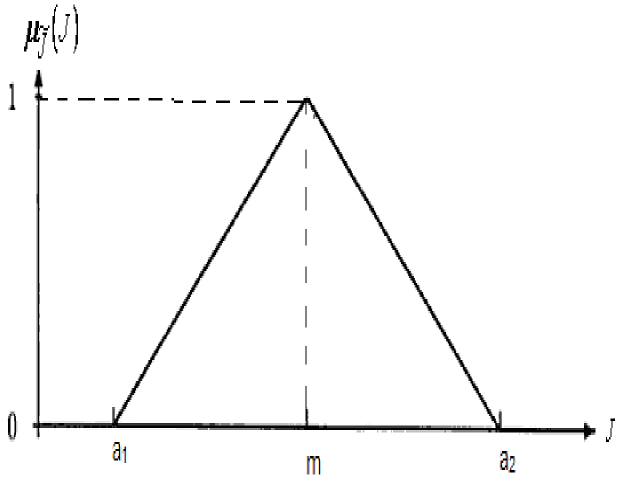

Fig. 3: Membership Function of Fuzzy Number *J*.

where *m* is a given value a_1 & a_2 denoting the lower and upper bounds. Sometimes, it is more convenient to use the notation explicitly highlighting the membership function parameters. In this case, we obtain

$$\mu(J;a_1,m,a_2) = \text{Max}\left\{\text{Min}\left[\frac{J-a_1}{m-a_1},\frac{a_2-J}{a_2-m}\right],0\right\} \qquad (19)$$

In what follows, the definition of the α-level set or α-cut of the fuzzy number $\tilde{J}$ is introduced.

Definition 2. [1] Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed non-empty universe. An intuitionistic fuzzy set IFS *A* in *X* is defined as

$$A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle | x \in X\} \qquad (20)$$

which is characterized by a membership function $\mu_A : X \to [0,1]$ and a non-membership function $\upsilon_A : X \to [0,1]$ with the condition $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ for all $x \in X$ where μ_A and υ_A represent ,respectively, the degree of membership and non-membership of the element *x* to the set *A*. In addition, for each IFS *A* in X, $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$ for all $x \in X$ is called the degree of hesitation of the element *x* to the set A. Especially, if $\pi_A(x) = 0$, , then the IFS *A* is degraded to a fuzzy set.

Definition 3. [4] The α-level set of the fuzzy parameters $\tilde{J}$ in problem (1) is defined as the ordinary set $L_\alpha\left(\tilde{J}\right)$ for which the degree of membership function exceeds the level, α, $\alpha \in [0,1]$, where:

$$L_\alpha\left(\tilde{J}\right) = \left\{J \in R \,\middle|\, \mu_{\tilde{J}}\left(J\right) \geq \alpha\right\} \tag{21}$$

For certain values α_j^* to be in the unit interval.

Definition 4. [10] Let X be a space of points (objects) and $x \in X$. A neutrosophic set A in X is defined by a truth-membership function (x), an indeterminacy-membership function (x) and a falsity-membership function $F(x)$. It has been shown in figure 2. (x), (x) and $F(x)$ are real standard or real nonstandard subsets of $]0^-,1^+[$. That is $T_A(x){:}X \to]0^-,1^+[$, $I_A(x){:}X \to]0^-,1^+[$ and $F_A(x){:}X \to]0^-,1^+[$. There is not restriction on the sum of (x), (x) and $F(x)$, so $0^- \leq \sup T_A(x) \leq \sup I_A(x) \leq F_A(x) \leq 3^+$.

In the following, we adopt the notations $\mu(x)$, $\sigma_A(x)$ and $v_A(x)$ instead of $T_A(x)$, $I_A(x)$ and $F_A(x)$, respectively. Also, we write SVN numbers instead of single valued neutrosophic numbers.

Definition 5. [10] Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form

$$A=\{\langle x, \mu_A(x), \sigma_A(x), v_A(x)\rangle : x \in X\},$$

where $\mu_A(x){:}X \to [0,1]$, $\sigma_A(x){:}X \to [0,1]$ and $v_A(x){:}X \to [0,1]$ with $0 \leq \mu_A(x)+\sigma_A(x)+v_A(x) \leq 3$ for all $x \in X$. The intervals $\mu(x)$, $\sigma_A(x)$ and $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

For convenience, a SVN number is denoted by $A=(a,b,c)$, where $a,b,c \in [0,1]$ and $a+b+c \leq 3$.

Definition 6 Let $\tilde{J}$ be a neutrosophic triangular number in the set of real numbers R, then its truth-membership function is defined as

$$T_{\tilde{J}}\left(J\right) = \begin{cases} \dfrac{J-a_1}{a_2-a_1}, & a_1 \leq J \leq a_2, \\ \dfrac{a_2-J}{a_3-a_2}, & a_2 \leq J \leq a_3, \\ 0, & otherwise. \end{cases} \tag{22}$$

its indeterminacy-membership function is defined as

$$I_{\tilde{J}}(J)=\begin{cases}\dfrac{J-b_1}{b_2-b_1}, & b_1\le J\le b_2,\\ \dfrac{b_2-J}{b_3-b_2}, & b_2\le J\le b_3,\\ 0, & otherwise.\end{cases} \tag{23}$$

and its falsity-membership function is defined as

$$F_{\tilde{J}}(J)=\begin{cases}\dfrac{J-c_1}{c_2-c_1}, & c_1\le J\le c_2,\\ \dfrac{c_2-J}{c_3-c_2}, & c_2\le J\le c_3,\\ 1, & otherwise.\end{cases} \tag{24}$$

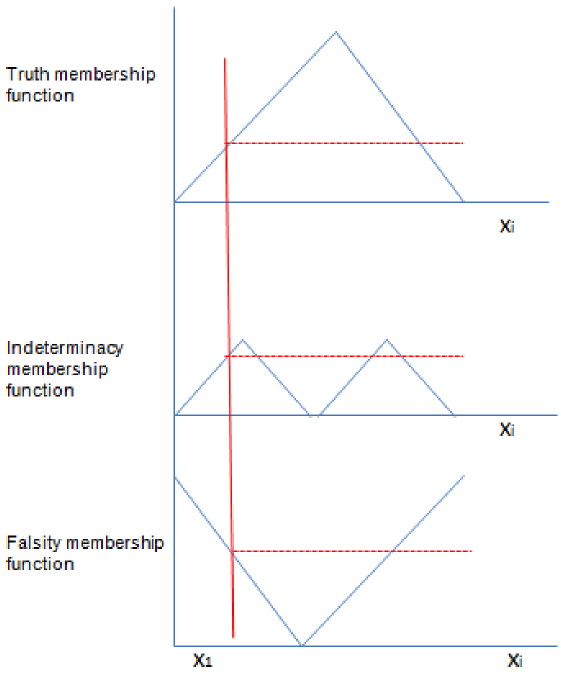

Fig. 4: Neutrosophication process [11]

3 Formation of The Problem

The multi-objective linear programming problem and the multi- objective neutrosophic linear programming problem are described in this section.

A. Multi-objective Programming Problem (MOPP)

In this chapter, the general mathematical model of the MOPP is as follows [6]:

$$\min/\max \left[z_1(x_1,...,x_n), z_2(x_1,...,x_n),...,z_p(x_1,...,x_n)\right] \quad (8)$$

$$\textit{subject to } x \in S, x \geq 0$$

$$S = \left\{ x \in R^n \middle| AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \quad X \geq 0. \right\} \quad (25)$$

B. Neutrosophic Multi-objective Programming Problem (NMOPP)

If an imprecise aspiration level is introduced to each of the objectives of MOPP, then these neutrosophic objectives are termed as neutrosophic goals.

Let $z_i \in \left[z_i^L, z_i^U\right]$ denote the imprecise lower and upper bounds respectively for the i^{th} neutrosophic objective function.

For maximizing objective function, the truth membership, indeterminacy membership, falsity membership functions can be expressed as follows:

$$\mu_i^I(z_i) = \begin{cases} 1, & if \quad z_i \geq z_i^U, \\ \dfrac{z_i - z_i^L}{z_i^U - z_i^L}, & if \quad z_i^L \leq z_i \leq z_i^U, \\ 0, & if \quad z_i \leq z_i^L \end{cases} \quad (26)$$

$$\sigma_i^I(z_i) = \begin{cases} 1, & if \quad z_i \geq z_i^U, \\ \dfrac{z_i - z_i^L}{z_i^U - z_i^L}, & if \quad z_i^L \leq z_i \leq z_i^U, \\ 0, & if \quad z_i \leq z_i^L \end{cases} \quad (27)$$

$$\upsilon_i^I(z_i) = \begin{cases} 0, & if \quad z_i \geq z_i^U, \\ \dfrac{z_i - z_i^L}{z_i^U - z_i^L}, & if \quad z_i^L \leq z_i \leq z_i^U, \\ 1, & if \quad z_i \leq z_i^L \end{cases} \quad (28)$$

For minimizing objective function, the truth membership, indeterminacy membership, falsity membership functions can be expressed as follows:

$$\mu_i^I(z_i)=\begin{cases}1, & if \quad z_i \le z_i^L,\\ \dfrac{z_i^U - z_i}{z_i^U - z_i^L}, & if \quad z_i^L \le z_i \le z_i^U,\\ 0, & if \quad z_i \ge z_i^U\end{cases} \tag{29}$$

$$\sigma_i^I(z_i)=\begin{cases}1, & if \quad z_i \le z_i^L,\\ \dfrac{z_i^U - z_i}{z_i^U - z_i^L}, & if \quad z_i^L \le z_i \le z_i^U,\\ 0, & if \quad z_i \ge z_i^U\end{cases} \tag{30}$$

$$\upsilon_i^I(z_i)=\begin{cases}0, & if \quad z_i \le z_i^L,\\ \dfrac{z_i^U - z_i}{z_i^U - z_i^L}, & if \quad z_i^L \le z_i \le z_i^U.\\ 1, & if \quad z_i \ge z_i^U\end{cases} \tag{31}$$

4 Algorithm for Neutrosophic Multi-Objective Programming Problem

The computational procedure and proposed algorithm of presented model is given as follows:

Step 1. Determine $x_i^* = \left(x_{i1}^*, x_{i2}^*, ..., x_{in}^*\right)$ that is used to maximize or minimize the i^{th} truth membership function $\mu_i^I(X)$, the indeterminacy membership $\sigma_i^I(X)$, and the falsity membership functions $\upsilon_i^I(X)$. *i=1,2,..,p* and *n* is the number of variables.

Step 2. Transform the truth membership, indeterminacy membership, falsity membership functions by using first-order Taylor polynomial series

$$\mu_i^I(x) \cong \mu_i^I\left(x_i^*\right) + \sum_{j=1}^{n}\left(x_j - x_{ij}^*\right)\frac{\partial \mu_i^I\left(x_i^*\right)}{\partial x_j} \tag{32}$$

$$\sigma_i^I(x) \cong \sigma_i^I\left(x_i^*\right) + \sum_{j=1}^{n}\left(x_j - x_{ij}^*\right)\frac{\partial \sigma_i^I\left(x_i^*\right)}{\partial x_j}$$

(33)

$$\upsilon_i^I\left(x\right) \cong \upsilon_i^I\left(x_i^*\right) + \sum_{j=1}^{n}\left(x_j - x_{ij}^*\right)\frac{\partial \upsilon_i^I\left(x_i^*\right)}{\partial x_j} \tag{34}$$

Step 3. Find satisfactory $x_i^* = \left(x_{i1}^*, x_{i2}^*, ..., x_{in}^*\right)$ by solving the reduced problem to a single objective for the truth membership, indeterminacy membership, falsity membership functions respectively.

$$p\left(x\right) = \sum_{i=1}^{p}\left[\mu_i^I\left(x_i^*\right) + \sum_{j=1}^{n}\left(x_j - x_{ij}^*\right)\frac{\partial \mu_i^I\left(x_i^*\right)}{\partial x_j}\right]$$

$$q\left(x\right) = \sum_{i=1}^{p}\left[\sigma_i^I\left(x_i^*\right) + \sum_{j=1}^{n}\left(x_j - x_{ij}^*\right)\frac{\partial \sigma_i^I\left(x_i^*\right)}{\partial x_j}\right] \tag{35}$$

$$h\left(x\right) = \sum_{i=1}^{p}\left[\upsilon_i^I\left(x_i^*\right) + \sum_{j=1}^{n}\left(x_j - x_{ij}^*\right)\frac{\partial \upsilon_i^I\left(x_i^*\right)}{\partial x_j}\right]$$

Thus neutrosophic multiobjective linear programming problem is converted into a new mathematical model and is given below:

Maximize or Minimize *p(x)*

Maximize or Minimize *q(x)*

Maximize or Minimize *h(x)*,

where $\mu_i^I\left(X\right)$, $\sigma_i^I\left(X\right)$ and $\upsilon_i^I\left(X\right)$ calculate using equations (10), (11), and (12) or equations (13), (14), and (15) according to type functions maximum or minimum respectively.

4.1 Illustrative Example

A multi-criteria supplier selection is selected from [2]. For supplying a new product to a market assume that three suppliers should be managed. The purchasing criteria are net price, quality and service. The capacity constraints of suppliers are also considered.

It is assumed that the input data from suppliers' performance on these criteria are not known precisely. The neutrosophic values of their cost, quality and service level are presented in Table 1.

The multi-objective linear formulation of numerical example is presented as min z_1, max z_2, z_3:

$$
\begin{aligned}
&\min\ z_1 = 5x_1 + 7x_2 + 4x_3,\\
&\max\ z_2 = 0.80x_1 + 0.90x_2 + 0.85x_3,\\
&\max\ z_3 = 0.90x_1 + 0.80x_2 + 0.85x_3,\\
&s.t.:\\
&x_1 + x_2 + x_3 = 800,\\
&x_1 \le 400,\\
&x_2 \le 450,\\
&x_3 \le 450,\\
&x_i \ge 0,\ i = 1,2,3.
\end{aligned}
$$

Table 1: Suppliers quantitative information

	Z1 Cost	Z2Quality (%)	Z3 Service (%)	Capacity
Supplier 1	5	0.80	0.90	400
Supplier 2	7	0.90	0.80	450
Supplier 3	4	0.85	0.85	450

The truth membership, indeterminacy membership, falsity membership functions were considered to be neutrosophic triangular. When they depend on three scalar parameters (a1, m, a2). z_1 depends on neutrosophic aspiration levels (3550,4225,4900), when z_2 depends on neutrosophic aspiration levels (660,681.5,702.5), and z3 depends on neutrosophic aspiration levels (657.5,678.75,700).

The truth membership functions of the goals are obtained as follows:

$$
\mu_1^I(z_1) = \begin{cases} 0, & if \quad z_1 \le 3550, \\ \dfrac{4225 - z_1}{4225 - 3550}, & if \quad 3550 \le z_1 \le 4225, \\ \dfrac{4900 - z_1}{4900 - 4225}, & if \quad 4225 \le z_1 \le 4900, \\ 0, & if \quad z_1 \ge 4900 \end{cases}
$$

$$
\mu_2^I(z_2) = \begin{cases} 0, & if \quad z_2 \ge 702.5, \\ \dfrac{z_2 - 681.5}{702.5 - 681.5}, & if \quad 681.5 \le z_2 \le 702.5, \\ \dfrac{z_2 - 660}{681.5 - 660}, & if \quad 660 \le z_2 \le 681.5, \\ 0, & if \quad z_2 \le 660. \end{cases}
$$

$$\mu_3^I(z_3)=\begin{cases}0, & if \quad z_3 \geq 700,\\ \dfrac{z_3-678.75}{700-678.75}, & if \quad 678.75 \leq z_3 \leq 700,\\ \dfrac{z_3-657.5}{678.75-657.5}, & if \quad 657.5 \leq z_3 \leq 678.75,\\ 0, & if \quad z_3 \leq 657.5.\end{cases}$$

If $\mu_1^I(z_1)=\max\left(\min\left(\dfrac{4225-(5x_1+7x_2+4x_3)}{675},\dfrac{4900-(5x_1+7x_2+4x_3)}{675},0\right)\right)$

$$\mu_2^I(z_2)=\min(\max(\frac{(0.8x_1+0.9x_2+0.85x_3)-681.5}{21}, \frac{(0.8x_1+0.9x_2+0.85x_3)-660}{21},1))$$

$$\mu_3^I(z_3)=\min(\max(\frac{(0.9x_1+0.8x_2+0.85x_3)-678.75}{21.25}, \frac{(0.9x_1+0.8x_2+0.85x_3)-657.5}{21.25},1))$$

Then $\mu_1^{I*}(350,0,450)$, $\mu_2^{I*}(0,450,350)$, $\mu_3^{I*}(400,0,400)$

The truth membership functions are transformed by using first-order Taylor polynomial series

$$\hat{\mu}_1^I(x)=\mu_1^I(350,0,450)+\left[(x_1-350)\frac{\partial\mu_1^I(350,0,450)}{\partial x_1}\right] +\left[(x_2-0)\frac{\partial\mu_1^I(350,0,450)}{\partial x_2}\right]+\left[(x_3-450)\frac{\partial\mu_1^I(350,0,450)}{\partial x_3}\right]$$

$$\hat{\mu}_1^I(x)=-0.00741x_1-0.0104x_2-0.00593x_3+5.2611$$

In the similar way, we get

$$\hat{\mu}_2^I(x)=0.0381x_1+0.0429x_2+0.0405x_3-33.405$$

$$\hat{\mu}_3^I(x)=0.042x_1+0.037x_2+0.0395x_3-32.512$$

The *p(x)* is

$$p(x)=\hat{\mu}_1^I(x)+\hat{\mu}_2^I(x)+\hat{\mu}_3^I(x)$$

$$P(x) = 0.07259x_1 + 0.0695x_2 + 0.0741x_3 - 60.6559$$

$$\begin{aligned} & s.t.: \\ & x_1 + x_2 + x_3 = 800, \\ & x_1 \le 400, \\ & x_2 \le 450, \\ & x_3 \le 450, \\ & x_i \ge 0,\ i = 1,2,3. \end{aligned}$$

The linear programming software LINGO 15.0 is used to solve this problem. The problem is solved and the optimal solution for the truth membership model is obtained is as follows: (x_1, x_2, x_3) = (350, 0,450) z_1=3550, z_2=662. 5, z_3=697. 5.

The truth membership values are $\mu_1 = 1,\ \mu_2 = 0.1163,\ \mu_3 = 0.894.$ The truth membership function values show that both goals z_1 , $z_3$and z_2 are satisfied with 100% , 11.63% and 89.4% respectively for the obtained solution which is $x1$=350; $x2$= 0, x3=450.

In the similar way, we get $\sigma_i^I(X)$, *q(x)*. Consequently, we get the optimal solution for the indeterminacy membership model is obtained is as follows: (x_1,x_2,x_3)=(350,0,450) z_1=3550, z_2=662.5, z_3=697.5 and the indeterminacy membership values are $\mu_1 = 1,\ \mu_2 = 0.1163,\ \mu_3 = 0.894.$ The indeterminacy membership function values show that both goals z_1 , $z_3$and z_2 are satisfied with 100% , 11.63% and 89.4% respectively for the obtained solution which is $x1$=350; $x2$= 0, x3=450.

In the similar way, we get $\upsilon_i^I(X)$ *and h(x)* Consequently, we get the optimal solution for the falsity membership model is obtained is as follows: (x_1,x_2,x_3)=(350,0,450) z_1=3550, z_2=662.5, z_3=697.5 and the falsity membership values are $\mu_1 = 0,\ \mu_2 = 0.8837,\ \mu_3 = 0.106.$ The falsity membership function values show that both goals z_1 , $z_3$and z_2 are satisfied with 0% , 88.37% and 10.6% respectively for the obtained solution which is $x1$=350; $x2$= 0, x3=450.

5 Conclusions and Future Work

In this chapter, we have proposed a solution to Neutrosophic Multiobjective Programming Problem (NMOPP). The truth membership, indeterminacy membership, falsity membership functions associated with each objective of the problem are transformed by using the first order Taylor polynomial series. The neutrosophic multi-objective programming problem is reduced to an equivalent multiobjective programming problem by the proposed method. The solution obtained from this method is very near to the solution of MOPP. Hence this method gives a more accurate solution as compared with other methods. Therefore, the complexity in solving NMOPP has reduced to easy computation. In the future studies, the proposed algorithm can be solved by metaheuristic algorithms.

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