关于F.Smarandache简单函数与Dirichlet除数和函数的混合均值

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摘 要:利用解析方法研究了F.Smarandache简单函数与Dirichlet除数和函数的混合均值,得出了两个较为精确的渐近公式.

关键词:F.Smarandache简单函数;Dirichlet除数和函数;混合均值;渐近公式

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1 引言及结论

1993年,美籍罗马尼亚著名数论专家F.Smarandache[1]在*Only problem, Not solutions*一书中提出了105个尚未解决的数论问题,其中大部分是与正整数n或素数p有关的问题.书中第42个问题定义了Smarandache简单函数如下:

定义1　设n为正整数,Smarandache简单函数定义为:满足$p^n \mid m!$的最小正整数$m\in N^+$,即$S_p(n)=\min\{m \mid p^n \mid m!, m\in N^+\}$.

文献[2]定义了Smarandache简单函数的加法类似函数如下:

定义2　设$\overline{S}_p(n)=\min\{m\in N^+: p^n\leqslant m!!\}$ $(n\in(1,\infty))$和$\overline{S}_p{}^*(n)=\max\{m\in N^+: m!!\leqslant p^n\}$ $(n\in(1,\infty))$,则称$\overline{S}_p(n)$和$\overline{S}_p^*(n)$为Smarandache简单函数的加法类似.

显然,若$(m-2)!!< p^n\leqslant m!!$,$\overline{S}_p(n)=m$,其中$m>2$.关于$\overline{S}_p(n)$的性质,许多学者都进行了研究[3-7].文献[6]研究了$d(\overline{S}_p(n))$的均值性质,得出渐近公式:$\sum\limits_{n\leqslant x} d(\overline{S}_p(n))=2x(\ln x-2\ln\ln x)+O(x\ln p)$.本文主要研究$\sigma_\alpha(\overline{S}_p(n))$的渐近性质,其中$\sigma_\alpha(n)$是除数和函数,并且得到了两个较为精确的渐近公式.

定理1　设p为一给定的素数,对任意实数$x\geqslant 1$,有

$$\sum_{n\leqslant x}\sigma_\alpha(\overline{S}_p(n))=\begin{cases}\dfrac{\pi^2}{3}\dfrac{x^2\ln p}{\ln^2 x}\ln\left(\dfrac{2x\ln p}{\ln x}\right)+O\left(\dfrac{x^2}{\ln^2 x}\right), & \text{如果 } \alpha=1,\\ \dfrac{\zeta(\alpha+1)}{\alpha+1}\dfrac{2^{\alpha+1}x^{\alpha+1}\ln^\alpha p}{\ln^{\alpha+1} x}\ln\left(\dfrac{2x\ln p}{\ln x}\right)+O\left(\dfrac{x^{\alpha+1}}{\ln^{\alpha+1} x}\right), & \text{如果 } \alpha\neq 1.\end{cases}$$

定理2　设p为一给定的素数,对任意实数$x\geqslant 1$,有

$$\sum_{n\leqslant x}\sigma_\alpha(\overline{S}_p{}^*(n))=\begin{cases}\dfrac{\pi^2}{3}\dfrac{x^2\ln p}{\ln^2 x}\ln\left(\dfrac{2x\ln p}{\ln x}\right)+O\left(\dfrac{x^2}{\ln^2 x}\right), & \text{如果 } \alpha=1,\\ \dfrac{\zeta(\alpha+1)}{\alpha+1}\dfrac{2^{\alpha+1}x^{\alpha+1}\ln^\alpha p}{\ln^{\alpha+1} x}\ln\left(\dfrac{2x\ln p}{\ln x}\right)+O\left(\dfrac{x^{\alpha+1}}{\ln^{\alpha+1} x}\right), & \text{如果 } \alpha\neq 1.\end{cases}$$

2 定理的证明

引理1　对任意实数$x\geqslant 1$,有

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$$\sum_{n\leqslant x}\sigma_1(n)=\frac{\pi^2}{12}x^2+O(x\ln x).$$

引理 2 对任意实数 $x\geqslant 1$ 和 $\alpha>0$, $\alpha\neq 1$, 有

$$\sum_{n\leqslant x}\sigma_\alpha(n)=\frac{\zeta(\alpha+1)}{\alpha+1}x^{\alpha+1}+O(x^\beta),$$

其中 $\beta=\max\{1,\alpha\}$.

引理 1 和引理 2 的证明见文献[8].

定理 1 的证明 由 $\overline{S}_p(n)$ 的定义知, 当 $n\in(\frac{\ln(m-2)!!}{\ln p},\frac{\ln m!!}{\ln p}]$, 有 $\overline{S}_p(n)=m$. 若 $(m-2)!!<p^x\leqslant m!!$, 则 $\left|m-\frac{2x\ln p}{\ln x}\right|\ll\ln\ln x$. 由引理 1 和阿贝尔恒等式[8], 有

$$\sum_{n\leqslant x}\sigma_1(\overline{S}_p(n))=\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\sum_{\frac{\ln(m-2)!!}{\ln p}<n\leqslant\frac{\ln m!!}{\ln p}}\sigma_1(m)+\sum_{\frac{2x\ln p}{\ln x}<m<\frac{2x\ln p}{\ln x}+\ln\ln x}\sum_{\frac{\ln(m-2)!!}{\ln p}<n\leqslant\frac{\ln m!!}{\ln p}}\sigma_1(m)=$$

$$\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\left[\frac{\ln m}{\ln p}\right]\sigma_1(m)+O\Big(\sum_{\frac{2x\ln p}{\ln x}<m<\frac{2x\ln p}{\ln x}+\ln\ln x}\frac{\ln x}{\ln p}\sigma_1(m)\Big)=$$

$$\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\left[\frac{\ln m}{\ln p}\right]\sigma_1(m)+O(x\ln\ln x)=$$

$$\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\frac{\ln m}{\ln p}\sigma_1(m)+O\Big(\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\sigma_1(m)\Big)+O(x\ln\ln x)=$$

$$\frac{1}{\ln p}\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\ln m\,\sigma_1(m)+O\Big(\frac{x^2}{\ln^2x}\Big)=$$

$$\frac{1}{\ln p}\left(\ln\Big(\frac{2x\ln p}{\ln x}\Big)\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\sigma_1(m)-\int_1^{\frac{2x\ln p}{\ln x}}\frac{1}{t}\sum_{m\leqslant t}\sigma_1(t)\mathrm{d}t\right)+O\Big(\frac{x^2}{\ln^2x}\Big)=$$

$$\frac{1}{\ln p}\ln\Big(\frac{2x\ln p}{\ln x}\Big)\frac{\pi^2}{12}\frac{4x^2\ln^2p}{\ln^2x}-\frac{1}{\ln p}\int_1^{\frac{2x\ln p}{\ln x}}\frac{1}{t}\Big(\frac{\pi^2}{12}t^2+O(t\ln t)\Big)\mathrm{d}t+O\Big(\frac{x^2}{\ln^2x}\Big)=$$

$$\frac{\pi^2}{3}\frac{x^2\ln p}{\ln^2x}\ln\Big(\frac{2x\ln p}{\ln x}\Big)+O\Big(\frac{x^2}{\ln^2x}\Big).$$

如果 $\alpha\neq 1$, 由引理 2 和阿贝尔恒等式[8], 有

$$\sum_{n\leqslant x}\sigma_\alpha(\overline{S}_p(n))=\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\sum_{\frac{\ln(m-2)!!}{\ln p}<n\leqslant\frac{\ln m!!}{\ln p}}\sigma_\alpha(m)+\sum_{\frac{2x\ln p}{\ln x}<m<\frac{2x\ln p}{\ln x}+\ln\ln x}\sum_{\frac{\ln(m-2)!!}{\ln p}<n\leqslant\frac{\ln m!!}{\ln p}}\sigma_\alpha(m)=$$

$$\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\frac{\ln m}{\ln p}\sigma_\alpha(m)+O\Big(\sum_{\frac{2x\ln p}{\ln x}<m<\frac{2x\ln p}{\ln x}+\ln\ln x}\frac{\ln x}{\ln p}\sigma_\alpha(m)\Big)=$$

$$\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\left[\frac{\ln m}{\ln p}\right]\sigma_\alpha(m)+O\Big(\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\sigma_\alpha(m)\Big)+O(x\ln\ln x)=$$

$$\frac{1}{\ln p}\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\ln m\,\sigma_\alpha(m)+O\Big(\frac{x^{\alpha+1}}{\ln^{\alpha+1}x}\Big)=$$

$$\frac{1}{\ln p}\left(\ln\Big(\frac{2x\ln p}{\ln x}\Big)\sum_{m\leqslant\frac{2x\ln p}{\ln x}}\sigma_\alpha(m)-\int_1^{\frac{2x\ln p}{\ln x}}\frac{1}{t}\sum_{m\leqslant t}\sigma_\alpha(t)\mathrm{d}t\right)+O\Big(\frac{x^{\alpha+1}}{\ln^{\alpha+1}x}\Big)=$$

$$\frac{1}{\ln p}\ln\Big(\frac{2x\ln p}{\ln x}\Big)\frac{\zeta(\alpha+1)}{\alpha+1}\frac{2^{\alpha+1}x^{\alpha+1}\ln^{\alpha+1}p}{\ln^{\alpha+1}x}-$$

$$\frac{1}{\ln p}\int_1^{\frac{2x\ln p}{\ln x}}\frac{1}{t}\left(\frac{\zeta(\alpha+1)}{\alpha+1}t^2+O(t^\alpha)\right)\mathrm{d}t+O\Big(\frac{x^{\alpha+1}}{\ln^{\alpha+1}x}\Big)=$$

$$\frac{\zeta(\alpha+1)}{\alpha+1}\frac{2^{\alpha+1}x^{\alpha+1}\ln^\alpha p}{\ln^{\alpha+1}x}\ln\Big(\frac{2x\ln p}{\ln x}\Big)+O\Big(\frac{x^{\alpha+1}}{\ln^{\alpha+1}x}\Big).$$

这就完成了定理 1 的证明. 用同样的方法, 可以证明定理 2.

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On the Hybrid Mean Value Involving the F.Smarandache Simple Function and the Dirichlet Divisor Function

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Abstract: The main purpose of this paper is using the analytic method to study the hybrid mean value involving the F.Smarandace simple function and the sum of Dirichlet divisor function. Two sharped asymptotic formulae are given.

Key words: F. Smarandache simple function; the sum of Dirichlet divisor function; hybrid mean value; asymptotic formula

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Convergence Theorems for a Vector Equilibrium Problem on Hilbert Spaces

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Abstract: Two algorithms for a vector equilibrium problem in Hilbert spaces are introduced. The convergence of algorithms is proved by using the nonlinear scalarization function. It is indicated that one of the algorithms is strongly convergent and the other is weakly convergent if the function in the vector equilibrium problem possesses the monotonicity, C-convexity and quasi lower semicontinuity.

Key words: nonlinear scalarization function; vector equilibrium problem; iterative algorithm

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