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Symmetry Measures of Simplified Neutrosophic Sets for Multiple Attribute Decision-Making Problems

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Abstract: A simplified neutrosophic set (containing interval and single-valued neutrosophic sets) can be used for the expression and application in indeterminate decision-making problems because three elements in the simplified neutrosophic set (including interval and single valued neutrosophic sets) are characterized by its truth, falsity, and indeterminacy degrees. Under a simplified neutrosophic environment, therefore, this paper firstly defines simplified neutrosophic asymmetry measures. Then we propose a normalized symmetry measure and a weighted symmetry measure of simplified neutrosophic sets and develop a simplified neutrosophic multiple attribute decision-making method based on the weighted symmetry measure. All alternatives can be ranked through the weighted symmetry measure between the ideal solution/alternative and each alternative, and then the best one can be determined. Finally, an illustrative example on the selection of manufacturing schemes (alternatives) in the flexible manufacturing system demonstrates the applicability of the proposed method in a simplified (interval and single valued) neutrosophic setting, and then the decision-making method based on the proposed symmetry measure is in accord with the ranking order and best choice of existing projection and bidirectional projection-based decision-making methods and strengthens the resolution/discrimination in the decision-making process corresponding to the comparative example.

Keywords: asymmetry measure; symmetry measure; simplified neutrosophic set; decision making

1. Introduction

To represent inconsistent and indeterminate information in the real world, Smarandache [1] introduced the neutrosophic set (NS) concept as the extension of the fuzzy set and (interval-valued) intuitionistic fuzzy sets. Because three function values of truth-membership, falsity-membership, and indeterminacy-membership in NS are defined in the real standard interval [0, 1] or nonstandard interval $]^{-}0, 1^{+}[$, the nonstandard interval shows its difficult application in the real world. As the subclass of NS, Ye [2] presented a simplified neutrosophic set (SNS), where its indeterminacy-membership, truth-membership, and falsity-membership functions are in the real standard interval [0, 1] to conveniently apply in engineering fields. SNS includes an interval neutrosophic set (INS) [3] and a single-valued neutrosophic set (SVNS) [4]. After that, Ye [5] introduced three simplified neutrosophic similarity measures in vector space and their multicriteria decision-making methods. Then, outranking approaches [6,7] were used for simplified neutrosophic and interval neutrosophic decision-making problems. Some researchers proposed correlation coefficients, cross entropy measures, similarity measures for INSs/ SVNSs/SNSs, and their multiple attribute decision-making (MADM) methods [8–12]. Some researchers presented various aggregation operators of SVNSs/INSs/SNSs for decision-making fields [13–19]. Furthermore, projection and bidirectional projection measures of INSs and SVNSs [20,21] were introduced for their decision-making. TOPSIS method [22] was presented for decision-making with SVNS information, also SVNS graphs [23] were used for decision-making problems. Then, some decision making methods were presented based on the neutrosophic MULTIMOORA, WASPAS-SVNS, and extended TOPSIS and VIKOR methods [24–26] under SNS environments.

As mentioned above, the measure method of SNSs is an important tool in decision-making. To develop new measures in simplified neutrosophic decision-making problems, this study proposes asymmetry measures of SNSs and their normalized symmetry measure of SNSs for the first time, and then develops a MADM method by using the weighted symmetry measure of SNSs. Therefore, this paper is presented as the following frame. Some definitions of asymmetry measures of SNSs are presented in Section 2. The normalized symmetry measure and weighted symmetry measure of SNSs are proposed in Section 3. A MADM method using the weighted symmetry measure is developed in Section 4. In Section 5, the practical example is provided in simplified neutrosophic environments to show the application, along with the sensitive analysis regarding to the attribute weight values, and then the feasibility and effectiveness are indicated by the comparative example. At last, Section 6 indicates conclusions and future work.

2. Asymmetry Measures of Simplified Neutrosophic Sets

In this section, asymmetry measures of SNSs are presented, including asymmetry measures of SVNSs and INSs.

Ye [2] presented an SNS as a subclass of a NS [1] and gave the following definition.

Definition 1 [2]. *A SNS is defined as* $A = \{\langle x, u_A(x), v_A(x), h_A(x)\rangle | x \in U\}$ *in the universe of discourse U, such that* $u_A(x)$: $U \to [0, 1]$, $v_A(x)$: $U \to [0,1]$, *and* $h_A(x) : U \to [0, 1]$, *which are described by the truth, indeterminacy and falsity-membership degrees, satisfying* $0 \le \sup u_A(x) + \sup v_A(x) + \sup h_A(x) \le 3$ *for INS or* $0 \le u_A(x) + v_A(x) + h_A(x) \le 3$ *for SVNS and* $x \in U$.

For convenience, an element in the SNS A is denoted by $a = (u_a, v_a, h_a)$, which is called the simplified neutrosophic number (SNN), including a single valued neutrosophic number (SVNN) and an interval neutrosophic number (INN).

First, asymmetry measures of SVNSs are defined in the following.

Definition 2. *Let* $B = \{b_1, b_2, \ldots, b_n\}$ *and* $A = \{a_1, a_2, \ldots, a_n\}$ *be two SVNSs, where* $b_j = (u_{bj}, v_{bj}, h_{bj})$ *and* $a_j = (u_{aj}, v_{aj}, h_{aj})$ *are the j-th SVNNs* ($j = 1, 2, \ldots, n$) *of B and A respectively. Then*

$$P_B(A) = \frac{A \cdot B}{\|B\|^2} = \frac{\sum_{j=1}^{n}(u_{aj}u_{bj} + v_{aj}v_{bj} + h_{aj}h_{bj})}{\sum_{j=1}^{n}(u_{bj}^2 + v_{bj}^2 + h_{bj}^2)} \quad (1)$$

$$P_A(B) = \frac{A \cdot B}{\|A\|^2} = \frac{\sum_{j=1}^{n}(u_{aj}u_{bj} + v_{aj}v_{bj} + h_{aj}h_{bj})}{\sum_{j=1}^{n}(u_{aj}^2 + v_{aj}^2 + h_{aj}^2)} \quad (2)$$

are called asymmetry measures of B and A.

If one considers the weight of each element b_j or a_j ($j = 1, 2, \ldots, n$), the weighted asymmetry measure of SNSs can be introduced below.

Definition 3. *Let B = {b_1, b_2, ..., b_n} and A = {a_1, a_2, ..., a_n} be two SVNSs, where $b_j = (u_{bj}, v_{bj}, h_{bj})$ and $a_j = (u_{aj}, v_{aj}, h_{aj})$ are the j-th SVNNs (j = 1,2, ..., n) of B and A respectively, and let the weight of an element b_j or a_j be w_j, $w_j \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$. Then*

$$Pw_B(A) = \frac{\sum_{j=1}^{n} w_j^2 (u_{aj}u_{bj} + v_{aj}v_{bj} + h_{aj}h_{bj})}{\sum_{j=1}^{n} w_j^2 (u_{bj}^2 + v_{bj}^2 + h_{bj}^2)} \tag{3}$$

$$Pw_A(B) = \frac{\sum_{j=1}^{n} w_j^2 (u_{aj}u_{bj} + v_{aj}v_{bj} + h_{aj}h_{bj})}{\sum_{j=1}^{n} w_j^2 (u_{aj}^2 + v_{aj}^2 + h_{aj}^2)} \tag{4}$$

are called the weighted asymmetry measures of B and A.

By the similar way, the two asymmetry measures of SVNSs can be further extended to the asymmetry measures of INSs, which are given by the following definition.

Definition 4. *Let B = {b_1, b_2, ..., b_n} and A = {a_1, a_2, ..., a_n} be two INSs, where $b_j = ([u_{bj}^L, u_{bj}^U], [v_{bj}^L, v_{bj}^U], [h_{bj}^L, h_{bj}^U])$ and $a_j = ([u_{aj}^L, u_{aj}^U], [v_{aj}^L, v_{aj}^U], [h_{aj}^L, h_{aj}^U])$ are the j-th INNs (j = 1, 2, ..., n) of B and A respectively. Then, two asymmetry measures of B and A are defined as*

$$P_B(A) = \frac{A \cdot B}{\|B\|^2} = \frac{\sum_{j=1}^{n} (u_{aj}^L u_{bj}^L + u_{aj}^U u_{bj}^U + v_{aj}^L v_{bj}^L + v_{aj}^U v_{bj}^U + h_{aj}^L h_{bj}^L + h_{aj}^U h_{bj}^U)}{\sum_{j=1}^{n} [(u_{bj}^L)^2 + (u_{bj}^U)^2 + (v_{bj}^L)^2 + (v_{bj}^U)^2 + (h_{bj}^L)^2 + (h_{bj}^U)^2]} \tag{5}$$

$$P_A(B) = \frac{A \cdot B}{\|A\|^2} = \frac{\sum_{j=1}^{n} (u_{aj}^L u_{bj}^L + u_{aj}^U u_{bj}^U + v_{aj}^L v_{bj}^L + v_{aj}^U v_{bj}^U + h_{aj}^L h_{bj}^L + h_{aj}^U h_{bj}^U)}{\sum_{j=1}^{n} [(u_{aj}^L)^2 + (u_{aj}^U)^2 + (v_{aj}^L)^2 + (v_{aj}^U)^2 + (h_{aj}^L)^2 + (h_{aj}^U)^2]} \tag{6}$$

Similarly, if one considers the weight of each element b_j or a_j (j = 1, 2, ..., n), the weighted asymmetry measures of INSs can be introduced below.

Definition 5. *Let B = {b_1, b_2, ..., b_n} and A = {a_1, a_2, ..., a_n} be two INSs, where $b_j = ([u_{bj}^L, u_{bj}^U], [v_{bj}^L, v_{bj}^U], [h_{bj}^L, h_{bj}^U])$ and $a_j = ([u_{aj}^L, u_{aj}^U], [v_{aj}^L, v_{aj}^U], [h_{aj}^L, h_{aj}^U])$ are the j-th INNs (j = 1, 2, ..., n) of B and A respectively, and let the weight of an element a_j or b_j be w_j, $w_j \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$. Thus, two weighted asymmetry measures of A on B are defined as*

$$Pw_B(A) = \frac{(A \cdot B)_w}{\|B\|_w^2} = \frac{\sum_{j=1}^{n} w_j^2 (u_{aj}^L u_{bj}^L + u_{aj}^U u_{bj}^U + v_{aj}^L v_{bj}^L + v_{aj}^U v_{bj}^U + h_{aj}^L h_{bj}^L + h_{aj}^U h_{bj}^U)}{\sum_{j=1}^{n} w_j^2 [(u_{bj}^L)^2 + (u_{bj}^U)^2 + (v_{bj}^L)^2 + (v_{bj}^U)^2 + (h_{bj}^L)^2 + (h_{bj}^U)^2]} \tag{7}$$

$$Pw_A(B) = \frac{(A \cdot B)_w}{\|A\|_w^2} = \frac{\sum_{j=1}^{n} w_j^2 (u_{aj}^L u_{bj}^L + u_{aj}^U u_{bj}^U + v_{aj}^L v_{bj}^L + v_{aj}^U v_{bj}^U + h_{aj}^L h_{bj}^L + h_{aj}^U h_{bj}^U)}{\sum_{j=1}^{n} w_j^2 [(u_{aj}^L)^2 + (u_{aj}^U)^2 + (v_{aj}^L)^2 + (v_{aj}^U)^2 + (h_{aj}^L)^2 + (h_{aj}^U)^2]} \tag{8}$$

3. Normalized Symmetry Measures of Simplified Neutrosophic Sets

A normalized symmetry measure of SNSs is proposed in this section.

Definition 6. *Let B = {b_1, b_2, . . . , b_n} and A = {a_1, a_2, . . . , a_n} be two SNSs, where $b_j = (u_{bj}, v_{bj}, h_{bj})$ and $a_j = (u_{aj}, v_{aj}, h_{aj})$ are the j-th SNNs (j = 1, 2, . . . , n) of B and A respectively. Thus*

$$M(B,A) = \frac{1}{1+\left|\frac{B\cdot A}{\|A\|^2}-\frac{B\cdot A}{\|B\|^2}\right|} = \frac{\|B\|^2\|A\|^2}{\|B\|^2\|A\|^2+\left|\|B\|^2-\|A\|^2\right|B\cdot A} \quad (9)$$

is called the normalized symmetry measure between B and A, where $\|B\| = \sqrt{\sum_{j=1}^{n}(u_{bj}^2+v_{bj}^2+h_{bj}^2)}$ *and* $\|A\| = \sqrt{\sum_{j=1}^{n}(u_{aj}^2+v_{aj}^2+h_{aj}^2)}$ *for SVNSs or* $\|B\| = \sqrt{\sum_{j=1}^{n}[(u_{bj}^L)^2+(u_{bj}^U)^2+(v_{bj}^L)^2+(v_{bj}^U)^2+(h_{bj}^L)^2+(h_{bj}^U)^2]}$ *and* $\|A\| = \sqrt{\sum_{j=1}^{n}[(u_{aj}^L)^2+(u_{aj}^U)^2+(v_{aj}^L)^2+(v_{aj}^U)^2+(h_{aj}^L)^2+(h_{aj}^U)^2]}$ *for INSs are the modules of B and A respectively, and* $B\cdot A = \sum_{j=1}^{n}(u_{aj}^L u_{bj}^L+u_{aj}^U u_{bj}^U+v_{aj}^L v_{bj}^L+v_{aj}^U v_{bj}^U+h_{aj}^L h_{bj}^L+h_{aj}^U h_{bj}^U)$ *is the inner product between B and A.*

Therefore, the closer the value of *M*(*B*, *A*) is to 1, the closer *B* is to *A*, and then there are *M*(*B*, *A*) = *M*(*A*, *B*) = 1 if *B* = *A*, and it satisfies $0 \le M(B,A) \le 1$ for any *B* and any *A*, which is a normalized symmetry measure.

The bellowing weighted symmetry measure between SNSs can be introduced if one considers the weight of each element b_j or a_j (*j* = 1, 2, . . . , *n*).

Definition 7. *Let B = {b_1, b_2, . . . , b_n} and A = {a_1, a_2, . . . , a_n} be two SNSs, where $b_j = (u_{bj}, v_{bj}, h_{bj})$ and $a_j = (u_{aj}, v_{aj}, h_{aj})$ are the j-th SNNs (j = 1, 2, . . . , n) of B and A respectively, and let the weight of an element b_j or a_j be w_j, $w_j \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$. Thus*

$$M_w(B,A) = \frac{1}{1+\left|\frac{(B\cdot A)_w}{\|A\|_w^2}-\frac{(B\cdot A)_w}{\|B\|_w^2}\right|} = \frac{\|B\|_w^2\|A\|_w^2}{\|B\|_w^2\|A\|_w^2+\left|\|B\|_w^2-\|A\|_w^2\right|(B\cdot A)_w} \quad (10)$$

is known as the weighted symmetry measure of B and A, where $\|B\|_w = \sqrt{\sum_{j=1}^{n} w_j^2(u_{bj}^2+v_{bj}^2+h_{bj}^2)}$ *and* $\|A\|_w = \sqrt{\sum_{j=1}^{n} w_j^2(u_{aj}^2+v_{aj}^2+h_{aj}^2)}$ *for SVNSs or* $\|B\|_w = \sqrt{\sum_{j=1}^{n} w_j^2[(u_{bj}^L)^2+(u_{bj}^U)^2+(v_{bj}^L)^2+(v_{bj}^U)^2+(h_{bj}^L)^2+(h_{bj}^U)^2]}$ *and* $\|A\|_w = \sqrt{\sum_{j=1}^{n} w_j^2[(u_{aj}^L)^2+(u_{aj}^U)^2+(v_{aj}^L)^2+(v_{aj}^U)^2+(h_{aj}^L)^2+(h_{aj}^U)^2]}$ *for INSs are the weighted modules of B and A, and then* $(B\cdot A)_w = \sum_{j=1}^{n} w_j^2(u_{aj}u_{bj}+v_{aj}v_{bj}+h_{aj}h_{bj})$ *or* $(B\cdot A)_w = \sum_{j=1}^{n} w_j^2(u_{aj}^L u_{bj}^L+u_{aj}^U u_{bj}^U+v_{aj}^L v_{bj}^L+v_{aj}^U v_{bj}^U+h_{aj}^L h_{bj}^L+h_{aj}^U h_{bj}^U)$ *for INNs is known as the weighted inner product of B and A.*

4. Decision-Making Method Using the Weighted Symmetry Measure

In this section, the proposed weighted symmetry measure is utilized for simplified neutrosophic MADM problems.

Set a set of alternatives as *S* = {S_1, S_2, . . . , S_m} and a set of attributes as *A* = {A_1, A_2, . . . , A_n} in a MADM problem. Assume that the weight of the attribute A_j is w_j, $w_j \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$.

Decision-Making Method Using the Weighted Symmetry Measure

In SNS setting, the satisfaction evaluations of an alternative S_i (*i* = 1, 2, . . . , *m*) for an attribute A_j (*j* = 1, 2, . . . , *n*) is expressed by an SNS $S_i = \{s_{i1}, s_{i2}, \ldots, s_{in}\}$, where $s_{ij} = (u_{ij}, v_{ij}, h_{ij})$ satisfies u_{ij}, v_{ij}, h_{ij}

$\in$ [0, 1] and $0 \leq u_{ij} + v_{ij} + h_{ij} \leq 3$ for SVNN or $u_{ij}, v_{ij}, h_{ij} \subseteq [0, 1]$ and $0 \leq u_{ij}^U + v_{ij}^U + h_{ij}^U \leq 3$ for INN. Thus, the decision matrix of SNSs can be established as $D = (s_{ij})_{m \times n}$.

In the MADM problem, the similarity measure between the ideal solution/alternative and an alternative can be used for determining the best one among all alternatives. By considering $s_j^* = (u_j^*, v_j^*, h_j^*) = (\max_i(u_{ij}), \min_i(v_{ij}), \min_i(h_{ij}))$ for SVNNs or $s_j^* = (u_j^*, v_j^*, h_j^*) = ([\max_i(u_{ij}^L), \max_i(u_{ij}^U)], [\min_i(v_{ij}^L), \min_i(v_{ij}^U)], [\min_i(h_{ij}^L), \min_i(h_{ij}^U))$ for INNs (j = 1, 2, ..., n; i = 1, 2, ..., m) as the ideal solution, an simplified neutrosophic ideal solution/alternative can be given as $S^* = \{s_1^*, s_2^*, \ldots, s_n^*\}$.

Then by applying Equation (10), the weighted symmetry measure between S^* and S_i (i = 1, 2, ..., m) is yielded by

$$M_w(S^*, S_i) = \frac{1}{1 + \left| \frac{(S^* \cdot S_i)_w}{\|S_i\|_w^2} - \frac{(S^* \cdot S_i)_w}{\|S^*\|_w^2} \right|} = \frac{\|S_i\|_w^2 \|S^*\|_w^2}{\|S_i\|_w^2 \|S^*\|_w^2 + \left| \|S_i\|_w^2 - \|S^*\|_w^2 \right| (S^* \cdot S_i)_w} \quad (11)$$

where $\|S_i\|_w = \sqrt{\sum_{j=1}^n w_j^2 (u_{ij}^2 + v_{ij}^2 + h_{ij}^2)}$, $\|S^*\|_w = \sqrt{\sum_{j=1}^n w_j^2 (u_j^{*2} + v_j^{*2} + h_j^{*2})}$, and $(S_i \cdot S^*)_w = \sum_{j=1}^n w_j^2 (u_{ij} u_j^* + v_{ij} v_j^* + h_{ij} h_j^*)$ for SVNNs or $\|S_i\|_w = \sqrt{\sum_{j=1}^n w_j^2 ((u_{ij}^L)^2 + (u_{ij}^U)^2 + (v_{ij}^L)^2 + (v_{ij}^U)^2 + (h_{ij}^L)^2 + (h_{ij}^U)^2)}$, $\|S^*\|_w = \sqrt{\sum_{j=1}^n w_j^2 ((u_j^{L*})^2 + (u_j^{U*})^2 + (v_j^{L*})^2 + (v_j^{U*})^2 + (h_j^{L*})^2 + (h_j^{U*})^2)}$, and $(S^* \cdot S_i)_w = \sum_{j=1}^n w_j^2 (u_{ij}^L u_j^{L*} + u_{ij}^U u_j^{U*} + v_{ij}^L v_j^{L*} + v_{ij}^U v_j^{U*} + h_{ij}^L h_j^{L*} + h_{ij}^U h_j^{U*})$ for INNs.

Thus, the greater the value of $M_w(S^*, S_i)$ is, the closer S_i is to S^*, and then the better the alternative S_i is.

5. Decision-Making Examples

A practical example about selecting the manufacturing schemes (alternatives) in the flexible manufacturing system is provided in SNS (SVNS and INS) environments to show the applications of the weighted symmetry measure-based MADM method in realistic scenarios, and then a comparative example with existing relative measures for SVNS is given to show the feasibility and effectiveness of the proposed method.

5.1. Practical Example

Assume that we consider a MADM problem in the flexible manufacturing system about the selection of manufacturing schemes (alternatives). Set a set of four alternatives for the flexible manufacturing system as $S = \{S_1, S_2, S_3, S_4\}$. They need to satisfy the three attributes: (i) A_1 is the improvement in quality; (ii) A_2 is the market response; (iii) A_3 is the manufacturing cost. In the decision-making problem, the decision maker/expert specifies the weight vector of the attributes as W = (0.36, 0.3, 0.34) corresponding to the importance of the three attributes.

Thus, the decision maker gives the satisfaction evaluation of an alternative S_i (i = 1, 2, 3, 4) for an attribute A_j (j = 1, 2, 3) by the evaluation information of SVNNs, and then single-valued neutrosophic decision matrix can be constructed as

$$D = \begin{bmatrix} (0.75, 0.2, 0.2) & (0.7, 0.24, 0.26) & (0.6, 0.2, 0.25) \\ (0.8, 0.1, 0.1) & (0.75, 0.2, 0.3) & (0.7, 0.3, 0.1) \\ (0.7, 0.2, 0.15) & (0.8, 0.2, 0.1) & (0.75, 0.25, 0.2) \\ (0.8, 0.1, 0.2) & (0.7, 0.15, 0.2) & (0.7, 0.2, 0.3) \end{bmatrix}$$

Thus, the developed approach is used for the MADM problem.

First, the single-valued neutrosophic ideal solution/alternative of $s_j^* = (u_j^*, v_j^*, h_j^*) = (\max_i(u_{ij}), \min_i(v_{ij}), \min_i(h_{ij}))$ for j = 1, 2, 3 and i = 1, 2, 3, 4 can be determined by

$$S^* = \{s_1^*, s_2^*, s_3^*\} = \{(0.8, 0.1, 0.1), (0.8, 0.15, 0.1), (0.75, 0.2, 0.1)\}$$

Then, according to Equation (11), the weighted symmetry measure values of S^* and S_i can be obtained as follows:

$$M_w(S^*, S_1) = 0.8945, M_w(S^*, S_2) = 0.9964, M_w(S^*, S_3) = 0.\ 9717, \text{ and } M_w(S^*, S_4) = 0.9730$$

Since the values of the weighted symmetry measure are $M_w(S^*, S_2) > M_w(S^*, S_4) > M_w(S^*, S_3) > M_w(S^*, S_1)$, the four alternatives are ranked as $S_2 > S_4 > S_3 > S_1$. Obviously, S_2 is the best one among the four alternatives.

If the fit judgments of the alternatives S_i (i = 1, 2, 3, 4) for the attributes are expressed by interval neutrosophic information, the interval neutrosophic decision matrix can be constructed as

$$D = \begin{bmatrix} ([0.7,0.8],[0.1,0.2],[0.15,0.3]) & ([0.7,0.8],[0.2,0.3],[0.1,0.3]) & ([0.6,0.7],[0,0.2],[0.1,0.4]) \\ ([0.75,0.9],[0.1,0.2],[0.1,0.2]) & ([0.7,0.8],[0.1,0.2],[0.1,0.3]) & ([0.6,0.7],[0.2,0.3],[0.1,0.3]) \\ ([0.6,0.8],[0.1,0.3],[0.1,0.2]) & ([0.7,0.8],[0.1,0.3],[0.1,0.2]) & ([0.7,0.8],[0.2,0.4],[0.1,0.3]) \\ ([0.8,0.9],[0.1,0.2],[0.1,0.2]) & ([0.7,0.8],[0.1,0.2],[0.1,0.3]) & ([0.6,0.8],[0.2,0.3],[0.2,0.4]) \end{bmatrix}$$

Then, an interval neutrosophic ideal solution/alternative of $s_j^* = (u_j^*, v_j^*, h_j^*) = ([\max_i(u_{ij}^L), \max_i(u_{ij}^U)], [\min_i(v_{ij}^L), \min_i(v_{ij}^U)], [\min_i(h_{ij}^L), \min_i(h_{ij}^U))$ for i = 1, 2, 3, 4 and j = 1, 2, 3 can be determined by

$$S^* = \{s_1^*, s_2^*, \ldots, s_n^*\} = \{([0.8,0.9],[0.1,0.2],[0.1,0.2]),([0.7,0.8],[0.1,0.2],[0.1,0.2]),([0.7,0.8],[0,0.2],[0.1,0.3])\}$$

By using Equation (11), the weighted symmetry measure values of S^* and S_i (i = 1, 2, 3, 4) can be obtained as

$$M_w(S^*, S_1) = 0.9053, M_w(S^*, S_2) = 0.9423, M_w(S^*, S_3) = 0.\ 9401, \text{ and } M_w(S^*, S_4) = 0.9762$$

Since the weighted symmetry measure values are $M_w(S_4, S^*) > M_w(S_2, S^*) > M_w(S_3, S^*) > M_w(S_1, S^*)$, the four alternatives are ranked as $S_4 > S_2 > S_3 > S_1$. Thus, S_4 is the best one among the four alternatives.

In this practical example, there is little difference of ranking orders under SVNS and INS environments.

To indicate the sensitivity of the proposed MADM method, this work only considers that the attribute weights may affect the ranking of alternatives as the sensitive analysis because the attribute weights are given by the decision maker's subjective judgment/preference in this decision-making problem. If the weights of the three attributes are not considered in this decision-making problem, the three weight values in Equation (11) are reduced to $w_j = 1/n = 1/3$ for j =1, 2, 3.

By using Equation (11) under a SVNS environment, the weighted symmetry measure values between S^* and S_i (i = 1, 2, 3, 4) can be obtained as

$$M_w(S^*, S_1) = 0.8933, M_w(S^*, S_2) = 0.9987, M_w(S^*, S_3) = 0.9811, \text{ and } M_w(S^*, S_4) = 0.9592$$

By using Equation (11) under an INS environment, the weighted symmetry measure values between S^* and S_i (i = 1, 2, 3, 4) can be obtained as

$$M_w(S^*, S_1) = 0.9207, M_w(S^*, S_2) = 0.9482, M_w(S^*, S_3) = 0.9550, \text{ and } M_w(S^*, S_4) = 0.9742$$

Thus, their ranking order is $S_4 > S_3 > S_2 > S_1$ according to the above measure values.

Clearly, there exists a little difference of the ranking orders with the given attribute weights and without the attribute weights under SVNS and INS environments, and then the best alternatives S_2 and S_4 in all ranking orders are still identical in these cases.

By the sensitive analysis regarding to the weights of the three attributes, it is obvious that the attribute weights can affect the ranking orders of four alternatives to some extent, which show some sensitivity to the attribute weights specified by the decision maker or expert.

5.2. Comparative Example with Existing Relative Measures for Single-Valued Neutrosophic Sets

For convenient comparison, let us adopt a MADM problem about selecting design schemes (alternatives) of punching machine from literature [21]. In the design schemes of punching machine [21], a set of four design schemes $S = \{S_1, S_2, S_3, S_4\}$ needs to satisfy a set of the five attributes $A = \{A_1, A_2, A_3, A_4, A_5\}$, where A_1, A_2, A_3, A_4, and A_5 are the manufacturing cost, structure complexity, transmission effectiveness, reliability, and maintainability respectively. The SVNS decision matrix of evaluating the four alternatives over the five attributes is adopted from literature [21], which is given as

$$D = \begin{bmatrix} (0.75, 0.1, 0.4) & (0.80, 0.1, 0.3) & (0.85, 0.1, 0.2) & (0.85, 0.1, 0.3) & (0.9, 0.1, 0.2) \\ (0.70, 0.1, 0.5) & (0.75, 0.1, 0.1) & (0.75, 0.2, 0.1) & (0.8, 0.1, 0.1) & (0.8, 0.2, 0.3) \\ (0.80, 0.2, 0.3) & (0.78, 0.1, 0.2) & (0.80, 0.1, 0.2) & (0.8, 0.2, 0.2) & (0.75, 0.1, 0.3) \\ (0.9, 0.1, 0.2) & (0.85, 0.1, 0.1) & (0.9, 0.1, 0.2) & (0.85, 0.1, 0.3) & (0.85, 0.2, 0.3) \end{bmatrix}$$

Then, the weight vector of the five attributes is given as $W = (0.25, 0.2, 0.25, 0.15, 0.15)$. The ideal solution/alternative in [21] is

$$S^* = \{s_1^*, s_2^*, s_3^*, s_4^*, s_5^*\} = \{(0.9, 0.1, 0.2), (0.85, 0.1, 0.1), (0.9, 0.1, 0.1), (0.85, 0.1, 0.1), (0.9, 0.1, 0.2)\}$$

Thus, we get the weighted symmetry measure values by Equation (11) as

$$M_w(S^*, S_1) = 0.9452,\ M_w(S^*, S_2) = 0.8390,\ M_w(S^*, S_3) = 0.8770, \text{ and } M_w(S^*, S_4) = 0.9803.$$

All the measure values of both the proposed weighted symmetry measure $M_w(S^*, S_i)$ and the various measures like the cosine measures of $Cos_w(S^*, S_i)$ and $C_w(S^*, S_i)$, the Dice measure of $D_w(S^*, S_i)$, the Jaccard measure of $J_w(S^*, S_i)$, the projection measure of $Projw_{s^*}(S_i)$, and the bidirectional projection measure of $BProj_w(S^*, S_i)$ in the literature [21] are shown in Table 1, where the average value (AV) and the standard deviation (SD) of S_i for $i = 1, 2, 3, 4$ are also given.

Table 1. Various measure results and ranking orders. AV: average value; SD: standard deviation.

Measure	S_1	S_2	S_3	S_4	AV	SD	Ranking Order
$Cos_w(S^*, S_i)$	0.9785	0.9685	0.9870	0.9942	0.9821	0.0096	$S_4 > S_3 > S_1 > S_2$
$C_w(S^*, S_i)$	0.9798	0.9750	0.9875	0.9929	0.9838	0.0069	$S_4 > S_3 > S_1 > S_2$
$D_w(S^*, S_i)$	0.9787	0.9696	0.9845	0.9927	0.9814	0.0084	$S_4 > S_3 > S_1 > S_2$
$J_w(S^*, S_i)$	0.9586	0.9427	0.9694	0.9857	0.9641	0.0157	$S_4 > S_3 > S_1 > S_2$
$Projw_{s^*}(S_i)$	0.3933	0.3632	0.3806	0.4158	0.3882	0.0192	$S_4 > S_1 > S_3 > S_2$
$BProj_w(S^*, S_i)$	0.9883	0.9636	0.9728	0.9958	0.9801	0.0126	$S_4 > S_1 > S_3 > S_2$
$M_w(S^*, S_i)$	0.9452	0.8390	0.8770	0.9803	0.9104	0.0555	$S_4 > S_1 > S_3 > S_2$

From Table 1, the ranking order based on the proposed symmetry measure is the same as the ones of the projection and bidirectional projection measures, but indicates a little difference of other ranking orders. However, the best one in all ranking orders is the same. It is obvious that the proposed symmetry measure is feasible and effective. According to their standard deviations, the SD value of the proposed symmetry measure is 0.0555, which is the biggest one among the SD values of various measures. In the MADM process, however, the cosine, Dice, and Jaccard measures indicate smaller SD values, which show lower resolution/discrimination; while the MADM methods using the projection and bidirectional projection measures also imply low resolution/discrimination due to their small SD values. Obviously, the new MADM method can strengthen the resolution/discrimination in the MADM process of the four alternatives so as to provide effective decision information for decision makers.

6. Conclusions

This paper firstly defined asymmetry measure of SNSs (SVNSs and INSs), and then developed the normalized symmetry measure and weighted symmetry measure of SNSs (SVNSs and INSs) and their MADM method with SNS information (interval and single valued neutrosophic information). Then the ranking of all alternatives and the best one can be given through the weighted symmetry measure between the ideal solution/alternative and each alternative. Finally, a practical example demonstrated the applications of the developed method for selecting the manufacturing schemes (alternatives) in the flexible manufacturing system under single-valued and interval neutrosophic environments, along with the sensitive analysis regarding to the attribute weights, and then the feasibility and effectiveness of the proposed method were indicated by the comparative example in single-valued neutrosophic setting.

Since the MADM method proposed in this study contains the biggest standard deviations among these existing related MADM methods, the higher resolution/discrimination given in the decision-making process is its main advantage. However, this study only proposes the simplified neutrosophic symmetry measure and its MADM method with the given (subjective) attribute weights for the first time, but it cannot handle simplified neutrosophic group decision-making problems. Therefore, in the future this study will be extended to simplified neutrosophic or simplified neutrosophic cubic group decision-making problems with given/unknown weights.

Author Contributions: Angyan Tu proposed the asymmetry and symmetry measures of SNSs and their MADM method; Bing Wang and Jun Ye presented the decision-making example and comparative analysis; we wrote this paper together.

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