q_i),\ \beta_{N_Q}(q_i),\ \gamma_{N_Q}(q_i)\rangle\}$. *Then* N_Q *is a strong subsystem of* M *if and only if* $\forall q_i, q_j \in Q$, *if* $\exists\ x \in \Sigma^*$ *such that* $\alpha_{N^*}(q_i, x, q_j) > [0, 0]$, $\beta_{N^*}(q_i, x, q_j) < [1, 1]$,*and* $\gamma_{N^*}(q_i, x, q_j) < [1, 1]$, *then* $\alpha_{N_Q}(q_j) \geq \alpha_{N_Q}(q_i)$, $\beta_{N_Q}(q_j) \leq \beta_{N_Q}(q_i)$, *and* $\gamma_{N_Q}(q_j) \leq \gamma_{N_Q}(q_i)$.

Proof. Suppose N_Q is a strong subsystem. We prove the result by induction on $|x| = n$. If $n = 0$, then $x = \epsilon$. Now if $q_i = q_j$, then $\alpha_{N^*}(q_i, \epsilon, q_j) = [1, 1]$, $\beta_{N^*}(q_i, \epsilon, q_j) = [0, 0]$, and $\gamma_{N^*}(q_i, \epsilon, q_j) = [0, 0]$.

Therefore, $\alpha_{N_Q}(q_i) = \alpha_{N_Q}(q_j)$, $\beta_{N_Q}(q_i) = \beta_{N_Q}(q_j)$, and $\gamma_{N_Q}(q_i) = \gamma_{N_Q}(q_j)$.

Suppose $q_i \neq q_j$, then $\alpha_{N^*}(q_i, \epsilon, q_j) = [0, 0]$, $\beta_{N^*}(q_i, \epsilon, q_j) = [1, 1]$, $\gamma_{N^*}(q_i, \epsilon, q_j) = [1, 1]$. Thus the result is true if $n = 0$.

Suppose the result is true $\forall y \in \Sigma^*$ such that $|y| = n - 1, n > 0$. Let $x = ya$, $|y| = n - 1, y \in \Sigma^*, a \in \Sigma$. Suppose $\alpha_{N^*}(q_i, x, q_j) > [0, 0]$, $\beta_{N^*}(q_i, x, q_j) < [1, 1]$ and $\gamma_{N^*}(q_i, x, q_j) < [1, 1]$. Then

$\alpha_{N^*}(q_i, x, q_j) = \alpha_{N^*}(q_i, ya, q_j) > [0, 0]$,
$\vee_{q_k \in Q} \{\alpha_{N^*}(q_i, y, q_k) \wedge \alpha_{N^*}(q_k, a, q_j)\} > [0, 0]$.
$\beta_{N^*}(q_i, x, q_j) = \beta_{N^*}(q_i, ya, q_j) < [1, 1]$,
$\wedge_{q_k \in Q} \{\beta_{N^*}(q_i, y, q_k) \vee \beta_{N^*}(q_k, a, q_j)\} < [1, 1]$
and
$\gamma_{N^*}(q_i, x, q_j) = \gamma_{N^*}(q_i, ya, q_j) < [1, 1]$,
$\wedge_{q_k \in Q} \{\gamma_{N^*}(q_i, y, q_k) \vee \gamma_{N^*}(q_k, a, q_j)\} < [1, 1]$.

Thus $\exists q_k \in Q$ s.t. $\alpha_{N^*}(q_i, y, q_k) > [0, 0]$, $\alpha_{N^*}(q_k, a, q_j) > [0, 0]$, $\beta_{N^*}(q_i, y, q_k) < [1, 1]$, and $\beta_{N^*}(q_k, a, q_j) < [1, 1]$, $\gamma_{N^*}(q_i, y, q_k) < [1, 1]$, and $\gamma_{N^*}(q_k, a, q_j) < [1, 1]$.

Hence, $\alpha_{N_Q}(q_j) \geq \alpha_{N_Q}(q_k)$, and $\alpha_{N_Q}(q_k) \geq \alpha_{N_Q}(q_i)$, $\beta_{N_Q}(q_j) \leq \beta_{N_Q}(q_k)$, and $\beta_{N_Q}(q_k) \leq \beta_{N_Q}(q_i)$, and $\gamma_{N_Q}(q_j) \leq \gamma_{N_Q}(q_k)$, and $\gamma_{N_Q}(q_k) \leq \gamma_{N_Q}(q_i)$.

Thus, $\alpha_{N_Q}(q_j) \geq \alpha_{N_Q}(q_i)$, $\beta_{N_Q}(q_j) \leq \beta_{N_Q}(q_i)$, and $\gamma_{N_Q}(q_j) \leq \gamma_{N_Q}(q_i)$.

The converse is obvious. □

Theorem 4.2. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. Let $N_Q = \{\langle \alpha_{N_Q}, \beta_{N_Q}, \gamma_{N_Q} \rangle\}$ be an interval neutrosophic subset in Q. If N_Q is a strong subsystem of M, then N_Q is a subsystem of M. The converse is need not be true.*

Proof. Let N_Q be a strong subsystem. Then $\forall\ q_i, q_j \in Q$, if $\exists\ a \in \Sigma$ such that $\alpha_{N_Q}(q_i, a, q_j) > [0, 0]$, $\beta_{N_Q}(q_i, a, q_j) < [1, 1]$, $\gamma_{N_Q}(q_i, a, q_j) < [1, 1]$. Further,

$$\alpha_{N_Q}(q_j) \geq \alpha_{N_Q}(q_i), \quad \beta_{N_Q}(q_j) \leq \beta_{N_Q}(q_i), \quad \text{and } \gamma_{N_Q}(q_j) \leq \gamma_{N_Q}(q_i). \tag{4.1}$$

Now, $\alpha_{N_Q}(q_j) \geq \alpha_{N_Q}(q_i) \wedge \alpha_{N_Q}(q_i, a, q_j)$, $\beta_{N_Q}(q_j) \leq \beta_{N_Q}(q_i) \vee \beta_{N_Q}(q_i, a, q_j)$, and $\gamma_{N_Q}(q_j) \leq \gamma_{N_Q}(q_i) \vee \gamma_{N_Q}(q_i, a, q_j)$, by (4.1). Hence, N_Q is a subsystem of M. □

Theorem 4.3. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automata. Let N_{Q_1} and N_{Q_2} be strong subsystems of M. Then the following conditions hold.*

(i) *$N_{Q_1} \wedge N_{Q_2}$ is a strong subsystem of M.*
(ii) *$N_{Q_1} \vee N_{Q_2}$ is a strong subsystem of M.*

Proof.

(i) Since N_{Q_1} and N_{Q_2} are strong subsystem of an interval neutrosophic automata M. Then $\forall\ q_i, q_j \in Q$ and $x \in \Sigma$ such that
$\alpha_{N_{Q_1}}(q_j) \geq \vee_{q_i \in Q}\{\alpha_{N_{Q_1}}(q_i)\}$,

$\beta_{N_{Q_1}}(q_j) \leq \wedge_{q_i \in Q}\{\beta_{N_{Q_1}}(q_i)\}$,
$\gamma_{N_{Q_1}}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{N_{Q_1}}(q_i)\}$,
$\alpha_{N_{Q_2}}(q_j) \geq \vee_{q_i \in Q}\{\alpha_{N_{Q_2}}(q_i)\}$,
$\beta_{N_{Q_2}}(q_j) \leq \wedge_{q_i \in Q}\{\beta_{N_{Q_2}}(q_i)\}$,
$\gamma_{N_{Q_2}}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{N_{Q_2}}(q_i)\}$.

Now we have to prove $N_{Q_1} \wedge N_{Q_2}$ is a subsystem of M. It is enough to prove
$(\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_j) \geq \vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_i)\}$,
$(\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_j) \leq \wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_i)\}$,
$(\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_j) \leq \wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_i)\}$.

Now,
$(\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_j) = (\alpha_{N_{Q_1}}(q_j) \wedge \alpha_{N_{Q_2}}(q_j))$
$\geq \{\vee_{q_i \in Q}\{(\alpha_{N_{Q_1}}(q_i)\}\} \wedge \{\vee_{q_i \in Q}\{\alpha_{N_{Q_2}}(q_i))\}\}$
[Since N_{Q_1} and N_{Q_2} are strong subsystem]
$= \vee_{q_i \in Q}\{\alpha_{N_{Q_1}}(q_i) \wedge \alpha_{N_{Q_2}}(q_i)\}$
$= \vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_i)\}$.

Thus,

$$\begin{aligned} (\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_j) &\geq \vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_i)\}, \\ (\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_j) &= (\beta_{N_{Q_1}}(q_j) \wedge \beta_{N_{Q_2}}(q_j)) \\ &\leq \{\wedge_{q_i \in Q}\{\beta_{N_{Q_1}}(q_i)\}\} \wedge \{\wedge_{q_i \in Q}\{\beta_{N_{Q_2}}(q_i)\}\} \\ &= \wedge_{q_i \in Q}\{\beta_{N_{Q_1}}(q_i) \wedge \beta_{N_{Q_2}}(q_i)\} \\ &= \wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_i)\}. \end{aligned} \tag{4.2}$$

Thus,

$$\begin{aligned} (\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_j) &\leq \wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_i)\}, \\ (\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_j) &= (\gamma_{N_{Q_1}}(q_j) \wedge \gamma_{N_{Q_2}}(q_j)) \\ &\leq \{\wedge_{q_i \in Q}\{\gamma_{N_{Q_1}}(q_i)\}\} \wedge \{\wedge_{q_i \in Q}\{\gamma_{N_{Q_2}}(q_i)\}\} \\ &= \wedge_{q_i \in Q}\{\gamma_{N_{Q_1}}(q_i) \wedge \gamma_{N_{Q_2}}(q_i)\} \\ &= \wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_i)\}. \end{aligned} \tag{4.3}$$

Thus,

$$(\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_j) \leq \wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_i)\}. \tag{4.4}$$

From (4.2), (4.3) and (4.4), $N_{Q_1} \wedge N_{Q_2}$ is a strong subsystem of an interval neutrosophic automaton M.

(ii) Now we have to prove $N_{Q_1} \vee N_{Q_2}$ is a strong subsystem of interval neutrosophic automaton M. It is enough to prove

$(\alpha_{N_{Q_1}} \vee \alpha_{N_{Q_2}})(q_j) \geq \vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \vee \alpha_{N_{Q_2}})(q_i)\}$,
$(\beta_{N_{Q_1}} \vee \beta_{N_{Q_2}})(q_j) \leq \wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \vee \beta_{N_{Q_2}})(q_i)\}$,
$(\gamma_{N_{Q_1}} \vee \gamma_{N_{Q_2}})(q_j) \leq \wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \vee \gamma_{N_{Q_2}})(q_i)\}$.

Now,

$(\alpha_{N_{Q_1}} \vee \alpha_{N_{Q_2}})(q_j) = (\alpha_{N_{Q_1}}(q_j) \vee \alpha_{N_{Q_2}}(q_j))$
$\geq \{\vee_{q_i \in Q}\{(\alpha_{N_{Q_1}}(q_i)\}\} \vee \{\vee_{q_i \in Q}\{\alpha_{N_{Q_2}}(q_i))\}\}$

[Since N_{Q_1} and N_{Q_2} are strong subsystem]

$= \vee_{q_i \in Q}\{\alpha_{N_{Q_1}}(q_i) \vee \alpha_{N_{Q_2}}(q_i)\}$
$= \vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \vee \alpha_{N_{Q_2}})(q_i)\}$.

Thus,

$$\begin{aligned} (\alpha_{N_{Q_1}} \vee \alpha_{N_{Q_2}})(q_j) &\geq \vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \vee \alpha_{N_{Q_2}})(q_i)\}, \\ (\beta_{N_{Q_1}} \vee \beta_{N_{Q_2}})(q_j) &= (\beta_{N_{Q_1}}(q_j) \vee \beta_{N_{Q_2}}(q_j)) \\ &\leq \{\wedge_{q_i \in Q}\{\beta_{N_{Q_1}}(q_i)\}\} \vee \{\wedge_{q_i \in Q}\{\beta_{N_{Q_2}}(q_i)\}\} \\ &= \wedge_{q_i \in Q}\{\beta_{N_{Q_1}}(q_i) \vee \beta_{N_{Q_2}}(q_i)\} \\ &= \wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \vee \beta_{N_{Q_2}})(q_i)\}. \end{aligned} \tag{4.5}$$

Thus,

$$\begin{aligned} (\beta_{N_{Q_1}} \vee \beta_{N_{Q_2}})(q_j) &\leq \wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \vee \beta_{N_{Q_2}})(q_i)\}, \\ (\gamma_{N_{Q_1}} \vee \gamma_{N_{Q_2}})(q_j) &= (\gamma_{N_{Q_1}}(q_j) \vee \gamma_{N_{Q_2}}(q_j)) \\ &\leq \{\wedge_{q_i \in Q}\{\gamma_{N_{Q_1}}(q_i)\}\} \vee \{\wedge_{q_i \in Q}\{\gamma_{N_{Q_2}}(q_i)\}\} \\ &= \wedge_{q_i \in Q}\{\gamma_{N_{Q_1}}(q_i) \vee \gamma_{N_{Q_2}}(q_i)\} \\ &= \wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \vee \gamma_{N_{Q_2}})(q_i)\}. \end{aligned} \tag{4.6}$$

Thus,

$$(\gamma_{N_{Q_1}} \vee \gamma_{N_{Q_2}})(q_j) \leq \wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \vee \gamma_{N_{Q_2}})(q_i)\}. \tag{4.7}$$

From (4.5), (4.6), and (4.7), $N_{Q_1} \vee N_{Q_2}$ is a strong subsystem.

□

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