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Open-Pit Mine Slope Stability Clustering Analysis and Assessment Models Based on an Inverse Hyperbolic Sine Similarity Measure of SVNSs

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Abstract: Slope instability is a common and typical problem of geological hazards, often accompanied by significant losses. So, it is necessary to provide some simple and effective methods to avoid the potential geological hazards of slope instability. It is obvious that the clustering and assessment of slope stability are very crucial. However, the existing clustering and assessment methods in the scenario of single-valued neutrosophic sets (SVNSs) imply some difficulties in engineering applications, such as a lot of collective sampling work, the complex training process, and the selection issue of different types of membership functions. Regarding these problems, this paper proposes an inverse hyperbolic sine similarity measure (IHSSM) of SVNSs and its netting clustering and assessment models for slope stability clustering analysis and evaluation based on the fuzzification process of the true, false, and uncertain Gaussian membership functions for slope sample data. Finally, the proposed clustering and assessment models are applied to the clustering analysis and assessment of 20 slope samples as the case study, and then comparing the results of clustering analysis and stability evaluation of the proposed models with those of the existing relative methods by the 20 slope samples, we verify the validity, consistency, and rationality of the proposed netting clustering and evaluation models.

Keywords: single-valued neutrosophic set; netting clustering method; Gaussian membership function; similarity measure; slope stability clustering analysis; slope stability evaluation

1. Introduction

Slope instability is a common phenomenon in geological hazards. Then, the occurrence of such hazards is often accompanied by significant losses. Therefore, it is crucial to establish effective methods to eliminate the potential risk of slope instability. Slope stability evaluation methods can be divided into two main categories: deterministic and uncertain methods. The traditional limit equilibrium method or analytical calculations using numerical methods are still commonly used in modern engineering [1]. The Sweden slice method [2] is the theoretical basis of the limit equilibrium method. After the refinement and improvement of the method by Janbu [3], Bishop [4], and Morgenstern & Price [5], the calculation results of the limit equilibrium method are more reasonable and accurate. The limit equilibrium method usually assumes that the mechanical properties of the slope rock mass are rigid bodies and the potential slip surface is flat or curved. However, due to the complex geological conditions of most slopes, the use of this method to assess slopes requires technicians or researchers with extensive experience in slope engineering. The finite element method [6], the strength discounting method [7], the discrete element method [8] and so on are all relatively common methods in numerical analysis.

Since the affecting factors of slope stability contain both the internal features of slopes and the external conditions of slopes, there is the uncertain, inconsistent, and incomplete information of the affecting factors. However, they cannot be well described by traditional analogical approaches. Therefore, to avoid this deficiency of the traditional methods, some researchers proposed uncertainty methods for the analysis and assessment of slope stability. For example, a FAHP (fuzzy analytic hierarchy process) efficiency coefficient approach was used for the stability classification of rock slope [9], the artificial neural network (ANN) and fuzzy clustering methods were applied to the estimation of rainfall-induced landslides [10]. Then, clustering methods using the adaptive neuro-fuzzy inference system (ANFIS) and k-means and fuzzy c-means were applied to the clustering analysis of slope stability [11, 12]. Recently, the similarity measures of interval-valued fuzzy credibility sets were presented and applied to the slope stability assessment [13]. Since a neutrosophic number (NN) $y = v + qI$ for $I \circledcirc\in [\inf I, \sup I]$ [14, 15, 16], which is composed of the certain part v and the uncertain part qI with uncertainty I, can better describe the uncertainty of a real thing, Li et al. [17] introduced a probabilistic method based on NNs for the assessment of rock slope stability. Subsequently, Zhou et al. [18] and Li et al. [19] proposed some similarity measures of NNs to assess the slope stability of open-pit mines. However, these clustering analysis/assessment methods only contain fuzzy/uncertain information, but do not take into account the true, false, and uncertain information about the factors that affect slope stability.

In view of an extension of the fuzzy set (FS) [20] and (interval-valued) intuitionistic FS [21, 22], a neutrosophic set (NS) concept was proposed by Smarandache [14] and described by the true, false, and uncertain membership functions. To better apply NSs in practical engineering, Wang et al. [23, 24] proposed single-valued NSs (SVNSs) and interval NSs (IVNSs) as the subclasses of NSs. Recently, Qin et al. [25] first applied SVNS and ANFIS to open-pit mine slope stability evaluation and proposed a SVNS-ANFIS evaluation approach for assessing slope stability. Moreover, Qin et al. [26] introduced a SVNS-GPR (gaussian process regression) approach for the aassessment of open-pit mine slope stability in terms of the potential relationships between the affecting factors and the slope stability. Ding and Ye [27] presented the clustering analysis and evaluation models of slope stability based on the hyperbolic sine similarity measure (HSSM) of SVNSs and its netting clustering and assessment approaches, then applied them to the stability clustering analysis and assessment of slope sample data.

Based on the previous studies, the SVNN-ANFIS and SVNS-GPR methods [25, 26] need a lot of slope sample data to train them, and their modeling algorithms imply complexity, which presents the difficult problems of extensive sampling work and a complex training process in engineering applications. Then in existing slope stability clustering analysis and evaluation models [27], it is difficult to select many different types of true, false, and uncertain membership functions in the fuzzified process of ample slope data, which will greatly increase difficulty in practical engineering applications. To solve these problems, the objective of this paper is to propose an inverse hyperbolic sine similarity measure (IHSSM) of SVNSs and its clustering analysis and evaluation models of slope stability by a unique type of Gaussian membership functions for fuzzifying slope sample data into SVNSs, including the true, false, and uncertain membership values.

In our study, we first propose the IHSSM of SVNSs and its netting clustering and evaluation models of slope stability based on the true, false, and uncertain Gaussian membership functions for fuzzifying slope sample data into SVNSs. Through 20 slope samples collected in the Zhejiang Province, China, the proposed models are used for the clustering analysis and evaluation of their stability. By comparing with existing relevant approaches, we verify the accuracy and rationality of the proposed models in the clustering and evaluation applications of the 20 slope samples.

The rest of this paper consists of the following sections. In Section 2, some preliminaries of SVNSs are introduced. Section 3 proposes the IHSSM of SVNSs and its netting clustering model for slope stability clustering analysis. In Section 4, an assessment model based on the IHSSM of SVNSs and the clustered results is proposed for the stability assessment of slopes. Section 5 applies the proposed clustering and assessment models to the stability clustering analysis and assessment of the 20 slope samples as the case study to verify the consistency and accuracy of the clustered and evaluated results by comparing the 20 slope samples with the existing approaches. Section 6 concludes the paper and provides further research directions in the future.

2. Some preliminaries of SVNSs

Wang et al. [24] introduced SVNS as a subclass of NS.

Definition 1 [24]. Let Ψ be a universal set. The SVNS Φ in Ψ can be represented as $\Phi = \{<\psi, X_\Phi(\psi), Y_\Phi(\psi), Z_\Phi(\psi)> \mid \psi \in \Psi\}$, where $X_\Phi(\psi), Y_\Phi(\psi), Z_\Phi(\psi) \in [0, 1]$ for $\psi \in \Psi$ are the true, uncertain, and false membership functions. Then, each element $\varphi = <\psi, X_\Phi(\psi), Y_\Phi(\psi), Z_\Phi(\psi)>$ in the SVNS Φ is represented as the single-valued neutrosophic number (SVNN) $\varphi = <X, Y, Z>$.

Definition 2 [24]. If there are two SVNSs $\varphi_1 = <X_1, Y_1, Z_1>$ and $\varphi_2 = <X_2, Y_2, Z_2>$, then they include the following relations:

(1) Mutual inclusion: $\varphi_1 \subseteq \varphi_2$ if and only if $X_1 \le X_2, Y_1 \ge Y_2, Z_1 \ge Z_2$;

(2) Mutual equality: $\varphi_1 = \varphi_2$ if and only if $\varphi_1 \subseteq \varphi_2$ and $\varphi_2 \subseteq \varphi_1$;

(3) The complement of φ_1: $\varphi_1^c = \langle Z_1, 1-Y_1, X_1 \rangle$;

(4) Union: $\varphi_1 \bigcup \varphi_2 = \langle X_1 \vee X_2, Y_1 \wedge Y_2, Z_1 \wedge Z_2 \rangle$;

(5) Intersection: $\varphi_1 \bigcap \varphi_2 = \langle X_1 \wedge X_2, Y_1 \vee Y_2, Z_1 \vee Z_2 \rangle$;

Definition 3 [28]. Assume that $\Phi_1 = \{\varphi_{11}, \varphi_{12}, \ldots, \varphi_{1n}\}$ and $\Phi_2 = \{\varphi_{21}, \varphi_{22}, \ldots, \varphi_{2n}\}$ are two SVNSs, where $\varphi_{1i} = <X_{1i}, Y_{1i}, Z_{1i}>$ and $\varphi_{2i} = <X_{2i}, Y_{2i}, Z_{2i}>$ ($i = 1, 2, \ldots, n$) are two SVNNs. Then, $\rho_i \in [0, 1]$ with $\sum_{i=1}^{n} \rho_i = 1$ is the weight of φ_{1i} and φ_{2i}. The weighted generalized distance between Φ_1 and Φ_2 is defined as follows:

$$V_\delta(\Phi_1, \Phi_2) = \left\{ \frac{1}{3} \sum_{i=1}^{n} \rho_i \left[\left| X_{1i} - X_{2i} \right|^\delta + \left| Y_{1i} - Y_{2i} \right|^\delta + \left| Z_{1i} - Z_{2i} \right|^\delta \right] \right\}^{1/\delta} \text{ for } \delta \ge 1. \quad (1)$$

The distance of Eq. (1) satisfies the following features:

(1) $0 \le V_\delta(\Phi_1, \Phi_2) \le 1$;

(2) $V_\delta(\Phi_1, \Phi_2) = 0$ if and only if $\Phi_1 = \Phi_2$;

(3) $V_\delta(\Phi_1, \Phi_2) = V_\delta(\Phi_2, \Phi_1)$;

(4) If $\Phi_1 \subseteq \Phi_2 \subseteq \Phi_3$ for any SVNS Φ_3, then $V_\delta(\Phi_1, \Phi_3) \ge V_\delta(\Phi_1, \Phi_2)$ and $V_\delta(\Phi_1, \Phi_3) \ge V_\delta(\Phi_2, \Phi_3)$.

Since the similarity measure and the distance are complementary, the similarity measure using the weighted generalized distance of SVNSs is presented as follows [28]:

$$W_\delta(\Phi_1, \Phi_2) = 1 - V_\delta(\Phi_1, \Phi_2) = 1 - \left\{ \frac{1}{3} \sum_{i=1}^{n} \rho_i \left[\left| X_{1i} - X_{2i} \right|^\delta + \left| Y_{1i} - Y_{2i} \right|^\delta + \left| Z_{1i} - Z_{2i} \right|^\delta \right] \right\}^{1/\delta}. \quad (2)$$

Thus, Eq. (2) also satisfies the following features [28]:

(1) $0 \le W_\delta(\Phi_1, \Phi_2) \le 1$;

(2) $W_\delta(\Phi_1, \Phi_2) = 1$ if and only if $\Phi_1 = \Phi_2$;

(3) $W_\delta(\Phi_1, \Phi_2) = W_\delta(\Phi_2, \Phi_1)$;

(4) If $\Phi_1 \subseteq \Phi_2 \subseteq \Phi_3$ for any SVNS Φ_3, then $W_\delta(\Phi_1, \Phi_2) \ge W_\delta(\Phi_1, \Phi_3)$ and $W_\delta(\Phi_2, \Phi_3) \ge W_\delta(\Phi_1, \Phi_3)$.

Recently, Ding and Ye [27] further presented the HSSM between the SVNSs Φ_1 and Φ_2:

$$N_\delta(\Phi_1, \Phi_2) = 1 - \sinh\left\{(\ln(1+\sqrt{2})V_\delta(\Phi_1, \Phi_2)\right\}$$
$$= 1 - \sinh\left\{\ln(1+\sqrt{2})\left(\frac{1}{3}\sum_{i=1}^{n}\rho_i\left[\left|X_{1i} - X_{2i}\right|^\delta + \left|Y_{1i} - Y_{2i}\right|^\delta + \left|Z_{1i} - Z_{2i}\right|^\delta\right]\right)^{1/\delta}\right\}. \quad (3)$$

Similarly, Eq. (3) also has the following features:

(1) $0 \le N_\delta(\Phi_1, \Phi_2) \le 1$;

(2) $N_\delta(\Phi_1, \Phi_2) = 1$ if and only if $\Phi_1 = \Phi_2$;

(3) $N_\delta(\Phi_1, \Phi_2) = N_\delta(\Phi_2, \Phi_1)$;

(4) If $\Phi_1 \subseteq \Phi_2 \subseteq \Phi_3$ for any SVNS Φ_3, then $N_\delta(\Phi_1, \Phi_2) \ge N_\delta(\Phi_1, \Phi_3)$ and $N_\delta(\Phi_2, \Phi_3) \ge N_\delta(\Phi_1, \Phi_3)$.

3. IHSSM between SVNSs and Its Netting Clustering Model of Slope Stability

Based on the weighted generalized distances of SVNSs, this section proposes the IHSSM of SVNSs and its netting clustering model of slope stability.

First, IHSSM for SVNSs is defined below.

Definition 4. Suppose that $\Phi_1 = \{\varphi_{11}, \varphi_{12}, \ldots, \varphi_{1n}\}$ and $\Phi_2 = \{\varphi_{21}, \varphi_{22}, \ldots, \varphi_{2n}\}$ are two SVNSs, where $\varphi_{1i} = \langle X_{1i}, Y_{1i}, Z_{1i}\rangle$ and $\varphi_{2i} = \langle X_{2i}, Y_{2i}, Z_{2i}\rangle$ ($i = 1, 2, \ldots, n$) are two SVNNs. Then, $\rho_i \in [0, 1]$ with $\sum_{i=1}^{n}\rho_i = 1$ is the weight of φ_{1i} and φ_{2i}. Thus, the weighted IHSSM between Φ_1 and Φ_2 is defined as follows:

$$M_\delta(\Phi_1, \Phi_2) = 1 - \frac{1}{\ln(1+\sqrt{2})}\sinh^{-1}\left\{V_\delta(\Phi_1, \Phi_2)\right\}$$
$$= 1 - \frac{1}{\ln(1+\sqrt{2})}\sinh^{-1}\left\{\frac{1}{3}\sum_{i=1}^{n}\rho_i\left[\left|X_{1i} - X_{2i}\right|^\delta + \left|Y_{1i} - Y_{2i}\right|^\delta + \left|Z_{1i} - Z_{2i}\right|^\delta\right]\right\}^{1/\delta}. \quad (4)$$

Then, the features of Eq. (4) are indicated as follows:

(1) $0 \le M_\delta(\Phi_1, \Phi_2) \le 1$;

(2) $M_\delta(\Phi_1, \Phi_2) = 1$ if and only if $\Phi_1 = \Phi_2$;

(3) $M_\delta(\Phi_1, \Phi_2) = M_\delta(\Phi_2, \Phi_1)$;

(4) If $\Phi_1 \subseteq \Phi_2 \subseteq \Phi_3$ for any SVNS Φ_3, then $M_\delta(\Phi_1, \Phi_2) \ge M_\delta(\Phi_1, \Phi_3)$ and $M_\delta(\Phi_2, \Phi_3) \ge M_\delta(\Phi_1, \Phi_3)$.

Proof: Since the features (1)–(3) are clearly valid, we only verify the feature (4).

According to the features of the distance $V_\delta(\Phi_1, \Phi_2)$, if $\Phi_1 \subseteq \Phi_2 \subseteq \Phi_3$, then $V_\delta(\Phi_1, \Phi_3) \ge V_\delta(\Phi_1, \Phi_2)$ and $V_\delta(\Phi_2, \Phi_3) \ge V_\delta(\Phi_1, \Phi_3)$ exist. Since $\sinh^{-1}(\alpha)$ for $\alpha \in [0, 1]$ is monotonically increasing, the inequalities $M_\delta(\Phi_1, \Phi_2) \ge M_\delta(\Phi_1, \Phi_3)$ and $M_\delta(\Phi_2, \Phi_3) \ge M_\delta(\Phi_1, \Phi_3)$ exist based on the complementary relationship between the distance and the similarity measure [28]. Hence, we verify that the feature (4) is correct.

In terms of the proposed weighted IHSSM of SVNSs, a netting clustering model is presented below to cluster slope sample data.

In the clustering analysis of slope sample data, $\Omega = \{\Omega_1, \Omega_2, \ldots, \Omega_m\}$ is a sample set of m slopes and $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ is a set of n indices/factors that impact slope stability. Then, each affecting factor λ_i needs to consider its weight ρ_i subject to $\rho_i \in [0, 1]$ and $\sum_{i=1}^{n}\rho_i = 1$.

In order to better express the uncertain and inconsistent information of the slop sample data, we use Gaussian membership functions (e.g. Figure 1) to fuzzify the true, uncertain, and false information of the sample data. In view of the true, false, and uncertain Gaussian membership functions, the affecting factors of slope stability are fuzzified into the SVNSs $\Phi_j = \{\varphi_{j1}, \varphi_{j2}, \ldots, \varphi_{jn}\}$, where $\varphi_{ji} = <X_{ji}, Y_{ji}, Z_{ji}>$ are SVNNs for $X_{ji}, Y_{ji}, Z_{ji} \in [0, 1]$ ($j = 1, 2, \ldots, m$; $i = 1, 2, \ldots, n$).

Then, the netting clustering model based on the IHSSM of SVNSs is used for the clustering analysis of the slope sample data and presented by the following clustering procedures.

Step 1: Obtain the IHSSM matrix $B = (\beta_{js})_{m\times m}$ (j, $s = 1, 2, \ldots, m$) by Eq. (4) (usually taking $\delta = 1$), where $\beta_{js} = M_\delta(\Phi_j, \Phi_s)$ with $\beta_{js} = \beta_{sj}$ and $\beta_{jj} = 1$.

Step 2: Replace all the diagonal elements in the IHSSM matrix B by the slope samples Ω_j.

Step 3: In terms of different confidence levels of ζ, the corresponding ζ-cutting matrices $B^\zeta=(\beta_{js}^\zeta)_{m\times m}$ are gained by the equation:

$$\beta_{js}^{\zeta} = \begin{cases} 0, \beta_{js} < \zeta \\ 1, \beta_{js} \geq \zeta \end{cases} (j, s = 1, 2, \ldots, m). \tag{5}$$

In the adjusted process for ζ, all '0' are funded in the ζ-cutting matrixes and deleted, then all '1' are replaced by '*' except for the diagonal elements. Then, '*' is connected to the corresponding diagonal elements by horizontal and vertical lines. The slope samples connected by '*' are formed as a classification according to the corresponding confidence level ζ. Next, the confidence level of ζ is adjusted from large to small until the expected clustering result for the slope sample data is achieved.

4. Slope Stability Assessment Using the IHSSM of SVNSs

Because the above clustered results cannot indicate their corresponding stability risk levels of slope samples, it is necessary to evaluate the stability risk levels of the slope samples so that we can decide which risk levels the slope samples belong to. In this section, we give an assessment model of slope stability based on the IHSSM of SVNSs.

In terms of the existing knowledge and the above clustered results of slope stability, we can classify the risk states of slope stability into the corresponding risk levels (risk patterns), which can be represented as the SVNSs $\Theta_k = \{\theta_{k1}, \theta_{k2}, \ldots, \theta_{kn}\}$ including the SVNNs $\theta_{ki} = <X_{ki}, Y_{ki}, Z_{ki}>$ for $X_{ki}, Y_{ki}, Z_{ki} \in [0, 1]$ ($k = 1, 2, \ldots, r$; $i = 1, 2, \ldots, n$).

Assume that $\Omega = \{\Omega_1, \Omega_2, \ldots, \Omega_m\}$ is a sample set of m slopes and a set of n affecting factors $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ affects slope stability. Based on the true, false and uncertain Gaussian membership functions, the slope sample data can be fuzzified into the SVNSs $\Phi_j = \{\varphi_{j1}, \varphi_{j2}, \ldots, \varphi_{jn}\}$ ($j = 1, 2, \ldots, m$), which contains the SVNNs $\varphi_{ji} = <X_{ji}, Y_{ji}, Z_{ji}>$ for $X_{ji}, Y_{ji}, Z_{ji} \in [0, 1]$ ($i = 1, 2, \ldots, n$).

Considering the weight of each affecting factor in the slope stability assessment, we assign the weight of each affecting factor by $\rho_i \in [0,1]$ with $\sum_{i=1}^{n}\rho_i = 1$. Thus, the weighted IHSSM between each SVNS Φ_j ($j = 1, 2, \ldots, n$) for each slope sample Ω_j and each slope stability risk level Θ_k ($k = 1, 2, \ldots, r$) is presented by the following equation:

$$\begin{aligned} M_\delta(\Phi_j, \Theta_k) &= 1 - \frac{1}{\ln(1+\sqrt{2})}\sinh^{-1}\left\{V_\delta(\Phi_j, \Theta_k)\right\} \\ &= 1 - \frac{1}{\ln(1+\sqrt{2})}\sinh^{-1}\left\{\frac{1}{3}\sum_{i=1}^{n}\rho_i\left[\left|X_{ji} - X_{ki}\right|^\delta + \left|Y_{ji} - Y_{ki}\right|^\delta + \left|Z_{ji} - Z_{ki}\right|^\delta\right]\right\}^{1/\delta}. \end{aligned} \tag{6}$$

Applying Eq. (6), we can obtain the IHSSM results, and then use $M_\delta(\Phi_j, \Theta_{k^*}) = \max_{1\leq k\leq r}\left(M_\delta(\Phi_j, \Theta_k)\right)$ to judge that Ω_j should belong to Θ_{k^*}.

5. Netting Clustering and Assessment applications of Practical Cases

5.1. Netting Clustering Analysis of Practical Cases

The mountainous terrain of Zhejiang Province in China, with its subtropical monsoon climate, accounts for 74.6%, making slope instability a very common geological hazard. Therefore, the clustering analysis and assessment of slope stability show their importance and necessity. In terms of rock and topographic features in Zhejiang Province, we take into account the lithological association (λ_1), slope structure (λ_2), weathering degree of rock (λ_3), slope height (λ_4), slope angle (λ_5), and vegetation coverage (λ_6) as the main factors affecting slope stability. According to the important degrees of these affecting factors, their weight vector is assigned as $\boldsymbol{\rho}$ = (0.25, 0.21, 0.19, 0.13, 0.11, 0.11) by experts/decision makers. Then, we collected 20 slope samples as actual cases in Zhejiang Province. According to the results of engineering investigation and measurement, we also provide the actual data of 20 slope samples Ω_j (j = 1, 2, …, 20) for the clustering analysis. We can score from 0 to 10 depending on the current situation of slope stability. Thus, λ_1, λ_2, λ_3, and λ_6 are assigned by the score values, and then λ_4, and λ_5 are given by the actual measured values, which are shown in Table 1.

Table 1. Actual data of 20 slope samples

Ω_j	λ_1	λ_2	λ_3	λ_4 (m)	λ_5 (°)	λ_6
Ω_1	2.0	5.0	4.0	15.0	84.0	3.0
Ω_2	4.0	3.0	4.0	8.0	84.0	3.0
Ω_3	8.0	7.0	6.0	15.0	55.0	4.0
Ω_4	3.0	4.0	5.0	9.0	76.0	4.0
Ω_5	2.0	4.0	4.0	8.0	84.0	5.0
Ω_6	10.0	8.0	10.0	36.0	63.0	5.0
Ω_7	7.0	7.0	6.0	23.0	76.0	5.0
Ω_8	8.0	8.0	7.2	32.0	63.0	3.0
Ω_9	5.0	9.0	4.0	15.0	63.0	5.0
Ω_{10}	3.0	3.0	6.0	11.0	76.0	4.0
Ω_{11}	3.0	6.0	5.0	21.0	71.0	5.0
Ω_{12}	7.0	10.0	7.0	26.8	76.0	5.0
Ω_{13}	9.0	10.0	10.0	28.0	76.0	5.0
Ω_{14}	5.0	10.0	3.0	19.0	73.0	3.0
Ω_{15}	4.0	2.0	3.0	18.0	74.0	3.0
Ω_{16}	2.0	6.0	3.0	24.0	45.0	4.0
Ω_{17}	7.0	9.0	6.0	60.0	70.0	4.0
Ω_{18}	7.0	8.0	8.0	32.0	72.0	4.0
Ω_{19}	4.0	4.0	4.0	19.0	39.0	4.0
Ω_{20}	3.0	4.0	3.0	17.0	83.0	5.0

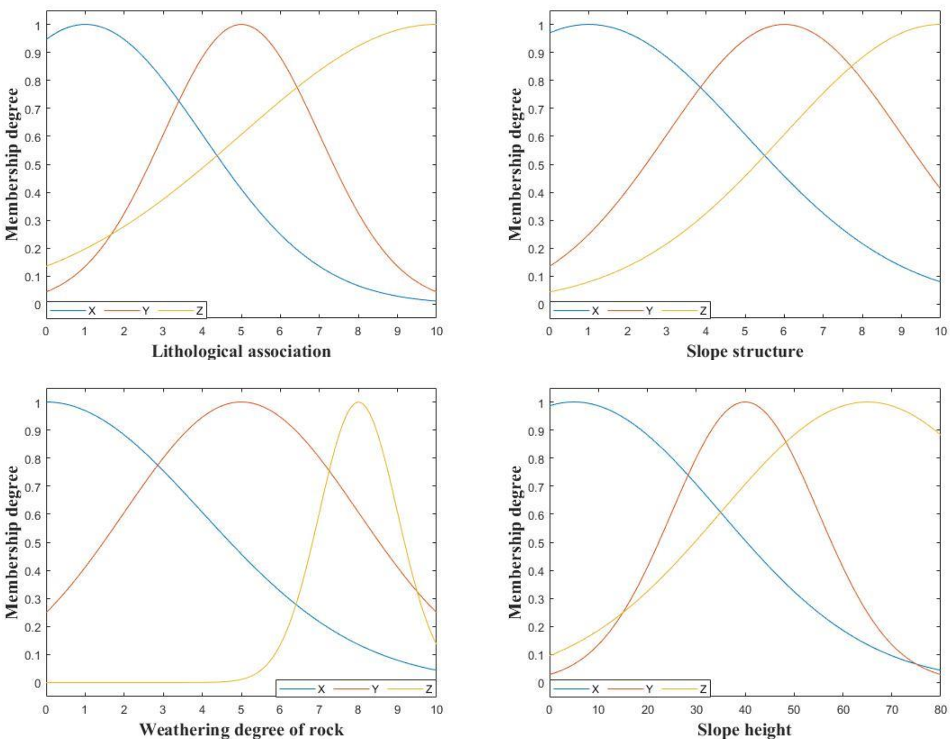
Table 2. Gaussian membership functions of the 6 affecting factors

Affecting factor	Gaussian membership function		
	X	Y	Z
(λ_1)	gaussmf[3 1]	gaussmf[2 5]	gaussmf[5 10]
(λ_2)	gaussmf[4 1]	gaussmf[3 6]	gaussmf[4 10]
(λ_3)	gaussmf[4 0]	gaussmf[3 5]	gaussmf[1 8]
(λ_4)	gaussmf[30 5]	gaussmf[15 40]	gaussmf[30 65]
(λ_5)	gaussmf[15 30]	gaussmf[25 50]	gaussmf[30 45]
(λ_6)	gaussmf[4 0]	gaussmf[3 5.5]	gaussmf[2 10]

In terms of the data of the 6 affecting factors, we use Gaussian membership functions in Table 2 to fuzzify them into the form of SVNNs. In Table 2, the true, uncertain, and false Gaussian membership functions of the six affecting factors are provided to fuzzify the slope sample data, and then the Gaussian membership degree curves of truth (X), uncertainty (Y), and falsity (*Z*) are shown in Figure 1. Thus, the 6 affecting factors of the 20 slope samples Ω_j (j = 1, 2, …, 20) are fuzzified into the corresponding SVNSs Φ_j, which are shown in Table 3.

Table 3. SVNSs of 20 slope samples

Φ_j	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
Φ_1	(0.946, 0.325, 0.278)	(0.607, 0.946, 0.458)	(0.607, 0.946, 0.000)	(0.946, 0.249, 0.249)	(0.002, 0.397, 0.430)	(0.755, 0.707, 0.002)
Φ_2	(0.607, 0.882, 0.487)	(0.882, 0.607, 0.216)	(0.607, 0.946, 0.000)	(0.995, 0.103, 0.164)	(0.002, 0.397, 0.430)	(0.755, 0.707, 0.002)
Φ_3	(0.066, 0.325, 0.923)	(0.325, 0.946, 0.755)	(0.325, 0.946, 0.135)	(0.946, 0.249, 0.249)	(0.249, 0.980, 0.946)	(0.607, 0.882, 0.011)
Φ_4	(0.801, 0.607, 0.375)	(0.755, 0.801, 0.325)	(0.458, 1.000, 0.011)	(0.991, 0.118, 0.175)	(0.009, 0.582, 0.586)	(0.607, 0.882, 0.011)
Φ_5	(0.946, 0.325, 0.278)	(0.755, 0.801, 0.325)	(0.607, 0.946, 0.000)	(0.995, 0.103, 0.164)	(0.002, 0.397, 0.430)	(0.458, 0.986, 0.044)
Φ_6	(0.011, 0.044, 1.000)	(0.216, 0.801, 0.882)	(0.044, 0.249, 0.135)	(0.586, 0.965, 0.627)	(0.089, 0.874, 0.835)	(0.458, 0.986, 0.044)
Φ_7	(0.135, 0.607, 0.835)	(0.325, 0.946, 0.755)	(0.325, 0.946, 0.135)	(0.835, 0.526, 0.375)	(0.009, 0.582, 0.586)	(0.458, 0.986, 0.044)
Φ_8	(0.066, 0.325, 0.923)	(0.216, 0.801, 0.882)	(0.216, 0.801, 0.607)	(0.667, 0.867, 0.546)	(0.089, 0.874, 0.835)	(0.755, 0.707, 0.002)
Φ_9	(0.411, 1.000, 0.607)	(0.135, 0.607, 0.969)	(0.607, 0.946, 0.000)	(0.946, 0.249, 0.249)	(0.089, 0.874, 0.835)	(0.458, 0.986, 0.044)
Φ_{10}	(0.801, 0.607, 0.375)	(0.882, 0.607, 0.216)	(0.325, 0.946, 0.135)	(0.980, 0.154, 0.198)	(0.009, 0.582, 0.586)	(0.607, 0.882, 0.011)
Φ_{11}	(0.801, 0.607, 0.375)	(0.458, 1.000, 0.607)	(0.458, 1.000, 0.011)	(0.867, 0.448, 0.341)	(0.024, 0.703, 0.687)	(0.458, 0.986, 0.044)
Φ_{12}	(0.135, 0.607, 0.835)	(0.080, 0.411, 1.000)	(0.216, 0.801, 0.607)	(0.768, 0.679, 0.445)	(0.009, 0.582, 0.586)	(0.458, 0.986, 0.044)
Φ_{13}	(0.029, 0.135, 0.980)	(0.080, 0.411, 1.000)	(0.044, 0.249, 0.135)	(0.745, 0.726, 0.467)	(0.009, 0.582, 0.586)	(0.458, 0.986, 0.044)
Φ_{14}	(0.411, 1.000, 0.607)	(0.080, 0.411, 1.000)	(0.755, 0.801, 0.000)	(0.897, 0.375, 0.309)	(0.016, 0.655, 0.647)	(0.755, 0.707, 0.002)
Φ_{15}	(0.607, 0.882, 0.487)	(0.969, 0.411, 0.135)	(0.755, 0.801, 0.000)	(0.910, 0.341, 0.293)	(0.014, 0.631, 0.627)	(0.755, 0.707, 0.002)
Φ_{16}	(0.946, 0.325, 0.278)	(0.458, 1.000, 0.607)	(0.755, 0.801, 0.000)	(0.818, 0.566, 0.393)	(0.607, 0.980, 1.000)	(0.607, 0.882, 0.011)
Φ_{17}	(0.135, 0.607, 0.835)	(0.135, 0.607, 0.969)	(0.325, 0.946, 0.135)	(0.186, 0.411, 0.986)	(0.029, 0.726, 0.707)	(0.607, 0.882, 0.011)
Φ_{18}	(0.135, 0.607, 0.835)	(0.216, 0.801, 0.882)	(0.135, 0.607, 1.000)	(0.667, 0.867, 0.546)	(0.020, 0.679, 0.667)	(0.607, 0.882, 0.011)
Φ_{19}	(0.607, 0.882, 0.487)	(0.755, 0.801, 0.325)	(0.607, 0.946, 0.000)	(0.897, 0.375, 0.309)	(0.835, 0.908, 0.980)	(0.607, 0.882, 0.011)
Φ_{20}	(0.801, 0.607, 0.375)	(0.755, 0.801, 0.325)	(0.755, 0.801, 0.000)	(0.923, 0.309, 0.278)	(0.002, 0.418, 0.448)	(0.458, 0.986, 0.044)



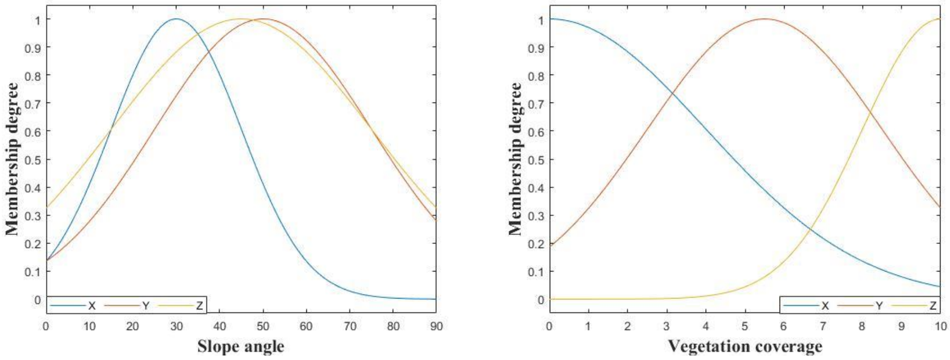


Figure 1. The Gaussian membership degree curves of the 6 affecting factors

Thus, we can calculate the IHSSM values between SVNSs of the slope samples by Eq. (4) for δ = 1, and then establish the IHSSM matrix B:

$$B=\begin{bmatrix}
1.0000 & 0.8144 & 0.7130 & 0.8608 & 0.9266 & 0.5175 & 0.7059 & 0.6064 & 0.6874 & 0.8159 & 0.8395 & 0.5655 & 0.5306 & 0.6784 & 0.7525 & 0.8356 & 0.5959 & 0.5543 & 0.7591 & 0.8629 \\
0.8144 & 1.0000 & 0.6336 & 0.8659 & 0.8361 & 0.4634 & 0.6775 & 0.5506 & 0.7618 & 0.8819 & 0.7536 & 0.5884 & 0.5042 & 0.7527 & 0.9095 & 0.6540 & 0.6190 & 0.5484 & 0.8483 & 0.8435 \\
0.7130 & 0.6336 & 1.0000 & 0.6995 & 0.6689 & 0.7632 & 0.8801 & 0.8300 & 0.7588 & 0.6914 & 0.7435 & 0.7345 & 0.7268 & 0.6841 & 0.6088 & 0.7345 & 0.7894 & 0.7380 & 0.7079 & 0.6548 \\
0.8608 & 0.8659 & 0.6995 & 1.0000 & 0.9073 & 0.5259 & 0.7441 & 0.5889 & 0.7256 & 0.9400 & 0.8862 & 0.6242 & 0.5390 & 0.6898 & 0.8080 & 0.7542 & 0.6551 & 0.6127 & 0.8420 & 0.9211 \\
0.9266 & 0.8361 & 0.6689 & 0.9073 & 1.0000 & 0.5186 & 0.6847 & 0.5593 & 0.6885 & 0.8554 & 0.8177 & 0.5666 & 0.5316 & 0.6303 & 0.7470 & 0.7905 & 0.5747 & 0.5334 & 0.7808 & 0.9085 \\
0.5175 & 0.4634 & 0.7632 & 0.5259 & 0.5186 & 1.0000 & 0.7491 & 0.8373 & 0.6403 & 0.5182 & 0.6025 & 0.7274 & 0.8837 & 0.5867 & 0.4738 & 0.5818 & 0.7326 & 0.7839 & 0.5318 & 0.5342 \\
0.7059 & 0.6775 & 0.8801 & 0.7441 & 0.6847 & 0.7491 & 1.0000 & 0.7923 & 0.7765 & 0.7360 & 0.8283 & 0.8527 & 0.7630 & 0.7396 & 0.6611 & 0.6958 & 0.8501 & 0.8172 & 0.7002 & 0.7325 \\
0.6064 & 0.5506 & 0.8300 & 0.5889 & 0.5593 & 0.8373 & 0.7923 & 1.0000 & 0.6825 & 0.5811 & 0.6443 & 0.8371 & 0.7716 & 0.6774 & 0.5613 & 0.6459 & 0.7834 & 0.8788 & 0.5949 & 0.5751 \\
0.6874 & 0.7618 & 0.7588 & 0.7256 & 0.6885 & 0.6403 & 0.7765 & 0.6825 & 1.0000 & 0.7116 & 0.7705 & 0.7475 & 0.6595 & 0.8994 & 0.7363 & 0.6784 & 0.7791 & 0.6723 & 0.7827 & 0.7257 \\
0.8159 & 0.8819 & 0.6914 & 0.9400 & 0.8554 & 0.5182 & 0.7360 & 0.5811 & 0.7116 & 1.0000 & 0.8335 & 0.6455 & 0.5598 & 0.6758 & 0.8307 & 0.7101 & 0.6766 & 0.6049 & 0.7973 & 0.8759 \\
0.8395 & 0.7536 & 0.7435 & 0.8862 & 0.8177 & 0.6025 & 0.8283 & 0.6443 & 0.7705 & 0.8335 & 1.0000 & 0.6840 & 0.5974 & 0.7259 & 0.7445 & 0.8430 & 0.7079 & 0.6647 & 0.7960 & 0.8667 \\
0.5655 & 0.5884 & 0.7345 & 0.6242 & 0.5666 & 0.7274 & 0.8527 & 0.8371 & 0.7475 & 0.6455 & 0.6840 & 1.0000 & 0.8417 & 0.7748 & 0.6212 & 0.5821 & 0.8340 & 0.8622 & 0.5816 & 0.6327 \\
0.5306 & 0.5042 & 0.7268 & 0.5390 & 0.5316 & 0.8837 & 0.7630 & 0.7716 & 0.6595 & 0.5598 & 0.5974 & 0.8417 & 1.0000 & 0.6863 & 0.5361 & 0.5470 & 0.7489 & 0.7537 & 0.4976 & 0.5473 \\
0.6784 & 0.7527 & 0.6841 & 0.6898 & 0.6303 & 0.5867 & 0.7396 & 0.6774 & 0.8994 & 0.6758 & 0.7259 & 0.7748 & 0.6863 & 1.0000 & 0.8155 & 0.6672 & 0.7538 & 0.6751 & 0.7300 & 0.7183 \\
0.7525 & 0.9095 & 0.6088 & 0.8080 & 0.7470 & 0.4738 & 0.6611 & 0.5613 & 0.7363 & 0.8307 & 0.7445 & 0.6212 & 0.5361 & 0.8155 & 1.0000 & 0.6856 & 0.6102 & 0.5591 & 0.8390 & 0.8371 \\
0.8356 & 0.6540 & 0.7345 & 0.7542 & 0.7905 & 0.5818 & 0.6958 & 0.6459 & 0.6784 & 0.7101 & 0.8430 & 0.5821 & 0.5470 & 0.6672 & 0.6856 & 1.0000 & 0.6009 & 0.5855 & 0.7831 & 0.7831 \\
0.5959 & 0.6190 & 0.7894 & 0.6551 & 0.5747 & 0.7326 & 0.8501 & 0.7834 & 0.7791 & 0.6766 & 0.7079 & 0.8340 & 0.7489 & 0.7538 & 0.6102 & 0.6009 & 1.0000 & 0.8005 & 0.6345 & 0.6212 \\
0.5543 & 0.5484 & 0.7380 & 0.6127 & 0.5334 & 0.7839 & 0.8172 & 0.8788 & 0.6723 & 0.6049 & 0.6647 & 0.8622 & 0.7537 & 0.6751 & 0.5591 & 0.5855 & 0.8005 & 1.0000 & 0.5849 & 0.5988 \\
0.7591 & 0.8483 & 0.7079 & 0.8420 & 0.7808 & 0.5318 & 0.7002 & 0.5949 & 0.7827 & 0.7973 & 0.7960 & 0.5816 & 0.4976 & 0.7300 & 0.8390 & 0.7831 & 0.6345 & 0.5849 & 1.0000 & 0.8294 \\
0.8629 & 0.8435 & 0.6548 & 0.9211 & 0.9085 & 0.5342 & 0.7325 & 0.5751 & 0.7257 & 0.8759 & 0.8667 & 0.6327 & 0.5473 & 0.7183 & 0.8371 & 0.7831 & 0.6212 & 0.5988 & 0.8294 & 1.0000
\end{bmatrix}$$

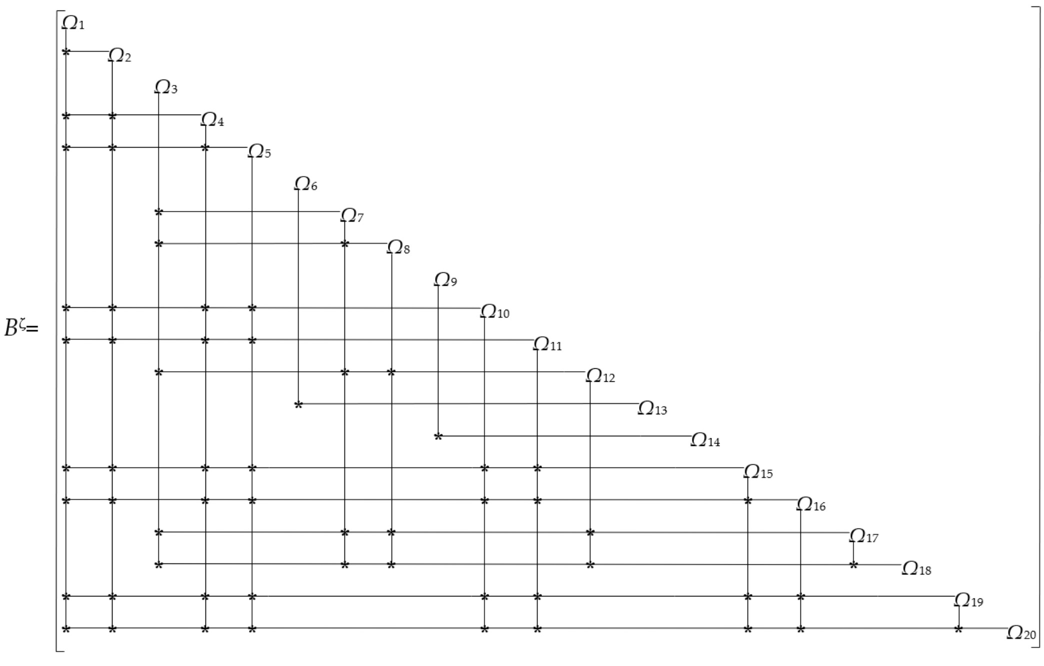


Figure 2. Clustering results based on the proposed clustering model

Using Eq. (5), we can perform the netting clustering analysis and indicate the clustering results in Figure 2, where when the confidence level takes $0.8430 \le \zeta \le 1$, the 20 slope samples are divided into four risk levels. In Figure 2, it is seen that $\{\Omega_1, \Omega_2, \Omega_4, \Omega_5, \Omega_{10}, \Omega_{11}, \Omega_{15}, \Omega_{16}, \Omega_{19}, \Omega_{20}\}$, $\{\Omega_3, \Omega_7, \Omega_8, \Omega_{12}, \Omega_{17}, \Omega_{18}\}$, $\{\Omega_6, \Omega_{13}\}$, and $\{\Omega_9, \Omega_{14}\}$ belong to four different risk classifications, respectively.

5.2. Slope Stability Assessment of Practical Cases

According to the Chinese standard for the engineering classification of rock mass (GB/T 50218 – 2014) and the historical experience of slope stability, we provide six affecting factors and four risk levels of slope stability. Furthermore, according to the actual situation of slope engineering, we can assess the four affecting factors λ_1, λ_2, λ_3, and λ_6 in the four risk levels from 0 to 10 slope stability scores: the stable score is 0–3; the basic stable score is 3–6; the relatively unstable score is 6–8; the unstable score is 8–10. Then, the two affecting factors λ_4 and λ_5 are the actual measured values. Subsequently, all the obtained results are shown in Table 4.

Table 4. Affecting factors and risk levels of slope stability

Affecting factor	Stable state	Basic stable state	Relatively unstable state	Unstable state
Lithological association	Hard rock (0–3)	Sub-hard rock (3–6)	Sub-soft rock (6–8)	Soft rock (8–10)
Slope structure	Homogeneous structure (0–3)	Blocky structure (3–6)	Stratified structure (6–8)	Loose structure (8–10)
Weathering degree of rock	Weak weathering (0–3)	Moderate weathering (3–6)	Intense weathering (6–8)	Complete weathering (8–10)
Slope height (m)	0–20	20–40	40–60	60–80
Slope angle (°)	0–10	10–30	30–60	60–90
Vegetation coverage	Very high (0–3)	High (3–6)	Low (6–8)	Very low (8–10)

From the above classified results of slope stability, we can classify the risk states of slope stability into four categories/levels: the unstable state (Θ_1), the relatively stable state (Θ_2), the basically stable state (Θ_3), and the stable state (Θ_4), where the unstable state implies that the slope is damaged or prone to damage, the basically stable and relatively unstable states reflect that the slope is between safety and damaged, then the stable state means that the slope is safe.

Based on the knowledge and experience of slope stability, the 6 affecting factors of the 4 risk levels are fuzzified into SVNNs in Table 5, then their SVNSs are represented as follows:

$\Theta_1 = \{s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}\} = \{<0.02, 0.90, 0.99>, <0.15, 0.61, 0.94>, <0.04, 0.25, 0.14>, <0.67, 0.85, 0.55>, <0.05, 0.73, 0.71>, <0.46, 0.99, 0.04>\}$;

$\Theta_2 = \{s_{21}, s_{22}, s_{23}, s_{24}, s_{25}, s_{26}\} = \{<0.11, 0.51, 0.86>, <0.22, 0.73, 0.86>, <0.26, 0.84, 0.44>, <0.68, 0.60, 0.52>, <0.07, 0.74, 0.72>, <0.58, 0.89, 0.02>\}$;

$\Theta_3 = \{s_{31}, s_{32}, s_{33}, s_{34}, s_{35}, s_{36}\} = \{<0.41, 1.00, 0.61>, <0.11, 0.51, 0.98>, <0.39, 0.97, 0.07>, <0.92, 0.31, 0.28>, <0.05, 0.76, 0.74>, <0.68, 0.79, 0.01>\}$;

$\Theta_4 = \{s_{41}, s_{42}, s_{43}, s_{44}, s_{45}, s_{46}\} = \{<0.79, 0.61, 0.38>, <0.73, 0.78, 0.35>, <0.59, 0.91, 0.02>, <0.93, 0.28, 0.26>, <0.15, 0.60, 0.62>, <0.61, 0.86, 0.02>\}$.

Table 1. SVNNs of the 4 risk levels for slope stability

θ_{ki}	Θ_1			Θ_2			Θ_3			Θ_4		
	X_{k1}	Y_{k1}	Z_{k1}	X_{k2}	Y_{k2}	Z_{k2}	X_{k3}	Y_{k3}	Z_{k3}	X_{k4}	Y_{k4}	Z_{k4}
θ_{k1}	0.02	0.90	0.99	0.11	0.51	0.86	0.41	1.00	0.61	0.79	0.61	0.38
θ_{k2}	0.15	0.61	0.94	0.22	0.73	0.86	0.11	0.51	0.98	0.73	0.78	0.35
θ_{k3}	0.04	0.25	0.14	0.26	0.84	0.44	0.39	0.97	0.07	0.59	0.91	0.02
θ_{k4}	0.67	0.85	0.55	0.68	0.60	0.52	0.92	0.31	0.28	0.93	0.28	0.26
θ_{k5}	0.05	0.73	0.71	0.07	0.74	0.72	0.05	0.76	0.74	0.15	0.60	0.62
θ_{k6}	0.46	0.99	0.04	0.58	0.89	0.02	0.68	0.79	0.01	0.61	0.86	0.02

Table 2. Assessed results of the risk levels for 20 slope samples

Ω_j	$M_\delta(\Phi_j, \Theta_1)$	$M_\delta(\Phi_j, \Theta_2)$	$M_\delta(\Phi_j, \Theta_3)$	$M_\delta(\Phi_j, \Theta_4)$	Risk level
Ω_1	0.494570	0.623006	0.677886	**0.873638**	Θ_4
Ω_2	0.564537	0.610318	0.752149	**0.853195**	Θ_4
Ω_3	0.713772	**0.832699**	0.756454	0.711081	Θ_2
Ω_4	0.551858	0.665090	0.728133	**0.951406**	Θ_4
Ω_5	0.495114	0.605004	0.642938	**0.886262**	Θ_4
Ω_6	**0.865189**	0.797835	0.626400	0.542880	Θ_1
Ω_7	0.776073	**0.875305**	0.782408	0.746375	Θ_2
Ω_8	0.785857	**0.904950**	0.704720	0.607567	Θ_2
Ω_9	0.743878	0.749130	**0.931893**	0.754172	Θ_3
Ω_{10}	0.572259	0.667849	0.728380	**0.910704**	Θ_4
Ω_{11}	0.628578	0.725045	0.761082	**0.882138**	Θ_4
Ω_{12}	0.803960	**0.884157**	0.773955	0.634420	Θ_2
Ω_{13}	**0.872155**	0.782644	0.685452	0.548984	Θ_1
Ω_{14}	0.714061	0.729656	**0.922054**	0.724081	Θ_3
Ω_{15}	0.575320	0.615914	0.752868	**0.834918**	Θ_4
Ω_{16}	0.534628	0.662498	0.646964	**0.791051**	Θ_4
Ω_{17}	0.793529	**0.865970**	0.808385	0.665402	Θ_2
Ω_{18}	0.810222	**0.888952**	0.700942	0.624027	Θ_2
Ω_{19}	0.582546	0.649881	0.750508	**0.869809**	Θ_4
Ω_{20}	0.559702	0.648989	0.690287	**0.935418**	Θ_4

Calculating the IHSSM values between Θ_k (k=1, 2, 3, 4) and Φ_j (j=1, 2, …, 20) by Eq. (6) for $\delta = 1$, we can gain all the IHSSM values, as shown in Table 6. Therefore, we can use the maximum measure value between Φ_j and Θ_k to decide the risk levels of these slope samples. The assessed results in Table 6 show that the 20 slope samples are clustered into the following four types of risk levels:

(1) {Ω_1, Ω_2, Ω_4, Ω_5, Ω_{10}, Ω_{11}, Ω_{15}, Ω_{16}, Ω_{19}, Ω_{20}} belongs to the risk level Θ_4;

(2) {Ω_3, Ω_7, Ω_8, Ω_{12}, Ω_{17}, Ω_{18}} belongs to the risk level Θ_2;

(3) {Ω_6, Ω_{13}} belongs to the risk level Θ_1;

(4) {Ω_9, Ω_{14}} belongs to the risk level Θ_3.

Clearly, the assessed results and the netting clustering results of the 20 slope samples are identical. Therefore, the slope stability evaluation model based on the netting clustering analysis verifies its rationality and accuracy.

5.3. Relative Comparison

In our comparative analysis, we apply the weighted similarity measures of Eqs. (2) and (3) [27, 28] to the risk level assessment of the 20 slope samples to verify the validity and accuracy of our new slope stability evaluation model.

Regarding the 20 slope samples, we calculate the weighted generalized distance-based similarity measure values between Θ_k (k = 1, 2, 3, 4) and Φ_j (j = 1, 2, …, 20) by Eq. (2) for δ = 1 [28], and then the assessed results are shown in Table 7. It can be clearly seen that there is consistency between the assessed results of Eq. (2) and the assessed results of the proposed IHSSM of SVNSs for the 20 slope samples Ω_j.

Table 7. Assessed results using Eq. (2)

Ω_j	$W_\delta(\Phi_j, \Theta_1)$	$W_\delta(\Phi_j, \Theta_2)$	$W_\delta(\Phi_j, \Theta_3)$	$W_\delta(\Phi_j, \Theta_4)$	Risk level
Ω_1	0.539646	0.661580	0.712268	**0.888397**	Θ_4
Ω_2	0.606702	0.649753	0.779809	**0.870249**	Θ_4
Ω_3	0.745041	**0.852010**	0.783693	0.742593	Θ_2
Ω_4	0.594669	0.700514	0.758084	**0.957157**	Θ_4
Ω_5	0.540175	0.644786	0.680074	**0.899586**	Θ_4
Ω_6	**0.880902**	0.820873	0.664736	0.586118	Θ_1
Ω_7	0.801353	**0.889876**	0.807043	0.774595	Θ_2
Ω_8	0.810137	**0.916128**	0.736800	0.647182	Θ_2
Ω_9	0.772339	0.777083	**0.939936**	0.781635	Θ_3
Ω_{10}	0.614006	0.703051	0.758308	**0.921215**	Θ_4
Ω_{11}	0.666760	0.755283	0.787864	**0.895933**	Θ_4
Ω_{12}	0.826355	**0.897722**	0.799449	0.672183	Θ_2
Ω_{13}	**0.887082**	0.807254	0.719201	0.591934	Θ_1
Ω_{14}	0.745305	0.759465	**0.931247**	0.754408	Θ_3
Ω_{15}	0.616897	0.654974	0.780458	**0.853987**	Θ_4
Ω_{16}	0.578235	0.698128	0.683798	**0.814795**	Θ_4
Ω_{17}	0.817016	**0.881594**	0.830311	0.700801	Θ_2
Ω_{18}	0.831953	**0.901969**	0.733355	0.662529	Θ_2
Ω_{19}	0.623709	0.686493	0.778328	**0.885001**	Θ_4
Ω_{20}	0.602119	0.685669	0.723624	**0.943048**	Θ_4

Table 8. Assessed results using Eq. (3)

Ω_j	$N_\delta(\Phi_j, \Theta_1)$	$N_\delta(\Phi_j, \Theta_2)$	$N_\delta(\Phi_j, \Theta_3)$	$N_\delta(\Phi_j, \Theta_4)$	Risk level
Ω_1	0.581600	0.697253	0.742503	**0.901390**	Θ_4
Ω_2	0.646720	0.688072	0.805233	**0.886962**	Θ_4
Ω_3	0.774547	**0.871636**	0.807972	0.769010	Θ_2
Ω_4	0.635925	0.735100	0.786431	**0.962682**	Θ_4
Ω_5	0.583890	0.683484	0.714810	**0.912949**	Θ_4
Ω_6	**0.896410**	0.838217	0.701436	0.627532	Θ_1
Ω_7	0.822104	**0.902188**	0.830551	0.800261	Θ_2
Ω_8	0.824908	**0.926346**	0.767125	0.684412	Θ_2
Ω_9	0.796648	0.802231	**0.945901**	0.806259	Θ_3
Ω_{10}	0.655512	0.738880	0.786225	**0.929453**	Θ_4
Ω_{11}	0.699053	0.781021	0.812508	**0.908627**	Θ_4
Ω_{12}	0.841484	**0.909690**	0.821160	0.705050	Θ_2
Ω_{13}	**0.901922**	0.824129	0.747833	0.629507	Θ_1
Ω_{14}	0.770840	0.784476	**0.936499**	0.781144	Θ_3
Ω_{15}	0.653407	0.689608	0.801997	**0.869884**	Θ_4
Ω_{16}	0.614510	0.726679	0.716873	**0.837449**	Θ_4
Ω_{17}	0.840246	**0.899429**	0.858115	0.740423	Θ_2
Ω_{18}	0.843693	**0.909883**	0.759968	0.694463	Θ_2
Ω_{19}	0.659384	0.718694	0.802222	**0.899089**	Θ_4
Ω_{20}	0.639969	0.718193	0.750376	**0.948368**	Θ_4

Regarding the 20 slope samples, we also calculate the weighted HSSM values between Θ_k (k = 1, 2, 3, 4) and Φ_j (j = 1, 2, …, 20) by Eq. (3) for δ = 1 [27], and then all the assessed results are shown in Table 8. It is obvious that the assessed risk results using Eq. (3) in Table 8 are the same risk results as using the proposed IHSSM, which proves the validity, rationality, and accuracy of our new assessment model in the scenario of SVNSs.

6. Conclusions

This article first presented the IHSSM of SVNSs, and then proposed its netting clustering and slope stability evaluation models to realize the risk level clustering analysis and assessment of slope stability in the scenario of SVNSs. Next, the proposed netting clustering and slope stability evaluation models are applied in the case study of the 20 slope samples. The assessment and comparison results verified the validity, rationality, and accuracy of the proposed new models in the scenario of SVNSs. However, the proposed new models can avoid the defects of the existing ANN, ANFIS, and SVNN-ANFIS evaluation methods [10, 11, 25] because they need a lot of training samples and the complex modeling process. Therefore, the new models proposed in this paper are not only simpler and more convenient in the evaluation process, but also more suitable for practical applications, which is the main advantage.

In future research, we will further investigate the slope stability clustering and evaluation problems with big sample data and verify the reasonableness and validity of the netting clustering results and the evaluation results. In addition, we shall also research on clustering and assessment methods with multi-layer affecting factors in neutrosophic scenarios and verify their accuracy and validity under the environment of big sample data.

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