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Study on neutrosophic graph with application in wireless network

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Abstract: A neutrosophic network is an extension of an intuitionistic fuzzy network that provides more precision compatibility and flexibility than an intuitionistic fuzzy graph in structuring and modelling many real-life problems. The authors have explored the use of a neutrosophic network for modelling the passive optical network, mobile *ad hoc* network (MANET), and wireless sensor graph. They have presented the idea of strong arc, weak arc strong domination numbers, and strong perfect domination of neutrosophic network. They have described the method to find the values of strong and strong perfect domination of neutrosophic network. Finally, they use the idea of a strong arc strong domination number in MANET and wireless sensor graphs.

1 Introduction

Graph theories [1–3] have several real-life applications in the area of computer science, transportation, physics, systems analysis, biology, economics, astronomy, and operations research. In those problems, the graph emerges as a mathematical graphical model of the observed real-life problems. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a graph, where $\mathcal{V}$ represents the non-empty set of all nodes/vertices and $\mathcal{E}$ represents the set of all arcs/edges of the graph $\mathcal{G}$. Two nodes k and l in a graph $\mathcal{G}$ are defined as an adjacent node in $\mathcal{G}$ if and only if (k, l) is an arc of $\mathcal{G}$. The graph $\mathcal{G}$ is a simple graph if $\mathcal{G}$ has no multiple arcs and loops. A simple graph $\mathcal{G}$ is said to be a complete graph if and only if every pair of distinct nodes of $\mathcal{G}$ is joined by an arc. If the complete graph $\mathcal{G}$ has n number of nodes then it has always $n(n-1)/2$ arcs. In this study, we have worked on different types of neutrosophic networks. Many decision-making problems, e.g. traffic planning, texture mapping, shortest path problem, spanning tree problem etc. can be modelled as a graph, which consists of objects/items/elements and relationships. Experts use the nodes of the graph to represent the objects/items/elements and edges to represent the relationship between objects of the problem. In almost every decision-making problem, the usable information is generally approximate or imprecise due to many distinct reasons such as lack of evidence, imperfect statistical information, loss of information, inaccurate data, and insufficient information.

The basic idea of the crisp set theory has been introduced by Cantor, which is used in classical graph theory. In a graph, for any node or arc, there exist two distinct possibilities: the first one either present in the graph or is not present in the graph. Owing to this reason, the simple graph theory is unable to model uncertain real-life problems properly. Fuzzy set [4] is an upper version of a crisp set, where each and every item/elements have varying membership grade. It can show that its elements have distinct membership grades between intervals 1 and 0. The idea of membership degrees is different from probability.

The idea of a fuzzy graph has been presented by Kaufmann [5] using the fuzzy relation. In [6], Rosenfeld has described the various ideas of fuzzy bridges, fuzzy cycles, fuzzy paths, fuzzy trees, and fuzzy connectedness to a fuzzy graph and presented some of the properties of them. Several mathematicians, such as Rashmanlou and Pal [7], Samanta and Pal [8], Ghorai and Pal [9], Paramik *et al.* [10], Rashmanlou *et al.* [11, 12], Nandhini and Nandhini [13], and Borzooei *et al.* [14] have researched a lot in the area of the fuzzy graph and its applications in real-life problems. Atanassov [15–17] has presented the new type of fuzzy set, i.e. intuitionistic fuzzy set, as a modification of type 1 fuzzy set. The type 1 fuzzy sets have only a single membership grade; however, the intuitionistic fuzzy set always considers two independent membership grades: membership grade and non-membership grade for each and every element. Shannon and Atanassov [18] have for the first time described the concept of intuitionistic fuzzy set relationship and intuitionistic fuzzy graphs. They have presented several proprieties, theorems, and proved in [18]. For further study on intuitionistic fuzzy graphs, please refer to [19–25]. However, the fuzzy graph and intuitionistic fuzzy graph both are employed to represent many real-life problems, but uncertainty due to the inconsistent information and indeterminate information of any real-life decision-making problem cannot be handled precisely by a fuzzy graph or intuitionistic fuzzy graph. Therefore, an expert requires other new concepts to handle these scenarios.

In [26], Smarandache has described the neutrosophic set, by extending the idea of a fuzzy set. It can manage with indeterminate, vague, uncertain, and inconsistent data of any real-world problem. The neutrosophic set is mainly an extension of the classical set, fuzzy set, and intuitionistic fuzzy set. A neutrosophic set [27–34] has three membership grades: truth, indeterminate, and false of each and every element. Those three membership grades are always independent and lie between the interval]0, 1[. In [27], the authors introduced a modified score function to find the rank of the single-valued neutrosophic set as well as the interval-valued neutrosophic set. They have also described a decision-making method based on the proposed function. Garg and Nancy [28] introduced a non-linear programming model to find the solutions to a decision-making problem in which parameters are represented by interval neutrosophic numbers. Garg and Nancy [29] describe some new aggregating operators for neutrosophic information. They have used those operators to find the solution to the multi-criteria decision-making problems. Garg and Nancy [28] introduced some hybrid aggregation operators using arithmetic aggregation operators and geometric aggregation operators. They have also presented a decision-making approach to solve the multi-criteria decision-making problem.

The single-valued neutrosophic sets are applied to graph theory and present a new type of graphical structure which is defined as a single-valued neutrosophic graph. The idea of neutrosophic graph theory (NGT) is based on neutrosophic relationships. The NGT can be applied to model the relationships between several objects/individuals in real-life problems. A decision-maker may use the NGT in numerous real-life applications in diverse areas such as modern engineering and sciences, database theory, image processing, data mining, artificial neural networks, cluster analysis, expert systems, and control systems. It is an extension of crisp graph theory, fuzzy graph theory, and intuitionistic fuzzy graph theory. The idea of NGT is a more efficient graphical representing method for dealing with inconsistency, indeterminacy, and uncertainties in the real-life information compared to fuzzy graph and intuitionistic fuzzy graph. Neutrosophic graph [9, 35] can efficiently represent the real-life problem. Several researchers have recently researched neutrosophic network theory, for instance, Yang *et al.* [36], Borzooei *et al.* [37], Naz *et al.* [38], Ye [39], Wang [40], Arkam *et al.* [41], Arkam [42], Prasertpong and Siripitukdet [43], Akram and Siddique [44], and Akrama [45].

The domination number is useful information to analyse any graph. Borzooei *et al.* [46] described the new ideas of dominating sets for vague graphs and described the idea of strong vague domination numbers for vague graphs. The neutrosophic graph is more precise and flexible to model the decision-making problem when it compares with the vague graph model. Thus, the importance of considering neutrosophic networks are inevitable to model the many decision-making problems [39, 44, 47–52] in real-world scenarios and it is required to introduce the idea of strong arc, weak arc strong domination numbers, and strong perfect domination of the neutrosophic network. In this study, we have described several ideas of the neutrosophic network. Then, we have introduced the idea of strong arc, weak arc strong domination numbers, and strong perfect domination of the neutrosophic network. We have explored the use of a neutrosophic network for modelling the passive mobile *ad hoc* graph [mobile *ad hoc* network (MANET)] and wireless sensor graph. We have described the method to find the values of strong and strong perfect domination of the neutrosophic network.

2 Preliminaries

Definition 1: Let U be a universal set. The single-valued neutrosophic set D on the universal U is denoted as follows:

$$A = \{\langle x{:}T_{\mathcal{L}}(k), I_{\mathcal{L}}(k), F_{\mathcal{L}}(k) | k \in U \rangle\} \tag{1}$$

The functions $T_{\mathcal{L}}(k) \in [0, 1]$, $I_A(k) \in [0, 1]$, and $F_A(k) \in [0, 1]$ are named as the degree of truth, indeterminacy, and falsity membership of k in A, and satisfy the following condition:

$$^{-}0 \le \sup T_{\mathcal{L}}(k) + \sup I_{\mathcal{L}}(k) + \sup F_{\mathcal{L}}(k) \le 3^{+} \tag{2}$$

Definition 2: A neutrosophic network $\mathcal{G} = (\mathcal{K}, \mathcal{L})$ is defined as a strong neutrosophic network if

$$\begin{aligned} T_{\mathcal{L}}(k, l) &= T_{\mathcal{K}}(k) \wedge T_{\mathcal{K}}(l), \\ I_{\mathcal{L}}(k, l) &= I_{\mathcal{K}}(k) \wedge I_{\mathcal{K}}(l), \\ F_{\mathcal{L}}(k, l) &= F_{\mathcal{K}}(k) \vee F_{\mathcal{K}}(l) \quad \forall k, l \in E \end{aligned} \tag{3}$$

Definition 3: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network. An edge kl in $\mathcal{G}$ is called an effective edge if

$$\begin{aligned} \mathcal{T}_{\mathcal{L}}(kl) &= \mathcal{F}_{\mathcal{K}}(k) \wedge \mathcal{F}_{\mathcal{K}}(l) \\ \mathcal{T}_{\mathcal{L}}(kl) &= \mathcal{F}_{\mathcal{K}}(k) \wedge \mathcal{F}_{\mathcal{K}}(l) \\ \mathcal{F}_{\mathcal{L}}(kl) &= \mathcal{F}_{\mathcal{K}}(k) \vee \mathcal{F}_{\mathcal{K}}(l) \end{aligned} \tag{4}$$

Definition 4: A neutrosophic network $\mathcal{G}$ is called a strong neutrosophic network if

$$\begin{aligned} \mathcal{T}_{\mathcal{L}}(kl) &= \mathcal{F}_{\mathcal{K}}(k) \wedge \mathcal{F}_{\mathcal{K}}(l) \\ \mathcal{T}_{\mathcal{L}}(kl) &= \mathcal{F}_{\mathcal{K}}(k) \wedge \mathcal{F}_{\mathcal{K}}(l) \\ \mathcal{F}_{\mathcal{L}}(kl) &= \mathcal{F}_{\mathcal{K}}(k) \vee \mathcal{F}_{\mathcal{K}}(l) \quad \forall kl \in E \end{aligned} \tag{5}$$

A neutrosophic network is a strong neutrosophic network if each and every edge has effective edges.

Definition 5: A neutrosophic number $\mathcal{G}$ is called a complete neutrosophic network if for every $k, l \in V$,

$$\begin{aligned} \mathcal{T}_{\mathcal{L}}(kl) &= \mathcal{F}_{\mathcal{K}}(k) \wedge \mathcal{F}_{\mathcal{K}}(l) \\ \mathcal{T}_{\mathcal{L}}(kl) &= \mathcal{F}_{\mathcal{K}}(k) \wedge \mathcal{F}_{\mathcal{K}}(l) \\ \mathcal{F}_{\mathcal{L}}(kl) &= \mathcal{F}_{\mathcal{K}}(k) \vee \mathcal{F}_{\mathcal{K}}(l) \end{aligned} \tag{6}$$

A complete neutrosophic network with n vertices is represented by K_n.

3 Strong and weak neutrosophic edges

Definition 6: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network.

(i) A neutrosophic path $\tilde{p}$ in $\mathcal{G}$ is a collection of different vertices $y_0, y_1, y_2, \ldots, y_{l-1}, y_l$ such that

$$\left(\mathcal{T}_{\mathcal{L}}\left(y_{i-1}y_i\right), \mathcal{I}_{\mathcal{L}}\left(y_{i-1}y_i\right), \mathcal{F}_{\mathcal{L}}\left(y_{i-1}y_i\right)\right) > 0, \quad i = 1, \ldots, l. \tag{7}$$

Here l describes the length of the path $\tilde{p}$. The two nodes k and l are connected by a path with length l such as $\tilde{p}{:}x = x_0, x_1, x_2, \ldots, x_{l-1}, x_k = y$ then $\mathcal{T}^k_{\mathcal{L}}(kl)$, $\mathcal{I}^k_{\mathcal{L}}(kl)$, and $\mathcal{F}^k_{\mathcal{L}}(kl)$ are determined by

$$\begin{aligned} &\mathcal{T}^l_{\mathcal{L}}(kl), \mathcal{I}^l_{\mathcal{L}}(kl), \mathcal{F}^l_{\mathcal{L}}(kl) \\ &\quad = \sup\left\{\mathcal{T}_{\mathcal{L}}\left(x, x_1\right) \wedge \mathcal{T}_{\mathcal{L}}\left(x_1, x_2\right) \wedge \ldots \wedge \mathcal{T}_{\mathcal{L}}\left(x_{l-1}, y\right)\right\}, \\ &\quad \inf\left\{\mathcal{I}_{\mathcal{L}}\left(x, x_1\right) \vee \mathcal{I}_{\mathcal{L}}\left(x_1, x_2\right) \vee \ldots \vee \mathcal{I}_{\mathcal{L}}\left(x_{l-1}, y\right)\right\}, \\ &\quad \inf\left\{\mathcal{F}_{\mathcal{L}}\left(x, x_1\right) \vee \mathcal{F}_{\mathcal{L}}\left(x_1, x_2\right) \vee \ldots \vee \mathcal{F}_{\mathcal{L}}\left(x_{l-1}, y\right)\right\} \end{aligned} \tag{8}$$

The strength of neutrosophic connectedness between any two neutrosophic nodes (k and l) within a neutrosophic network $\mathcal{G}$ is defined as follows:

$$\begin{aligned} &\left(\mathcal{T}^{\infty}_{\mathcal{L}}(kl), \mathcal{I}^{\infty}_{\mathcal{L}}(kl), \mathcal{F}^{\infty}_{\mathcal{L}}(kl)\right) \\ &\quad = \sup_{l \in N}\left\{\mathcal{T}^l_{\mathcal{L}}(kl)\right\}, \ \inf_{l \in N}\left\{\mathcal{F}^l_{\mathcal{L}}(kl)\right\}, \ \inf_{l \in N}\left\{\mathcal{F}^l_{\mathcal{L}}(kl)\right\} \end{aligned} \tag{9}$$

Definition 7: An edge kl in $\mathcal{G}$ is called a strong edge if and only if $\mathcal{T}_{\mathcal{L}}(kl) \ge \mathcal{T}^{\infty}_{\mathcal{L}}(kl)$, $\mathcal{I}_{\mathcal{L}}(kl) \le \mathcal{I}^{\infty}_{\mathcal{L}}(kl)$, and $\mathcal{F}_{\mathcal{L}}(kl) \le \mathcal{F}^{\infty}_{\mathcal{L}}(kl)$. For any k, $l \in V$, if and only if there exists any strong edge between the nodes k and l then the node k is dominated by the node l in $\mathcal{G}$.

Definition 8: Let NDS be a neutrosophic subset of the vertices V in $\mathcal{G}$. NDS is said to be a neutrosophic dominating set if, for all $y \in V \backslash NDS$, there exist some neutrosophic nodes $x \in$ NDS such that node k dominates node y. A neutrosophic dominating set NDS in $\mathcal{G}$ is called a minimal neutrosophic dominating set if there exists no proper subset of NDS in a neutrosophic dominating set.

4 Strong (neighbourhood) domination number

In this section, the neutrosophic strong and neutrosophic strong neighbourhood domination numbers are defined for the neutrosophic network. We present the ideas of neutrosophic strong size and neutrosophic strong order of a neutrosophic network.

Definition 9: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network and node $k \in V$. Then the neutrosophic node $y \in V$ is said to be a neutrosophic strong neighbour node of neutrosophic node x, if and only if neutrosophic edge kl is a strong edge. The set of all neutrosophic strong neighbour node of k is represented by $N_s(k)$. The neutrosophic closed strong neighbourhood vertex of k is described as $N_s[x] = N_s(k) \cup \{x\}$.

Definition 10: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network and the node $k \in V$. The neutrosophic strong degree of a node l is defined as follows:

$$d_s(l) = \sum_{k \in N_s(l)} \mathcal{T}_{\mathcal{L}}(kl), \sum_{k \in N_s(l)} \mathcal{I}_{\mathcal{L}}(kl), \sum_{k \in N_s(l)} \mathcal{F}_{\mathcal{L}}(kl) \quad (10)$$

Definition 11: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network and the node $k \in V$. The neutrosophic strong neighbourhood degree of a neutrosophic node l is defined as follows:

$$d_{sN}(l) = \left(\sum_{k \in N_s(l)} \mathcal{T}_{\mathcal{K}}(x), \sum_{k \in N_s(l)} \mathcal{T}_{\mathcal{K}}(x), \sum_{k \in N_s(l)} \mathcal{F}_{\mathcal{K}}(x) \right) \quad (11)$$

Definition 12: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network and the node $k \in V$. The neutrosophic strong degree cardinality of node l is defined as follows:

$$|d_s(l)| = \sum_{k \in N_s(l)} \frac{\mathcal{T}_{\mathcal{L}}(kl) + \mathcal{I}_{\mathcal{L}}(kl) + (3 - \mathcal{F}_{\mathcal{L}}(kl))}{3} \quad (12)$$

Definition 13: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network and the node $k \in V$. The neutrosophic strong neighbourhood degree cardinality of node l is defined as follows:

$$|d_{sN}(l)| = \sum_{k \in N_s(l)} \frac{\mathcal{T}_{\mathcal{K}}(k) + \mathcal{I}_{\mathcal{K}}(k) + (3 - \mathcal{F}_{\mathcal{K}}(k))}{3} \quad (13)$$

Definition 14: The maximum and minimum neutrosophic strong degree of $\mathcal{G} = (\mathcal{K}, \mathcal{L})$ are determined as follows:

$$\begin{aligned} \Delta_s(\mathcal{G}) &= \vee\{|d_s(l)| \| \forall y \in V\} \\ \delta_s(\mathcal{G}) &= \wedge\{|d_s(l)| \| \forall y \in V\}. \end{aligned} \quad (14)$$

The minimum and maximum strong neighbourhood degree of $\mathcal{G} = (\mathcal{K}, \mathcal{L})$ are determined as follows:

$$\begin{aligned} \Delta_{Ns}(\mathcal{G}) &= \vee\{|d_{Ns}(l)| \| \forall y \in V\} \\ \delta_{Ns}(\mathcal{G}) &= \wedge\{|d_{Ns}(l)| \| \forall y \in V\} \end{aligned} \quad (15)$$

Remark 1: Let $\mathcal{G} = (\mathcal{K}, \mathcal{L})$ be a vague graph. If t_A and f_A are constant functions, then $\delta_s(\mathcal{G}) = \delta_{sN}(\mathcal{G})$ and $\Delta_s(\mathcal{G}) = \Delta_{sN}(\mathcal{G})$.

Definition 15: The neutrosophic strong size of the neutrosophic network $\mathcal{G}$ is defined as follows:

$$S_s(\mathcal{G}) = \left\{ \sum_{kl \in E} \frac{\mathcal{T}_{\mathcal{L}}(kl) + \mathcal{I}_{\mathcal{L}}(kl) + (1 - \mathcal{F}_{\mathcal{L}}(kl))}{3} \right\} \quad (16)$$

Here kl is a strong arc. The neutrosophic strong order of the neutrosophic network $\mathcal{G}$ is defined as follows:

$$O_s(\mathcal{G}) = \left\{ \sum_{kl \in E} \frac{\mathcal{T}_{\mathcal{K}}(l) + \mathcal{I}_{\mathcal{K}}(kl) + (1 - \mathcal{F}_{\mathcal{K}}(k))}{3} \right\} \quad (17)$$

Here, l is an end vertex of a strong edge.

Definition 16: Let D be a neutrosophic dominating set in a neutrosophic network $\mathcal{G}$. The node weight and arc weight of D are defined as follows:

$$\begin{aligned} W_x(D) = \sum_{k \in D, y \in N_s(k)} &\wedge\{\mathcal{T}_{\mathcal{K}}(k)\} + \vee\{\mathcal{I}_{\mathcal{K}}(k)\} \\ &+ \left(3 - \vee\left\{\frac{\mathcal{F}_{\mathcal{K}}(k)}{3}\right\}\right) \end{aligned} \quad (18)$$

$$\begin{aligned} W_e(D) = \sum_{k \in D, y \in N_s(k)} &\wedge\{\mathcal{T}_{\mathcal{L}}(kl)\} + \vee\{\mathcal{I}_{\mathcal{L}}(kl)\} \\ &+ \left(3 - \vee\left\{\frac{\mathcal{F}_{\mathcal{L}}(kl)}{3}\right\}\right) \end{aligned} \quad (19)$$

The neutrosophic strong domination number of a neutrosophic network $\mathcal{G}$ is defined as the minimum neutrosophic arc cost in $\mathcal{G} = (\mathcal{K}, \mathcal{L})$ and it is denoted by $\gamma_s(\mathcal{G})$.

The neutrosophic strong neighbourhood domination number of $\mathcal{G}$ is defined as minimum neutrosophic node cost in $\mathcal{G} = (\mathcal{K}, \mathcal{L})$ and it is denoted by $\gamma_{sN}(\mathcal{G})$.

Definition 17: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network. The graph $\mathcal{G}$ is a complete bipartite neutrosophic network if the vertex set V can be partitioned into two non-empty sets p_1 and p_2 such that $(\mathcal{T}_{\mathcal{L}}(p_1p_2), \mathcal{I}_{\mathcal{L}}(p_1p_2), \mathcal{F}_{\mathcal{L}}(p_1p_2)) = (0, 1)$ for $p_1, p_2 \in p_1$ or $p_1, p_2 \in p_2$.
Moreover

$$\begin{aligned} &(\mathcal{T}_{\mathcal{L}}(kl), \mathcal{I}_{\mathcal{L}}(kl), \mathcal{F}_{\mathcal{L}}(kl)) \\ &\quad = (\mathcal{T}_{\mathcal{K}}(k) \wedge \mathcal{T}_{\mathcal{K}}(l), \mathcal{I}_{\mathcal{K}}(k) \wedge \mathcal{I}_{\mathcal{K}}(l), \mathcal{F}_{\mathcal{K}}(k) \vee \mathcal{F}_{\mathcal{K}}(l)) \end{aligned} \quad (20)$$

for $k \in y_1$ and $y \in V2$.

5 Strong perfect domination number

In this section, the idea of neutrosophic perfect dominating set and neutrosophic strong perfect domination number of a neutrosophic network are introduced. Then we show that under some specific criteria, the neutrosophic strong domination number and the neutrosophic strong perfect domination number in a neutrosophic network $\mathcal{G} = (\mathcal{K}, \mathcal{L})$ are always equalled. Finally, we find a maximum bound for strong perfect domination numbers in neutrosophic networks.

Definition 18: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network. A neutrosophic subset D of vertex set V is said to be perfect neutrosophic dominating set $(\text{or } D^p)$ in $\mathcal{G} = (\mathcal{K}, \mathcal{L})$, if all the nodes of $y \in V \backslash D$, there is one and only node $k \in D$ such that node k dominates node y. The D^p is called the minimal perfect neutrosophic dominating set if for each $y \in D^p$, $D^p \backslash \{v\}$ is not a perfect dominating set in $\mathcal{G}$.

Definition 19: The strong perfect domination number of a vague graph $\mathcal{G}$ is defined as the minimum arc weight of perfect dominating sets of $\mathcal{G} = (\mathcal{K}, \mathcal{L})$, which is denoted by $\gamma_{sp}(\mathcal{G})$.

6 Strong semi-global domination number

The idea of a semi-complementary neutrosophic network and semi-neutrosophic dominating set in a neutrosophic network $\mathcal{G}$ are presented in this section.

Definition 20: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network, then semi-complementary neutrosophic network of $\mathcal{G} = (\mathcal{K}, \mathcal{L})$, i.e. $\mathcal{G}^{\mathrm{SC}} = (\mathcal{K}^{\mathrm{SC}}, \mathcal{L}^{\mathrm{SC}})$, is described by

(i) $\mathcal{T}_{\mathcal{K}}^{\mathrm{SC}}(l) = \mathcal{T}_{\mathcal{K}}(l)$, $\mathcal{I}_{\mathcal{K}}^{\mathrm{SC}}(l) = \mathcal{I}_{\mathcal{K}}(l)$, and $\mathcal{F}_{\mathcal{K}}^{\mathrm{SC}}(l) = \mathcal{F}_{\mathcal{K}}(l)$.
(ii) $E^{\mathrm{SC}} = \{uv \notin E, \exists w, uw, wv \in E\}$ where for any $kl \in E^{\mathrm{SC}}$, $\mathcal{T}_{\mathcal{L}}^{\mathrm{SC}}(kl) = \mathcal{T}_{\mathcal{K}}(k) \wedge \mathcal{T}_{\mathcal{K}}(l)$, $I^{\mathrm{SC}}(kl) = \mathcal{I}_{\mathcal{K}}(k) \vee \mathcal{I}_{\mathcal{K}}(l)$, and $F^{\mathrm{SC}}(kl) = \mathcal{F}_{\mathcal{K}}(k) \vee \mathcal{F}_{\mathcal{K}}(l)$.

Definition 21: Let $\mathcal{G}(\mathcal{K}, \mathcal{L})$ be a neutrosophic network.

(i) The neutrosophic subset D is defined as a semi-neutrosophic dominating set in a neutrosophic network $\mathcal{G} = (\mathcal{K}, \mathcal{L})$ if and only if the neutrosophic subset D is a neutrosophic dominating set for both neutrosophic network $(\mathcal{G} = (\mathcal{K}, \mathcal{L}))$ and semi-complementary neutrosophic network $\mathcal{G}^{\mathrm{SC}}$.
(ii) A semi-neutrosophic dominating set is defined as minimal semi-neutrosophic dominating set if and only if $\forall y \in D^S$, $D^S \backslash \{v\}$ is not a global dominating set in $\mathcal{G}$.

Definition 22: The strong semi-neutrosophic domination number of a neutrosophic network $\mathcal{G}$ is described as the minimum edge cost of semi-neutrosophic dominating set in $\mathcal{G}$.

7 Application of the neutrosophic network in the passive optical network (PON)

The PON is a fibre optic telecommunication graph that uses the single point to multipoint technology and optical splitter to send data between a single source point to multiple end-users. A PON system mainly consists of a single optical line terminal (OLT), which is placed in the main branch of the service provider's (hub) and some optical network terminals and optical network units (ONUs), near to end-user points. A PON system can reduce the cost of the main branch equipment and fibre needed compared with a point to point technology. It is a special type of fibre optic access telecommunication graph. PON is generally used as a tree structure. A tree is nothing but a connected circuit less graph. A service provider company wants to provide a PON to a city. We can represent the city map using a neutrosophic network. The roads are used as the communication links (arc) and the road junctions together with the ONU and OLTs are considered as the nodes of the neutrosophic network. The decision maker considers the weights of the communication links based on the length of the corresponding roads. In the PON system, the OLT is located in the main branch and it is used as a root of the tree. The ONU is considered as the leaves of the tree and they are placed near the end-users. The OLT and ONUs both are considered as passive elements of the fibres and the optical splitters are spread. In the PON, user access points are linked to the ONUs via classical engineering such as coax lines or copper. An example of a PON is shown in Fig. 1. Most of the project cost has to be used for digging the line ducts to install a new PON in a city. The decision maker has to find the shortest paths, which connect all the ONUs with the single OLT in the PON. It helps to find minimise the project cost. From the neutrosophic network representation, this neutrosophic optimisation problem can be converted into the neutrosophic minimum Steiner tree problem. The decision-maker wants to determine a neutrosophic tree of the neutrosophic network, which consists of a subset of neutrosophic vertices whose sum of the weights of the selected arcs becomes minimal.

8 Some applications of strong domination numbers

A neutrosophic set is generalised of Atanassov's intuitionistic fuzzy set which consists of three membership grades: truth membership, indeterminate membership, and false membership. The neutrosophic network is an extension of a vague graph and intuitionistic fuzzy graph which provides more flexibility, precision, and compatibility to design the real-life problem when

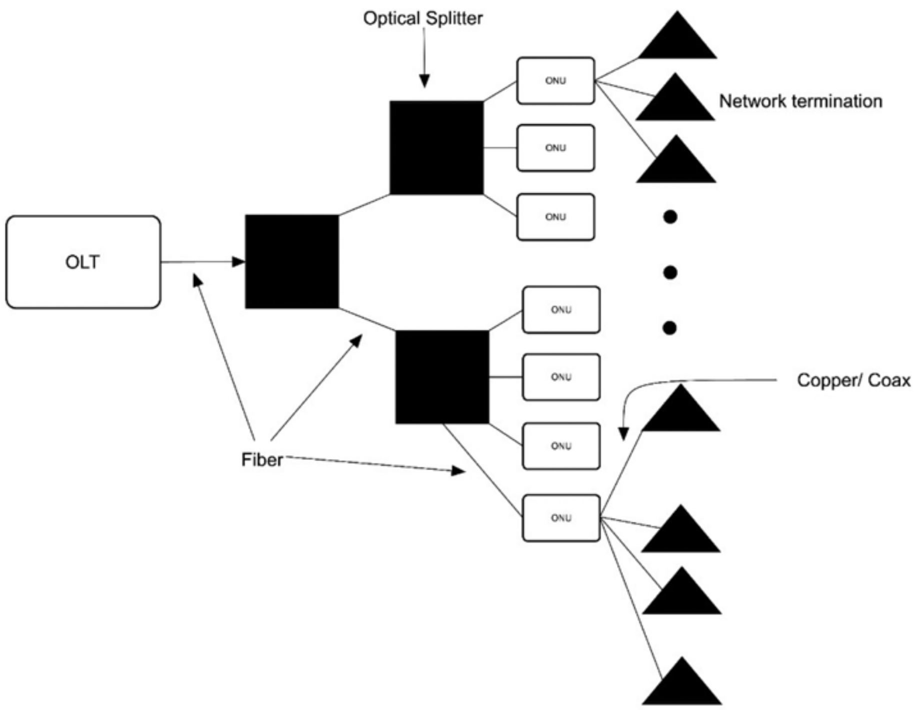

Fig. 1 *Neutrosophic network to model the PONs*

compared with the intuitionistic fuzzy graphs. Neutrosophic graph models are recently using to model many real-life problems in several different areas of engineering and science. In this study, we introduce the idea of a dominating set in neutrosophic network theory. The domination in the neutrosophic network can be used to solve much real-life problem. Here, in society, as well as in administration, the influence of the individual depends on the strength that he derives from his supporters, and these effects may be not effective. Besides, the individual has to depend more on his supporters than on himself.

Now, we express an application of dominating set. An office consists of seven employees and elections are being held to determine the new head. We show that a few employees can select a person $\mathcal{G} = (\mathcal{K}, \mathcal{L})$ (who does not have considerable influence on all employees) as the head by using a domination set of a vague graph. First, we represent the office with a vague graph G as in Fig. 2.

In this vague graph, the nodes and arcs represent employees and friendships between them, respectively. True membership function for each node is considered as the significance of the node in the office, including level of education, work experience etc., and false membership function for each node is evaluated as lack of compatibility between educational major and occupation, lack of ability, and other cases. In this example, we see that *ac*, *cd*, *bd*, *de*, *cf*, and *cg* are strong arcs, and there is a strong relationship between them. Hence, $D_1 = \{c, d\}$, $D_2 = \{c, b, d\}$, and $D_3 = \{c, d, e\}$ are dominating sets in this vague graph and weights are

$$W_\mathrm{e}(D_1) = \left(\frac{0}{2} + \frac{0}{5}\right) + \left(\frac{0}{5} + \frac{0}{6}\right) = \frac{1}{8}$$

$$W_\mathrm{e}(D_2) = \left(\frac{0}{2} + \frac{0}{5}\right) + \left(\frac{0}{5} + \frac{0}{6}\right) + \left(\frac{0}{5} + \frac{0}{8}\right) = \frac{3}{1}$$

$$W_\mathrm{e}(D_3) = \left(\frac{0}{2} + \frac{0}{5}\right) + \left(\frac{0}{5} + \frac{0}{6}\right) + \left(\frac{0}{5} + \frac{0}{6}\right) = \frac{2}{9}$$

Also, so D_1 is a minimal domination set in this example. Since the nodes in dominating set have the most influence on the other members who are not in D_1, therefore, by this influence, they can select one of their own as $\mathcal{G} = (\mathcal{K}, \mathcal{L})$ or anyone else as head of office. (However, we see that the crisp graph, which is made up of employees and relationships between them, $D = \{c, e\}$ is a minimal domination set that has less influence on the other members as compared to D_1)

In the following, we have some more applications of strong domination numbers in everyday life.

8.1 *Mobile* ad hoc *graph (MANET)*

A mobile *ad hoc* graph (MANET) is a decentralise mobile graph, which is continuously self-organising, self-configuring, and infrastructure less graph used to communicate over wireless channel. In this MANET, each mobile device can freely move in any direction and the links between other mobile devices will change frequently. Each device in a MANET must send the traffic to their own use. The main challenge in modelling a MANET is to equip each mobile device to provide the exact information needed to handle the route traffic properly. We can represent the MANET as a neutrosophic network where the vertices represent the mobile device and edges describe pattern of messaging among the mobile devices. The idea of neutrosophic network can handle the uncertainty of MANET. We can use the dominating set to find the routing table in MANET. The small dominating set is considered as a backbone for MANET. If a device is not present in the dominating set then the device transmits the messages using the neighbour devices in the set. The strong edges are only considered in the dominating set of a neutrosophic network and if any device is not present in dominating set of graph (neutrosophic network) then it has always a strong neighbour in the dominating set. We can transmit message faster to its neighbour in the set. The strong domination number describes the smallest number of mobile device in dominating set of the neutrosophic network. The transmission speed and routing of message can be improved using dominating set, and it can help to minimise the construction cost of the MANET.

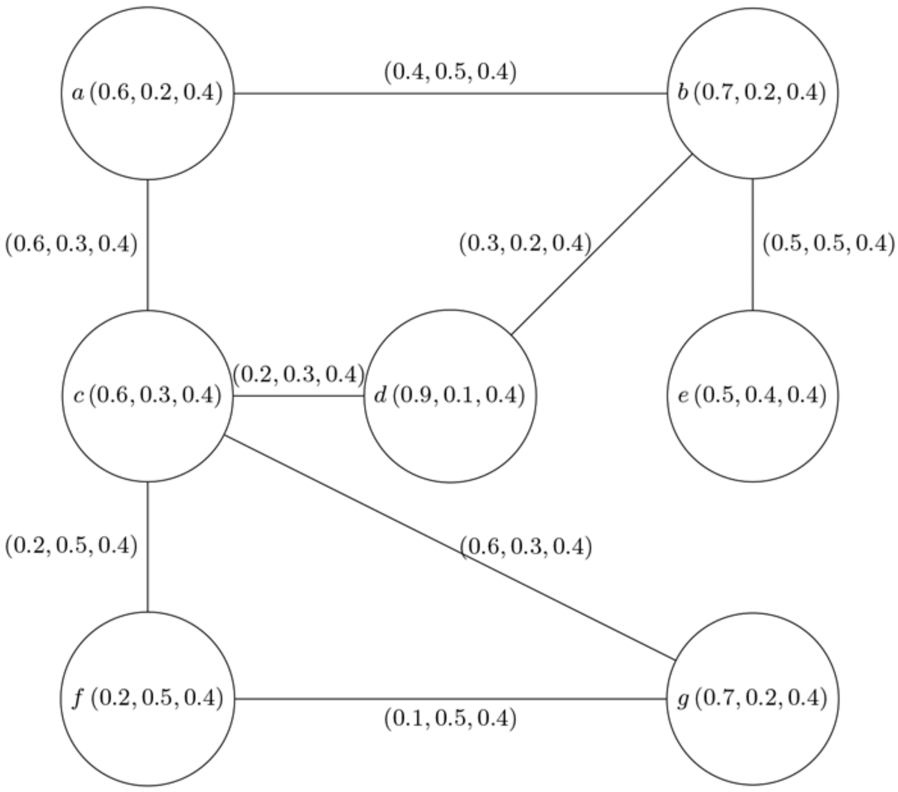

Fig. 2 *Application of neutrosophic network to model the networks*

8.2 Computer communication graphs

The idea of neutrosophic dominating set plays an important role in computer and communication graphs to route the information between the nodes. Neutrosophic graph can be used to model the computer graph where the each node represents the computer and two nodes are connected by a directed arc if there exists any direct communication line between the two computers. Information is collected from the computer and each computer can transmit the information to other computers, which have direct communication line with it. In this problem, we need to find a small set of computers in the graph, which directly communicate with all other computers. We can solve this problem using the idea of minimum neutrosophic dominating set of the corresponding neutrosophic network. The information may be collected by making each processor to route its information to one of the small sets of collecting processor, which is constructed by a dominating set of the corresponding graph.

8.3 Wireless sensor graph

A wireless sensor graph [wireless sensor network (WSN)] is an infrastructure less and self-configure wireless graph. It is collection of transducers, which can monitor and record the conditions of different locations. It generally monitors the parameters such as pressure, temperature, humidity, pollutant levels, wind speed, power line voltage, wind direction, chemical concentrations, sound intensity, and illumination intensity. The WSN is mainly constructed of sensor nodes, which are generally spread in the graph. The sensor nodes gather the information from other sensor nodes and graphs. The topology control is the most basic problem in WSN. We can model the WSN as a neutrosophic network since the place of each node is indefinite and can convey every piece of information, even destructive, to other nodes and, therefore induce intervention in the graph. As a result, we can define true (regarding level of importance, necessity, effect, pace of conveyance etc.) and false (regarding the degree of intervention, vagueness etc.) membership functions and also valuates each arc considering importance and necessity of conveyance of information etc. Accordingly, since each dominating set in a vague graph is gained using strong arcs, we can, in so doing, make the smallest and the most effective minimal backbone set by gaining minimal dominating set (by using minimum arc weight). Virtual backbone is necessary for fault tolerance and routing flexibility.

9 Conclusion

In this paper, we have explored the use of neutrosophic network for modelling the PON, mobile *ad hoc* graph (MANET), and wireless sensor graph. We have presented the idea of strong arc, weak arc strong domination numbers, and strong perfect domination of neutrosophic network. We have described the method to find the values of strong and strong perfect domination of neutrosophic network. Finally, we use the idea of strong arc strong domination number in MANET and wireless sensor graph.

10 References

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