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The algebraic structure on the neutrosophic triplet set

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Abstract. The notion of the neutrosophic triplet was introduced by Smarandache and Ali. This notion is based on the fundamental law of neutrosophy that for an idea X , we have neutral of X denoted as $\text{neut}(X)$ and anti of X denoted as $\text{anti}(X)$. This paper studied a neutrosophic triplet set which is a collection of all triple of three elements that satisfy certain properties with some binary operation. Also given some interesting properties related to them. Further, in this paper investigated that from the neutrosophic triplet group can construct a classical group under multiplicative operation for \mathbb{Z}_n , for some specific n . These neutrosophic triplet groups are built using only modulo integer $2p$, with p is an odd prime or Cayley table.

1. Introduction

The concept of neutrosophic triplet firstly introduced by F. Smarandache and M. Ali in [1]. According to them the neutrosophic triplet is defined as a 3-tuple of an element of any classical groupoid with its neutral and its opposite that satisfies certain properties. In [1] by using the basic idea of the neutrosophic theory as given in [2–4], the notion of neutrosophic triplet group is introduced. This algebraic structure is completely different from the classical group in the structural properties. In further developments, in 2017, Florentin et.al continued their research related to the triplet as given in [5]. In this reference, given some algebraic structures and their properties and also given their applications to mathematical modeling. In [5], the collection of all triplets neutrosophic is called a triplet neutrosophic set.

As we known the neutrosophic triplet set is adopted from the concept of neutrosophic set and the concept of neutrosophic perspective. The first concept is introduced by Smarandache in [6] and noted that, the neutrosophic set is a generalization of intuitionistic fuzzy set. The neutrosophic set theory has widely applications to algebraic structures, decision making, and so on. The second concept proposed by Smarandache in [7], exactly the concept of the triplet. As stated in [1, 5], we utilizing the neutrosophic triplet set to built another neutrosophic algebraic structure that called the neutrosophic triplet group.

This paper is focused on the neutrosophic triplet, neutrosophic triplet set, and neutrosophic triplet group. Several new properties related with them are studied and investigated an algebraic structure that can be constructed from the collection of neutrosophic triplets. Explained that the set of all non-trivial neutrosophic triplets of semigroup \mathbb{Z}_n forms a classical group under multiplicative operation on \mathbb{Z}_n , for some specific n . These neutrosophic triplet groups are built using Cayley table or only modulo integers $2p$, with p is an odd prime.

2. Elementary properties of neutrosophic triplet

This section stated for emphasis some basic definitions and result about neutrosophic triplets. For details about the neutrosophic triplet, the reader should see [2] or [1].



Definition 2.1[1, 2] Let $N = (N, *)$ be a groupoid. An element $x \in N$ forms a neutrosophic triplet if there exists a neutral element of x , denoted $neut(x) \in N$, different from the classical identity element of N and an opposite element of x , denoted $anti(x) \in N$ such that

$$x * neut(x) = x = neut(x) * x \text{ and } x * anti(x) = neut(x) = anti(x) * x.$$

The triplet $(x, neut(x), anti(x))$ is called a neutrosophic triplet.

We illustrate this by the following examples.

Example 2.2 The set $\{a, b\}$ is a groupoid with classical unit element a , given by the following table.

Table 1. Binary operation “*” on $\{a, b\}$

*	a	b
a	a	b
b	b	b

In this example, can not be taken (a, a, a) as a neutrosophic triplet, because a is a classical groupoid unit. Then (b, b, b) is the only neutrosophic triplet herein.

Example 2.3 Let $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ be the set of integers modulo 4 together with binary operation " \times_4 ". Can be seen that none of both elements 1 and 3 contribute to neutrosophic triplets as they are units under multiplication modulo 4 in \mathbb{Z}_4 . Furthermore, 2 cannot contribute to a neutrosophic triplet with 3, because $neut(2) = 3$ and $anti(2)$ does not exist in \mathbb{Z}_4 . Finally, 0 gives rise to a neutrosophic triplet, because $neut(0) = 0 = anti(0)$.

The neutrosophic triplet is called a trivial neutrosophic triplet and denoted by $(0, 0, 0)$.

Now we proceed to prove some properties of the neutrosophic triplet.

Theorem 2.4 Let $(N, *)$ be a cancellable semigroup. If $(x, neut(x), anti(x))$ is a neutrosophic triplet, then $(anti(x), neut(x), x)$ is also a neutrosophic triplet.

Proof : Let $(x, neut(x), anti(x))$ be a neutrosophic triplet. Obviously, $anti(x) * x = neut(x)$. Need to be proven that $anti(x) * neut(x) = anti(x)$. From the definition of a neutrosophic triplet and use the left-cancellable law we have

$$\begin{aligned} x * neut(x) = x &\Leftrightarrow [x * neut(x)] * neut(x) = x * neut(x) \\ &\Leftrightarrow x * neut(x) * neut(x) = x * neut(x) \\ &\Leftrightarrow neut(x) * neut(x) = neut(x) \\ &\Leftrightarrow [x * anti(x)] * neut(x) = neut(x) \\ &\Leftrightarrow x * anti(x) * neut(x) = x * anti(x) \\ &\Leftrightarrow anti(x) * neut(x) = anti(x) \end{aligned}$$

It follows that $(anti(x), neut(x), x)$ is a neutrosophic triplet. ■

Definition 2.5 Let $(N, *)$ be a semigroup. An element x in a semigroup N is called an idempotent element if $x * x = x$.

Furthermore, we proceed to prove that every non-trivial idempotent element give rise to a neutrosophic triplet as given in the following theorem.

Theorem 2.6 If x is a non-trivial idempotent in semigroup $(N, *)$, then it forms a neutrosophic triplet.

Proof : Let x be a non-trivial idempotent in N . Then, by Definition 2.5, we have $x * x = x$. Obviously, $neut(x) = x = anti(x)$. So x gives rise to a neutrosophic triplet (x, x, x) . ■

Example 2.7 In semigroup (\mathbb{Z}_6, \times_6) of integer modulo 6, 3 and 4 are the only non-trivial idempotents in \mathbb{Z}_6 . Clearly $3 \times_6 3 = 9 \equiv 3(\text{mod } 6)$ and 3 is not a neutral element, so $(3, 3, 3)$ is a trivial neutrosophic triplet. Similarly $4 \times_6 4 = 16 \equiv 4(\text{mod } 6)$ and $(4, 4, 4)$ is also a neutrosophic triplet.

Now given the following definition.

Definition 2.8 Let $N = (N, *)$ be a groupoid. Then N is called a neutrosophic triplet set (briefly, NT-set) if for every $x \in N$ forms a neutrosophic triplet.

Example 2.9 Consider the groupoid $(\{1, 2\}, *)$, given by the following table.

Table 2. Binary operation “*” on $\{1, 2\}$

*	1	2
1	2	1
2	1	2

Clearly $\{1, 2\}$ is a groupoid under binary operation “*” without a classical unit element. Can be seen that 1 contributes to a neutrosophic triplet, because of $neut(1) = 2$, as $1 * 2 = 1 = 2 * 1$. $anti(1) = 1$, since $1 * 1 = 2$. Thus $(1, 2, 1)$ is a neutrosophic triplet. Similarly, 2 contributes to a neutrosophic triplet, because of $neut(2) = 2 = anti(2)$. So $(2, 2, 2)$ is a neutrosophic triplet. Hence the NT-set is $\{1, 2\}$.

Lemma 2.10 Every classical monoid is not a neutrosophic triplet set.

Proof : Let e be a classical monoid unit. Clearly, we do not take (e, e, e) as a neutrosophic triplet. Hence the monoid never is a neutrosophic triplet set. ■

3. Group on the NT-group

In this section and further, unless otherwise state all neutrosophic triplet groups are built using only modulo integer \mathbb{Z}_{2p} , with p is an odd prime or Cayley tables.

Will be defined about the neutrosophic triplet group (NT-group)

Definition 3.1[1] Let $(N, *)$ be a NT-set. Then, N is called a NT-group, if it satisfies:

- The operation $*$ is closed and
- The operation $*$ is associative.

Furthermore, if the operation $*$ is commutative, then $(N, *)$ is called a commutative NT-group.

Now given some examples of a triplet neutrosophic group.

Example 3.2 Consider the set $N = \{a, b, c, d\}$ and the operation $*$ on N is defined by the table below

Table 3. Binary operation “*” on N

*	a	b	c	d
a	a	a	a	a
b	a	b	a	b
c	a	a	c	c
d	a	b	c	d

Then, $(N,*)$ is a commutative NT-group, and

$$\text{neut}(a) = a = \text{anti}(a), \text{neut}(b) = b = \text{anti}(b), \text{neut}(c) = c = \text{anti}(c), \text{ and } \text{neut}(d) = d = \text{anti}(d)$$

Example 3.3 Let $M = \{a, b, c, d, e\}$ endowed with a binary operation $*$, given by the following table.

Table 4. Binary operation “*” on M

*	a	b	c	d	e
a	a	a	a	a	a
b	b	b	b	b	b
c	d	d	c	d	e
d	d	d	d	d	d
e	d	d	e	d	c

Then, $(M,*)$ is a non-commutative NT-group, and

$$\text{neut}(a) = a = \text{anti}(a), \text{neut}(b) = b = \text{anti}(b), \text{neut}(c) = c = \text{anti}(c), \\ \text{neut}(d) = d, \{\text{anti}(d)\} = \{c, d\}, \text{neut}(e) = c, \text{ and } \text{anti}(e) = e.$$

Now we give some basic properties of the triplet neutrosophic group, and the following result will be needed.

Theorem 3.4 Let $(N,*)$ be a neutrosophic triplet group and $x \in N$. The following are true:

- $\text{neut}(x)$ is unique
- $\text{neut}(x) * \text{neut}(x) = \text{neut}(x)$

Proof : Assume that $\text{neut}(x) = r$ and $\text{neut}(x) = s$. Then $x * r = x = r * x$ and $x * s = x = s * x$, and there exists $p, q \in N$ such that $x * p = r = p * x$ and $x * q = s = q * x$. Thus

$$r * s = (p * x) * s = p * (x * s) = p * x = r.$$

On the other hand $r * s = (x * p) * (x * q) = [x * (p * x)] * q = (x * r) * q = x * q = s$.

Therefore, $r = r * s = s$. This means that $\text{neut}(x)$ is unique.

Further, from $\text{anti}(x) * \text{neut}(x) = \text{anti}(x)$, multiply by x to the left, we get

$$x * [\text{anti}(x) * \text{neut}(x)] = x * \text{anti}(x) \Rightarrow [x * \text{anti}(x)] * \text{neut}(x) = \text{neut}(x) \\ \Rightarrow \text{neut}(x) * \text{neut}(x) = \text{neut}(x).$$

This proves that $\text{neut}(x) * \text{neut}(x) = \text{neut}(x)$. ■

Remark 3.5 For any x in a NT-group $(N,*)$, although $\text{neut}(x)$ is unique, but we can see from Example 3.3 that $\text{anti}(x)$ is usually not unique.

Consider the following example.

Example 3.6 Let (\mathbb{Z}_5, \times_5) be a multiplicative semigroup under multiplication modulo 5. Can be seen that none of the elements 1, 2, 3, and 4 in \mathbb{Z}_5 contributes to neutrosophic triplet as they are units under multiplicative operation in \mathbb{Z}_5 . Therefore, \mathbb{Z}_5 has no non-trivial neutrosophic triplet. We further see a triplet $(0, 0, 0)$ is the only one trivial neutrosophic triplet of \mathbb{Z}_5 .

From this example, we have the following result.

Lemma 3.7 Let (\mathbb{Z}_p, \times_p) be a semigroup under product modulo p . If p is a prime, then \mathbb{Z}_p has no non-trivial neutrosophic triplets.

Proof : Straightforward from the consequence of Example 3.6. ■

Next considered another example below.

Example 3.8 Let $S = (\mathbb{Z}_{10}, \times_{10})$ be a multiplicative semigroup. Clearly, both 5 and 6 are the only non-trivial idempotents of \mathbb{Z}_{10} , $5 \times_{10} 5 = 25 \equiv 5 \pmod{10}$ and $6 \times_{10} 6 = 36 \equiv 6 \pmod{10}$. Furthermore, it can be seen that $(0, 0, 0)$ and $(5, 5, 5)$ are trivial neutrosophic triplets. Only elements 2, 4, 6, and 8 are the probable ones in \mathbb{Z}_{10} which contributes to the non-trivial neutrosophic triplet. The neutrosophic triplets with respect to these elements are $(2, 6, 8)$, $(4, 6, 4)$, $(6, 6, 6)$, and $(8, 6, 2)$, respectively. It can also be seen that $H = \{2, 4, 6, 8\} \subseteq \mathbb{Z}_{10}$ is such that it form a classical multiplicative group (under product modulo 10), with 6 as the identity given by the following table.

Table 5. Binary operation “ \times_{10} ” on H

\times_{10}	2	4	6	8
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

Clearly, H is a cyclic group and $H = \langle 2 \rangle$, because
 $2 = 2^1$, $4 = 2^2$, $8 = 2^3$, and $2^4 = 16 \equiv 6 \pmod{10}$.

From this example, the following result is obtained.

Theorem 3.9 If $(\mathbb{Z}_{2p}, \times_{2p})$ is a multiplicative semigroup with p is an odd prime, then \mathbb{Z}_{2p} contains a proper classical group which elements are the non-trivial neutrosophic triplets of \mathbb{Z}_{2p} .

Proof : Let $(\mathbb{Z}_{2p}, \times_{2p})$ be the multiplicative semigroup with p is an odd prime. We see the idempotents in \mathbb{Z}_{2p} are p and $p + 1$. Only elements $2k$, with $k = 1, 2, \dots, p - 1$ which contribute to the non-trivial neutrosophic triplet. So the collection of these elements is a proper subset of \mathbb{Z}_{2p} and it forms a classical group under product modulo $2p$, with element $p + 1$ as the identity. ■

Theorem 3.10 Let $(\mathbb{Z}_{2p}, \times_{2p})$ be a semigroup under product modulo $2p$, with p is an odd prime and H is a classical multiplicative group of the non-trivial neutrosophic triplets in \mathbb{Z}_{2p} . Then H is cyclic and $H = \langle 2 \rangle$.

Proof : Clearly H is cyclic with 2 as a generator of H . ■

4. Conclusion

From the discussion can be concluded that a neutral of any element in a NT-group is unique but not true for the opposite element. Furthermore from the semigroup of modulo $2p$ can be constructed a classical group with the identity is $p + 1$ and the group is cyclic with the generator is 2.

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