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Application of Referee Functions to the Vehicle-Born Improvised Explosive Device Problem

Frédéric Dambreville
DGA/Ingénierie de Projet/EORD
7 – 9 rue des Mathurins
92221 Bagneux cedex, France.
Email form: http://email.fredericdambreville.com

***Abstract*—We propose a solution to the Vehicle-Born Improvised Explosive Device problem. This solution is based on a modelling by belief functions, and involves the construction of a combination rule dedicated to this problem. The construction of the combination rule is made possible by a tool developped in previous works, which is a generic framework dedicated to the construction of combination rules. This tool implies a tripartite architecture, with respective parts implementing the logical framework, the combination definition (referee function) and the computation processes. Referee functions are decisional arbitrament conditionally to basic decisions provided by the sources of information, and allows rule definitions at logical level adapted to the application. We construct a referee function for the Vehicle-Born Improvised Explosive Device problem, and compare it to reference combinaton rules.**

Keywords: Threat Assessment, Referee Function, Dezert Smarandache Theory, Belief function.

Notations

- $I[boolean]$ is defined by $I[\mathbf{true}] = 1$ and $I[\mathbf{false}] = 0$. Typically, $I[x = y] = 1$ when $x = y$, and $= 0$ when $x \neq y$,
- $x_{1:n}$ is an abbreviation for the sequence $x_1, \cdots, x_n$. Similarly, $\sum_{i=1:n} x_i$ means $x_1 + \cdots + x_n$. This abbreviation also holds for other operators.

I. Introduction

Belief Functions [1], [2] are often promoted as alternative approaches for fusing information, when the hypotheses for a Bayesian approach cannot be precisely stated. When manipulating belief-based information, the interpretation of the belief combination rules may be difficult, when conflicts are notably involved. In the recent literature, there has been a large amount of work devoted to the definition of new fusion rules [3]–[12], in order to handle the conflict efficiently. The choice for a rule is often dependent of the applications and there is not a systematic approach for this task. Somehow, it appears also that this choice of a rule implies the choice of decision paradigm in order to handle the conflict.

This paper will present belief based solution to the Vehicle-Born Improvised Explosive Device problem (VBIED). The VBIED is a challenge which has been proposed to the community of uncertain reasoning during the conference Fusion'2010. The VBIED results in a decision issue between conflictual information. Thus, it is an interesting problem for challenging belief combinations rules. There has already been a proposal for a belief-based approach to VBIED [22], and our approach will be inspired from this work. However, while our approach is notably simplified, it also addresses the problem with a different philosophy. More precisely, while in [22] the authors apply different existing rules (typically, Dempster-Shafer and PCR rules) to the problem, in our approach we propose a construction of a rule on the basis of the problem setting.

This rule construction is made possible by the use of a framework for generic implementation of combination rules. This approach implies a tripartite architecture, with respective parts implementing the logical framework, the combination rule definition (referee function) and the computation processes. The combination rule definition is obtained by implementing a *referee function*. Referee functions are decisional arbitraments conditionally to basic decisions provided by the sources of information. It is defined at logical level. It is shown that referee functions [13] are sufficient for a definition of most combination rules. Then, a generic implementation of the rule is made possible on the basis of an algorithmic extension implementing the referee function. In this paper, we propose the construction of a referee function specially for the VBIED.

Section II makes a quick introduction of belief functions. Section III presents the Vehicle-Born Improvised Explosive Device problem (VBIED) and how it may be interpreted by means of belief functions. Section IV recalls the principle of the referee function approach. Section V defines a referee function for the VBIED. Section VI presents the results of an actual implementation. Section VII concludes.

II. A short introduction to belief functions

Belief functions are formalisms for defining and manipulating information with uncertainty and imprecision. On some aspects, belief functions could be seen as a generalization of probabilistic representations of the information. Belief functions are defined on logical structures, such as powersets [1], [2] or equivalently on *Boolean algebras*. However, some extensions of the theory have been proposed [20], [21], by considering distributive lattices. For this quick introduction, our presentation is restricted to the powerset $G = 2^\Omega$, *i.e.* the set of subsets of Ω. In this case, the set Ω is the frame of discernment of our problem. These definition are generalized to other lattice structures by replacing 2^Ω by lattices.

a) Basic Belief Assignment: A *basic belief assignment*(bba) m represents both the uncertainty and the imprecision of the information. It is the assignment of belief mass to each non-$\emptyset$ propositions of $G = 2^\Omega$, that is the mapping from elements of G onto $[0, 1]$ such that:

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{X \in G} m(X) = 1 \ . \tag{1}$$

A focal element X is an element of G such that $m(X) \neq 0$. Bba seems quite similar to a probabilistic distribution. A probabilistic distribution are defined on points (or singleton), while a bba may put basic belief on any proposition. This is how a bba manipulate both the uncertainty and the imprecision of the information. In particular, it is possible to define total ignorance by setting $m(\Omega) = 1$ and $m(X) = 0$ otherwise.

From basic belief assignments, other belief functions can be defined such as credibility and plausibility.

b) Credibility: The credibility represents the intensity that the information given by one expert supports an element of G, this is a minimal belief function given from a bba for all $X \in G$ by:

$$\text{bel}(X) = \sum_{Y \subseteq X, Y \neq \emptyset} m(Y). \tag{2}$$

c) Plausibility: The plausibility represents the intensity with which there is no doubt on one element. This function is given from a bba for all $X \in G$ by:

$$\text{pl}(X) = \sum_{Y \in G, Y \cap X \neq \emptyset} m(Y). \tag{3}$$

Being given several pieces of information encoded in the belief function formalism, it is often necessary to combine these pieces of information in order to produce a fused information and make a decision. In the literature, several combination rules exist, and we present in this short introduction the rule of Dempster-Shafer.

d) Combination rules: Today there is a lot of combination rules in the belief functions framework. Most of them are based on the conjunction of the focal elements in order to increase the belief on the most precise elements of the power set. Dempster-Shafer rule is given for two basic belief assignments m_1 and m_2 and for all $X \in G$ by:

$$m_{\text{DS}}(X) = \frac{I[X \neq \emptyset] m_{\text{c}}(X)}{1 - Z} \ , \tag{4}$$

$$\text{where:} \quad \left\{ \begin{array}{l} Z = m_{\text{c}}(\emptyset) \ , \\ m_{\text{c}}(X) = \sum_{Y \cap Z = X} m_1(Y) m_2(Z) \ . \end{array} \right. \tag{5}$$

Z can be interpreted as a non-expected solution and is generally called the global conflict of the combination or the inconsistence of the combination. The interpretation of Z and the transfer of this belief on other elements of the powerset gave birth to several combination rules [3]–[12].

In the next section, we present the Vehicle-Born Improvised Explosive Device problem and propose a formalization of this problem by means of Belief functions. In the forthcoming sections IV and V, we will present an approach for defining a combination rule for this problem.

III. THE VEHICLE-BORN IMPROVISED EXPLOSIVE DEVICE PROBLEM

This problem (VBIED) has been proposed during conference *FUSION'2010* as a challenge for evaluating various approximate reasoning approaches.

e) Description of the VBIED problem: We will only describe the first part of this problem.

- **Concern:** Vehicle-Born Improvised Explosive Device attack on an administrative building B.
- **Prior information:** We consider an Individual A under surveillance due to previous unstable behaviour, who drives customized white Toyota vehicle.
- **Observation done at time** $t - 10min$ **:** From a video sensor on road that leads to building B, one has observed a White Toyota $200m$ from the building B travelling in normal traffic flow toward building B. We consider the following two sources of information based on this video observation:
  - **Source 1:** An Analyst 1 (10 years experience) analyses the video and concludes that individual A is now probably near building B.
  - **Source 2:** An Automatic Number Plate Recognition (ANPR) system analyzing same video outputs 30% probability that the vehicle is individual A's white Toyota.
- **Observation done at time** $t - 5\text{min}$ **:** From a video sensor on road $15km$ from building B, one gets a video that indicates a white Toyota with some resemblance to individual A's white Toyota. We consider the following third source of information available:
  - **Source 3:** An Analyst 2 (new in post) analyses the video and concludes that it is improbable that individual A is near building B.
- **Question:** Should building B be evacuated?

NOTE: Deception (*e.g.* individual A using different car, false number plates, *etc.*) and biasing (on the part of the analysts) are often a part of reality, but they are not part of this example.

Dezert and Smarandache proposed [22] a solution to this problem based on belief functions. Our approach will be based on a similar, but simplified, formalization of the problem by means of basic belief assignment. However, our answer to this problem will be based on the construction of a new combination rule. This rule will be constructed from the logical structure of the VBIED problem.

f) The logical structure: The logical framework of our problem is the free Boolean algebra generated by the atomic propositions A, V, B.

- The proposition A means that the suspicious individual has been observed, while $\overline{A}$ means the negation of this fact,

- The proposition V means that the White Toyota Vehicle has been observed, while $\overline{V}$ means the negation of this fact,
- The proposition B means an observation near the building B, while $\overline{B}$ means the negation of this fact.

We may consider a pessimistic, temperate and optimistic attitude for diagnosing a potential danger:

- **Pessimistic attitude:** A danger is diagnosed by the proposition:

$$(A \cap V \cap B) \cup (A \cap \overline{V} \cap B) \cup (\overline{A} \cap V \cap B) ,$$

  which means one of the following cases:
  - $(A \cap V \cap B)$: The suspicious individual has been seen with a white Toyota near the building,
  - $(A \cap \overline{V} \cap B)$: The suspicious individual has been seen near the building,
  - $(\overline{A} \cap V \cap B)$: The white Toyota has been seen near the building.
- **Temperate attitude:** A danger is diagnosed by the proposition:

$$(A \cap V \cap B) \cup (\overline{A} \cap V \cap B)$$

- **Optimistic attitude:** A danger is diagnosed by the proposition $A \cap V \cap B$

g) Defining the belief assignments: We propose an encoding of the hypotheses of the problem into basic belief assignments as follows:

- **Prior information:** It is assumed that the suspicious individual is driving the white Toyota, *i.e.* $A \cap V$. The bba m_0 for the prior is defined by:

$$m_0(A \cap V) = 1 .$$

  This information is assumed to be known with a great reliability and we will characterize this reliability by a probability p_0 near 1 that the information is reliable: $p_0 = 1 - \alpha$ with $\alpha = 5\%$. The quite small value of α means that this information is considered as almost sure.
- **Source 1:** Source 1 is considering probable that the individual is near the Building. This information will be modelled as follows:

$$m_1(A \cap B) = 75\% \quad \text{and} \quad m_1(\Omega) = 25\% .$$

  This information is considered as highly reliable, and this reliability is characterized by $p_1 = 90\%$.
- **Source 2:** Source 2 is an automated system (ANPR) which is assumed to provide a probabilistic information. The provided information is that individual and vehicle has been seen with a probability of 30%, which is encoded as follows:

$$\left\{ \begin{array}{l} m_2(A \cap V) = 30\% \\ m_2((\overline{A} \cap V) \cup (A \cap \overline{V}) \cup (\overline{A} \cap \overline{V})) = 70\% . \end{array} \right.$$

  This information is considered as fully reliable, and this reliability is characterized by $p_2 = 100\%$.
- **Source 3:** Source 3 is considering as improbable that the individual is near the Building. This information will be modelled as follows:

$$m_3(A \cap B) = 25\% \quad \text{and} \quad m_3(\Omega) = 75\% .$$

  The Analyst is new in the post and this information will be characterized by a temperate reliability $p_3 = 50\%$.

Our purpose now is to define a combination rule for fusing this information, to fuse it, and to evaluate the fused belief on proposition:

$$\left\{ \begin{array}{l} (A \cap V \cap B) \cup (A \cap \overline{V} \cap B) \cup (\overline{A} \cap V \cap B) , \\ (A \cap V \cap B) \cup (\overline{A} \cap V \cap B) , \\ (A \cap V \cap B) . \end{array} \right.$$

From this information, we will decide for an evacuation or not.

Our work is based on the java toolbox *Referee toolbox*, which allows the implementation of new fusion rules by defining them by means of referee functions. The next section will quickly introduce the toolbox by explaining its architecture, the notion of referee functions, and how it is possible to build new rules for combining belief functions.

IV. Referee functions

The java toolbox *Referee toolbox* is the implementation of previous works [13], [14] in the domain of belief functions. It is downloadable at address:

http://refereefunction.fredericdambreville.com/releases

It implements the concept of referee function, which allows a possible generic implementation of combination rules. The toolbox is based on a tripartite architecture, with respective parts implementing the logical framework *(logical component)*, the combination definition *(referee component)*, and the belief-related processes *(belief component)*; figure 1. The

Figure 1. A tripartite architecture

logical framework represents the information while considered without its uncertainty. Typical logical frameworks are powersets, which are Boolean algebras. However, lattices are also useful frameworks and actually generalize the Boolean algebras; hyperpowersets are examples of lattices. The combination definition is obtained by implementing *referee functions*. Referee functions are decisional arbitraments at the logical level, conditionally to basic decisions provided by the sources of information. It is shown that a definition of the referee function is sufficient for defining most combination rules. The belief-related processes deal with all belief computations. That means the computation of the plausibility and the credibility from the basic belief assignments, as well as the computation of the combination rules defined by the referee functions. By the way, the belief-related processes also deal with the complexity of the combination rules and implements approximation methods for resolving these issues. A more detailed description of this architecture can be found in [14].

A. Logical component

The logical component constitutes one of the three part of our system. While the combination of Dempster-Shafer [1], [2] is generally defined over powersets, the combination rules are defined on the basis of logical operators and properties which are not necessary inherent to powersets, nor Boolean algebras. Extensions of the theory have been proposed on the basis of distributive lattices [20]. Hyperpowersets [21] are also distributive lattices. Most variants of evidence theory are built on logical structures which are instances of *complete* (in general finite) *distributive lattices*. In the *Referee Toolbox*, the logical component handles these structures in a generic manner, especially considering the logical notions from the general viewpoint of *complete distributive lattices*. A short definition of these notions is given in appendix A.

In the present work, the logical component in use is a Boolean algebra, which is a structure equivalent to the powersets.

B. Belief component

The belief component is the main algorithmic part of our system. It is the part of main interest in this works, since it will allow us the definition of combination rules from the setting of the VBIED problem. It interacts with both the logical and referee component. It implements all the belief-related processes. A detailed description of the component is available in [22] and we will not present in details this component, since it is not directly useful here. However, it is noticed that this component implements the following tasks:

- Computation of the credibility from the bba,
- Computation of the plausibility from the bba,
- Generic combination processes for computing combination rules defined by a referee function.

The generic combination processes handle the combinatorics of the combination. In order to do that, two approximation paradigms are implemented:

- Generic combination processes based on a sampling. The principle is then to approximate the bba by a Monte-Carlo method,
- Generic combination processes based on a summarization [16]–[19]. The principle is to reduce the size of the set of focal elements by simple approximations which decease the information. Typically, one may transfer the negligible basic belief assignments of two propositions to the union of these propositions. It is this implementation which will be used in the present work.

The previous components are the algorithmic framework of the toolbox. Now, in order to build a new rule, it is only necessary to define and encode a referee function. The forthcoming section explains this notion.

C. Referee component

The referee component constitutes the part of our system, which implements the definition of the combination rules. The definition of the combination rules is made by means of *referee functions*. Referee functions have been defined in [13] and allow a simple, general and computational interpretation of the combination rules. This interpretation was inspired first by works on probabilistic restrictions of evidence combinations [13]. This work led to a conditional interpretation of the combination, in terms of *Referee Function*.

1) Referee function:

a) Definition: A referee function over G for s sources of information and with context γ is a mapping $X, Y_{1:s} \mapsto F(X|Y_{1:s};\gamma)$ defined on propositions $X, Y_{1:s} \in G$, which satisfies for any $X, Y_{1:s} \in G$:

$$F(X|Y_{1:s};\gamma) \geq 0 \quad \text{and} \quad \sum_{X\in G} F(X|Y_{1:s};\gamma) = 1 \ ,$$

The context γ is a parameter used for an adaptive control of the arbitrament. It may be a function of $Y_{1:s}$ and subsequently, it is considered $\gamma = m_{1:s}$.

A referee function for s sources of information is also called a s-ary referee function. The quantity $F(X|Y_{1:s};\gamma)$ is called a *conditional arbitrament* between $Y_{1:s}$ in favour of X. Notice that X is not necessary one of the propositions $Y_{1:s}$; typically, it could be a combination of them. The case $X = \emptyset$ is called the *rejection case*.

b) Fusion rule: Let be given s basic belief assignments (bba) $m_{1:s}$ and a s-ary referee function F with context $m_{1:s}$. Then, the fused bba $m_1 \oplus \cdots \oplus m_s[F] \overset{\Delta}{=} \oplus[m_{1:s}|F]$ based on the referee F is constructed as follows:

$$\begin{aligned}\oplus[m_{1:s}|F](X) = \frac{I[X\neq\emptyset]}{1-z}& \\ \times \sum_{Y_{1:s}\in G} F(X|Y_{1:s};m_{1:s}) \prod_{i=1:s} m_i(Y_i) \ ,& \\ \text{where } z = \sum_{Y_{1:s}\in G} F(\emptyset|Y_{1:s};m_{1:s}) \prod_{i=1:s} m_i(Y_i) \ .&\end{aligned} \tag{6}$$

The value z is called the *rejection rate*.

c) Property: The function $\oplus[m_{1:s}|F]$ defined on G is actually a basic belief assignment.

2) Examples of referee functions:

a) Dempster-shafer: The definition of a referee function for Dempster-Shafer combination is immediate:

$$m_{\rm DS} = \oplus[m_{1:s}|F_{\rm DS}] \;,$$

$$\text{where } F_{\rm DS}(X|Y_{1:s};m_{1:s}) = I\left[X = \bigcap_{k=1:s} Y_k\right] \;.$$

b) Disjunctive: The definition of a referee function for the disjunctive combination is:

$$m_{\rm d} = \oplus[m_{1:s}|F_{\rm d}] \;,$$

$$\text{where } F_{\rm d}(X|Y_{1:s};m_{1:s}) = I\left[X = \bigcup_{k=1:s} Y_k\right] \;.$$

c) Dubois&Prade: The definition of a referee function for Dubois&Prade combination on two sources is:

$$m_{\rm DP} = m_1 \oplus m_2[|F_{\rm DP}] \;,$$

$$\text{where } F_{\rm DP}(X|Y_{1:2};m_{1:2}) = I\left[X = Y_1 \cap Y_2 \neq \emptyset\right] + I\left[Y_1 \cap Y_2 = \emptyset\right] I\left[X = Y_1 \cup Y_2\right] \;.$$

d) PCR6: The proportional conflict redistribution rules (PCRn) have been introduced By Smarandache and Dezert [10]. The rule PCR6 has been proposed by Martin and Osswald in [8]. The original definition of the rule could be found there. A formulation of PCR6 by means of a referee function is derived in [13]:

$$m_{\rm PCR6} = \oplus[m_{1:s}|F_{\rm PCR6}] \;,$$

where the referee function $F_{\rm PCR6}$ is defined by:

$$F_{\rm PCR6}(X|Y_{1:s};m_{1:s}) = I\left[X = \bigcap_{k=1:s} Y_k \neq \emptyset\right] + I\left[\bigcap_{k=1:s} Y_k = \emptyset\right] \frac{\sum_{j=1:s} I[X=Y_j]\, m_j(Y_j)}{\sum_{j=1:s} m_j(Y_j)} \;. \quad (7)$$

This referee function implies an interpretation of PCR6 as a two-cases process (PCR6 only considers full consensus or no-consensus cases):

- The inputs are compatible; then, the conjunctive consensus is decided,
- The inputs are not compatible; then, a mean decision is decided, weighted by the relative beliefs of the entries.

Then, it is noticed that a referee function could be interpreted as a stochastic process. Typically, the referee function $F_{\rm PCR6}$ consists in the following process:

1) If $\bigcap_{i\in\gamma} Y_i \neq \emptyset$, then set return $\bigcap_{i=1:s} Y_i$ and **stop**,
2) Otherwise, compute:

$$p_i = \frac{m_i(Y_i)}{\sum_{i=1:s} m_i(Y_i) m_i(Y_i)} \;,$$

3) Generate the integer i_o randomly according to the distribution $(p_{1:s})$,
4) Return Y_{i_o} and **stop**,

In fact, the *Referee Toolbox* requests this stochastic process to be implemented in the alternate deterministic form:

$$\{(X_k,\omega_k)\}_{k=1:K} = \textbf{refereeFunction}\,(Y_{1:s},m_{1:s}) \;.$$

The actual implementation of the referee function $F_{\rm PCR6}$ in the toolbox is thus:

$\{(X_k,\omega_k)\}_{k=1:K} = \textbf{refereeFunction}\,(Y_{1:s},m_{1:s})$:

1) If $\bigcap_{i\in\gamma} Y_i \neq \emptyset$, then set return $\{(\bigcap_{i=1:s} Y_i, 1)\}$ and **stop**,
2) Otherwise, compute:

$$p_i = \frac{m_i(Y_i)}{\sum_{i=1:s} m_i(Y_i) m_i(Y_i)} \;,$$

3) Return $\{(Y_i,p_i)\}_{i=1:s}$ and **stop**,

It is not difficult to jump from a form to another. In this paper, we will choose the most convenient form for the paper presentation.

e) Conclusion: The last example has illustrated that a referee function has a direct interpretation as a stochastic process. This stochastic process just implements the way we make the arbitrament between the logical entries $Y_{1:s}$. Then, we have an intuitive way to build a combination rule by defining this arbitrament process (or referee function). We propose now to build such a referee function for the VBIED.

V. Definition of a referee function for the VBIED

f) Proposal of a Referee function: Subsequently, we propose a combination rule for fusing the bba m_0 (the prior), m_1, m_2, m_3 (sources 1, 2 and 3) defined in section III, in order to answer to the VBIED. This computation rule will be defined by means of a referee function, F_{VBIED}, and the obtained fused bba is denoted m_{VBIED}. Subsequently, F_{VBIED} is defined as a stochastic process.

In the VBIED, we have first to take into account the reliability of the different sources (including the prior). Typically, the way we handle the input information depends on how they are reliable. Then our arbitrament process works in two steps. A first step will simulate which entries are reliable. The second step will arbitrate between the reliable entries.

The first step is implemented by the process $J = \textbf{getReliable}()$ which produce a random set $J \subset [\![0,3]\!]$ accordingly to the reliability parameters $p_{0:3}$:

$J = \textbf{getReliable}()$:

1) Set $J = \emptyset$,
2) For $i = 0:3$ do:
    a) With probability p_i, set $J = J \cup \{i\}$,
    b) Otherwise, do nothing,
3) Return J and **stop**.

When the first step is done, we have to run a subprocess $X = \textbf{subReferee}(Y_{1:s}, m_{1:s}, J)$ which arbitrates for the best consensus with the prior (if the prior is actually reliable). In this example, we consider that the best consensus is the consensus which involves the greater number of entries. As a consequence, it may exists several best consensus. In such a case, the best consensus are chosen randomly according to their respective weights. The subprocess is sketched as follows:

$X = \mathbf{subReferee}(Y_{1:s}, m_{1:s}, J)$:

1) If $0 \in J$, set $Z = Y_0$,
2) Otherwise, set $Z = \Omega$,
   // The two previous steps decide to use entry Y_0 or not, depending on J,
3) Find $H \subset J \setminus \{0\}$ with maximal size such that $Z \cap \bigcap_{i \in H} Y_i \neq \emptyset$,
4) Define M the set of subset $K \subset J \setminus \{0\}$ such that $Z \cap \bigcap_{i \in K} Y_i \neq \emptyset$ and $\mathrm{size}(K) = \mathrm{size}(H)$,
   // The two previous steps build the set M of maximal consensus with the prior $Z = Y_0$ (if the prior is actually reliable),
5) If $M = \emptyset$, return Z and **stop**,
   // If there is no consensus between the sources of information, the prior is considered as prevalent,
6) For any $K \in M$, build the weight:
$$\omega_K = \prod_{i \in K} m_i(Y_i) \;,$$
7) Generate $K_o \in M$ randomly, according to probability:
$$\frac{\omega_{K_o}}{\sum_{L \in M} \omega_L} \;,$$
8) Return $Z \cap \bigcap_{i \in K} Y_i$ and **stop**,
   // The two previous steps result in generating a best consensus randomly according to their respective belief.

We may notice the specific role of the prior Y_0. As soon as the prior is considered reliable, its is always involved in the decision. This is a choice in our problem modelling. What is important here, is the fact that we are able to tune our combination rule according to our modelling choices. This is a significant contribution of the referee functions.

Now at last, we are able to define the full arbitrament process for F_{VBIED} which is a combination of the two previous process:

$\{(X_k, \omega_k)\}_{k=1:K} = \mathbf{refereeFunction}\,(Y_{1:s}, m_{1:s})$:

1) Set $J = \mathbf{getReliable}()$,
2) Return $X = \mathbf{subReferee}(Y_{1:s}, m_{1:s}, J)$ and **stop**,

The next section gives some numerical results of our approach.

VI. Results.

For simplicity, it is denoted:

- $a = A \cap \overline{V} \cap B$,
- $b = \overline{A \cap V}$,
- $c = A \cap V \cap B$,
- $d = A \cap V$,
- $e = A \cap B$

The fused bba m_{VBIED} computed by means of the referee function F_{VBIED} is as follows:

$$m_{VBIED}(a) = 0.025\,,\; m_{VBIED}(b) = 0.010\,,$$
$$m_{VBIED}(c) = 0.691\,,\; m_{VBIED}(d) = 0.274\,.$$

It is then derived:

- **Pessimistic:** $\mathrm{bel}(c \cup a \cup (\overline{A} \cap V \cap B)) = 0.716 > 0.5$,
- **Temperate:** $\mathrm{bel}(c \cup (\overline{A} \cap V \cap B)) = 0.691 > 0.5$,
- **Optimistic:** $\mathrm{bel}(c) = 0.691 > 0.5$.

means clearly an alert and an evacuation of B even in case of an optimistic attitude.

It may be interesting to see how this result evolves when changing the reliability parameters p_i and to compare this combination rule with reference rules like Dempster-Shafer and PCR6. The following tests are done with $p_2 = 1$ and different values of p_0, p_1 and p_2. In order to take into account the reliability in the Dempster-Shafer and PCR6 combination, the basic belief assignments are weakened as follows:

$$m_i^w(X) = p_i m_i(X) + I[X = \Omega](1 - p_i)\;.$$

Then, the rule of Dempster-Shafer and PCR6 are applied to m_i^w, while the rule VBIED is applied to m_i directly.

α) $p_0 = 0,\; p_1 = 0.9,\; p_2 = 1,\; p_3 = 0.5$:

X	a	b	c	d	e	Ω	Z
$m_{VBIED}(X)$	.501	.199	.215	.085			
$m_{PCR6}(X)$	.501	.199	.215	.085			
$m_{DS}(X)$	.501	.199	.215	.085			0

β) $p_0 = 0,\; p_1 = 0.5,\; p_2 = 1,\; p_3 = 0.9$:

X	a	b	c	d	e	Ω	Z
$m_{VBIED}(X)$	.361	.339	.155	.145			
$m_{PCR6}(X)$	.361	.339	.155	.145			
$m_{DS}(X)$	.361	.339	.155	.145			0

In the case α) and β), the prior is considered as unreliable. By removing this prior, the possible conflict vanishes, and their is a possible full consensus between the sources of information. As a consequence, the results are identical for the three rules.

γ) $p_0 = 0.5,\; p_1 = 0.9,\; p_2 = 1,\; p_3 = 0.5$:

X	a	b	c	d	e	Ω	Z
$m_{VBIED}(X)$	.25	.1	.465	.185			
$m_{PCR6}(X)$	.25	.198	.215	.155	.064	.118	
$m_{DS}(X)$	.386	.153	.330	.131			.35

δ) $p_0 = 0.5,\; p_1 = 0.5,\; p_2 = 1,\; p_3 = 0.9$:

X	a	b	c	d	e	Ω	Z
$m_{VBIED}(X)$	.18	.17	.335	.314			
$m_{PCR6}(X)$	.18	.274	.155	.22	.031	.14	
$m_{DS}(X)$	.278	.261	.238	.223			.35

ϵ) $p_0 = 0.95,\; p_1 = 0.9,\; p_2 = 1,\; p_3 = 0.5$:

X	a	b	c	d	e	Ω	Z
$m_{VBIED}(X)$	.025	.01	.691	.274			
$m_{PCR6}(X)$	.025	.167	.215	.299	.103	.191	
$m_{DS}(X)$	.075	.03	.641	.254			.665

ζ) $p_0 = 0.95,\; p_1 = 0.5,\; p_2 = 1,\; p_3 = 0.9$:

X	a	b	c	d	e	Ω	Z
$m_{VBIED}(X)$	.018	.017	.498	.467			
$m_{PCR6}(X)$	.018	.183	.155	.37	.049	.225	
$m_{DS}(X)$	.054	.051	.462	.434			.665

$\eta)$ $p_0 = 1,\ p_1 = 0.9,\ p_2 = 1,\ p_3 = 0.5$:

X	a	b	c	d	e	Ω	Z
$m_{VBIED}(X)$			.716	.284			
$m_{PCR6}(X)$		.163	.215	.318	.106	.198	
$m_{DS}(X)$			.716	.284			.7

$\theta)$ $p_0 = 1,\ p_1 = 0.5,\ p_2 = 1,\ p_3 = 0.9$:

X	a	b	c	d	e	Ω	Z
$m_{VBIED}(X)$			.516	.484			
$m_{PCR6}(X)$		.172	.155	.39	.051	.233	
$m_{DS}(X)$			.516	.484			.7

Dempster-Shafer and VBIED rules produce the same results in cases $\eta)$ and $\theta)$. This result is probably a fortuitous consequence of the VBIED problem setting. Dempster-Shafer and VBIED are inherently different rules.

PCR6 seems to be the most undecided rule. We have seen that PCR6 does not handle partial consensus, and produces a mean of its inputs when a full consensus is not possible. Such result is perhaps a consequence of that.

It is noticed for both Dempster-Shafer and VBIED, that $\mathrm{bel}(c \cup a \cup (\overline{A} \cap V \cap B)) > 0.5$ in all cases, which means an evacuation according to the cautious attitude. It is not the case however, when considering the temperate or optimistic attitudes. In these cases, the results depends increasingly on the reliability of the prior and on the high reliability of source 1 in regards to source 3. An evacuation is clear in the following cases, by considering the score of c ($\mathrm{bel}(c) > 0.5$):

VBIED $\epsilon)$, $\eta)$, $\theta)$,
DS $\epsilon)$, $\eta)$, $\theta)$.

An evacuation is possible in the following cases, by considering the score of both c and d ($\mathrm{pl}(c) > 0.5$):

VBIED $\gamma)$, $\delta)$, $\zeta)$,
DS $\zeta)$.

It appears generally that the rule VBIED provides a more accentuated answer than DS and PCR6. The reason is that it takes into account any intermediate consensus, in case the full consensus fails.

VII. Conclusion

This paper has proposed a belief function approach for solving the Vehicle-Born Improvised Explosive Device problem. This approach made use of a previously produced library, the java toolbox *Referee Toolbox* which allows generic implementations of combination rules for belief function. The definition the combination rule is implemented by means of referee functions, which are arbitrament processes conditionally to the contributions of the sources of information. This allowed us to construct a combination rule adapted to the VBIED, by especially taking into account the modelled reliability of the sources and the particular role of the prior. Moreover, by always considering the best consensus between the entries, the obtained fusion process closely processes the input information. While this draft paper only handles the first part of the VBIED challenge, the final paper should consider the full problem.

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Appendix

A. Complete distributive lattice

This section provides a theoretical description of the logical structures which are implemented in the toolbox *Referee Toolbox*.

a) Distributive lattice: An algebraic structure G with operators $\cap$ and $\cup$ is a distributive lattice if it verifies the properties:

- $X \cup (Y \cup Z) = (X \cup Y) \cup Z$
  $X \cap (Y \cap Z) = (X \cap Y) \cap Z$ (associativity)
- $X \cup Y = Y \cup X$
  $X \cap Y = Y \cap X$ (commutativity)
- $X \cup (X \cap Y) = X$
  $X \cap (X \cup Y) = X$ (absorption)
- $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ (distributivity)

for any $X, Y, Z \in G$.

Implied order: The relation $\subseteq$ defined by:

$$X \subseteq Y \overset{\Delta}{\iff} X \cap Y = X \iff X \cup Y = Y \;,$$

is a partial order on G.

b) Complete distributive lattice: A distributive lattice G is *complete* if any subset $\Xi \subset G$ has a greatest lower bound and a least upper bounds:

- There is $\bigcap_{X \in \Xi} X \in G$ such that:
  - $\bigcap_{X \in \Xi} X \subseteq Y$ for any $Y \in \Xi$,
  - If $Z \in G$ is such that $Z \subseteq Y$ for any $Y \in \Xi$, then $Z \subseteq \bigcap_{X \in \Xi} X$.
- There is $\bigcup_{X \in \Xi} X \in G$ such that:
  - $Y \subseteq \bigcup_{X \in \Xi} X$ for any $Y \in \Xi$,
  - If $Z \in G$ is such that $Y \subseteq Z$ for any $Y \in \Xi$, then $\bigcup_{X \in \Xi} X \subseteq Z$.

In particular are defined the lower bound $\emptyset = \bigcap_{X \in G} X$ and upper bound $\Omega = \bigcup_{X \in G} X$ of G.

Examples:

- A finite distributive lattice is necessary complete.
- An *hyperpower set*, D^Θ, generated by a *finite* Θ is a complete distributive lattice,

c) Complement and co-complement: The referee toolbox implements a complement notion for the complete distributive lattices, the complement and co-complement operators. These operators are of course identical for Boolean algebras.

The complement, $\overline{X}$, of $X \in G$ is defined by:

$$\overline{X} = \bigcup_{Y : Y \cap X = \emptyset} Y \;;$$

The co-complement, $\underline{X}$, of $X \in G$ is defined by:

$$\underline{X} = \bigcap_{Y : Y \cup X = \Omega} Y \;.$$

Examples:

- For a *closed* hyperpower set (*i.e.* with property $\bigcup_{Y \in \Theta} Y = \Omega$), the complement and co-complement are defined by:
  - $\overline{X} = \emptyset$ for $X \neq \emptyset$, and $\overline{\emptyset} = \Omega$,
  - $\underline{X} = \bigcup_{Y \in \Theta : Y \not\subset X} Y$.