Weakly Generalized Continuous Mappings in Neutrosophic Topological Spaces

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Abstract: In this paper we have introduced weakly generalized continuous mappings in neutrosophic topological spaces and analyzed some of their properties. We have discussed some characterizations of weakly generalized continuous mappings in neutrosophic topological spaces. **Mathematics Subject Classification (2010):** 54A40, 03E72

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I. Introduction

The concept of fuzzy sets and fuzzy logic was first introduced by L.Zadeh [17] in 1965. It shows the degree of membership of elements in a set. Later on, fuzzy topology was introduced by Chang [4] in 1968. K.Atanassov [2] in 1983, introduced intuitionistic fuzzy sets where the degree of both membership and non-membership of elements are discussed. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. In 1997, Coker [5] introduced the concept of intuitionistic fuzzy topological space. Atanassov (1999) [3] defined the interval-valued intuitionistic fuzzy set on a universe X as an object A = { $< (x, \mu_A(x), \sigma_A(x), \nu_A(x) >: x \in X$ }. Neutrosophic set theory was proposed in 1998 by Florentin Smarandache [7], who also developed the concept of single valued neutrosophic set, oriented towards real world scientific and engineering applications. A.A. Salama and S.A. Alblowi [13] introduced neutrosophic topological spaces by using the neutrosophic sets. ,A.A.Salma, Floretin Smarandache and Valeri Kromov[16] introduced the concept of continuous functions in neutrosophic topological spaces. S.Abinaya and D.Jayanthi [1] introduced weakly generalized closed sets in neutrosophic topological spaces. In this paper, we have introduced neutrosophic weakly generalized continuous mappings and neutrosophic weakly generalized irresolute mappings and discussed some of their characterizations.

II. Preliminaries

Here in this paper the neutrosophic topological space is denoted by (X,τ) . Also the neutrosophic interior, neutrosophic closure of a neutrosophic set A are denoted by NInt(A) and NCl(A). The complement of a neutrosophic set A is denoted by C(A) and the empty and whole sets are denoted by 0_N and 1_N respectively.

Definition 2.1: [7] Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form A = { $\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$: $x \in X$ } where $\mu_A(x), \sigma_A(x), \nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A. A neutrosophic set A = { $\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$: $x \in X$ } can be identified as an ordered triple $\langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ in]⁻⁰, 1 ⁺[on X.

Definition 2.2: [7] Let A= $\langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ be a NS on X, then the complement C(A) may be defined as

- 1. $C(A) = \{ \langle x, 1 \mu_A(x), 1 \nu_A(x) \rangle : x \in X \}$
- 2. $C(A) = \{ \langle x, v_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}$
- 3. C(A) = { $\langle x, v_A(x), 1 \sigma_A(x), \mu_A(x) \rangle$: $x \in X$ }

Note that for any two neutrosophic sets A and B,

- 4. $C(A \cup B) = C(A) \cap C(B)$
- 5. $C(A \cap B) = C(A) \cup C(B)$

Definition 2.3: [7] For any two neutrosophic sets A = { $\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$: $x \in X$ } and B = { $\langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle$: $x \in X$ } we may have

- 1. $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x) and \nu_A(x) \ge \nu_B(x) \forall x \in X$
- 2. $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x) and \nu_A(x) \ge \nu_B(x) \forall x \in X$
- 3. A \cap B = $\langle x, \mu_A(x) \land \mu_B(x), \sigma_A(x) \land \sigma_B(x), \nu_A(x) \lor \nu_B(x) \rangle$
- 4. A $\cap B = \langle x, \mu_A(x) \land \mu_B(x), \sigma_A(x) \lor \sigma_B(x), \nu_A(x) \lor \nu_B(x) \rangle$
- 5. A \cup B = $\langle x, \mu_A(x) \lor \mu_B(x), \sigma_A(x) \lor \sigma_B(x), \nu_A(x) \land \nu_B(x) \rangle$

6. AU $B = \langle x, \mu_A(x) \lor \mu_B(x), \sigma_A(x) \land \sigma_B(x), \nu_A(x) \land \nu_B(x) \rangle$

Definition 2.4: [13] A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

 $\begin{array}{ll} (\mathrm{NT}_1) & 0_{\mathrm{N}}, 1_{\mathrm{N}} \in \tau \\ (\mathrm{NT}_2) & G_1 \cap G_2 \in \tau \text{ for any } G_1, G_2 \in \tau \\ (\mathrm{NT}_3) & \cup G_i \in \tau \forall \{G_i : i \in J\} \subseteq \tau \end{array}$

In this case the pair (X,τ) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (NOS) in X. A neutrosophic set A is a neutrosophic closed set (NCS) if and only if its complement C(A) is a neutrosophic open set in X. Here the empty set (0_N) and the whole set (1_N) may be defined as follows:

 (0_1) $0_{\rm N} = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$ (0_2) $0_{\mathbb{N}} = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$ (0_3) $0_{N} = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$ (0_4) $0_{\rm N} = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$ $1_{N} = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$ (1_1) $1_{\mathbb{N}} = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$ (1_2) $1_{N} = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$ (1_3) $1_{N} = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$ (1_4)

Definition 2.5: [13] Let (X,τ) be a NTS and A= { $\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$: $x \in X$ } be a NS in X. Then the neutrosophic interior and the neutrosophic closure of A are defined by

NInt(A) = \cup { G: G is an NOS in X and G \subseteq A} NCl(A) = \cap { K: K is an NCS in X and A \subseteq K}

Note that for any NS A, NCl(C(A)) = C(NInt(A)) and NInt(C(A)) = C(NCl(A)).

Definition 2.6: [9] A NS A of a NTS X is said to be

(i) a neutrosophic pre-open set if $A \subseteq NInt(NCl(A))$

(ii) a neutrosophic semi-open set if $A \subseteq NCl(NInt(A))$

(iii) a neutrosophic α - open set if A \subseteq NInt(NCl(NInt(A)))

(iv) a neutrosophic semi- α -open set if A \subseteq NCl(α NInt(A))

Definition 2.7: [9] A NS A of a NTS X is said to be

(i) a neutrosophic pre-closed set if $NCl(NInt(A)) \subseteq A$

(ii) a neutrosophic semi-closed set if $NInt(NCl(A)) \subseteq A$

(iii) a neutrosophic α - closed set if NCl(NInt(NCl(A))) \subseteq A

(iv) a neutrosophic semi- α -closed set if NInt(α NCl(A)) \subseteq A

Definition 2.8:[10] A neutrosophic set A in a neutrosophic topological space X is said to be a neutrosophic regular closed set if NCl(NInt(A)) = A and neutrosophic regular open set if NInt(NCl(A)) = A.

Definition 2.9:[12] A neutrosophic set A in a neutrosophic topological space X is said to be a neutrosophic generalized closed set if $NCl(A) \subseteq U$ whenever $A \subseteq U$, where U is a neutrosophic open set. The complement of a neutrosophic generalized closed set is a neutrosophic generalized open set.

Definition 2.10:[10] A neutrosophic set A in a neutrosophic topological space X is said to be a neutrosophic α generalized closed set if N α Cl(A) \subseteq U whenever A \subseteq U, where U is a neutrosophic open set. The complement of a neutrosophic α generalized closed set is a neutrosophic α generalized open set.

Definition 2.11:[15] A neutrosophic set A in a neutrosophic topological space X is said to be a neutrosophic generalized semi closed set if $NsCl(A) \subseteq U$ whenever $A \subseteq U$, where U is a neutrosophic closed set. The complement of a neutrosophic generalized semi closed set is a neutrosophic generalized semi open set.

Definition 2.12:[1] A neutrosophic set A in a neutrosophic topological space X is said to be a neutrosophic weakly generalized closed set if $NCl(Int(A)) \subseteq U$ whenever $A \subseteq U$, where U is a neutrosophic closed set. The complement of a neutrosophic weakly generalized closed set is a neutrosophic weakly generalized open set.

Definition 2.13:[1] A neutrosophic topological space (X,τ) is called a neutrosophic ${}_{w}T_{1/2}$ space if every neutrosophic weakly generalized closed set in X is neutrosophic closed set in X.

Definition 2.14:[1] A neutrosophic topological space (X,τ) is called a neutrosophic ${}_{wg}T_{q}$ space if every neutrosophic weakly generalized closed set in X is a neutrosophic pre-closed set in X.

Definition 2.15:[14] Let (X,τ) and (Y,σ) be two neutrosophic topological spaces.

A mapping $f: (X,\tau) \to (Y,\sigma)$ is called neutrosophic continuous if the inverse image of every neutrosophic closed set in (Y,σ) is a neutrosophic closed set in (X,τ) .

Definition 2.16: [16] Let (X,τ) and (Y,σ) be two neutrosophic topological spaces. Then the

- (i) mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is called a neutrosophic α continuous if the inverse image of every neutrosophic closed set in (Y,σ) is a neutrosophic α closed set in (X,τ) .
- (ii) mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is called a neutrosophic semi continuous if the inverse image of every neutrosophic closed set in (Y,σ) is a neutrosophic semi closed set in (X,τ)
- (iii) mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is called a neutrosophic pre continuous if the inverse image of every neutrosophic closed set in (Y,σ) is a neutrosophic pre closed set in (X,τ) .

Definition 2.17: [8] Let (X,τ) and (Y,σ) be two neutrosophic topological spaces. A mapping

 $f: (X,\tau) \to (Y,\sigma)$ is called a neutrosophic generalized continuous mapping if $f^{-1}(B)$ is a neutrosophic generalized closed set in (X,τ) for every neutrosophic closed set B of (Y,σ) .

Definition 2.18:[11] Let (X,τ) and (Y,σ) be two neutrosophic topological spaces. A mapping

 $f: (X,\tau) \to (Y,\sigma)$ is called a neutrosophic α generalized continuous mapping if $f^{-1}(B)$ is a neutrosophic α generalized closed set in (X,τ) for every neutrosophic closed set B of (Y,σ) . **Definition 2.18:**[6] Let f be a mapping from a neutrosophic topological space (X,τ) into a neutrosophic topological space (Y,σ) . Then f is said to be a neutrosophic generalized irresolute mapping if the inverse image of every neutrosophic generalized closed set in (Y,σ) is a neutrosophic generalized closed set in (X,τ) .

III. Neutrosophic weakly generalized continuous mappings

In this section we have introduced neutrosophic weakly generalized continuous mappings and analyzed some of their properties. Also we have established the relation between the newly introduced mapping and already existing mappings.

Definition 3.1: Let (X,τ) and (Y,σ) be two neutrosophic topological spaces. A mapping $f: (X,\tau) \to (Y,\sigma)$ is called a neutrosophic generalized semi-continuous mapping if $f^{-1}(B)$ is a neutrosophic generalized semi-closed set in (X,τ) for every neutrosophic closed set B of (Y,σ) .

Definition 3.2: Let (X,τ) and (Y,σ) be two neutrosophic topological spaces. A mapping $f: (X,\tau) \to (Y,\sigma)$ is called a neutrosophic weakly generalized continuous mapping if $f^{-1}(B)$ is a neutrosophic weakly generalized closed set in (X,τ) for every neutrosophic closed set B of (Y,σ) .

Example 3.3: Let X= {a,b} and Y= {u,v}. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosophic topologies on X and Y respectively, where U = $\langle x, (0.5, 0.5), (0.3, 0.3), (0.5, 0.5) \rangle$ and V= $\langle y, (0.7, 0.6), (0.3, 0.3), (0.3, 0.3) \rangle$. Define a mapping f : $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set V^c = $\langle y, (0.3, 0.3), (0.3, 0.3), (0.3, 0.3), (0.7, 0.6) \rangle$ is a neutrosophic closed set in Y. Then f⁻¹(V^c) = $\langle x, (0.3, 0.3), (0.3, 0.3), (0.3, 0.3), (0.3, 0.3), (0.3, 0.3), (0.7, 0.6) \rangle$ is a neutrosophic closed set in (X, τ) as f⁻¹(V^c) \subseteq U implies NCl(NInt(f⁻¹(V^c))) = 0_N \subseteq U where U is a neutrosophic open set in X. Therefore f is a neutrosophic weakly generalized continuous mapping.

Proposition 3.4: Every neutrosophic continuous mapping is a neutrosophic weakly generalized continuous mapping but not conversely in general.

Proof: Let $f: (X,\tau) \to (Y,\sigma)$ be a neutrosophic continuous mapping. Let A be a neutrosophic closed set in Y. Then $f^{-1}(A)$ is a neutrosophic closed set in X. Since every neutrosophic closed set is a neutrosophic weakly generalized closed set in X [1], $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X. Hence f is a neutrosophic weakly generalized continuous mapping.

Example 3.5: Let X= {a,b} and Y= {u,v}. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosophic topologies on X and Y respectively, where U = $\langle x, (0.6, 0.7), (0.3, 0.3), (0.3, 0.3) \rangle$ and V= $\langle y, (0.4, 0.4), (0.3, 0.3), (0.6, 0.6) \rangle$. Define a mapping f : $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set V^c = $\langle y, (0.6, 0.6), (0.3, 0.3), (0.4, 0.4) \rangle$ is a neutrosophic closed set in Y. Then f⁻¹(V^c) = $\langle x, (0.6, 0.6), (0.3, 0.3), (0.4, 0.4) \rangle$ is a neutrosophic closed set in X as f⁻¹(V^c) \subseteq U implies NCl(NInt(f⁻¹(V^c))) = 0_N \subseteq U where U is a neutrosophic continuous mapping since V^c is a closed set in Y but its inverse image f⁻¹(V^c) is not a neutrosophic closed set in X as NCl(f⁻¹(V^c)) = 1_N \neq f⁻¹(V^c).

Proposition 3.6: Every neutrosophic α continuous mapping is a neutrosophic weakly generalized continuous mapping but not conversely in general.

Proof: Let $f: (X,\tau) \to (Y,\sigma)$ be a neutrosophic α continuous mapping. Let A be a neutrosophic closed set in Y. Then $f^{-1}(A)$ is a neutrosophic α closed set in X. Since every neutrosophic α closed set is a neutrosophic weakly generalized closed set in X [1], $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X. Hence f is a neutrosophic weakly generalized continuous mapping.

Example 3.7: Let X= {a,b} and Y= {u,v}. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosophic topologies on X and Y respectively, where U = $\langle x, (0.5, 0.4), (0.3, 0.3), (0.5, 0.6) \rangle$ and V= $\langle y, (0.5, 0.7), (0.3, 0.3), (0.3, 0.2) \rangle$. Define a mapping f : $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set V^c = $\langle y, (0.3, 0.2), (0.3, 0.3), (0.5, 0.7) \rangle$ is a neutrosophic closed set in Y. Then f⁻¹(V^c) = $\langle x, (0.3, 0.2), (0.3, 0.3), (0.5, 0.7) \rangle$ is a neutrosophic open set in X as f⁻¹(V^c) \subseteq U implies NCl(NInt(f⁻¹(V^c))) = 0_N \subseteq U where U is a neutrosophic α continuous mapping since V^c is a closed set in Y but its inverse image f⁻¹(V^c) is not a neutrosophic α closed set in X as NCl(NInt(NCl(f⁻¹(V^c)))) = U^c $\not\subseteq$ f⁻¹(V^c).

Proposition 3.8: Every neutrosophic generalized continuous mapping is a neutrosophic weakly generalized continuous mapping but the converse is not true in general.

Proof: Let $f: (X,\tau) \to (Y,\sigma)$ be a neutrosophic generalized continuous mapping. Let A be a neutrosophic closed set in Y. Then $f^{-1}(A)$ is a neutrosophic generalized closed set in X. Since every neutrosophic generalized closed set is a neutrosophic weakly generalized closed set in X [1], $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X. Hence f is a neutrosophic weakly generalized continuous mapping.

Example 3.9: Let X= {a,b} and Y= {u,v}. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosophic Y respectively. where U = $\langle x, (0.5, 0.4), (0.3, 0.3), (0.5, 0.6) \rangle$ topologies on Х and and $V = \langle y, (0.5, 0.7), (0.3, 0.3), (0.3, 0.2) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set $V^c = \langle y, (0.3, 0.2), (0.3, 0.3), (0.5, 0.7) \rangle$ is a neutrosophic closed set in Y. Then f⁻¹(V^c) = $\langle x, (0.3, 0.2), (0.3, 0.3), (0.5, 0.7) \rangle$ is a neutrosophic weakly generalized closed set in X as $f^{-1}(V^c) \subseteq U$ implies NCl(NInt($f^{-1}(V^c)$)) = $0_N \subseteq U$ where U is a neutrosophic open set in X. Hence f is a neutrosophic weakly generalized continuous mapping. But f is not a neutrosophic generalized continuous mapping since V^c is a closed set in Y but its inverse image $f^{-1}(V^c)$ is not a neutrosophic generalized closed set in X as $NCl(f^{-1}(V^c)) =$ U^c⊈U.

Proposition 3.10: Every neutrosophic pre-continuous mapping is a neutrosophic weakly generalized continuous mapping but not conversely in general.

Proof: Let $f: (X,\tau) \to (Y,\sigma)$ be a neutrosophic pre-continuous mapping. Let A be a neutrosophic closed set in Y. Then $f^{-1}(A)$ is a neutrosophic pre-closed set in X. Since every neutrosophic pre-closed set is a neutrosophic weakly generalized closed set in X [1], $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X. Hence f is a neutrosophic weakly generalized continuous mapping.

Example 3.11: Let X= {a,b} and Y= {u,v}. Then $\tau = \{0_N, U_1, U_2, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $U_1 = \langle x, (0.5, 0.6), (0.3, 0.3), (0.5, 0.2) \rangle$;

 $V^{c} = \langle y, (0.5, 0.5), (0.3, 0.3), (0.5, 0.5) \rangle$ is a neutrosophic closed set in Y. Then

 $f^{-1}(V^c) = \langle x, (0.5, 0.5), (0.3, 0.3), (0.5, 0.5) \rangle$ is a neutrosophic weakly generalized closed set in X as $f^{-1}(V^c) \subseteq U_1$ implies NCl(NInt($f^{-1}(V^c)$)) = $U_2^c \subseteq U_1$ where U_1 is a neutrosophic open set in X. Hence f is a neutrosophic weakly generalized continuous mapping. But f is not a neutrosophic pre-continuous mapping since V^c is a closed set in Y but its inverse image $f^{-1}(V^c)$ is not a neutrosophic pre-closed set in X as NCl(NInt($f^{-1}(V^c)$)) = $U_2^c \not\subseteq f^{-1}(V^c)$.

Proposition 3.12: Every neutrosophic α generalized continuous mapping is a neutrosophic weakly generalized continuous mapping but not conversely in general.

Proof: Let $f: (X,\tau) \to (Y,\sigma)$ be a neutrosophic α generalized continuous mapping. Let A be a neutrosophic closed set in Y. Then $f^{-1}(A)$ is a neutrosophic α generalized closed set in X. Since every neutrosophic α generalized closed set is a neutrosophic weakly generalized closed set in X [1], $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X. Hence f is a neutrosophic weakly generalized continuous mapping.

Example 3.13: Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.5, 0.6), (0.3, 0.3), (0.5, 0.4) \rangle$ and $V = \langle y, (0.6, 0.5), (0.3, 0.3), (0.3, 0.5) \rangle$. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set $V^c = \langle y, (0.3, 0.5), (0.3, 0.3), (0.6, 0.5) \rangle$ is a neutrosophic closed set in Y. Then $f^{-1}(V^c) = \langle x, (0.3, 0.5), (0.3, 0.3), (0.6, 0.5) \rangle$ is a neutrosophic open set in X as $f^{-1}(V^c) \subseteq U$ implies NCl(NInt($f^{-1}(V^c)$)) = $0_N \subseteq U$ where U is a neutrosophic open set in X. Hence f is a neutrosophic weakly generalized continuous mapping. But f is not a neutrosophic α generalized closed set in X as $N_{\alpha}Cl(f^{-1}(V^c)) = f^{-1}(V^c) \cup NCl(NInt(NCl(f^{-1}(V^c)))) = 1_N \nsubseteq U$.

Remark 3.14: Neutrosophic semi continuous mapping and neutrosophic weakly generalized continuous mapping are independent to each other in general.

Example 3.15: Let X= {a,b} and Y= {u,v}. Then $\tau = \{0_N, U_1, U_2, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $U_1 = \langle x, (0.5, 0.5), (0.3, 0.3), (0.1, 0.3) \rangle$;

 $U_2 = \langle x, (0.3, 0.2), (0.3, 0.3), (0.5, 0.5) \rangle$ and $V = \langle y, (0.5, 0.5), (0.3, 0.3), (0.5, 0.4) \rangle$. Define a mapping

 $f: (X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set

 $V^{c} = \langle y, (0.5, 0.4), (0.3, 0.3), (0.5, 0.5) \rangle$ is a neutrosophic closed set in Y. Then

 $f^{-1}(V^c) = \langle x, (0.5, 0.4), (0.3, 0.3), (0.5, 0.5) \rangle$ is a neutrosophic semi closed set in X as

 $NInt(NCl(f^{-1}(V^c))) = U_2 \subseteq f^{-1}(V^c)$ where U_2 is a neutrosophic open set in X. Hence f is a neutrosophic semi continuous mapping. But f is not a neutrosophic weakly generalized continuous mapping since V^c is a closed set in Y but its inverse image $f^{-1}(V^c)$ is not a neutrosophic weakly generalized closed set in X as $f^{-1}(V^c) \subseteq U_1$ implies $NCl(NInt(f^{-1}(V^c))) = U_2^c \nsubseteq U_1$.

Example 3.16: Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.5, 0.4), (0.3, 0.3), (0.5, 0.6) \rangle$ and $V = \langle y, (0.5, 0.7), (0.3, 0.3), (0.3, 0.2) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set $V^c = \langle y, (0.3, 0.2), (0.3, 0.3), (0.5, 0.7) \rangle$ is a neutrosophic closed set in Y. Then $f^{-1}(V^c) = \langle x, (0.3, 0.2), (0.3, 0.3), (0.5, 0.7) \rangle$ is a neutrosophic veakly generalized closed set in X as $f^{-1}(V^c) \subseteq U$ implies NCl(NInt($f^{-1}(V^c)$)) = $0_N \subseteq U$ where U is a neutrosophic semi continuous mapping since NInt(NCl($f^{-1}(V^c)$))) = $U \not\subseteq f^{-1}(V^c)$ implies $f^{-1}(V^c)$ is not a neutrosophic semi closed set in X.

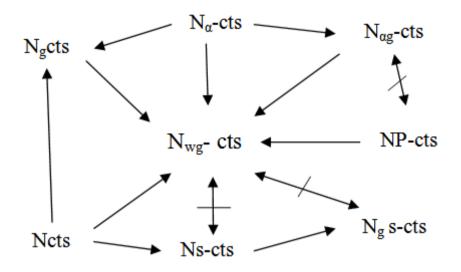
Remark 3.17: Neutrosophic generalized semi continuous mapping and neutrosophic weakly generalized continuous mapping are independent to each other in general.

Example 3.18: Let $X = \{a,b\}$ and $Y = \{u,v\}$. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosophic topologies on X and Y respectively, where $U = \langle x, (0.5, 0.4), (0.3, 0.3), (0.5, 0.5) \rangle$ and $V = \langle y, (0.5, 0.5), (0.3, 0.3), (0.5, 0.4) \rangle$. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set $V^c = \langle y, (0.5, 0.4), (0.3, 0.3), (0.5, 0.5) \rangle$ is a neutrosophic closed set in Y. Then $f^{-1}(V^c) = \langle x, (0.5, 0.4), (0.3, 0.3), (0.5, 0.5) \rangle$ is a neutrosophic set in X as $f^{-1}(V^c) \subseteq U$ implies NsCl(f^{-1}

 ${}^{1}(V^{c})) = f^{-1}(V^{c}) \cup NInt(NCl(f^{-1}(V^{c}))) = f^{-1}(V^{c}) \cup U \subseteq U$ where U is a neutrosophic open set in X. Hence f is a neutrosophic generalized semi continuous mapping. But f is not a neutrosophic weakly generalized continuous mapping since V^c is a closed set in Y but its inverse image $f^{-1}(V^{c})$ is not a neutrosophic weakly generalized continuous closed set in X as $f^{-1}(V^{c}) \subseteq U$ implies $NCl(NInt(f^{-1}(V^{c}))) = U^{c} \notin U$.

Example 3.19: Let X= {a,b} and Y= {u,v}. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosophic and respectively, where $= \langle \mathbf{x}, (0.8, 0.9), (0.3, 0.3), (0.2, 0.1) \rangle$ topologies on Х Y U and $V = \langle y, (0.3, 0.2), (0.3, 0.3), (0.7, 0.8) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here the neutrosophic set $V^c = \langle y, (0.7, 0.8), (0.3, 0.3), (0.3, 0.2) \rangle$ is a neutrosophic closed set in Y. Then f⁻¹(V^c) = $\langle x, x \rangle$ (0.7, 0.8), (0.3, 0.3), (0.3, 0.2) is a neutrosophic weakly generalized closed set in X as $f^{-1}(V^c) \subseteq U$ implies NCl(NInt($f^{-1}(V^c)$)) = $0_N \subseteq U$ where U is a neutrosophic open set in X. Hence f is a neutrosophic weakly generalized continuous mapping. But f is not a neutrosophic generalized semi continuous mapping since V^c is a closed set in Y but its inverse image $f^{-1}(V^c)$ is not a neutrosophic semi closed set in X as NsCl $(f^{-1}(V^c)) = f^{-1}(V^c)$ \cup NInt(NCl(f⁻¹(V^c))) = 1_N \nsubseteq U.

The relation between various types of neutrosophic continuous functions is diagrammatically represented below. In the diagram the reverse implications are not true in general.



Proposition 3.20: A mapping $f : (X,\tau) \to (Y,\sigma)$ is a neutrosophic weakly generalized continuous if and only if the inverse image of each neutrosophic open set in Y is a neutrosophic weakly generalized open set in X.

Proof: Necessity: Let A be a neutrosophic open set in Y. This implies A^c is neutrosophic closed set in Y. Since f is neutrosophic weakly generalized continuous, $f^{-1}(A^c)$ is a neutrosophic weakly generalized closed set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic weakly generalized open set in X.

Sufficiency: Let A be a neutrosophic closed set in Y. This implies A^c is neutrosophic open set in Y. By hypothesis, $f^{-1}(A^c)$ is a neutrosophic weakly generalized open set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X. Hence f is a neutrosophic weakly generalized continuous mapping.

Proposition 3.21: If $f : (X,\tau) \to (Y,\sigma)$ is a neutrosophic weakly generalized continuous mapping and $g : (Y,\sigma) \to (Z,\delta)$ is a neutrosophic continuous mapping, then $g \circ f : (X,\tau) \to (Z,\delta)$ is a neutrosophic weakly generalized continuous mapping.

Proof: Let A be a neutrosophic closed set in Z. Then $g^{-1}(A)$ is a neutrosophic closed set in Y, by hypothesis. Since f is a neutrosophic weakly generalized continuous mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic weakly generalized closed set in X. Hence $g \circ f$ is a neutrosophic weakly generalized continuous mapping.

Definition 3.22: Let (X,τ) be a neutrosophic topological space. The neutrosophic weakly generalized closure $(N_{wg}Cl(A))$ for any neutrosophic set A is defined as follows.

 $N_{wg}Cl(A) = \cap \{ K: K \text{ is a neutrosophic weakly generalized closed set in X and } A \subseteq K \}$. If A is a neutrosophic weakly generalized closed set, then $N_{wg}Cl(A) = A$.

Proposition 3.23: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic weakly generalized continuous mapping. Then the following conditions are hold.

- (i) $f(N_{wg}Cl(A)) \subseteq NCl(f(A))$, for every neutrosophic set A in X.
- (ii) $N_{wg}Cl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$, for every neutrosophic set B in Y.

Proof: (i) Since NCl(f(A)) is a neutrosophic closed set in Y and f is a neutrosophic weakly generalized continuous mapping, then $f^{-1}(NCl(f(A)))$ is a neutrosophic weakly generalized closed set in X. That is $N_{wg}Cl((f^{-1}(NCl(f(A)))) = f^{-1}(NCl(f(A))) =$

(ii)Replacing A by $f^{-1}(B)$ in (i), we get $f(N_{wg}Cl(f^{-1}(B))) \subseteq NCl(f(f^{-1}(B))) \subseteq NCl(B)$. Hence $N_{wg}Cl(f^{-1}(B)) \subseteq f^{-1}f(N_{wg}Cl(f^{-1}(B))) \subseteq f^{-1}(NCl(B))$, for every neutrosophic set B in Y.

Proposition 3.24: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic weakly generalized continuous mapping, then f is a neutrosophic continuous mapping, if X is a neutrosophic ${}_{w}T_{1/2}$ space.

Proof: Let A be a neutrosophic closed set in Y. Then $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X, by hypothesis. Since X is a neutrosophic ${}_{w}T_{1/2}$ space, $f^{-1}(A)$ is a neutrosophic closed in X. Hence f is a neutrosophic continuous mapping.

Proposition 3.25: Let $f: (X,\tau) \to (Y,\sigma)$ be a neutrosophic weakly generalized continuous mapping, then f is a neutrosophic pre-continuous mapping, if X is a neutrosophic $_{wg}T_q$ space.

Proof: Let A be a neutrosophic closed set in Y. Then $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X, by hypothesis. Since X is a neutrosophic $_{wg}T_q$ space, $f^{-1}(A)$ is a neutrosophic pre-closed set in X. Hence f is a neutrosophic pre-continuous mapping.

Proposition 3.26: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be a mapping from a neutrosophic topological space X into a neutrosophic topological space Y. Then the following conditions are equivalent if X is a neutrosophic ${}_{w}T_{1/2}$ space:

(i) f is a neutrosophic weakly generalized continuous mapping

(ii) If B is a neutrosophic open set in Y, then $f^{-1}(B)$ is a neutrosophic weakly generalized open set in X

(iii) $f^{-1}(NInt(B)) \subseteq NInt(NCl(f^{-1}(B)))$ for every neutrosophic set B in Y

Proof: (i) \Rightarrow (ii) is obviously true, since $f^{-1}(A^c) = (f^{-1}(A))^c$ for any neutrosophic set A in X and Y.

 $(ii) \Rightarrow (iii)$ Let B be any neutrosophic set in Y. Then NInt(B) is a neutrosophic open set in Y. Then f⁻¹(NInt(B)) is neutrosophic weakly generalized open set in X. Since X is a neutrosophic ${}_{w}T_{1/2}$ space, f⁻¹(NInt(B)) is a neutrosophic open set in X. Therefore, f⁻¹(NInt(B)) =NInt(f⁻¹(NInt(B))) \subseteq NInt(NCl(f⁻¹(B))).

(iii)⇒(i) Let B be a neutrosophic closed set in Y. Then its complement B^c is a neutrosophic open set in Y. By hypothesis, $f^1(NInt(B^c)) \subseteq NInt(NCl(f^1(B^c)))$. This implies

 $f^{-1}(B^c) \subseteq \text{NInt}(\text{NCl}(f^{-1}(B^c)))$. Hence $f^{-1}(B^c)$ is a neutrosophic pre-open set in X, since every neutrosophic pre-open set is a neutrosophic weakly generalized open set, $f^{-1}(B^c)$ is a neutrosophic weakly generalized open set in X. Therefore, $f^{-1}(B)$ is a neutrosophic weakly generalized closed set in X. Hence f is a neutrosophic weakly generalized continuous mapping.

Proposition 3.27: Let $f: (X,\tau) \to (Y,\sigma)$ be a mapping from a neutrosophic topological space X into a neutrosophic topological space Y. Then the following conditions are equivalent if X is a neutrosophic ${}_{w}T_{1/2}$ space:

(i) f is a neutrosophic weakly generalized continuous mapping

(ii) If $f^{-1}(B)$ is a neutrosophic weakly generalized closed set in X, for every neutrosophic closed set B in Y (iii) NCl(NInt($f^{-1}(A)$)) $\subseteq f^{-1}(NCl(A))$ for every neutrosophic set B in Y

Proof: (i) \Rightarrow (ii) is obviously true.

(ii) \Rightarrow (iii) Let A be any neutrosophic set in Y. Then NCl(A) is a neutrosophic closed set in Y. By hypothesis, f⁻¹ (NCl(A)) is a neutrosophic weakly generalized closed set in X. Since X is a neutrosophic $_{w}T_{1/2}$ space, f⁻¹ (NCl(A)) is a neutrosophic closed set in X. Therefore

 $NCl(f^{-1}(NCl(A)) = f^{1}(NCl(A)). Now NCl(NInt(f^{-1}(A))) \subseteq NCl(NInt(f^{-1}(NCl(A))) \subseteq NCl(f^{-1}(NCl(A))) = f^{1}(NCl(A)).$

 $(iii) \Rightarrow (i)$ Let A be a neutrosophic closed set in Y. Then by hypothesis

 $NCl(NInt(f^{-1}(A))) \subseteq f^{-1}(NCl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is a neutrosophic pre closed set in X and hence it is a neutrosophic weakly generalized closed set in X. Therefore f is a neutrosophic weakly generalized continuous mapping.

IV. Neutrosophic weakly generalized irresolute mappings

In this section we have introduced neutrosophic weakly generalized irresolute mappings and discussed some of their properties.

Definition 4.1: A mapping $f: (X,\tau) \to (Y,\sigma)$ is called a neutrosophic weakly generalized irresolute mapping if f ⁻¹(A) is a neutrosophic weakly generalized closed set in (X,τ) for every neutrosophic weakly generalized closed set A of (Y,σ) .

Example 4.2: Let X={a,b} and Y={u,v}. Then $\tau = \{0_N, U, 1_N\}$ and $\sigma = \{0_N, V, 1_N\}$ are neutrosphic topological spaces on X and Y respectively, where U= $\langle x, (0.5, 0.4), (0.3, 0.3), (0.5, 0.6) \rangle$ and

 $V = \langle y, (0.5, 0.6), (0.3, 0.3), (0.5, 0.6) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here $A = \langle y, (0.3, 0.2), (0.3, 0.3), (0.5, 0.7) \rangle$ is a neutrosophic weakly generalized closed set in Y since $A \subseteq V$ implies NCl(NInt(A)) = $0_N \subseteq V$. Then f is a neutrosophic weakly generalized irresolute mapping since $f^{-1}(A) \subseteq U$ implies NCl(NInt($f^{-1}(A)$)) = $0_N \subseteq U$, $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X.

Proposition 4.3: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic weakly generalized irresolute, then f is neutrosophic weakly generalized continuous mapping but not conversely.

Proof: Let f be a neutrosophic weakly generalized irresolute mapping. Let A be any neutrosophic closed set in Y. Since every neutrosophic closed set is a neutrosophic weakly generalized closed set [1], A is a neutrosophic weakly generalized closed set in Y. By hypothesis $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X. Hence f is a neutrosophic weakly generalized continuous mapping.

Example 4.4: Let X={a,b} and Y={u,v}. Then $\tau = \{0_N, U_1, U_2, 1_N\}$ and $\sigma =\{0_N, V, 1_N\}$ are neutrosphic topologies on X and Y respectively, where $U_1 = \langle x, (0.6, 0.7), (0.3, 0.3), (0.3, 0.3) \rangle$; $U_2 = \langle x, (0.2, 0.2), (0.3, 0.3), (0.5, 0.3) \rangle$ and $V = \langle y, (0.4, 0.4), (0.3, 0.3), (0.3, 0.2) \rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is a neutrosophic weakly generalized continuous mapping since for a neutrosophic closed set in Y, its inverse image $f^{-1}(V^c)$ is a neutrosophic weakly generalized closed set in X, as f $^{-1}(V^c) \subseteq U_1$ implies NCl(NInt($f^{-1}(V^c)$)) $\subseteq U_1$. But f is not a neutrosophic weakly generalized closed set in Y but $f^{-1}(A)$ is not a neutrosophic weakly generalized closed set in X as $f^{-1}(A) = \langle x, (0.3, 0.2), (0.3, 0.3), (0.4, 0.3) \rangle \subseteq U_1$ but NCl(NInt($f^{-1}(A)$)) = $U_2^c \notin U_1$. Hence f is not a neutrosophic weakly generalized irresolute mapping.

Proposition 4.5: If $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\delta)$ are any two neutrosophic weakly generalized irresolute mappings, then $g \circ f : (X,\tau) \to (Z,\delta)$ is a neutrosophic weakly generalized irresolute mapping.

Proof: Let A be a neutrosophic weakly generalized closed set in Z. Then $g^{-1}(A)$ is a neutrosophic weakly generalized closed set in Y, by hypothesis. Since f is a neutrosophic weakly generalized irresolute mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic weakly generalized closed set in X. Hence $g \circ f$ is a neutrosophic weakly generalized irresolute mapping.

Proposition 4.6: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic weakly generalized irresolute mapping and g: $(Y,\sigma) \to (Z,\delta)$ be a neutrosophic weakly generalized continuous mapping, then $g \circ f : (X,\tau) \to (Z,\delta)$ is a neutrosophic weakly generalized continuous mapping.

Proof: Let A be a neutrosophic closed set in Z. Then $g^{-1}(A)$ is a neutrosophic weakly generalized closed set in Y, by hypothesis. Since f is a neutrosophic weakly generalized irresolute mapping, $f^{-1}(g^{-1}(A))$ is a neutrosophic weakly generalized closed set in X. Hence $g \circ f$ is a neutrosophic weakly generalized continuous mapping.

Proposition 4.7: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic weakly generalized irresolute mapping and $g : (Y,\sigma) \to (Z,\delta)$ be a neutrosophic continuous mapping, then $g \circ f : (X,\tau) \to (Z,\delta)$ is a neutrosophic weakly generalized continuous mapping.

Proof: Let A be a neutrosophic closed set in Z. Then $g^{-1}(A)$ is a neutrosophic closed set in Y. Since every neutrosophic closed set is a neutrosophic weakly generalized closed set, $g^{-1}(A)$ is a neutrosophic weakly generalized closed set in Y. Therefore, $f^{-1}(g^{-1}(A))$ is a neutrosophic weakly generalized closed set in X, by hypothesis. Hence $g \circ f$ is a neutrosophic weakly generalized continuous mapping.

Proposition 4.8: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic weakly generalized irresolute mapping and g: $(Y,\sigma) \to (Z,\delta)$ be a neutrosophic α continuous mapping, then $g \circ f : (X,\tau) \to (Z,\delta)$ is a neutrosophic weakly generalized continuous mapping.

Proof: Let A be a neutrosophic closed set in Z. Then $g^{-1}(A)$ is a neutrosophic α closed set in Y, by hypothesis. Since every neutrosophic α closed set is a neutrosophic weakly generalized closed set, $g^{-1}(A)$ is a neutrosophic weakly generalized closed set in Y. Also, f is a neutrosophic weakly generalized irresolute mapping implies f⁻¹($g^{-1}(A)$) is a neutrosophic weakly generalized closed set in X. Hence $g \circ f$ is a neutrosophic weakly generalized closed set in C. Hence $g \circ f$ is a neutrosophic weakly generalized closed set in X.

Proposition 4.9: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic weakly generalized irresolute mapping and g: $(Y,\sigma) \to (Z,\delta)$ be a neutrosophic α generalized continuous mapping, then $g \circ f : (X,\tau) \to (Z,\delta)$ is a neutrosophic weakly generalized continuous mapping.

Proof: Let A be a neutrosophic closed set in Z. Then $g^{-1}(A)$ is a neutrosophic α generalized closed set in Y, by hypothesis. Since every neutrosophic α generalized closed set is a neutrosophic weakly generalized closed set, $g^{-1}(A)$ is a neutrosophic weakly generalized closed set in Y. Also, f is a neutrosophic weakly generalized irresolute mapping implies $f^{-1}(g^{-1}(A))$ is a neutrosophic weakly generalized closed set in X. Hence $g \circ f$ is a neutrosophic weakly generalized continuous mapping.

Proposition 4.10: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic weakly generalized irresolute mapping and g: $(Y,\sigma) \to (Z,\delta)$ be a neutrosophic generalized continuous mapping, then $g \circ f : (X,\tau) \to (Z,\delta)$ is a neutrosophic weakly generalized continuous mapping.

Proof: Let A be a neutrosophic closed set in Z. Then $g^{-1}(A)$ is a neutrosophic generalized closed set in Y, by hypothesis. Since every neutrosophic generalized closed set is a neutrosophic weakly generalized closed set, $g^{-1}(A)$ is a neutrosophic weakly generalized closed set in Y. Also, f is a neutrosophic weakly generalized irresolute mapping implies $f^{-1}(g^{-1}(A))$ is a neutrosophic weakly generalized closed set in X. Hence $g \circ f$ is a neutrosophic weakly generalized continuous mapping.

Proposition 4.11: Let $f : (X,\tau) \to (Y,\sigma)$ be a neutrosophic weakly generalized irresolute mapping and g: $(Y,\sigma) \to (Z,\delta)$ be a neutrosophic pre-continuous mapping, then $g \circ f : (X,\tau) \to (Z,\delta)$ is a neutrosophic weakly generalized continuous mapping.

Proof: Let A be a neutrosophic closed set in Z. Then $g^{-1}(A)$ is a neutrosophic pre-closed set in Y, by hypothesis. Since every neutrosophic pre-closed set is a neutrosophic weakly generalized closed set, $g^{-1}(A)$ is a neutrosophic weakly generalized closed set in Y. Also, f is a neutrosophic weakly generalized irresolute mapping implies f⁻¹($g^{-1}(A)$) is a neutrosophic weakly generalized closed set in X. Hence $g \circ f$ is a neutrosophic weakly generalized closed set in Z.

Proposition 4.12: A mapping $f : (X,\tau) \to (Y,\sigma)$ is a neutrosophic weakly generalized irresolute mapping if and only if the inverse image of each neutrosophic weakly generalized open set in Y is a neutrosophic weakly generalized open set in X.

Proof: Necessity: Let A be a neutrosophic weakly generalized open set in Y. Then A^c is a neutrosophic weakly generalized closed set in Y. Since f is neutrosophic weakly generalized irresolute, $f^{-1}(A^c)$ is neutrosophic weakly generalized closed set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic weakly generalized open set in X.

Sufficiency: Let A be a neutrosophic weakly generalized closed set in Y. This implies A^c is neutrosophic weakly generalized open set in Y. By hypothesis, $f^{-1}(A^c)$ is a neutrosophic weakly generalized open set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a neutrosophic weakly generalized closed set in X. Hence f is a neutrosophic weakly generalized irresolute mapping.

Proposition 4.13: Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping from a neutrosophic topological space X into a neutrosophic topological space Y. Then the following conditions are equivalent if X and Y are neutrosophic ${}_{w}T_{1/2}$ space:

- (i) f is a neutrosophic weakly generalized irresolute mapping
- (ii) f⁻¹(B) is a neutrosophic weakly generalized open set in X for each neutrosophic weakly generalized open set B in Y
- (iii) NCl($f^{-1}(B)$) $\subseteq f^{-1}(NCl(B))$ for each neutrosophic set B of Y

Proof: (i) \Rightarrow (ii) is obviously true from the proposition 4.12.

(ii) \Rightarrow (iii) Let B be any neutrosophic set in Y and B \subseteq NCl(B).Then f⁻¹(B) \subseteq f⁻¹(NCl(B)). Since NCl(B) is a neutrosophic closed set in Y, f⁻¹(NCl(B)) is a neutrosophic weakly generalized closed set in X, by hypothesis. Since X is a neutrosophic wT_{1/2} space, f⁻¹(NCl(B)) is a neutrosophic closed set in X. Hence NCl(f⁻¹(B)) \subseteq NCl(f⁻¹(NCl(B))) = f⁻¹(NCl(B)).

 $(iii) \Rightarrow (i)$ Let B be a neutrosophic weakly generalized closed set in Y. Since Y is a neutrosophic ${}_{w}T_{1/2}$ space, B is a neutrosophic closed set in Y and hence NCl(B) = B. Therefore, $f^{-1}(B) = f^{-1}(NCl(B)) \supseteq NCl(f^{-1}(B))$. But $f^{-1}(B) \subseteq NCl(f^{-1}(B))$. Therefore, $NCl(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is a neutrosophic closed set in X and hence it is a neutrosophic weakly generalized closed set in X. Thus f is a neutrosophic weakly generalized irresolute mapping.

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