Neutrosophic Treatment of Duality Linear Models and the Binary Simplex Algorithm

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Abstract
One of the most important theories in linear programming is the dualistic theory and its basic idea is that for every linear model has dual linear model, so that solving the original linear model gives a solution to the dual model. Therefore, when we solving the linear programming model, we actually obtain solutions for two linear models. In this research, we present a study of the models. The neutrosophic dual and the binary simplex algorithm, which works to find the optimal solution for the original and dual models at the same time. The importance of this algorithm is evident in that it is relied upon in several operations research topics, such as integer programming algorithms, some nonlinear programming algorithms, and sensitivity analysis in linear programming...
Keywords: Neutrosophic Science; Neutrosophic Linear Models; Dual Models; Neutrosophic dual Linear Models; binary Simplex Algorithm.

1. Introduction:

Most companies and institutions rely on studies provided by experts and researchers using operations research methods in order to ensure a safe work environment away from danger and danger and to give decision makers a margin of freedom, after the studies and research presented using the concepts of neutrosophic science in most fields of science have proven their ability to provide more accurate results. From the results that we were obtaining using classical data-driven studies, we reformulated many operations research topics using neutrosophic concepts [1-20], and to complement what we have done, we present in this research a study of dual linear models and the binary simplex algorithm that gives us a solution for the original and dual models at the same time so that we can provide a clear study that helps decision makers in companies and institutions develop plans and programs through which the greatest profit and lowest cost are achieved.

Discussion:

In our practical life, we encounter many problems that are formulated in the form of linear mathematical models consisting of an objective function and a set of constraints in the form of equations or inequalities. The linear model is written in many formulas that are distinguished by the type of the objective function and the form of the constraints. Many references have been provided that studies of linear and dual linear models are based on classical data, and the appropriate algorithms for solving each of them and how to find the dual model for any model [21-25]. Since classical data results in optimal solutions, which are classical values, restricted values appropriate to the conditions in which the data were collected, and for any change in these conditions, these solutions are inappropriate and may cause the facility a large unexpected loss, so we found it appropriate to reformulate these studies using concepts neutrosophic science is done by taking data with neutrosophic values suitable for all conditions. In previous research, we presented neutrosophic linear models, the direct neutrosophic simplex algorithm, and the graphical method for finding the optimal solution for linear models,[18,19,20] . As a

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continuation of what we have done previously, we present in this research a study whose purpose is to reformulate the binary simplex algorithm to find the optimal solution for the original model and the dual model at the same time.

We present this study in two stages:

The first stage:

Finding the neutrosophic dual models using the binary table.

When we want to find the model associated with any linear model, we start by placing this model in the symmetrical form, so we mention the symmetrical form.

The symmetrical form of the neutrosophic linear model: [19]

We say of a linear model that it is in the symmetrical form if all variables are constrained to be non-negative and if all constraints are given in the form of inequalities (and the inequalities of the maximization model constraints must be in the form (≤ less than or equal to) while the inequalities of the minimization model constraints must be in the form (≥ is greater than or equal to), then the linear model is written in the neutrosophic symmetric form in one of two cases:

First case:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Max$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \leq b_i \pm \delta_i \quad ; \quad i = 1,2, \dots, m$$

$$x_j \geq 0$$

Second case:

$$NL = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Min$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \geq b_i \pm \delta_i \quad ; \quad i = 1,2, \dots, m$$

$$x_j \geq 0$$

In both cases, we have $x_j \geq 0$, which are the decision variables, unknown values that we obtain after solving the linear model.

$Nc_j = c_j \pm \varepsilon_j$ and $Nb_i = b_i \pm \delta_i$ and $Na_{ij} = a_{ij} \pm \mu_{ij}$ where (j=1,2,…,n , i=1,2,…,m) are the data of the problem under study, and they are neutrosophic values, indefinite values that enjoy a margin of freedom and are taken according to The nature of the situation represented by the linear model

First method:

Constructing neutrosophic dual linear models using tables:

If the original model is given in detailed form, we put it in the symmetrical form as stated in the previous paragraph, then we draw a binary table for the original and dual models according to the following steps:

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1- The coefficients of the objective function in the original model are the constants column in the dual model, and the constants column in the original model are the coefficients of the objective function in the dual model.
2- We invert the signs of the inequalities of the constraints (if they were in the original model of type ≤ they become in the dual model of type ≥)
3- We change the objective from maximizing in the original model to minimizing in the dual model
4- We place each constraint (row) in the original model corresponding to a column in the dual model, meaning there is one variable for each constraint in the original model
5- The variables in the original model and the dual model satisfy the non-negativity constraints.

We explain the above through the following two cases:

The first case: The original model is symmetrical and of the maximization type

Find

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Max$$

Constants:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n \leq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \leq Nb_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \leq Nb_m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

The binary table for the original model and the accompanying model is as follows:

Table No. (1) Objective follower of the maximization type

			Original model		
		objective function / constants	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$		*Max* / Constants column
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n$	≤	Nb_1
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n$	≤	Nb_2
	…	…	… … … … … … … … … … … … … …	≤	…
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n$	≤	Nb_m
		Non-negativity constraints	$x_1, x_2, \ldots, x_n$	≥	0
			Dual model		
		Objective function / constraints	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m$		*Min* / Constants column
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m$	≥	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m$	≥	Nc_2
		…	… … … … … … … … … … … … … …	≥	…
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m$	≥	Nc_n

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Non-negativity constraints	$y_1, y_2, \dots, y_m$	$\geq$	0

The second case: The original model is symmetrical and of the reduction type:
Find

$$NL = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Min$$

Constants:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + \mathrm{Na}_{1n}x_n \geq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \geq Nb_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \geq Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

The binary table for the original model and the accompanying model is as follows:

Table No. (2) objective follower in the original model of the reduce type

		Original model			
		objective function / constants	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$		Min / Constants column
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n$	$\geq$	Nb_1
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n$	$\geq$	Nb_2
	…	…	… … … … … … … … … … … … … …	$\geq$	…
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n$	$\geq$	Nb_m
		Non-negativity constraints	$x_1, x_2, \dots, x_n$	$\geq$	0
		Dual model			
		objective function / constants	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m$		Max / Constants column
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m$	$\leq$	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m$	$\leq$	Nc_2
		…	… … … … … … … … … … … … … …	$\leq$	…
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m$	$\leq$	Nc_n
		Non-negativity constraints	$y_1, y_2, \dots, y_m$	$\geq$	0

The second stage: formulation of the binary neutrosophic algorithm.

The binary simplex algorithm (for the original and dual models) neutrosophic. Through this algorithm, we can find the two ideal solutions for both the original and dual models at the same time. Before starting the binary simplex algorithm, we must mention the modified simplex algorithm that we will use within the steps of the binary algorithm.

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Modified simplex algorithm:

In the modified Simplex algorithm, after converting the regular linear model to the basic form, we place the coefficients in a short table whose first column includes the basic variables and whose top row includes the non-basic variables only. We define the pivot column, which is the column corresponding to the largest positive value in the objective function row if the objective function is a maximization function (but if the objective function is a minimization function, it is the column corresponding to the most negative values). Let this column be the column of the variable x_s. We define the pivot row. The pivot row is determined. Through the following indicator:

$$N\theta = min\left[\frac{Nb_i}{Na_{is}}\right] = \frac{Nb_t}{Na_{ts}} > 0; \quad Na_{is} > 0, Nb_i > 0$$

Let this line be the line of the base variable y_t, then the anchor element is the element resulting from the intersection of the anchor column and the anchor line, i.e., Na_{ts}. Then we calculate the new elements corresponding to the anchor line and the anchor column as follows:

1- We put opposite the pivot element Na_{ts} the reciprocal of $\frac{1}{Na_{ts}}$
2- We calculate the elements of the row corresponding to the pivot row (except the pivot element) by dividing the elements of the pivot row by the pivot element Na_{ts}
3- We calculate all the elements of the column opposite the pivot (except the pivot element) by dividing the elements of the pivot column by the pivot element Na_{ts} and then multiplying them by (-1)
4- We calculate the other elements from the following relationships:

$$Nb_i' = Nb_i - Nb_t\frac{Na_{is}}{Na_{ts}} = \frac{Nb_iNa_{ts} - Nb_tNa_{is}}{Na_{ts}}$$
$$Na_{ij}' = Na_{ij} - Na_{tj}\frac{Na_{is}}{Na_{ts}} = \frac{Na_{ij}Na_{ts} - Na_{tj}Na_{is}}{Na_{ts}}$$
$$Nc_j' = Nc_j - Nc_s\frac{Na_{tj}}{Na_{ts}} = \frac{Nc_jNa_{ts} - Nc_sNa_{tj}}{Na_{ts}}$$

5- We apply the stopping criterion of the direct Simplex algorithm on the objective function row. If the objective function is of the maximize type, the objective function row in the table must not contain any positive value. (But if the objective function is of the minimization type, the objective function row in the new table must not be contains any negative value), if the criterion is not met, we repeat the same steps until the stopping criterion is met and we obtain the desired ideal solution.

Steps of the binary simplex algorithm:

a. We write the two models in basic form by adding or subtracting additional variables or using synthetic variables and isolating the non-restricting variables.

Basal form of the original model:

Find

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n + 0u_1 + 0u_2 + \cdots + 0u_m \longrightarrow Max$$

Constans:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n + u_1 = Nb_1$$
$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n + u_2 = Nb_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n + u_m = Nb_m$$
$$x_j \geq 0 \;; \mathrm{j} = 1,2, \ldots, \mathrm{n}$$
$$u_i \geq 0 \;; \mathrm{i} = 1,2, \ldots, \mathrm{m}$$

Here we do not require that $Nb_i \geq 0$.

Basic form of the dual model:

Find

$$NL = Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m + 0v_1 + 0v_2 + \cdots + 0v_n \longrightarrow Min$$

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Constans:

$$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m - v_1 = Nc_1$$
$$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m - v_2 = Nc_2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m - v_n = Nc_n$$
$$y_i \geq 0 \ ; i = 1,2,\dots,m$$
$$v_j \geq 0 \ ; \mathrm{j} = 1,2,\dots,\mathrm{n}$$

Here we do not require that $Nc_j \geq 0$

The two models have the same coefficients, and the matrix of instances of the dual model is the transpose of the matrix of instances of the original model. We write the two models in the following binary table:

Table No. (3) Standard format for the original and companion models

		Original model			
		objective function / constants	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n + 0u_1 + 0u_2 + \cdots + 0u_m$		Max / Constants column
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n + u_1$	$=$	Nb_1
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n + u_2$	$=$	Nb_2
	…	…	… … … … … … … … … … … … … …	$=$	…
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n + u_m$	$=$	Nb_m
		Non-negativity constraints	$x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_m$	$\geq$	0
		Dual model			
		objective function / constants	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m + 0v_1 + 0v_2 + \cdots + 0v_n$		Min / Constants column
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m - v_1$	$=$	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m - v_2$	$=$	Nc_2
		…	… … … … … … … … … … … … … …	$=$	…
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m - v_n$	$=$	Nc_n
		Non-negativity constraints	$y_1, y_2, \dots, y_m, v_1, v_2, \dots, v_n$	$\geq$	0

b. We place the variables and coefficients of the original model in the modified simplex table, and we place the variables of the dual model outside the table as follows:

Table No. (4): The binary table for the original and dual models according to the modified Simplex algorithm

			Basic variables with a (-) sign in the dual model				
			$-v_1$	$-v_2$	…	$-v_n$	
		Non-basic vibrable / basic vibrable	x_1	x_2	…	x_n	**Follow the objective of the dual model** NB_i
No n-	y_1	u_1	Na_{11}	Na_{12}	…	Na_{1n}	Nb_1

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	y_2	u_2	Na_{21}	Na_{22}	…	Na_{2n}	Nb_2
	…	…	…	…	…	…	…
	y_m	u_m	Na_{m1}	Na_{m2}	…	Na_{mn}	Nb_m
		objective of the original model	Nc_1	Nc_2	…	Nc_n	$L - 0 \to Min$ $Z - 0 \to Max$

Binary simplex algorithm for the original and dual models:

From the modified simplex algorithm of the original model, we obtain the optimal solution of the original model when all the elements are in the last row (the objective function row of the original model) $Nc_j \leq 0$; $\mathrm{j} = 1,2,\dots,\mathrm{n}$ and at the same time all the elements are in the column The last (associated objective function column) $Nb_i \geq 0$; $i = 1,2,\dots,m$ and we get the optimal solution for the dual model when all elements in the last column (associated objective function column) are $Nb_i \geq 0$; $i = 1,2,\dots,m$ and at the same time the last row (the objective function row of the original model) $Nc_j \leq 0$; $\mathrm{j} = 1,2,\dots,\mathrm{n}$ (because it will correspond to $Nc_j = -v_j$) which are the two conditions Same for both models. Therefore, when searching for the optimal solution for both models together, we must work to make all elements $Nb_i \geq 0$; $i = 1,2,\dots,m$ and to make all elements $Nc_j \leq 0$; $\mathrm{j} = 1,2,\dots,\mathrm{n}$ to achieve this, we rely on one of the two models, put its variables and coefficients in a table, and place the dual model in an external frame of that table. In general, we find that the necessity of placing the two models in a short table does not allow us to get rid of the negative constants on the right side, and therefore the general case of the previous binary table can it must include negative constants $Nb_i < 0$, and the elements of the last row can include positive elements $Nc_j > 0$, so when searching for the optimal solution for the two models, we must work to address these elements based on one of the two models.

Depending on the original model, we do this in two stages:

The first stage:

We make the constant Nb_i non-negative, which corresponds to obtaining a non-negative basic solution for the original model.

The second stage:

We make all elements of the objective function row non-positive (in the case of the objective function, maximization), and this corresponds to obtaining the optimal solution required for the original model.

Based on the dual model, we do this in two stages:

The first stage:

We must make the elements of the dual model objective function column $Nb_i \geq 0$; $\mathrm{i} = 1,2,\dots,\mathrm{m}$,The last row is non-negative

The second stage:

We must make the free constants for the dual model $-Nc_j$ non-positive, and this corresponds to obtaining the optimal solution for the dual model. We explain the above through the following example:

Find the optimal solution for both the following neutrosophic linear model and its dual using the binary algorithm

Example:

Find:

$$[5,8]\, x_1 + [3,6]\, x_2 \longrightarrow Max$$

Constans:

$$2x_1 + 3x_2 \leq [14,20]$$

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$$2x_1 + x_2 \leq [10,16]$$

$$3x_2 \leq [12,18]$$
$$3x_1 \leq [15,21]$$

$$x_1 \geq 0, x_2 \geq 0$$

We form the binary table of the model and the dual model:

Table No. (5) The original model and its dual mode

	Original model				
	objective function / constants	$[5,8]\ x_1 + [3,6]\ x_2$		Max / Constants column
Dual vibrable	y_1	1	$2x_1 + 3x_2$	$\leq$	[14,20]
	y_2	2	$2x_1 + x_2$	$\leq$	[10,16]
	y_3	3	$3x_2$	$\leq$	[12,18]
	y_4	4	$3x_1$	$\leq$	[15,21]
		Non-negativity constraints	x_1, x_2	$\geq$	0
		Dual model			
		objective function / constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4$		Min / Constants column
		1	$2y_1 + 2y_2 + 3y_4$	$\geq$	[5,8]
		2	$3y_1 + y_2 + 3y_4$	$\geq$	[3,6]
		Non-negativity constraints	y_1, y_2, y_3, y_4	$\geq$	0

In the following table, we wrote the two models in standard form:

		Original model			
		objective function / constants	$[5,8]\ x_1 + [3,6]\ x_2 + 0u_1 + 0u_2 + 0u_3 + 0u_4$		Max / Constants column
Dual vibrable	y_1	1	$2x_1 + 3x_2 + u_1$	$=$	[14,20]
	y_2	2	$2x_1 + x_2 + u_2$	$=$	[10,16]
	y_3	3	$3x_2 + u_3$	$=$	[12,18]
	y_4	4	$3x_1 + u_4$	$=$	[15,21]
		Non-negativity constraints	$x_1, x_2, u_1, u_2, u_3, u_4$	$\geq$	0
		Dual model			
		objective function / constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4 + 0v_1 + 0v_2$		Min / Constants column
		1	$2y_1 + 2y_2 + 3y_4 - v_1$	$=$	[5,8]
		2	$3y_1 + y_2 + 3y_4 - v_2$	$=$	[3,6]

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Non-negativity constraints	$y_1, y_2, y_3, y_4, v_1, v_2$	$\geq$	0

Table No. (6) Standard format for the original model and the dual model

We notice from the table that the standard form of the original model includes a ready-made base of additional variables u_1, u_2, u_3, u_4, but for the dual model there is no ready-made base. Therefore, we multiply the two restrictions by (-1) and we obtain the basic form of the dual model.

The following table shows the basic form of the original and dual models:

Table No. (7): The basic shape of the original model and the dual model

		Original model			
		objective function / constants	$[5,8]\, x_1 + [3,6]\, x_2 + 0u_1 + 0u_2 + 0u_3 + 0u_4$		Max / Constants column
Dual vibrable	y_1	1	$2x_1 + 3x_2 + u_1$	$=$	[14,20]
	y_2	2	$2x_1 + x_2 + u_2$	$=$	[10,16]
	y_3	3	$3x_2 + u_3$	$=$	[12,18]
	y_4	4	$3x_1 + u_4$	$=$	[15,21]
		Non-negativity constraints	$x_1, x_2, u_1, u_2, u_3, u_4$	$\geq$	0
		Dual model			
		objective function / constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4 + 0v_1 + 0v_2$		Min / Constants column
		1	$-2y_1 - 2y_2 - 3y_4 + v_1$	$=$	$-[5,8]$
		2	$-3y_1 - y_2 - 3y_4 + v_2$	$=$	$-[3,6]$
		Non-negativity constraints	$y_1, y_2, y_3, y_4, v_1, v_2$	$\geq$	0

We put the two models in the modified Simplex algorithm table and we get the following table:

Table No. (8): The binary table for the original and dual models according to the modified Simplex algorithm

			According to the original model		
			$-v_1$	$-v_2$	
		Non-basic vibrable / **basic vibrable**	x_1	x_2	objective function NB_i
Non-basic vibrable	y_1	u_1	2	3	[14,20]
	y_2	u_2	2	1	[10,16]
	y_3	u_3	0	3	[12,18]
	y_4	u_4	3	0	[15,21]
		objective function Nc_i	[5,8]	[3,6]	$L-0$ / $Z-0$

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			According to the dual model				
			u_1	u_2	u_3	u_4	
		Non-basic vibrable basic vibrable	y_1	y_2	y_3	y_4	objective function Original model Nc_i
Non-basic	x_1	v_1	-2	-2	0	-3	$-[5,8]$
	x_2	v_2	-3	-1	-3	0	$-[3,6]$
		objective function Dual model NB_i	$[14,20]$	$[10,16]$	$[12,18]$	$[15,21]$	$Z-0$ $L-0$

The first stage:

1- For the original model:

Since the values in the constant's column are all positive, we study the values in the objective function row and determine the largest positive value. We find:

$$\max([5,8],[3,6]) = [5,8]$$

It is an expression of the variable x_1. This means that it will enter the base. To determine the element that will exit from the base, we calculate the index $N\theta$, where:

$$N\theta \in min\left[\frac{[14,20]}{2},\frac{[10,16]}{2},\frac{[15,21]}{3}\right] = \frac{[15,21]}{3} = [5,7]$$

We find that the pivot column is the column of the non-base variable x_1, meaning that the variable x_1 will enter the base instead of the variable u_4, and the pivot element is the element resulting from the intersection of the pivot row and the pivot column, which is (3)

We perform the switching between variables using a modified simplex algorithm.

2- For the dual model:

We study the elements of the objective function row. We notice that all the values are positive. Therefore, we study the elements of the constant's column. We find that they are all negative values. We choose the most negative of them, which is $(-[5,8])$ which is the row of the base variable v_1, so its row is the pivot row. To determine the pivot column and the pivot element, we calculate the index $N\theta'$where:

$$N\theta' \in Max\left[\frac{[14,20]}{-2},\frac{[10,16]}{-2},\frac{[15,21]}{-3}\right] = \frac{[15,21]}{-3}$$

So, the column of the non-base variable u_4 is the pivot column, meaning that the variable u_4 will enter the base instead of the variable v_1, and the pivot element is the element resulting from the intersection of the pivot row and the pivot column, which is (-3). We perform the switching between the variables using the modified simplex algorithm, from (1) and (2) We get the following double table:

Table No. (9): The binary table for the first stage, the solution according to the original and dual models

	According to the original model		
	$-y_4$	$-v_2$	
Non-basic vibrable basic vibrable	u_4	x_2	objective function Original model NB_i

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Non-basic vibrable	y_1	u_1	$\frac{-2}{3}$	3	$[4,6]$
	y_2	u_2	$\frac{-2}{3}$	1	$[0,4]$
	y_3	u_3	0	3	$[12,18]$
	v_1	x_1	$\frac{1}{3}$	0	$[5,7]$
		objective function Original model Nc_i	$\left[\frac{-8}{3}, \frac{-5}{3}\right]$	$[3,6]$	$L-[25,56]$ $Z-[25,56]$

			According to the dual model				
			u_1	u_2	u_3	x_1	
		Non-basic vibrable / basic vibrable	y_1	y_2	y_3	v_1	objective function Original model Nc_i
Non-basic vibrable	u_4	y_4	$\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{-1}{3}$	$\left[\frac{8}{3}, \frac{5}{3}\right]$
	x_2	v_2	-3	-1	-3	0	$-[3,6]$
		objective function Original model Nc_i	$[4,6]$	$[0,4]$	$[12,18]$	$[5,7]$	$Z-[25,56]$ $L-[25,56]$

The second phase:

We apply the stopping criterion of the algorithm

For the original model:

Since the values in the constant's column are all positive, we study the values in the objective function row. We notice that there is a positive value, which is [3,6], meaning that we have not yet reached the optimal solution. Therefore, we specify the pivot column, which is the column of the variable x_2 corresponding to the only positive value in the objective function row. [3,6] In order to determine the pivot row and the pivot element, we calculate the index $N\theta$, where:

$$N\theta \in \min\left[\frac{[4,6]}{3}, \frac{[0,4]}{1}, \frac{[12,18]}{3}\right] = \frac{[4,6]}{3}$$

It corresponds to the base element u_1, so its row is the pivot row and the pivot element is $(\mathbf{3})$. We swap between the variables using the modified simplex algorithm.

For the dual model:

We study the elements of the objective function row. We notice that all the values are positive. Therefore, we study the elements of the constant's column. We find that there is a single negative value, which is $(-[3,6])$, which is the line of the base variable v_2, so its row is the pivot row. To determine the pivot column and the pivot element, we calculate the index $N\theta'$ where:

$$N\theta' \in Max\left[\frac{[4,6]}{-3}, \frac{[0,4]}{-1}, \frac{[12,18]}{-3}\right] = \frac{[4,6]}{-3}$$

So, the column of the non-base variable y_1 is the pivot column, meaning that the variable y_1 will enter the base instead of the variable v_2, and the pivot element is the element resulting from the intersection of the pivot row and

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the pivot column, which is (-3). We perform the switching between the variables using the modified simplex algorithm, from (1) and (2) We get the following double table:

Table No. (10): The binary algorithm table for the second stage

			According to the original model		
			$-y_4$	$-y_1$	
		Non-basic vibrable / basic vibrable	u_4	u_1	objective function Dual model NB_i
Non-basic vibrable	v_2	x_2	$\frac{-2}{9}$	$\frac{1}{3}$	$\left[\frac{4}{3},2\right]$
	y_2	u_2	$\frac{-4}{9}$	$\frac{-1}{3}$	$\left[\frac{4}{3},2\right]$
	y_3	u_3	$\frac{2}{3}$	-1	$[8,12]$
	v_1	x_1	1	0	$[5,7]$
		objective function Original model Nc_i	$[-6,-1]$	$[-2,-1]$	$L-[29,68]$ / $Z-[29,68]$

			According to the dual model				
			x_2	u_2	u_3	x_1	
		Non-basic vibrable / basic vibrable	v_2	y_2	y_3	v_1	objective function Original model Nc_i
Non-basic vibrable	u_4	y_4	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{-2}{3}$	-1	$[-6,-1]$
	u_1	y_1	$\frac{-1}{3}$	$\frac{1}{3}$	1	0	$[-2,-1]$
		objective function Dual model NB_i	$\left[\frac{4}{3},2\right]$	$\left[\frac{4}{3},2\right]$	$[8,12]$	$[5,7]$	$Z-[29,68]$ / $L-[29,68]$

We apply the stopping criterion of the algorithm:

1- For the original model, we study the elements of the objective function row until the criterion for stopping the algorithm is met, which is the absence of any positive element
2- For the dual model, we also study the elements of the constants column until the criterion for stopping the algorithm is met, which is the absence of any negative element
3- We find that the criterion has been met and thus we have reached the optimal solution

The optimal solution of the original model is:

$$x_2^* \in \left[\frac{4}{3},2\right] \ ,u_2^* \in \left[\frac{4}{3},2\right] \ ,u_3^* \in [8,12]\ ,x_1^* \in [5,7] \ ,u_1^* = u_4^* = 0$$

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The value of the objective function corresponds to:

$$Z^* = max\, Z \in [29,68]$$

The optimal solution of the dual model is:

$$y_1^* \in [1,2]\ , y_4^* \in [1,6]\ , v_2^* = y_2^* = y_3^* = v_1^* = 0$$

The value of the objective function corresponds to:

$$L^* = MinL \in [29,166]$$

We note that:

$$Z^* = Max\, Z \in [29,68] \leq L^* = MinL \in [29,166]$$

This solution is acceptable according to the following theory:

If $(x_1, x_2, \dots, x_n)$is an acceptable solution to the original model of type Max and $(y_1, y, \dots, y_m)$was an acceptable solution for the dual model of type Min, so the value of the objective function of the original model does not exceed the value of the objective function of the dual model for these two solutions, that is, it is

$$\sum_{j=1}^{n} Nc_j x_j \leq \sum_{i=1}^{m} Nb_i y_i$$

This is for all acceptable solutions for both models (including the optimal solution)

5. Conclusion and Results:

From the previous study, we arrived at a solution for the original and dual models at the same time, which are neutrosophic values from which we know the minimum and maximum profit that we can obtain, because the interpretation of the optimal solution for the original model is that it gives us the best production plan that makes the value of that production as large as possible, within Available capabilities. As for the optimal solution for the dual model, it gives us the best values for the prices of raw materials, which, if used without waste, will also give us the best production plan, and the result is the maximum profit.

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