HyperUncertain, SuperUncertain, and SuperHyperUncertain Sets/Logics/Probabilities/Statistics

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Abstract

In this paper, we define for the first time the HyperUncertain, SuperUncertain, and SuperHyperUncertain Sets/Logics/Probabilities/Statistics, the classical case (when the appurtenance degree of a generic element belongs to the unit interval [0, 1]), and the non-classical case (when the appurtenance degree of a generic element is outside on the interval [0, 1]; such theories are called Over/Under/Off-Sets/Logics/Probabilities/Statistics).

We prove that the Unary SuperHyperFunction is a generalization of all Uncertain Sets/Logics/Probabilities.

Keywords

SuperHyperStructure, Unary SuperHyperFunction, (h-ary, k-ary) SuperHyperFunction,

HyperUncertain Sets, HyperUncertain Logics, HyperUncertain Probabilities, HyperUncertain Statistics, SuperUncertain Sets, SuperUncertain Logics, SuperUncertain Probabilities, SuperUncertain Statistics, SuperHyperUncertain Sets, SuperHyperUncertain Logics, SuperHyperUncertain Probabilities, SuperHyperUncertain Statistics.

1. Denominations of "Uncertain" and "Theory"

By "Uncertain" we mean the following types: Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Inconsistent Intuitionistic Fuzzy (Picture Fuzzy, Ternary Fuzzy), Pythagorean Fuzzy (Atanassov's Intuitionistic Fuzzy of second type), Fermatean Fuzzy, q-Rung Orthopair Fuzzy, Spherical Fuzzy, n-HyperSpherical Fuzzy, Refined Neutrosophic, Plithogenic, etc.

By "Theory" we mean the following: Set, Logic, Probability, and Statistics.

2. Definition of SuperHyperStructure

A SuperHyperStructure [4] is a structure built on the n-th PowerSet of a Set A, for n ≥ 1, as in our real world
{because a set (or system) A (that may be a set of items, an organization, a country, etc.) is composed by sub-sets that are parts of P(A), which in their turn are organized in sub-sub-sets that are parts of $P(P(A)) = P^2(A)$, then in sub-sub-sub-sets that are parts of $P^3(A)$, and so on, $P^n(A) = P(P^{n-1}(A)$ }.
As part of this most general notion, called SuperHyperStructure or (m, n)-SuperHyperStructure, and its associated terms such as SuperHyperAxiom, SuperHyperOperator, SuperHyperAlgebra etc. all introduced by Smarandache [1] in 2016, let's recall the definitions of the n-PowerSet of a Set, and of the SuperHyperFunctions that will be used in this paper.

3. Definition [1, 4] of the n-th PowerSet of a Set A

Let $\mathcal{U}$ be a universe of discourse, and A be a non-empty subset of $\mathcal{U}$, with $P(A)$ *the powerset of A*, and $n \geq 0$ an integer.
Then:

$$P^0(A) \overset{def}{=} A$$

$$P^1(A) = P(A)$$

$$P^2(A) = P(P(A))$$

....

$$P^n(A) = P(P^{n-1}(A))$$

4. Definition of the (h, k)-SuperHyperFunction

The most general form of a function is the (h-ary, k-ary)-SuperHyperFunction:

$$f_{SH}^{SH}: P^{m_1}(A_1) \times P^{m_2}(A_2) \times \ldots \times P^{m_h}(A_h) \rightarrow P^{n_1}(B_1) \times P^{n_2}(B_2) \times \ldots \times P^{n_k}(B_k)$$

where $A_1, A_2, \ldots, A_h, B_1, B_2, \ldots, B_k$ are subsets of $\mathcal{U}$, and h, k are integers ≥ 0.
For each $x_1 \in P^{m_1}(A_1), x_2 \in P^{m_2}(A_2), \ldots, x_h \in P^{m_h}(A_h)$ one has

$$f_{SH}^{SH}(x_1, x_2, \ldots, x_h) \in P^{n_1}(B_1) \times P^{n_2}(B_2) \times \ldots \times P^{n_k}(B_k),$$

where "*SH*" on the bottom of f_{SH}^{SH} stands for "SuperHyper" (which means that one has *m_i-powersets,* $1 \leq i \leq h$) in the domain of the function, while "*SH*" on the top of f_{SH}^{SH} stands for "SuperHyper" (which similarly means that one has *n_j-powersets,* $1 \leq j \leq k$)

in the range of the function.

5. Unary SuperHyperFunction

The Unary SuperHyperFunction is a particular case of the (*h-ary, k-ary*)-SuperHyperFunction when $h = k = 1$.
Let $\mathcal{U}$ be a universe of discourse, and A, B two non-empty subsets of $\mathcal{U}$.
Let $m, n \geq 0$ be integers,
and $P^m(A)$ be the *m-th powerset of A*, while similarly, $P^n(B)$ be the *n-th powerset of B*.
Then: $f_{SH}^{SH}: P^m(A) \to P^n(B)$
is called Unary SuperHyperFunction.

For an $x \in P^m(A)$ one has an $f_{SH}^{SH}(x) \in P^n(B)$.

6. Theorem:

The Unary SuperHyperFunction is a generalization of all classical and non-classical Uncertain Sets/Logics/Probabilities/Statistics.
By "classical" we mean those theories whose codomain is the unit interval,
$B = [0, 1]$.
By "non-classical" we mean the theories whose codomain is different from [0, 1], i.e.
$B = [\varphi, \psi] \neq [0,1]$, where $\varphi \leq 0 < 1 \leq \psi$,
as in Over/Under/Off-Sets/Logics/Probability/Statistics {Smarandache, [2, 3]}.

Proof:

Let $\mathcal{U}$ be a universe of discourse, and A a non-empty subset of $\mathcal{U}$.
Let A be the domain of an Uncertain Set/Logic/Probability/Statistics, where the Uncertain Statistics is the statistical characterization of its corresponding Uncertain Probability, therefore it may have the same domain as its Uncertain Probability – no matter if one has a classical or non-classical theory (see above).

Let $B = [0, 1]$ be the real unit interval, as codomain.

(a) Then, as a particular case of the Unitary SuperHyperFunction, one considers the function:

$$f_{SH}^{SH}: P^m(A) \to P^n([0,1]) \qquad (*)$$

This function can easily represent any type of <u>classical</u> HyperUncertain SuperUncertain, and SuperHyperUncertain Set/Logic/Probability/Statistics.

(b) While the function:

$f_{SH}^{SH}: P^m(A) \to P^n([\varphi, \psi])$, where $\varphi \leq 0 < 1 \leq \psi$,

represents any type of <u>non-classical</u> { i.e. Over/Under/Off- } HyperUncertain, SuperUncertain, and

SuperHyperUncertain Set/Logic/Probability/Statistics [2, 3].

7. Number of Degree-Types

Let $\tau: A \rightarrow N$, where A is the domain of a given Uncertain Theory, while N is the set of natural (positive integer) numbers, with $\tau(x)$ = the number of degree types of this given Uncertain Theory (i.e. Uncertain Set/Logic/Probability/Statistics).

Let $\tau(x)$ = r ≥ 1 be the number of degree types ($d_1, d_2, \ldots, d_r$) of some Uncertain Theory, for example, if r = 4 one may have:
d_1 = degree of membership/truth/chance_of_occuring,
d_2 = degree of nonmembership/falsehood/chance_of_not_occuring
d_3 = degree of indeterminacy/neutrality
d_4 = degree of contradiction.
Then:
$d_1, d_2, \ldots, d_r : P^m(A) \rightarrow P^n([0,1])$, for classical Uncertain Theories.
Or:
$d_1, d_2, \ldots, d_r : P^m(A) \rightarrow P^n([\varphi,\psi])$, $\varphi \leq 0 < 1 \leq \psi$, for un-classical Uncertain Theories.
As theories, we especially refer to "Set" and "Logic" and "Probability" (since "Statistics" is just characterization of the Probability events), which are the most used ones with respect to uncertain(ty).

(i) For several Uncertain Theories, for any $x \in A$, $\tau(x)$ is a single degree – degree of membership (or truth, or chance of occurring), normally denoted by $t(x)$, therefore $\tau(x) = t(x)$,
or $\tau: A \rightarrow [0,1]$, especially in classical fuzzy set/logic/probability.

(ii) For other Uncertain Theories, $\tau(x)$ is a double degree: degree of membership (or truth), denoted by $t(x)$, and degree of nonmembership (or falsehood), denoted by $f(x)$. Therefore: $\tau(x) = (t(x), f(x))$, or $\tau: A \rightarrow [0,1]^2$.
As an example: for intuitionistic fuzzy set/logic/probability and its derivatives.

(iii) For Neutrosophic Set/Logic/Probability and some of its derivatives, τ(x) is a triple degree:
- degree of membership (or truth), $t(x)$;
- degree of indeterminacy (or neutrality), $i(x)$;
- and degree of nonmembership (or falsehood), $f(x)$.

As such, $\tau(x) = (t(x), i(x), f(x))$, or $\tau: A \rightarrow [0,1]^3$.

(iv) For Refined Neutrosophic Set/Logic/Probability, and its derivatives, τ(x) is an q-tuple degree ($q \geq 1$):

$\tau(x) = \big(t_1(x), t_2(x), \ldots, t_p(x);\ i_1(x), i_2(x), \ldots, i_r(x);\ f_1(x), f_2(x), \ldots, f_s(x)\big)$

where $t_j(x), 1 \leq j \leq p$, is type-j of membership (truth);
$i_k(x), 1 \leq k \leq r$, is type-k of indeterminacy (neutrality);
$f_l(x), 1 \leq l \leq s$, is type-l of nonmembership (falsehood),
with $p + r + s = q$,
or $\tau: A \rightarrow [0,1]^q$.

8. HyperUncertain Classical Set/Logic/Probability/Statistics

$\tau : A \rightarrow P^n([0,1]^r)$

9. SuperUncertain Classical Set/Logic/Probability/Statistics

$\tau : P^m(A) \rightarrow [0,1]^r$

10. SuperHyperUncertain Classical Set/Logic/Probability/Statistics

$\tau : P^m(A) \rightarrow P^n([0,1]^r)$

*

For non-classical Uncertain Theories, the unit interval [0, 1] is replaced by the non-unit interval $[\varphi, \psi]$, where $\varphi \leq 0 < 1 \leq \psi$.

11. HyperUncertain Non-Classical Set/Logic/Probability/Statistics

$\tau : A \rightarrow P^n([\varphi, \psi]^r)$

12. SuperUncertain Non-Classical Set/Logic/Probability/Statistics

$\tau : P^m(A) \rightarrow [\varphi, \psi]^r$

13. SuperHyperUncertain Non-Classical Set/Logic/Probability/Statistics

$\tau : P^m(A) \rightarrow P^n([\varphi, \psi]^r)$

14. Examples of HyperUncertain Classical Set/Logic/Probability/Statistics

$\tau : A \rightarrow P^n([0,1]^r)$

"Hyper" means the degree of an element $x \in A$ is a subset (not a single element) of $[0, 1]$.

(i) **HyperFuzzy** is equivalent to Subset-Valued Fuzzy Set

Let $t: A \rightarrow P([0,1])$,
where for any $x \in A$, t(x) represents the degree of membership/truth/chance_of_occuring of the element x with respect to the set A.
t(x) is a subset (not a single element) of [0,1].

- *The Interval-Valued Fuzzy* is part of Hyperfuzzy, for examples:

$t(a_1) = [0.2, 0.3]$, $t(a_2) = (0.4, 0.6)$.

- *The Subset-Valued Fuzzy* is also part of Hyperfuzzy, for example

$t(a_1) = (0.4, 0.5) \cup [0.6, 0.8]$.

- *The Hesitant Fuzzy* is also a part of Hyper Fuzzy, for example

$t(a_1) = \{0.4, 0.7, 0.9\}$.

(ii) **HyperIntuitionistic Fuzzy Set** is equivalent to Subset-Valued Intuitionistic Fuzzy Set

Let $\mathcal{U}$ be a universe of discourse and A a non-empty subset of $\mathcal{U}$.
$t: A \rightarrow P([0,1])$ is the truth/membership function,
while $f: A \rightarrow P([0,1])$ is the falsehood/nonmembership function.
As particular cases of the HyperIntuitionistic Fuzzy Set, one has:

- *Interval-Valued Intuitionistic Fuzzy Set*

where $t, f: A \rightarrow P([0,1])$ are intervals, for example:
$t(a_1) = [0.1,0.4]$,
$f(a_1) = [0.2,0.5]$.

- *Hesitant Intuitionistic Fuzzy Set*

For example:
$t(a_1) = \{0.2, 0.3, 0.4\}$,
$f(a_1) = \{0.1, 0.7\}$.

(iii) **HyperNeutrosophic Set** is equivalent to Subset-Valued Neutrosophic Set

Let $\mathcal{U}$ be a universe of discourse and A a non-empty subset of $\mathcal{U}$.
$t, i, f: A \to P([0,1])$,
where $t(a), i(a), f(a)$ are subsets in [0,1], for all $a \in A$.
As particular cases one has:

- Subset-Valued Neutrosophic Set

When $t(a), i(a), f(a)$ are any types of subsets of [0, 1]. For example:
$t(a_1) = [01., 0.2] \cup (0.5, 0.6)$,
$i(a_1) = \{0.6, 0.7, 0.9\}$,
$f(a_1) = [03., 0.4] \cup \{0.50, 0.51, 0.52\}$.
This was the general definition [1995] of the Standard Neutrosophic Set. (The Non-Standard Neutrosophic Set is not the subject of this paper.)

- Interval-Valued Neutrosophic Set

where $t(a), i(a), f(a)$ are intervals included in [0,1], for all $a \in A$.
For example: $t(a_1) = [0.6, 0.7]$, $i(a_1) = [0.5, 0.6]$, $f(a_1) = [0.8, 1.0]$.

- Hesitant-Valued Neutrosophic Set

When $t(a), i(a), f(a)$ are hesitant discrete subsets of [0, 1]. For example:
$t(a_1) = \{0.2, 0.3, 0.4\}$,
$i(a_1) = \{0.70, 0.75\}$,
$f(a_1) = \{0.5, 0.6, 0.7, 0.8\}$.

(iv) **HyperRefinedNeutrosophic Set** is equivalent to Subset-Valued Refined Neutrosophic Set

- Interval-Valued (2,3,1)-Refined Neutrosophic Set/Logic/Probability is a Hyper (2,3,1)-Refined Neutrosophic Set/Logic/Probability

  $\tau: A \to P([0,1]^6)$, because $2 + 3 + 1 = 6$.
  $\tau(a^1) = \left(t^1(a^1), t^2(a^1);\ i^1(a^1), i^2(a^1), i^3(a^1);\ f^1(a^1)\right) =$
  $[0.1, 0.2], [0.3, 0.4]; (0.2, 0.3]; (0.2, 0.4), [0.3, 0.5); (0.7, 0.9])$,
  for example.

15. Examples of SuperUncertain Classical Set/Logic/Probability/Statistics

$\tau : P^m(A) \to [0,1]^r$

"Super" means that one considers the degree of a subset (not of a single element) of A.

- SuperFuzzy Set/Logic/Probability

τ: P(A) → [0,1]

but $\tau(\{a_1, a_2\}) = t(\{a_1, a_2\}) = 0.9$, for example, which means that the membership/truth degree of the whole subset $\{a_1, a_2\}$ together being as a team is 0.9.

- SuperIntuitionistic Fuzzy Set/Logic/Probability

$\tau: P(A) \to [0,1]^2)$

$\tau(\{a_1, a_2\}) = \{t(\{a_1, a_2\}), f(\{a_1, a_2\})\} = \{ 0.5, 0.7 \}$, for example.

Or, $t(\{a_1, a_2\}) = 0.5$, and $f(\{a_1, a_2\}) = 0.7$.

- SuperNeutrosophic Set/Logic/Probability

$\tau: P(A) \to [0, 1]^3$

$\tau(\{a_1, a_2\}) = \{ t(\{a_1, a_2\}), i(\{a_1, a_2\}), f(\{a_1, a_2\}) \} = \{ 0.8, 0.1, 0.3 \}$, for example.

Or, $t(\{a_1, a_2\}) = 0.8$, $i(\{a_1, a_2\}\} = 0.1$, $f(\{a_1, a_2\}) = 0.3$.

16. Examples of SuperHyperUncertain Classical Set/Logic/Probability/Statistics

$\tau : P^m(A) \to P^n([0,1]^r)$

"Super" means that one considers the degree of a subset (not of a single element) of A.

While "Hyper" means that that the degree is a subset (not of a single element) of [0, 1].

- SuperHyperFuzzy Set/Logic/Probability

τ: P(A) → P([0,1])

but $\tau(\{a_1, a_2\}) = t(\{a_1, a_2\}) = [0.7, 0.8]$, for example.

- SuperHyperIntuitionistic Fuzzy Set/Logic/Probability

$\tau: P(A) \to P([0,1]^2)$

$\tau(\{a_1, a_2\}) = \{t(\{a_1, a_2\}), f(\{a_1, a_2\})\}$ = { [0.7, 0.8], [0.9, 1.0] }, for example.

- SuperHyperNeutrosophic Set/Logic/Probability

$\tau: P(A) \to P([0,1]^3)$
$\tau(\{a_1, a_2\}) = \{ t(\{a_1, a_2\}), i(\{a_1, a_2\}), f(\{a_1, a_2\}) \}$ = {[0.7,0.8], [0.0,0.2], [0.9,1.0]}, for example.

17. HyperUncertain, SuperUncertain, and SuperHyperUncertain Over/Under/Off-Sets/Logics/Probability/Statistics

Let A be an uncertain (set/logic/probability) domain, and $B = [\varphi, \psi]$ be a real interval, where $\varphi \leq 0 < 1 \leq \psi$.

Then, as a particular case, one considers the function: $f_{SH}^{SH} : P^m(A) \to P^n([\varphi, \psi])$, which similarly represent the HyperUncertain and SuperHyperUncertain Over/Under/Off-Sets/Logics/Probability/Statistics.

18. Examples of HyperUncertain, SuperUncertain, and SuperHyperUncertain Over/Under/Off-Sets/Logics/Probability/Statistics

- HyperFuzzy Non-Classical Set/Logic/Probability

$\tau: A \to P([-0.2, 1])$
$\tau(a_1) = t(a_1) = [-0.1, 0.3]$, for example.

- SuperFuzzy Non-Classical Set/Logic/Probability

$\tau: P(A) \to P([0, 1.1])$
$\tau(\{a_1, a_2\}) = t(\{a_1, a_2\}) = 1.1$, for example.

- SuperHyperFuzzy Non-Classical Set/Logic/Probability

τ: P(A) → P([-0.3, 1.2])
but $\tau(\{a_1, a_2\}) = t(\{a_1, a_2\})$ = [0.9, 1.1], for example.

19. Conclusion

We have defined all Classical (and Non-Classical) HyperUncertain, SuperUncertain, and

SuperHyperUncertain Set/Logic/Probability/Statistics. We also presented many numerical examples that occur in our everyday world.

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