Transforming Neutrosophic Fuzzy Set into Fuzzy Set by Imprecision Method

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ABSTRACT

Determining the proper knowledge management strategies is important to make sure that the alignment of organizational Procedures and the knowledge management-related Information produces effective creation, sharing and utilization of knowledge. Data sets in the form of neutrosophic fuzzy values sometimes make the decision process very complicated and unstructured. Besides the theory from fuzzy sets, vague sets and intuitionistic fuzzy sets, neutrosophic fuzzy set is one of the methods used to deal with uncertain information and it can provide more information than fuzzy sets. The purpose of this research is determining the knowledge management strategy of transforming neutrosophic fuzzy values into fuzzy values using imprecision method and techniques in defuzzification proposed in the literature and to propose a new method to calculate the correlation coefficient between neutrosophic fuzzy sets. Numerical illustration is given to support the proposed theory.

Keywords: Vague set, Intuitionistic fuzzy set, Correlation coefficient of neutrosophic fuzzy sets.

INTRODUCTION

Fuzzy set theory has long been introduced to handle inexact and imprecise data, since in the real world there is vague information about different applications, we can formalize the measurements from different sensors to a vague set. In fuzzy set theory, each object \( u \in U \) is
assigned a single real value, called the grade of membership, between zero and one. (Here U is a classical set of objects, called the universe of discourse). Gau & Buehrer [1994] point out that the drawback of using the single membership value in fuzzy set theory is that the evidence for \( u \in U \) and the evidence against \( u \notin U \) are in fact mixed together. In order to tackle this problem, they proposed the notion of Vague Sets (VSs), which allow using interval-based membership instead of using point-based membership as in FSs. The interval-based membership generalization in VSs is more expressive in capturing vagueness of data. However, VSs are shown to be equivalent to that of Intuitionistic Fuzzy Sets (IFSs). For this reason, the interesting features for handling vague data that are unique to VSs are largely ignored. In this paper, we attempt to make a more detailed comparison between VSs and IFSs from various perspectives of algebraic properties, graphical representations and practical applications. We find that there are many interesting features of VSs from a data modelling point of view. Essentially, due to the fact that a VS corresponds to a more intuitive graphical view of data sets, it is much easier to define and visualize the relationship of vague data objects. The classical nulls representing incompleteness can be viewed as a special case of a vague set and then generalized to vague data. In addition, we show that the notions of crisp and imprecision in vague data can be captured by interval relationships.

Since fuzzy set (FSs) theory was introduced, several new concepts of higher-order FSs have been proposed. Among them, intuitionistic fuzzy sets (IFSs), proposed by Atanassov (1989), provide a flexible mathematical framework to cope, besides the presence of vagueness, with the hesitancy originating from imperfect or imprecise information. IFSs use two characteristic functions to express the degree of membership (belongingness) and the degree of non-membership (non-belongingness) of elements of the universe to the IFSs. Therefore, the idea of using positive and (independently) negative information becomes the core of IFSs. This idea is natural in real life human discourse and action, and as an obvious consequence, is well-known and widely studied in psychology and other social sciences. In fact, IFSs, interval-valued fuzzy sets (IVFSs) and vague sets can be viewed as three equivalent generalizations of fuzzy sets. However, they are different as IFSs force a user to explicitly consider positive and negative information independently. On the other hand, while employing IVFSs, the user’s attention is forced on positive information (in an interval) only. So the two concepts, IFSs and IVFSs, are different in application.

In the real world there are vaguely specified data values in many applications, such as sensor information. Fuzzy set theory has been proposed to handle such vagueness by generalizing the notion of membership in a set. Essentially, in a Fuzzy Set (FS) each element is associated with a point-value selected from the unit interval \([0,1]\), which is termed the grade of membership in the set. A Vague Set (VS), as well as an Intuitionistic Fuzzy Set (IFS), is a further generalization of an FS. Instead of using point-based membership as in FSs, interval-based membership is used in a VS. The interval-based membership in VSs is more expressive in capturing vagueness of data. In the literature, the notions of IFSs and VSs are regarded as equivalent, in the sense that an IFS is isomorphic to a VS. Furthermore, due to such equivalence and IFSs being earlier known as a tradition, the interesting features for handling
vague data that are unique to VSs are largely ignored. In this paper, we attempt to make a comparison between VSs and IFSs from various perspectives of algebraic properties, graphical representations and practical applications.

Gau & Buehrer (1994) pointed out that this single value combines the evidence for \( u_i \) in \( U \) and the evidence against \( u_i \) in \( U \), without indicating how much there is of each. They also pointed out that the single number tells us nothing about its accuracy. Thus they presented the concepts of vague sets. They used a truth-membership function \( t_A \) and false-membership function \( f_A \) to characterize the lower bound on \( \mu_A \). These lower bounds are used to create a subinterval on \([0, 1]\), namely \([t_A(u_i), 1-f_A(u_i)]\), to generalize the \( \mu_A(u_i) \) of fuzzy sets, where \( t_A(u_i) \leq \mu_A(u_i) \leq 1-f_A(u_i) \). For example, let \( A \) be a vague set with truth-membership function \( t_A \) and false-membership function \( f_A \), respectively. If \([t_A(u_i), 1-f_A(u_i)] = [0.5, 0.8]\), then we can see that \( t_A(u_i)=0.5; 1-f_A(u_i)=0.8; f_A(u_i)=0.2 \). It can be interpreted as ,the vote for resolution is 5 in favor, 2 against, and 3 abstentions.

The contributions in VSs and IFSs, which has so far been done in the literature only by few authors (Gau & Buehrer, 1994), which leads to the undermining of the development of VSs. Second, the discussion of similarity measures for vague sets. Third, transformation of vague sets into Fuzzy sets using diverse techniques (Liu et al., 2008). Fourth, numerical illustration for transforming vague sets into fuzzy sets and fifth, proposing a new method for correlation coefficient for vague sets. They (2008) proposed different methods for transforming vague sets into fuzzy sets. Some of them are given below: Chiang & Lin, (1999), Kao & Liu, (2002), Park et al., (2009), Robinson & Amirtharaj, (2011, 2012, 2013), and Power (2013) proposed correlation coefficients for different applications of decision making problems. In the following we present a new approach of correlation coefficient for vague sets.

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**SECTION 2 – DEFINITIONS AND PRELIMINARIES**

**Definition 2.1: (Fuzzy Set):** A fuzzy set \( T \) on a set \( U \) is \( \{(u, T(u)): u \in U\} \) where \( T: U \rightarrow [0, 1] \).

**Definition 2.2: (Vague Set) \{VS\} ((Lu & Ng, 2005;2009): A vague set \( V \) in a set \( U \) is \( \{u, T(u), F(u): u \in U\} \) where \( U \) is characterized by a true membership function \( T \), and a false membership function \( F \) such that \( T: U \rightarrow [0, 1] \) and \( F: U \rightarrow [0, 1] \) with \( 0 \leq T(u) + F(u) \leq 1 \). Here \( T(u) \) is a lower bound on the grade of membership of \( u \) derived from the evidence for \( u \), and \( F(u) \) is a lower bound on the grade of membership of the negation of \( u \) derived from the evidence against \( u \).

**Definition 2.3: (Intuitionistic fuzzy set) \{IFS\}:** An intuitionistic fuzzy set \( S \) in a set \( U \) is \( \{u, T(u), F(u): u \in U\} \) where \( U \) is characterized by a membership function \( T \), and a non-membership function \( F \) such that \( T: U \rightarrow [0, 1] \) and \( F: U \rightarrow [0, 1] \) with \( 0 \leq T(u) + F(u) \leq 1 \).

Definition 2.4: A neutrosophic fuzzy set A on the universe of discourse X characterized by a truth membership function $T_{A}(x)$, an indeterminacy function $I_{A}(x)$ and a falsity membership function $F_{A}(x)$ is defined as $A = \{<x, T_{A}(x), I_{A}(x), F_{A}(x)> : x \in X\}$, where $T_{A}, I_{A}, F_{A} : X \rightarrow [0, 1]$ and $0 \leq T_{A}(x) \leq 1; 0 \leq I_{A}(x) \leq 1; 0 \leq F_{A}(x) \leq 1$, for all $x \in X$.

Operations in neutrosophic fuzzy sets

Definition 2.5: Let $A = (T_{A}, I_{A}, F_{A})$ and $B = (T_{B}, I_{B}, F_{B})$ be two neutrosophic fuzzy sets on an universe set X. The following definitions are defined.

(a). $A \cup B = \{<x, T_{A}(x) \lor T_{B}(x), I_{A}(x) \lor I_{B}(x), F_{A}(x) \land F_{B}(x)> : x \in X\}$;
(b). $A \cap B = \{<x, T_{A}(x) \land T_{B}(x), I_{A}(x) \land I_{B}(x), F_{A}(x) \lor F_{B}(x)> : x \in X\}$;
(c). $A'(x) = \{<x, 1 - T_{A}(x), 1 - I_{A}(x), 1 - F_{A}(x)> : x \in X\}$;
(d). $(A / B)(x) = \{<x, T_{A}(x) \land \overline{T_{B}(x)}, I_{A}(x) \lor I_{B}(x), F_{A}(x) \lor F_{B}(x)> : x \in X\}$;
(e). $A \subseteq B$ if $T_{A}(x) \leq T_{B}(x)$, $I_{A}(x) \leq I_{B}(x)$, and $F_{A}(x) \geq F_{B}(x)$;
(f). $0_{N} = \{(x, 0, 0, 1): x \in X\}; 1_{N} = \{(x, 1, 0, 0): x \in X\}$;

Results 2.6: The following properties are hold:

(i). $0_{N} \subseteq A \subseteq 1_{A}; C(0_{N}) = 1_{N}; C(1_{N}) = 0_{A};$ (ii). $A \subseteq B$ iff $C(B) \subseteq C(A);$ (iii). $C(C(A)) = A;$
(iv). $C(A \cup B) = C(A) \cap C(B);$ (v). $C(A \cap B) = C(A) \cup C(B);$ (vi). $A \subseteq B$ and $E \subseteq D$ iff $A \cup E \subseteq B \cup D;$ (vii). $A \subseteq B$ and $E \subseteq D$ iff $A \cap E \subseteq B \cap D;$
(viii). $A \subseteq B$ and $A \subseteq E$ iff $A \subseteq B \cap E,$ and $A \subseteq B \cup E;$ (ix). $A \subseteq E$ and $B \subseteq E$ iff $A \cap B \subseteq E,$ and $A \cup B \subseteq E; (x). A \subseteq B$ and $B \subseteq D$ implies $A \subseteq D,$

Properties 2.7: For $A$, $B \in NFS(X)$, (i). $0 \leq (1/3n)\ C(A, B) \leq 1;$ (ii). $C(A, B) = C(B, A);$ (iii). $C(A, B) = 1$ if $A = B.$

SECTION 3- TRANSFORMING NEUTROSOPHIC FUZZY VALUES INTO FUZZY VALUES

There are two methods for transforming neutrosophic fuzzy values (sets) into fuzzy values (sets).

Method I (Imprecision membership): Any neutrosophic fuzzy set $A = (T_{A}, I_{A}, F_{A})$ containing neutrosophic fuzzy values are converted into intuitionistic fuzzy values or vague values as $\Gamma(A) = (T_{A}, f_{A})$ where $f_{A}$ is calculated the formula mentioned below which is called as Impression membership method.

$$D_{A} = F_{A} + \frac{1 - F_{A} - I_{A}}{I_{A} + F_{A}}$$

if $F_{A} = 0$;

$$= F_{A} + \frac{1 - F_{A} - I_{A}}{F_{A} + I_{A}}$$

if $0 < F_{A} \leq 0.5$;

$$= F_{A} + \frac{1 - F_{A} - I_{A}}{0.5 + (F_{A} - 0.5) / (F_{A} + I_{A})}$$

if $0.5 < F_{A} \leq 1$.

Method II (Defuzzification): After Method I (Median membership), intuitionistic (vague) fuzzy values of the form $\Gamma(A) = (T_{A}, f_{A})$ are converted into fuzzy set containing fuzzy values as $<D(A)> = <T_{A} / (T_{A} + f_{A})>$. There are some unreasonable problems for some cases when
we use method two to transform vague sets into fuzzy sets. For example vague value [0, 0.2] in this voting model, there are 0 votes in favor, 8 against. The abstention persons voting attitude tends to vote against instead of in favor, since there are more negative voter than affirmative votes. However, the abstention person in favor voting attitude in this model is 0. It means that an abstentions person voting attitude is absolutely against obviously, it is unreasonable. For this reason, we go to new transforming method namely method three.

SECTION 4 - PROPOSED MODEL FOR CORRELATION COEFFICIENT BETWEEN NEUTROSOFPIC FUZZY SETS

Algorithm 4.1: The required is obtained through the following algorithm having 8 steps

Step 1: Finite Neutrosophic fuzzy values are formed into a neutrosophic fuzzy decision s x t-matrix R having s rows and t columns, such that s neutrosophic fuzzy attributes (each row) corresponding to t neutrosophic fuzzy alternatives (each column). There are n number of such fuzzy decision matrices (A_i) [i = 1 to n] are assumed.

Step 2: The above fuzzy matrix A_i is changed into a single fuzzy matric \( \Gamma(A_i) \) having two membership functions in which \( T_{A_i} \) is unchanged , and \( f_{A_i} \) is found from \( I_{A_i} \), and \( F_{A_i} \) using impression membership (Method 3).

Step 3: After step 2, and for each i = 1 to n, the single fuzzy decision matrix \( \Gamma(A_i) \) is defuzzyfied as a single fuzzy decision matrix \( D(A_i) \) having only on membership by the method 2.

Step 4: For every \( A_i \) (i = 1 to n), the energy of \( A_i \) is found from the formula

\[
E_{\text{NFS}}(A_i) = \left( \sum_{i=1}^{n} D_{A_i}^2(x) \right)
\]

Step 5: The covariance \( C_{\text{NFS}}(A_i, A_j) \) is derived \( \left| \sum_{i=1}^{n} [D_{A_i}(x)D_{A_j}(x)] \right| \).

Step 6: The correlation coefficient \( R_{\text{NFS}} \) is calculated by equation \( R_{\text{NFS}} (A_i, A_j) = \frac{C_{\text{NFS}}(A_iA_j)}{\sqrt{E_{\text{NFS}}(A_i)E_{\text{NFS}}(A_j)}} \).

Step 7: Ranking the correlation coefficients.

Step 8: Select the best pair.

Example 4.2: The above algorithm is investigated in the explanations stated below:

Let us take the initial assumption as given below:

Step 1: Five Neutrosophic fuzzy values (\( r_{ij} \)) are formed into a neutrosophic fuzzy decision 4 x 4-matrix R having 4 rows and 4 columns, such that 4 neutrosophic fuzzy attributes (each row) corresponding to 4 neutrosophic fuzzy alternatives (each column). They are as follows:

\[
R^1 = \begin{bmatrix}
0.25 & 0.54 & 0.8 & 0.3 & 0.4 & 0.9 & 0.7 & 0.35 & 0.5 & 0.9 & 0.2 & 0.8 \\
0.6 & 0.5 & 0.5 & 0.6 & 0.2 & 0.3 & 0.2 & 0.4 & 0.9 & 0.6 & 0.23 & 0.7 \\
0.3 & 0.45 & 0.9 & 0.7 & 0.1 & 0.4 & 0.6 & 0.5 & 0.5 & 0.4 & 0.2 & 0.9 \\
0.45 & 0.38 & 0.27 & 0.37 & 0.68 & 0.16 & 0.6 & 0.25 & 0.3 & 0.1 & 0.4 & 0.8
\end{bmatrix}
\]

\[
R^2 = \begin{bmatrix}
0.3 & 0.55 & 0.37 & 0.75 & 0.42 & 0.1 & 0.32 & 0.67 & 0.56 & 0.35 & 0.56 & 0.72 \\
0.5 & 0.4 & 0.32 & 0.65 & 0.25 & 0.32 & 0.6 & 0.3 & 0.1 & 0.75 & 0.25 & 0.55 \\
0.27 & 0.9 & 0.81 & 0.31 & 0.4 & 0.6 & 0.75 & 0.65 & 0.55 & 0.3 & 0.7 & 0.9
\end{bmatrix}
\]
Step 2: Using Method I, the neutrosophic fuzzy values are converted into intuitionistic or vague fuzzy values as follows:

\[
R_1 = \begin{bmatrix}
<0.25,0.554> & <0.3,0.658> & <0.7,0.588> & <0.9,0.8> \\
<0.6,0.5> & <0.6,0.6> & <0.2,0.658> & <0.6,0.750> \\
<0.3,0.621> & <0.7,0.8> & <0.6,0.5> & <0.4,0.814> \\
<0.45,0.415> & <0.37,0.191> & <0.6,0.545> & <0.1,0.65> \\
<0.1,0.7> & <0.6,0.455> & <0.4,0.333> & <0.3,0.427>
\end{bmatrix}
\]

\[
R_2 = \begin{bmatrix}
<0.3,0.402> & <0.75,0.192> & <0.32,0.434> & <0.35,0.532> \\
<0.5,0.444> & <0.65,0.561> & <0.6,0.25> & <0.75,0.663> \\
<0.27,0.316> & <0.31,0.6> & <0.75,0.442> & <0.3,0.45> \\
<0.32,0.559> & <0.9,0.75> & <0.6,0.555> & <0.3,0.567> \\
<0.12,0.604> & <0.17,0.379> & <0.5,0.25> & <0.45,0.293> \\
<0.50,0.277> & <0.56,0.271> & <0.3,0.143> & <0.57,0.512> \\
<0.54,0.370> & <0.73,0.410> & <0.5,0.143> & <0.3,0.137> \\
<0.7,0.25> & <0.5,0.5> & <0.2,0.793> & <0.7,0.317> \\
<0.3,0.471> & <0.57,0.294> & <0.23,0.390> & <0.53,0.293> \\
<0.32,0.649> & <0.56,0.381> & <0.1,0.733> & <0.57,0.512> \\
<0.72,0.265> & <0.13,0.4> & <0.55,0.474> & <0.7,0.793> \\
<0.52,0.182> & <0.57,0.213> & <0.76,0.261> & <0.57,0.512> \\
<0.3,0.504> & <0.7,0.2> & <0.3,0.430> & <0.5,0.6> \\
<0.2,0.4> & <0.6,0.714> & <0.1,0.495> & <0.3,0.325> \\
<0.27,0.582> & <0.75,0.561> & <0.32,0.415> & <0.35,0.532>
\end{bmatrix}
\]

Step 3: Using Model II, the following are defuzzified from the given NFSs.

\[
R_1 = \begin{bmatrix}
<0.311> & <0.313> & <0.544> & <0.529> \\
<0.545> & <0.5> & <0.233> & <0.444> \\
<0.326> & <0.467> & <0.545> & <0.330> \\
<0.520> & <0.660> & <0.524> & <0.133> \\
<0.125> & <0.513> & <0.382> & <0.413> \\
<0.427> & <0.796> & <0.424> & <0.467> \\
<0.53> & <0.537> & <0.706> & <0.524> \\
<0.461> & <0.341> & <0.657> & <0.4>
\end{bmatrix}
\]
Step 4: Energy for fuzzy set is found.
Energy (R1) = 3.297472; Energy (R2) = 4.071829; Energy (R3) = 4.790154
Energy (R4) = 4.233525; Energy (R5) = 4.359342.

Step 5: Covariance for fuzzy sets are calculated.
Cov (R1, R2) = 3.406081; Cov (R1, R3) = 3.582698
Cov (R1, R4) = 3.278317; Cov (R1, R4) = 3.408115.

Step 6: Now the correlation coefficients between NFSs are derived.
Correlation coefficient (R1, R2) = 3.406081 / \sqrt{(3.297472)(4.071829)} = 0.929543349.
Correlation coefficient (R1, R3) = 3.582698 / \sqrt{(3.297472)(4.790154)} = 0.90145677
Correlation coefficient (R1, R4) = 3.278317 / \sqrt{(3.297472)(4.233525)} = 0.877423623
Correlation coefficient (R1, R5) = 3.408115 / \sqrt{(3.297472)(4.359342)} = 0.89890379.

Step 7: R1 and R2 are highly correlated.

CONCLUSION

The general models for transforming neutrosophic fuzzy sets into fuzzy sets are also discussed and the validity of the transformation models is analysed. In this paper an approach to find correlation coefficient in the situations where the attribute values are characterized by neutrosophic fuzzy values is presented. From this study, it can be seen that correlation coefficient for neutrosophic fuzzy sets needs to be exclusively defined using its special properties, even though in the literature it is believed that NFSs are indeed VSs / IFSs. It is also seen that correlation coefficient of NFSs shows a greater variation from the correlation coefficient of the FSs derived from the same NFSs, thereby giving more credits for the correlation of NFSs defined exclusively. In future, the relationship between correlation coefficient of neutrosophic fuzzy sets can be studied more exclusively. This proposed approach provides us an effective and practical way to deal when the information about attribute weights is partially known (vague values) and has greater applications in decision making problems.
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