



# Neutrosophic Optimization of Industry 4.0 Production Inventory Model

#### NivethaMartin<sup>1</sup>., M.Kasi Mayan<sup>2</sup> and Florentin Smarandache<sup>3</sup>

- 1. Department of Mathematics, Arul Anandar College (Autonomous), Karumathur, India, Email: nivetha.martin710@gmail.com
  - 2. Department of Mathematics, S.B.K.College, Aruppukottai, India, Email: 1988kasu@gmail.com
  - 3. Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA, Email: smarand@unm.edu

\* Correspondence: nivetha.martin710@gmail.com

**Abstract:** In this age of information, the industrial sectors are embedding its functioning principles with the components of Industry 4.0. This article proposes a production inventory model discussing the paradigm shift towards smart production process involving many new cost parameters in addition to the conventional inventory costs. The proposed Industry 4.0 production inventory model is discoursed and compared in both deterministic and neutrosophic environments. The trapezoidal neutrosophic number representation of the parameters enhances the efficiency of the model in determining the optimal order time that minimizes the total costs. The model is highly comprehensive in nature and it is validated with a numerical example.

Keywords: Neutrosophic sets, Industry 4.0, production inventory model, optimization, decision making.

#### 1. Introduction

Presently the industrial sectors are incorporating the techniques of digitalization to meet the requirements of the customer's demands at all ends. The production sectors practice new production methods to ease the process of production that comprises of several sequential steps and new cost parameters. The optimizing principle of manufacturing companies is costs minimization and profit maximization and the inventory models are utilized to make optimal decisions on order time and quantity. The Economic Production Quantity (EPQ) model proposed by Taft [1], a basic production inventory model to manage the levels of inventory by the production sectors. This model is the underlying model and it was developed and extended based on decision-making situations. The fundamental EPQ model was further modified with the integration of the cost parameters of shortages, trade discount, imperfect items, supply chain, deteriorating items, remanufacturing, waste disposal and so on. The production inventory models are extended to cater the requirements of the production sectors. Presently, the fourth industrial revolution is gaining significance amidst the developed and developing nations. Industry 4.0 will certainly bring a paradigm transition at all the levels of organization and control over the different stages of the product's life. The entire process of product production beginning from product conception, product design, product development,

initialization of product production, manufacturing of the product, product delivery and ending with product rework, recycle and disposal will get into the digitalized mode based on customer- centric approach.

The elements of Industry 4.0 are stepping into the production sectors of large, medium and smallsized and at all phases of production processes. Christian Decker et al [2] introduced a cost-benefit model for smart items in the supply chain which is an initial initiative in calculating the advantages of introducing smart items into the network of the supply chain. Andrew Kusiak [3] presented the benefits of smart manufacturing; its core components and the production pattern in future. Fei et al [4] developed IT -driven assistance arranged shrewd assembling with its structure and attributes. Sameer et al [5] introduced a basic audit on keen assembling and Industry 4.0 development models and the suggestions for the entrance of medium and little enterprises. Xiulong et al [6] developed CPS-based smart production system for Industry 4.0 based on the review of the existing literature on smart production systems. Pietro et al [7] built up a digital flexibly chain through the powerful stock and smart agreements. Marc Wins[8] introduced a wide depiction of the highlights of a smart stock administration framework. Souvik et al [9] investigated the savvy stock administration framework dependent on the web of things (Iot). Poti et al [10] introduced the prerequisite examination for shrewd flexibly chain the board for SMEs. Ghadge et al [11] tended to the effect of Industry 4.0 execution on flexibly chains; introduced the benefits and confinements of industry 4.0 in supply chain arrange alongside its cutting-edge headings; clarified the core Industry 4.0 innovations and their business applications and investigated the ramifications of Industry 4.0 with regards to operational and gainful proficiency. Iqra Asghar et al [12] presented a digitalized smart EPQ-based stock model for innovation subordinate items under stochastic failure and fix rate. The above examined stock models are deterministic in nature and the costs boundaries are traditional in nature and they don't mirror the real costs boundaries identified with industry 4.0 components.

In this paper, manufacturing inventory model incorporating a new range of smart costs is formulated, also in this industry 4.0 model, the cost parameters are characterized as neutrosophic sets. This is the novelty of this research work and as for as the literature is concerned, industry 4.0 neutrosophic production inventory models have not been discussed so far and related literature does not exist. Smarandache [13] introduced neutrosophic sets that deal with truth, indeterminacy and falsity membership functions. Neutrosophic sets are widely applied to handle the situations of indeterminacy and it has extensive applications in diverse fields. Sahidul et al [14] developed neutrosophic goal programming for choosing the optimal green supplier, Abdel Nasser [15] used an integrated neutrosophic approach for supplier selection, Lyzbeth [16] constructed neutrosophic decision- making model to determine the operational risks in financial management, Ranjan Kumar et al [17,18] developed neutrosophic multi-objective programming for finding the solution to shortest path problem, Vakkas et al [19] proposed MADM method with bipolar neutrosophic sets.

Abdel-Basst, Mohamed et al [20] has developed neutrosophic decision-making models for effective identification of COVID-19; constructed bipolar neutrosophic MCDM for professional selection [21]; formulated a model to solve supply chain problem using best-worst method [22] and to measure the

financial performance of the manufacturing industries [23]. Also, Abdel-Basset proposed presented a new framework for evaluating the innovativeness of the smart product – service systems [24]. As neutrosophic sets are highly viable, neutrosophic inventory models are formulated by many researchers. Chaitali Kar et al[25] developed inventory model with neutrosophic geometric programming approach. Mullai and Broumi[26] discussed neutrosophic inventory model without shortages, Mullai [27] developed neutrosophic model with price breaks. Mullai et al [28] constructed neutrosophic inventory model dealing with single-valued neutrosophic representation.

In all these neutrosophic inventory models, the cost parameters of the conventional inventory models are represented as neutrosophic sets or numbers, but these models did not discuss any new kind of cost parameters reflecting the transitions in the production processes. But the proposed model reflects the paradigm shift towards smart production process and incorporates new kinds of costs to cater the requirements of smart production inventory model. The industry 4.0 neutrosophic production inventory model with the inclusion of the respective costs to the core elements of smart production systems is highly essential as the existing production sectors are adapting to the environment of smart production set up, but to the best of our knowledge such models are still uncovered. This model primarily focuses on increase productivity and high quality of the product within low investment of finance. The composition of several components of industry 4.0 production inventory model result in diverse costs parameters such as smart ordering cost, internet connectivity initialization cost, holding costs ,smart product design cost, data management cost, customer data analysis cost, supplier data analysis cost, smart technology cost, production monitoring cost, reworking cost, smart training work personnel cost, smart tools purchase cost, smart disposal costs, smart environmental costs, holding cost. The term smart refers to the costs incurred with the integration of digital gadgets to the respective production departments.

The article is structured into the following sections: section 2 consists of the preliminary definitions of neutrosophic sets and its arithmetic operation; section 3 presents the industry 4.0 production inventory model; section 4 validates the proposed model with neutrosophic parameters; section 5 discusses the results and the last section concludes the paper.

### 2. Basics of Neutrosophic sets and operations

This section presents the fundamentals of neutrosophic sets, arithmetic operations and defuzzification

#### 2.1 Neutrosophic set [13]

A neutrosophic set is characterized independently by a truth-membership function  $\alpha(x)$ , an indeterminacy-membership function  $\beta(x)$ , and a falsity-membership function $\gamma(x)$  and each of the function is defined from  $X \to [0,1]$ 

### 2.2Single valued Trapezoidal Neutrosophic Number

A single valued trapezoidal neutrosophic number  $\tilde{A} = \langle (a, b, c, d) : \rho_{\overline{A}}, \sigma_{\overline{A}}, \tau_{\overline{A}} \rangle$  is a special neutrosophic set on the real number set R, whose truth –membership, indeterminacy-membership, and a falsity –membership is given as follows.

$$\mu_{\overline{A}}(\mathbf{x}) = \begin{cases} (\mathbf{x} - \mathbf{a}) \rho_{\overline{A}} / (\mathbf{b} - \mathbf{a}) & (\mathbf{a} \le \mathbf{x} < \mathbf{b}) \\ \rho_{\overline{A}} & (\underline{b} \le \mathbf{x} \le \mathbf{c}) \\ (\mathbf{d} - \mathbf{x}) \rho_{\overline{A}} / (\mathbf{d} - \mathbf{c}) & (\mathbf{c} < \mathbf{x} \le d) \\ 0 & \text{otherwise} \end{cases}$$

$$\pi_{\overline{A}}(\mathbf{x}) = \begin{cases} (\mathbf{b} - \mathbf{x} + \sigma_{\overline{A}}(\mathbf{x} - a))) / (\mathbf{b} - a) & (\underline{\mathbf{a}} \le \mathbf{x} < \mathbf{b}) \\ \sigma_{\overline{A}} & (\underline{\mathbf{b}} \le \mathbf{x} \le \mathbf{c}) \\ (\underline{\mathbf{x}} - \mathbf{c} + \sigma_{\overline{A}}(\mathbf{d} - \mathbf{x})) / (\mathbf{d} - \mathbf{c}) & (\mathbf{c} < \mathbf{x} \le d) \\ 1 & \text{otherwise} \end{cases}$$

$$\varphi_{\overline{A}}(\mathbf{x}) = \begin{cases} (\mathbf{b} - \mathbf{x} + \tau_{\overline{A}} \ (\mathbf{x} - a))) / \ (\mathbf{b} - a) & (\underline{\mathbf{a}} \leq \mathbf{x} < \mathbf{b} \ ) \\ \tau_{\overline{A}} & | (\underline{\mathbf{b}} \leq \mathbf{x} \leq \mathbf{c} \ ) \\ (\underline{\mathbf{x}} - \mathbf{c} + \tau_{\overline{A}} \ (\mathbf{d} - \mathbf{x})) / (\mathbf{d} - \mathbf{c}) & (\mathbf{c} < \mathbf{x} \leq \mathbf{d} \ ) \\ 1 & \text{otherwise} \end{cases}$$

## 2.3. Operations on Single valued Trapezoidal Neutrosophic Numbers

Let  $\tilde{A} = \langle (a_1, b_1, c_1, d_1) : \rho_{\overline{A}}, \sigma_{\overline{A}}, \tau_{\overline{A}} \rangle$  and  $\tilde{B} = \langle (a_2, b_2, c_2, d_2) : \rho_{\overline{B}}, \sigma_{\overline{B}}, \tau_{\overline{B}} \rangle$  be two single valued trapezoidal neutrosophic numbers and  $\mu \neq 0$ , then

1. 
$$\tilde{A} + \tilde{B} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2) : \rho_{\overline{A}} \wedge \rho_{\overline{B}}, \sigma_{\overline{A}} \vee \sigma_{\overline{B}}, \tau_{\overline{A}} \vee \tau_{\overline{B}} \rangle$$

2. 
$$A - B = \langle (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2) : \rho_{\overline{A}} \wedge \rho_{\overline{B}}, \sigma_{\overline{A}} \vee \sigma_{\overline{B}}, \tau_{\overline{A}} \vee \tau_{\overline{B}} \rangle$$

$$\begin{aligned} 2. \quad \tilde{A} - \tilde{B} &= \langle (\mathbf{a}_1 - \ d_2, b_1 - c_2, c_1 - b_2, d_1 - \ \mathbf{a}_2) \colon \rho_{\overline{A}} \wedge \rho_{\overline{B}}, \sigma_{\overline{A}} \vee \sigma_{\overline{B}}, \tau_{\overline{A}} \vee \tau_{\overline{B}} \rangle \\ 3. \quad \tilde{A} \quad \tilde{B} &= \begin{cases} \langle (\mathbf{a}_1 \mathbf{a}_2, b_1 b_2, c_1 c_2, d_1 d_2) \colon \rho_{\overline{A}} \wedge \rho_{\overline{B}}, \sigma_{\overline{A}} \vee \sigma_{\overline{B}}, \tau_{\overline{A}} \vee \tau_{\overline{B}} \rangle & (d_1 > 0, d_2 > 0) \\ \langle (\mathbf{a}_1 \mathbf{d}_2, b_1 c_2, c_1 b_2, d_1 a_2) \colon \rho_{\overline{A}} \wedge \rho_{\overline{B}}, \sigma_{\overline{A}} \vee \sigma_{\overline{B}}, \tau_{\overline{A}} \vee \tau_{\overline{B}} \rangle & (d_1 < 0, d_2 > 0) \\ \langle (\mathbf{d}_1 \mathbf{d}_2, c_1 c_2, b_1 b_2, a_1 a_2) \colon \rho_{\overline{A}} \wedge \rho_{\overline{B}}, \sigma_{\overline{A}} \vee \sigma_{\overline{B}}, \tau_{\overline{A}} \vee \tau_{\overline{B}} \rangle & (d_1 < 0, d_2 < 0) \end{cases} \end{aligned}$$

4. 
$$\widetilde{A}/\widetilde{B} =$$

$$\begin{cases} \langle (\mathbf{a}_{1}/\mathbf{d}_{2},b_{1}/c_{2},c_{1}/b_{2},d_{1}/a_{2}) \colon \rho_{\overline{A}} \wedge \rho_{\overline{B}},\sigma_{\overline{A}} \vee \sigma_{\overline{B}},\tau_{\overline{A}} \vee \tau_{\overline{B}} \rangle & (d_{1} > 0,d_{2} > 0) \\ \langle (\mathbf{d}_{1}/\mathbf{d}_{2},c_{1}/c_{2},b_{1}/b_{2},a_{1}/a_{2}) \colon \rho_{\overline{A}} \wedge \rho_{\overline{B}},\sigma_{\overline{A}} \vee \sigma_{\overline{B}},\tau_{\overline{A}} \vee \tau_{\overline{B}} \rangle & (d_{1} < 0,d_{2} > 0) \\ \langle (\mathbf{d}_{1}/a_{2},c_{1}/b_{2},b_{1}/c_{2},a_{1}/d_{2}) \colon \rho_{\overline{A}} \wedge \rho_{\overline{B}},\sigma_{\overline{A}} \vee \sigma_{\overline{B}},\tau_{\overline{A}} \vee \tau_{\overline{B}} \rangle & (d_{1} < 0,d_{2} < 0) \\ \langle (\mathbf{d}_{1}/a_{2},c_{1}/b_{2},b_{1}/c_{2},a_{1}/d_{2}) \colon \rho_{\overline{A}},\sigma_{\overline{A}},\tau_{\overline{A}} \rangle & (\mu > 0) \\ \delta. \quad \mu \tilde{A} = \begin{cases} \langle (\mu a_{1},\mu b_{1},\mu c_{1},\mu d_{1}) \colon \rho_{\overline{A}},\sigma_{\overline{A}},\tau_{\overline{A}} \rangle & (\mu < 0) \\ \langle (\mu d_{1},\mu c_{1},\mu b_{1},\mu a_{1}) \colon \rho_{\overline{A}},\sigma_{\overline{A}},\tau_{\overline{A}} \rangle & (\mu < 0) \end{cases}$$

5. 
$$\mu \tilde{A} = \begin{cases} \langle (\mu a_1, \mu b_1, \mu c_1, \mu d_1) : \rho_{\overline{A}}, \sigma_{\overline{A}}, \tau_{\overline{A}} \rangle & (\mu > 0) \\ \langle (\mu d_1, \mu c_1, \mu b_1, \mu a_1) : \rho_{\overline{A}}, \sigma_{\overline{A}}, \tau_{\overline{A}} \rangle & (\mu < 0) \end{cases}$$

6. 
$$\tilde{A}^{-1} = \langle (1/d_1, 1/c_1, 1/b_1, 1/a_1) : \rho_{\overline{e}}, \sigma_{\overline{e}}, \tau_{\overline{e}} \rangle$$
  $(\tilde{A} \neq 0)$ .

## 2.4 Defuzzification of Neutrosophic set

A single valued trapezoidal neutrosophic numbers of the form  $\tilde{A} = \langle (a, b, c, d); \rho, \sigma, \tau \rangle$  can be defuzzified by finding its respective score value  $K(\tilde{A})$ 

$$K(\tilde{A}) = \frac{1}{16}[a+b+c+d] \times (2 + \mu_{\overline{A}} - \pi_{\overline{A}} - \varphi_{\overline{A}}).$$

## 3. Model Development

### 3.1 Assumptions

Shortages are not allowed.

Demand is not deterministic in nature.

The products are not of deteriorating type.

Planning horizon is infinite.

#### 3.2 Notations

The below notations are used throughout this paper.

P – Smart production rate per cycle

D → Uniform demand rate per cycle

## **General Costs**

Os - Smart Ordering cost

 $I_{\longleftarrow} Internet\ Connectivity\ Initialization\ Cost$ 

## Costs for time period $0 \le t \le t_1$

PDs - Smart Product design cost

DM - Data management Cost

CD - Customer Data Analysis cost

SD - Supplier Data Analysis cost

Ts- Smart Technology Cost

M – Production Monitoring cost

r - defective rate

R - Reworking Cost

TRs - Smart training work personnel cost

TOs – Smart tools purchase cost

## Costs for time period $t_1 \le t \le T$

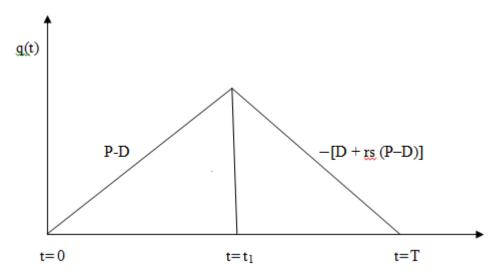
s - disposal rate

D<sub>s</sub>– Smart disposal costs

Es- Smart Environmental costs

## Costs common for both the time periods

## H - Holding costs



If q(t) represents the inventory level at time  $t \in [0, T]$ , so the differential equation for the instantaneous inventory q(t) at any time t over [0, T] is

$$\frac{dq(t)}{dt} = P - D \qquad 0 \le t \le t_1 \rightarrow (1)$$
$$= -[D + rs(P-D)] \qquad t_1 \le t \le T(2)$$

With initial condition q(0) = 0 and

Boundary condition q(T) = 0

$$\frac{dq(t)}{dt} = P - D$$

$$dq(t) = (P - D) dt$$

$$q(t) = (P - D) t + c$$

with initial condition q(0) = 0

$$q(0) = (P - D) 0 + c$$

$$0 = c$$

$$q(t) = (P - D) t 0 \le t \le t_1 \rightarrow (3)$$

solving equation (3)

$$\frac{dq(t)}{dt} = - [D + rs (P-D)]$$
  $t_1 \le t \le T$ 

$$dq(t) = - [D + rs (P-D)] dt$$

$$q(t) = -[D + rs (P-D)] t + c$$

With boundary condition q(T) = 0

$$q(T) = - [D + rs(P-D)] T + c$$

$$0 = - [D + rs(P-D)] T + c$$

$$c = [D + rs(P-D)] T$$

$$q(t) = -[D + rs (P-D)] t + [D + rs (P-D)] T$$
  $\rightarrow$  (4)

using equation (3),(4), we get

$$I_{max} = (P-D) t_1$$

$$I_{max} = [D + rs (P-D)] (T-t_1)$$

$$t_1 = \frac{I_{max}}{P - D}$$

$$T-t_1 = \frac{I_{max}}{D+rs(P-D)}$$

We adding, we get

$$t_1 + T - t_1 = I_{max} \left[ \frac{1}{(P-D)} + \frac{1}{D + rs(P-D)} \right]$$

$$T = I_{max} \left[ \frac{1}{(P-D)} + \frac{1}{D+rs(P-D)} \right]$$

$$T = I_{\text{max}} \left[ \left( \frac{P + (P - D)rs}{P - D[D + rs(P - D)]} \right) \right]$$

$$I_{max} = \left[\frac{P - D[D + rs(P - D)]}{P + (P - D)rs}\right] T$$

Smart product design cost =  $PD_s \int_0^{t_1} q(t) dt$ 

$$= PD_s \int_0^{t_1} (P - D) t dt$$

$$= \frac{PD_s}{2} \left[ P-D \left( \frac{D+rs(P-D)}{P+(P-D)rs} T \right)^2 \right]$$

Data management cost = DM  $\int_0^{t_1} q(t) dt$ 

$$= DM \int_0^{t_1} (P - D)t dt$$

$$= \frac{DM}{2} \left[ P-D \left( \frac{D+rs(P-D)}{P+(P-D)rs} T \right)^2 \right]$$

Customer data analysis cost = CD  $\int_0^{t_1} q(t)dt$ 

$$= CD \int_0^{t_1} (P - D)t dt$$

$$= \frac{CD}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} T \right)^2 \right]$$

Supplier data analysis cost = SD  $\int_0^{t_1} q(t)dt$ 

$$= SD \int_0^{t_1} (P - D)t dt$$

$$= \frac{SD}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} T \right)^2 \right]$$

Smart Technology cost =  $T_s \int_0^{t_1} q(t) dt$ 

$$= T_{s} \int_{0}^{t_{1}} (P - D) t dt$$

$$= \frac{T_{s}}{2} [P-D (\frac{D+rs(P-D)}{P+(P-D)rs} T)^{2}]$$

Production Monitoring cost = M  $\int_0^{t_1} q(t)dt$ 

= M 
$$\int_0^{t_1} (P - D)t dt$$
  
=  $\frac{M}{2} [ P-D (\frac{D+rs(P-D)}{P+(P-D)rs}T)^2 ]$ 

Reworking cost = R  $\int_0^{t_1} q(t)dt$ 

= R 
$$\int_0^{t_1} (P - D)t dt$$
  
=  $\frac{R}{2} [ P-D ( \frac{D+rs(P-D)}{P+(P-D)rs} T)^2 ]$ 

Smart training work personal cost =  $TR_s \int_0^{t_1} q(t) dt$ 

$$= TR_s \int_0^{t_1} (P - D) t dt$$

$$= \frac{TR_s}{2} \left[ P-D \left( \frac{D+rs(P-D)}{P+(P-D)rs} T \right)^2 \right]$$

Smart tools purchase cost =  $TO_s \int_0^{t_1} q(t) dt$ 

$$= TO_s \int_0^{t_1} (P - D)t dt$$

$$= \frac{TO_s}{2} [P-D \left( \frac{D+rs(P-D)}{P+(P-D)rs} T \right)^2]$$

Smart disposal cost =  $D_s \int_{t_1}^{T} q(t) dt$ 

$$= D_s \int_{t_1}^T D + rs(P - D) t dt$$

$$= \frac{D_S}{2} [D + rs(P - D) (\frac{(P - D)}{P + (P - D)rs}T)^2]$$

Smart Environmental cost =  $E_s \int_{t_1}^{T} q(t) dt$ 

$$= E_{s} \int_{t_{1}}^{T} D + rs(P - D) t dt$$

$$= \frac{E_{s}}{2} \left[ D + rs(P - D) \left( \frac{(P - D)}{P + (P - D)rs} T \right)^{2} \right]$$

::Holding cost =C<sub>1</sub>  $\left[\int_0^{t_1} q(t)dt + \int_{t_1}^T q(t)dt\right]$ 

= C<sub>1</sub> 
$$[(P-D) \frac{t_1^2}{2} + [D + rs(P-D)] \frac{(T-t_1)^2}{2}]$$
  
=  $\frac{C_1}{2} [\frac{P-D[D+rs(P-D)]}{P+(P-D)rs}] T^2$ 

∴Total Cost = Smart Ordering cost + Internet Connectivity Initialization Cost+ Holding Costs +Smart Product design cost+ Data management Cost+ Customer Data Analysis cost+ Supplier Data Analysis cost+ Smart Technology Cost+ Production Monitoring cost+ Reworking Cost+ Smart training work personnel cost+ Smart tools purchase cost + Smart disposal costs + Smart Environmental costs

$$= O_{s} + I_{c} + \frac{c_{1}}{2} \left[ \frac{P - D[D + rs(P - D)]}{P + (P - D)rs} \right] T^{2} + \frac{PD_{s}}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} T \right)^{2} \right] + \frac{DM}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} T \right)^{2} \right] + \frac{CD}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} T \right)^{2} \right] + \frac{SD}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} T \right)^{2} \right] + \frac{T_{s}}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} T \right)^{2} \right] + \frac{M}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)VV} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)V} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{V + (V - V)V} V \right)^{2} \right] + \frac{V}{2} \left[ P - D \left( \frac{V + VV(V - V)}{$$

$$\frac{\star \star_{\star}}{2} \left[ P-D \left( \frac{\star + \star \star_{\star}(\star - \star)}{\star_{\star} + (\star - \star) \star_{\star}} \star \right)^{2} \right] + \frac{\star \star_{\star}}{2} \left[ P-D \left( \frac{\star + \star_{\star}(\star - \star)}{\star_{\star} + (\star - \star) \star_{\star}} \star \right)^{2} \right] +$$

$$\frac{\mathbf{v}_{\tau}}{2} \left[ \mathbf{v} + \mathbf{v} \mathbf{v} \left( \mathbf{v} - \mathbf{v} \right) \left( \frac{(\mathbf{v} - \mathbf{v})}{\mathbf{v} + (\mathbf{v} - \mathbf{v}) \mathbf{v}} \mathbf{v} \right)^{2} \right] + \frac{\mathbf{v}_{\tau}}{2} \left[ \mathbf{v} + \mathbf{v} \mathbf{v} \left( \mathbf{v} - \mathbf{v} \right) \left( \frac{(\mathbf{v} - \mathbf{v})}{\mathbf{v} + (\mathbf{v} - \mathbf{v}) \mathbf{v}} \mathbf{v} \right)^{2} \right]$$

$$= O_s + I_c + \frac{v_1}{2} \left[ \frac{v - v \left[v + v v \left(v - v\right)\right]}{v + \left(v - v\right) r v} \right] T^2 + \left[ P - D \left( \frac{v + v v \left(v - v\right)}{v + \left(v - v\right) v v} \right)^2 \right] \left[ \frac{v v}{2} + \frac{v v}{2} +$$

$$\frac{\mathbf{v}_{\mathbf{v}_{\mathbf{v}}}}{2}] + \left[\mathbf{v} + \mathbf{v}_{\mathbf{v}}(\mathbf{v} - \mathbf{v}) - \left(\frac{(\mathbf{v} - \mathbf{v})}{\mathbf{v} + (\mathbf{v} - \mathbf{v})\mathbf{v}\mathbf{v}}\right)^{2}\right] \left[\frac{\mathbf{v}_{\mathbf{v}}}{2} + \frac{\mathbf{v}_{\mathbf{v}}}{2}\right]$$

$$=O_s+I_c+\frac{\mathbf{v}_1}{2}\left[\frac{\mathbf{v}-\mathbf{v}[\mathbf{v}+\mathbf{v}\cdot(\mathbf{v}-\mathbf{v})]}{\mathbf{v}+(\mathbf{v}-\mathbf{v})\mathbf{v}\mathbf{v}}\right]T^2+\frac{1}{2}\left[P-D\left(\frac{\mathbf{v}+\mathbf{v}\cdot(\mathbf{v}-\mathbf{v})}{\mathbf{v}+(\mathbf{v}-\mathbf{v})\mathbf{v}\mathbf{v}}\right)^2\left(PD_s+\mathbf{v}\mathbf{v}+\mathbf{v}\mathbf{v}\mathbf{v}+\mathbf{v}\mathbf{v}\mathbf{v}+\mathbf{v}\mathbf{v}\mathbf{v}+\mathbf{v}\mathbf{v}\mathbf{v}\right)^2$$

$$[++-(--)(\frac{(P-D)}{P+(P-D)rs}T)^2]$$
 (D<sub>s</sub>+ E<sub>s</sub>)]

$$SD+T_s+M+R+TR_s+TO_s)+[D+rs(P-D) (\frac{(P-D)}{P+(P-D)rs}T)^2](D_s+E_s)]$$

$$= \frac{O_s}{T} + \frac{I_c}{T} + \frac{C_1}{2} \left[ \frac{P - D[D + rs(P - D)]}{P + (P - D)rs} \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right)^2 T \left( P D_s + DM + CD + SD + T_s + M + R + TR_s \right. \\ + T O_s \right) + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right)^2 T \left( P D_s + DM + CD + SD + T_s + M + R + TR_s \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right)^2 T \left( P D_s + DM + CD + SD + T_s + M + R + TR_s \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right)^2 T \left( P D_s + DM + CD + SD + T_s + M + R + TR_s \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right)^2 T \left( P D_s + DM + CD + SD + T_s + M + R + TR_s \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right)^2 T \left( P D_s + DM + CD + SD + T_s + M + R + TR_s \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right)^2 T \left( P D_s + DM + CD + SD + T_s + M + R + TR_s \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right) \right] T + \frac{1}{2} \left[ P - D \left( \frac{D + rs(P - D)}{$$

$$[D + rs(P - D) (\frac{(P-D)}{P+(P-D)rs})^2 T](D_s + E_s)]$$

So the Classical EPQ model is

$$T_s + M + R + TR_s + TO_s + [D + rs(P - D) (\frac{(P-D)}{P + (P-D)rs})^2 T](D_s + E_s)]$$

Such that T > 0

We can show that TAC(T) will be minimum for

$$T^* = \sqrt{\frac{2(O_S + I_C)}{C_1 \left[\frac{P - D[D + r_S(P - D)]}{P + (P - D)r_S}\right] + \left[P - D\left(\frac{D + r_S(P - D)}{P + (P - D)r_S}\right)^2(PD_S + DM + CD + SD + T_S + M + R + TR_S + TO_S) + \left[D + r_S(P - D)\left(\frac{(P - D)}{P + (P - D)r_S}\right)^2\right](D_S + E_S)]} }$$

$$\begin{aligned} & \text{TAC}^*(\text{T}^*) \\ & \sqrt{2(O_S + I_C) + C_1 \left[ \frac{P - D[D + rs(P - D)]}{P + (P - D)rs} \right] + \left[ P - D \left( \frac{D + rs(P - D)}{P + (P - D)rs} \right)^2 (PD_S + DM + CD + SD + DS) \right] } \\ & \sqrt{T_S + M + R + TR_S + TO_S} + \left[ D + rs(P - D) \left( \frac{(P - D)}{P + (P - D)rs} \right)^2 \right] (D_S + E_S) } \end{aligned}$$

#### 4.Illustration

To validate the developed model, an inventory system with the below characteristics is taken into consideration

Smart production rate per cycle = Rs.500unit/per month , Uniform demand rate per cycle = Rs.250/month, Smart Ordering cost = Rs.310/run, Internet Connectivity Initialization Cost = Rs.370/year, Smart Product design cost = Rs.25/unit, Data management Cost = Rs.50/unit, Customer Data Analysis cost = Rs.45/unit, Supplier Data Analysis cost = Rs.25/unit, Smart Technology Cost = Rs.15/unit, Production Monitoring cost = Rs.45/unit, defective rate = Rs.1, Reworking Cost = Rs.22/run, Smart training work personnel cost = Rs.30/unit, Smart tools purchase cost= Rs.10/unit, disposal rate = Rs. 3/unit, Smart disposal costs = Rs. 5/unit, Smart Environmental costs = Rs.7/unit, Holding costs = Rs.1/unit/year. Find the time interval and find the total average cost.

The value of T\* and TAC(T\*) is 0.179 and Rs.208.39 respectively

This model can be validated with the single valued neutrosophic trapezoidal fuzzy value representations as follows,

 $D = \langle (250,350,450,550):0.7,0.2,0.1 \rangle$ 

Os= \((350,450,550,650):0.9,0.3,0.1\)

Ic= \((550,650,750,850):0.8,0.3,0.4\)

PDs = ((25,35,45,55):0.7,0.3,0.2)

DM = ((65,75,85,95):0.9,0.3,0.4)

 $CD = \langle (55,65,75,85):0.8,0.1,0.2 \rangle$ 

 $SD = \langle (20,30,40,50):0.8,0.3,0.2 \rangle$ 

 $Ts = \langle (15,18,22,24):0.7,0.1,0.2 \rangle$ 

```
\begin{split} M &= & \langle (60,70,80,90) : 0.7,0.2,0.4 \rangle \\ r &= \langle (1,1.5,2.5,3) : 0.9,0.1,0.2 \rangle \\ R &= \langle (20,25,35,40) : 0.8,0.2,0.1 \rangle \\ TRs &= \langle (35,45,55,65) : 0.7,0.1,0.3 \rangle \\ TOs &= & \langle (8,12,16,20) : 0.7,0.1,0.4 \rangle \\ S &= & \langle (3,4,6,8) : 0.8,0.1,0.4 \rangle \\ Ds &= & \langle (5,7,9,11) : 0.7,0.2,0.3 \rangle \\ Es &= & \langle (6,9,12,15) : 0.8,0.2,0.3 \rangle \\ C_1 &= & \langle (1,1.5,2.5,3) : 0.9,0.3,0.2 \rangle \\ The value of T* &= 0.178 and TAC*(T*) &= 210.29 \end{split}
```

#### 5. Discussion

A neutrosophic production inventory model incorporating the costs parameters of industry 4.0 is developed together with the presentation of its conceptual framework. Several key benefits of neutrosophic production inventory model have been emphasized in this paper, together with the additional cost parameters. Another point of discussion is the usage of the production inventory model to find the feasible time to place orders that confines the total expenses. The representation of these costs parameters as single valued trapezoidal neutrosophic number tackles the conditions of uncertainty.

The constructed manufacturing inventory model is validated with deterministic parameters and neutrosophic parameters. The optimal time that yields minimum costs is nearly equal in both the cases of deterministic and neutrosophic validation. The neutrosophic representation makes this model more comprehensive. In this paper shortages are not allowed, the products are not of deteriorating type, planning horizon is infinite. The developed model can be extended to neutrosophic production inventory model with shortages and deteriorating items. This model primarily focuses on increases productivity of high-quality products within low investment of finance. The discussion is summarized as follows, a novel neutrosophic production inventory model is developed with the cost parameters pertaining to the fourth industrial revolution. This proposed model will certainly assist the production sectors to incorporate new types of costs. A deeper investigation on the effects of our decision making is clearly an obligation for upcoming work.

#### 6. Conclusion

The proposed industry 4.0 inventory model is a novel approach integrating the concept of smart production principles, and neutrosophic representations of cost parameters. This model is an underlying smart production model and this model can be further developed based on the needs of the production sectors. The proposed model is pragmatic in nature and it can be extended by including the concepts of customer acquisition and product propagation with additional cost parameters. These models will certainly unveil the new requirements of production scenario to meet the demands of the customers of this information age. The model constructed in this paper presents the present need of the production environment and it will certainly assist the decision makers to

optimize profit. The cost parameters of this model can be scaled to the requirements of small and medium sized enterprises which could be the future work.

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Received: June 10, 2020. Accepted: Nov 20, 2020