International J.Math. Combin. Vol.2(2024), 100-110

4-Total Mean Cordial Graphs Derived From Star, Jellyfish and Fan

R. Ponraj[1], S.Subbulakshmi[2] and M. Sivakumar[3]

1. Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India
2. Research Scholar, Reg.No: 19124012092011, Department of Mathematics, Manonmaniam Abishekapatti, Sundarnar University, Tirunelveli-627012, Tamilnadu, India
3. Department of Mathematics, Government Arts and Science College,Tittagudi-606106,Tamilnadu, India

E-mail: ponrajmaths@gmail.com, ssubbulakshmis@gmail.com, sivamaths1975@gmail.com

Abstract: Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \cdots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \ldots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph.

Key Words: Star, bistar, jellyfish, fan, total mean cordial labeling, Smarandachely total mean cordial labeling.

AMS(2010): 05C78.

§1. Introduction

All graphs in this paper are finite, simple and undirected. Cordial labeling was introduced by Cahit [1]. Subsequently cordial related labeling was studied by several authors [9, 10, 11, 12, 13]. The notation of k-total mean cordial labeling have been introduced in [4]. Also 4-total mean cordial labeling of certain graphs like path, cycle, star, bistar, comb, crown, square of path, double comb, double crown, double fan, subdivision of star, subdivision of comb, subdivision of ladder, helm, flower graph, gear graph and web graph have been investigated [4, 5, 6, 7, 8]. In this paper, we investigate the 4-total mean cordial behavior of $K_{1,n} \odot K_1$, $K_{1,n} \odot \overline{K_2}$, $K_{1,n} \odot \overline{K_3}$, $B_{n,n} \odot \overline{K_2}$, $B_{n,n} \odot \overline{K_3}$, $J_{n,n} \odot K_1$, $J_{n,n} \odot \overline{K_2}$, $J_{n,n} \odot \overline{K_3}$, $F_n \odot K_1$, $F_n \odot \overline{K_3}$. Let x be any real number. Then $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms not defined here follow from Harary [3] and Gallian [2].

.

§2. k-Total Mean Cordial Graph

Definition 2.1 *Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean*

[1]Received December 6, 2023, Accepted May 31,2024.

cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, \cdots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \cdots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph.

Conversely, such a labeling f is called a Smarandachely k-total mean cordial labeling of G if there are integers $i, j \in \{0, 1, 2, \cdots, k-1\}$ hold with $|t_{mf}(i) - t_{mf}(j)| \geq 2$ and G is called a Smarandachely k-total mean cordial graph.

§3. Preliminaries

Definition 3.1 *A complete bipartite graph $K_{1,n}$ is called a star.*

Definition 3.2 *A bistar $B_{m,n}$ is the graph obtained by joining the two central vertices of $K_{1,m}$ and $K_{1,n}$.*

Definition 3.3 *A jellyfish graph $J(m, n)$ is obtained from a cycle $C_4 : uxvwu$ by joining x and w with an edge and appending m pendent edges to u and n pendent edges to v.*

Definition 3.4 *A graph $F_n = P_n + K_1$ is called a fan graph, where P_n is a path.*

Definition 3.5 *Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. A corona of G_1 with G_2 is the graph $G_1 \odot G_2$ obtained by taking one copy of G_1 , p_1 copies of G_2 and joining the i^{th} vertex of G_1 by an edge to every vertex in the i^{th} copy of G_2 where $1 \leq i \leq p_1$.*

Definition 3.6 *The complement $\overline{G}$ of a graph G also has $V(G)$ as its vertex set, but two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G.*

§4. Main Results

Theorem 4.1 *A graph $K_{1,n} \odot K_1$ is 4-total mean cordial for all n.*

Proof Let $V(K_{1,n}) = \{u, u_i : 1 \leq i \leq n\}$, $E(K_{1,n}) = \{uu_i : 1 \leq i \leq n\}$. Let v_1, v_2, $\cdots$, v_n be the pendent vertices adjacent to u_1, u_2, $\cdots$, u_n and v be the pendent vertex adjacent to u.

Clearly,
$$|V(K_{1,n} \odot K_1)| + |E(K_{1,n} \odot K_1)| = 4n + 3.$$

Assign the labels 1, 3 to the vertices u, v respectively. Consider the vertices u_1, u_2, $\cdots$, u_n. Now we assign the label 0 to the n vertices u_1, u_2, $\ldots$, u_n. We now consider the vertices v_1, v_2, $\cdots$, v_n. Finally we assign the label 3 to the n vertices v_1, v_2, $\ldots$, v_n.

Obviously, $t_{mf}(0) = n$, $t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = n + 1$. □

Theorem 4.2 *A graph $K_{1,n} \odot \overline{K_2}$ is 4-total mean cordial for all n.*

Proof Let $V(K_{1,n}) = \{u, u_i : 1 \leq i \leq n\}$, $E(K_{1,n}) = \{uu_i : 1 \leq i \leq n\}$. Let v, w be the pendent vertices adjacent to u and v_1, v_2, $\cdots$, v_n, w_1, w_2, $\cdots$, w_n be the pendent vertices adjacent to $u_1, u_2, \cdots, u_n$. Obviously,

$$\left|V\left(K_{1,n} \odot \overline{K_2}\right)\right| + \left|E\left(K_{1,n} \odot \overline{K_2}\right)\right| = 6n + 5.$$

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, where $r \in N$. Assign the labels 0, 2, 3 to the vertices u, v, w respectively.

Now, we assign the label 0 to the $3r$ vertices $u_1, u_2, \cdots, u_{3r}$. We now assign the label 1 to the r vertices $u_{3r+1}, u_{3r+2}, \cdots, u_{4r}$. Next we assign the label 1 to the $2r$ vertices $v_1, v_2, \cdots, v_{2r}$. We now assign the label 3 to the $2r$ vertices $v_{2r+1}, v_{2r+2}, \cdots, v_{4r}$. Finally we assign the label 3 to the $4r$ vertices $w_1, w_2, \cdots, w_{4r}$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, where $r \in N$. Assign the labels 0, 2, 3 to the vertices u, v, w respectively.

We now assign the label 0 to the $3r+1$ vertices $u_1, u_2, \cdots, u_{3r+1}$. Now we asisgn the label 1 to the r vertices $u_{3r+2}, u_{3r+3}, \cdots, u_{4r+1}$. Next we assign the label 1 to the $2r + 1$ vertices $v_1, v_2, \cdots, v_{2r+1}$. Now we assign the label 3 to the $2r$ vertices $v_{2r+2}, v_{2r+3}, \cdots, v_{4r+1}$. Finally we assign the label 3 to the $4r + 1$ vertices $w_1, w_2, \cdots, w_{4r+1}$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, where $r \geq 0$. Assign the labels 0, 3, 3 to the vertices u, v, w respectively.

We now assign the label 0 to the $3r+2$ vertices $u_1, u_2, \cdots, u_{3r+2}$. Now we asisgn the label 1 to the r vertices $u_{3r+3}, u_{3r+4}, \cdots, u_{4r+2}$. Next we assign the label 1 to the $2r + 2$ vertices $v_1, v_2, \cdots, v_{2r+2}$. Now we assign the label 3 to the $2r$ vertices $v_{2r+3}, v_{2r+4}, \cdots, v_{4r+2}$. Finally we assign the label 3 to the $4r + 2$ vertices $w_1, w_2, \ldots, w_{4r+2}$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r + 3$, where $r \geq 0$. Assign the labels 0, 3, 3 to the vertices u, v, w respectively.

Now we assign the label 0 to the $3r + 2$ vertices $u_1, u_2, \cdots, u_{3r+2}$. We now asisgn the label 1 to the $r + 1$ vertices $u_{3r+3}, u_{3r+4}, \cdots, u_{4r+3}$. Next we assign the label 1 to the $2r + 2$ vertices $v_1, v_2, \cdots, v_{2r+2}$. We now assign the label 3 to the $2r + 1$ vertices $v_{2r+3}, v_{2r+4}, \cdots, v_{4r+3}$. Finally we assign the label 3 to the $4r + 3$ vertices $w_1, w_2, \cdots, w_{4r+3}$.

This vertex labeling f is a 4-total mean cordial labeling follows from the Table 1.

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$6r + 1$	$6r + 1$	$6r + 2$	$6r + 1$
$n = 4r + 1$	$6r + 3$	$6r + 3$	$6r + 3$	$6r + 2$
$n = 4r + 2$	$6r + 5$	$6r + 4$	$6r + 4$	$6r + 4$
$n = 4r + 3$	$6r + 5$	$6r + 6$	$6r + 6$	$6r + 6$

Table 1.

This completes the proof. □

Theorem 4.3 *A graph $K_{1,n} \odot \overline{K_3}$ is 4-total mean cordial for all n.*

Proof Let $V(K_{1,n}) = \{u, u_i : 1 \leq i \leq n\}$, $E(K_{1,n}) = \{uu_i : 1 \leq i \leq n\}$. Let x, y, z be the pendent vertices adjacent to u and $x_1, x_2, \cdots, x_n, y_1, y_2, \ldots, y_n, z_1, z_2, \cdots, z_n$ be the pendent vertices adjacent to $u_1, u_2, \cdots, u_n$ respectively. Note that

$$\left|V\left(K_{1,n} \odot \overline{K_3}\right)\right| + \left|E\left(K_{1,n} \odot \overline{K_3}\right)\right| = 8n + 7.$$

Assign the labels 0, 1, 3, 3 to the vertices u, x, y, z respectively. Now, we assign the label 0 to the n vertices $u_1, u_2, \cdots, u_n$. We now assign the label 1 to the n vertices $x_1, x_2, \cdots, x_n$. Next we assign the label 3 to the n vertices $y_1, y_2, \cdots, y_n$. Finally we assign the label 3 to the n vertices $z_1, z_2, \cdots, z_n$.

Clearly,

$$t_{mf}(0) = 2n + 1, \ t_{mf}(1) = 2n + 2, \ \ t_{mf}(2) = 2n + 2 \text{ and } t_{mf}(3) = 2n + 2. \qquad \square$$

Example 4.1 A 4 - total mean cordial labeling of $K_{1,3} \odot \overline{K_3}$ is given in Figure 1.

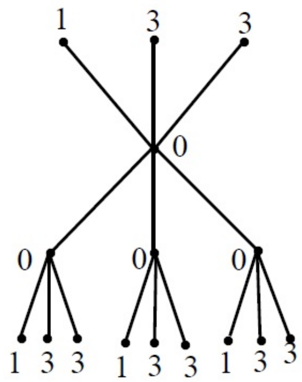

Figure 1. $K_{1,3} \odot \overline{K_3}$

Theorem 4.4 *A graph $B_{n,n} \odot \overline{K_2}$ is 4-total mean cordial for all n.*

Proof Let $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(B_{n,n}) = \{uv, uu_i, vv_i : 1 \leq i \leq n\}$. Let x, y be the pendent vertices adjacent to u and $x_1, x_2, \ldots, x_n, y_1, y_2, \cdots, y_n$ be the pendent vertices adjacent to $u_1, u_2, \cdots, u_n$. Let p, q be the pendent vertices adjacent to v and $p_1, p_2, \cdots, p_n, q_1, q_2, \cdots, q_n$ be the pendent vertices adjacent to $v_1, v_2, \cdots, v_n$.

Clearly,

$$\left|V\left(B_{n,n} \odot \overline{K_2}\right)\right| + \left|E\left(B_{n,n} \odot \overline{K_2}\right)\right| = 12n + 11.$$

Assign the labels 1, 3, 0, 0, 2, 2 to the vertices u, v, x, y, p, q respectively. Now, we assign the label 0 to the n vertices $u_1, u_2, \cdots, u_n$. We now assign the label 2 to the n vertices $v_1, v_2, \cdots, v_n$. Next we assign the label 0 to the n vertices $x_1, x_2, \cdots, x_n$. We now assign the label 1 to the n vertices $y_1, y_2, \cdots, y_n$. Now we assign the label 2 to the n vertices $p_1, p_2, \cdots, p_n$. Finally, we assign tne label 3 to the n vertices $q_1, q_2, \cdots, q_n$.

Thus,

$$t_{mf}(0) = 3n + 2, \quad t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = 3n + 3. \qquad \square$$

Example 4.2 A 4-total mean cordial labeling of $B_{2,2} \odot \overline{K_2}$ is shown in Figure 2.

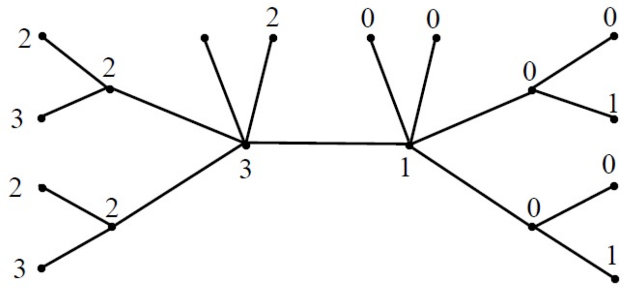

Figure 2. $B_{2,2} \odot \overline{K_2}$

Theorem 4.5 *A graph $B_{n,n} \odot \overline{K_3}$ is 4-total mean cordial for all n.*

Proof Let $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(B_{n,n}) = \{uv, uu_i, vv_i : 1 \leq i \leq n\}$. Let x, y, z be the pendent vertices adjacent to u and x_1, x_2, $\cdots$, x_n, y_1, y_2, $\cdots$, y_n, z_1, z_2, $\cdots$, z_n be the pendent vertices adjacent to u_1, u_2, $\cdots$, u_n. Let p, q, r be the pendent vertices adjacent to v and p_1, p_2, $\cdots$, p_n, q_1, q_2, $\cdots$, q_n, r_1, r_2, $\cdots$, r_n be the pendent vertices adjacent to v_1, v_2, $\cdots$, v_n. Note that

$$\left|V\left(B_{n,n} \odot \overline{K_3}\right)\right| + \left|E\left(B_{n,n} \odot \overline{K_3}\right)\right| = 16n + 15.$$

Assign the labels 0, 1, 3, 3, 0, 1, 3, 3 to the vertices u, x, y, z, v, p, q, r respectively. Now we assign the label 0 to the $2n$ vertices u_1, u_2, $\ldots$, u_n, v_1, v_2, $\cdots$, v_n. We now assign the label 1 to the $2n$ vertices x_1, x_2, $\cdots$, x_n, p_1, p_2, $\ldots$, p_n. Finally, we assign the label 3 to the $4n$ vertices y_1, y_2, $\ldots$, y_n, z_1, z_2, $\cdots$, z_n, q_1, q_2, $\cdots$, q_n, r_1, r_2, $\cdots$, r_n.

Clearly,

$$t_{mf}(0) = 4n + 3, \; t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = 4n + 4. \qquad \square$$

Example 4.3 A 4 - total mean cordial labeling of $B_{3,3} \odot \overline{K_3}$ is given in Figure 3.

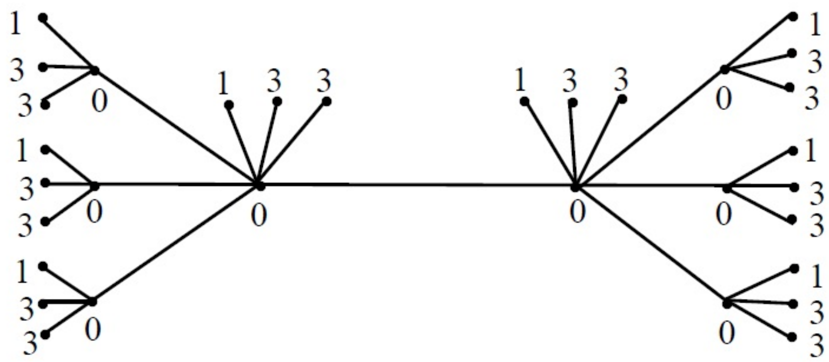

Figure 3. $B_{3,3} \odot \overline{K_3}$

Theorem 4.6 *A graph $J_{n,n} \odot K_1$ is 4-total mean cordial for all n.*

Proof Let $V(J_{n,n} \odot K_1) = \{u, v, x, y, p, q, r, s\} \cup \{u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$, $E(J_{n,n} \odot K_1) = \{ux, xv, vy, yu, xy, pu, qx, rv, sy\} \cup \{uu_i, u_ix_i, vv_i, v_iy_i : 1 \leq i \leq n\}$. Clearly,

$$|V(J_{n,n} \odot K_1)| + |E(J_{n,n} \odot K_1)| = 8n + 17.$$

Assign the labels 0, 2, 1, 3, 0, 0, 0, 3 to the vertices u, v, x, y, p, q, r, s respectively. We now assign the label 0 to the n vertices u_1, u_2, $\cdots$, u_n. Now we assign the label 1 to the n vertices x_1, x_2, $\cdots$, x_n. Next we assign the label 2 to the n vertices v_1, v_2, $\cdots$, v_n. Finally we assign the label 3 to the n vertices y_1, y_2, $\cdots$, y_n.

Clearly,

$$t_{mf}(0) = 2n + 5, \quad t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = 2n + 4. \qquad \square$$

Example 4.4 A 4 - total mean cordial labeling of $J_{3,3} \odot K_1$ is shown in Figure 4.

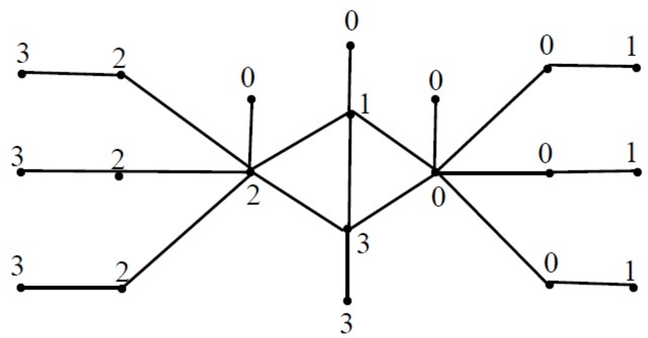

Figure 4. $J_{3,3} \odot K_1$

Theorem 4.7 *A graph $J_{n,n} \odot \overline{K_2}$ is 4-total mean cordial for all n.*

Proof Let $V(J_{n,n} \odot \overline{K_2}) = \{u, v, x, y, a, b, c, d, e, f, g, h\} \cup \{u_i, v_i, x_i, y_i, p_i, q_i : 1 \leq i \leq n\}$, $E(J_{n,n} \odot \overline{K_2}) = \{ux, xv, vy, yu, xy, au, bu, cx, dx, ev, fv, gy, hy\} \cup \{uu_i, u_ix_i, u_iy_i, vv_i, v_ip_i, v_iq_i : 1 \leq i \leq n\}$.

Obviously,

$$|V(J_{n,n} \odot \overline{K_2})| + |E(J_{n,n} \odot \overline{K_2})| = 12n + 25.$$

Assign the labels 1, 3, 1, 0, 0, 2, 3, 3, 3, 3, 0, 0 to the vertices u, v, x, y, a, b, c, d, e, f, g, h respectively. Now we assign the label 0 to the $2n$ vertices u_1, u_2, $\cdots$, u_n, x_1, x_2, $\cdots$, x_n. We now assign the label 1 to the n vertices y_1, y_2, $\cdots$, y_n. Next we assign the label 2 to the $2n$ vertices v_1, v_2, $\cdots$, v_n, p_1, p_2, $\cdots$, p_n. Finally we assign the label 3 to the n vertices q_1, q_2, $\cdots$, q_n.

Clearly,

$$t_{mf}(0) = t_{mf}(1) = t_{mf}(2) = 3n + 6, \; t_{mf}(3) = 3n + 7. \qquad \square$$

Example 4.5 A 4 - total mean cordial labeling of $J_{2,2} \odot \overline{K_2}$ is given in Figure 5.

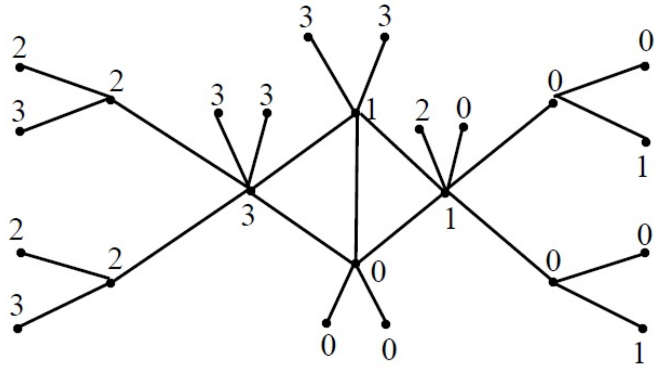

Figure 5. $J_{2,2} \odot \overline{K_2}$

Theorem 4.8 *A graph $J_{n,n} \odot \overline{K_3}$ is 4-total mean cordial.*

Proof Let $V\left(J_{n,n} \odot \overline{K_3}\right) = \{u, v, x, y\} \cup \{u_i, v_i, x_i, y_i : 1 \leq i \leq 3\} \cup \{z_j, p_j, q_j, r_j, w_j, a_j, b_j, c_j : 1 \leq i \leq n\}$, $E\left(J_{n,n} \odot \overline{K_3}\right) = \{ux, xv, vy, yu, xy\} \cup \{uu_i, xx_i, vv_i, yy_i : 1 \leq i \leq 3\} \cup \{uz_j, z_jp_j, z_jq_j, z_jr_j, vw_j, w_ja_j, w_jb_j, w_jc_j : 1 \leq j \leq n\}$.

Clearly,

$$\left|V\left(J_{n,n} \odot \overline{K_3}\right)\right| + \left|E\left(J_{n,n} \odot \overline{K_3}\right)\right| = 16n + 33.$$

Assign the label 0, 0, 3, 2 to the vertices u, v, x, y respectively. We now assign the label 0 to the vertices u_1, u_2, u_3. Now we assign the label 1 to the vertices v_1, v_2, v_3. Next we assign the label 3 to the vertices x_1, x_2, x_3. Now we assign the label 2 to the vertices y_1, y_2, y_3. We now assign the label 0 to the $2n$ vertices z_1, z_2, $\cdots$, z_n, w_1, w_2, $\cdots$, w_n. Next we assign the label 1 to the $2n$ vertices p_1, p_2, $\cdots$, p_n, a_1, a_2, $\cdots$, a_n. Finally we assign the label 3 to the $4n$ vertices q_1, q_2, ..., q_n, r_1, r_2, $\cdots$, r_n, b_1, b_2, $\cdots$, b_n, c_1, c_2, $\cdots$, c_n.

Clearly,

$$t_{mf}(0) = t_{mf}(1) = t_{mf}(3) = 4n + 8, \ t_{mf}(2) = 4n + 9. \qquad \square$$

Example 4.6 A 4 - total mean cordial labeling of $J_{2,2} \odot \overline{K_3}$ is shown in Figure 6.

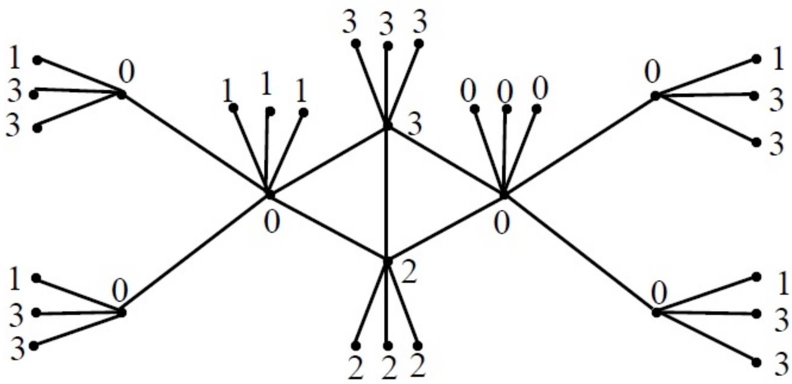

Figure 6. $J_{2,2} \odot \overline{K_3}$

Theorem 4.9 *A graph $F_n \odot K_1$ is 4-total mean cordial for all $n \geq 2$.*

Proof Let P_n be the path $u_1u_2 \cdots u_n$. Let $V(F_n \odot K_1) = \{u, x\} \cup V(P_n) \cup \{x_i : 1 \leq i \leq n\}$ and

$$E(F_n \odot K_1) = \{uu_i : 1 \leq i \leq n\} \cup E(P_n) \cup \{u_ix_i : 1 \leq i \leq n\}.$$

Clearly $|V(F_n \odot K_1)| + |E(F_n) \odot K_1| = 5n + 2$.

Assign the labels 2, 0 to the vertices u, x respectively.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, where $r \in N$. Consider the path vertices $u_1, u_2, \cdots, u_n$. Assign the labels 0, 0, 2, 3 respectively to the vertices u_1, u_2, u_3, u_4. Now we assign the labels 0, 0, 2, 3 to the vertices u_5, u_6, u_7, u_8 respectively. We now assign the labels 0, 0, 2, 3 respectively to the vertices u_9, u_{10}, u_{11}, u_{12}. Proceeding like this until reach the vertex u_{4r}. Obviously the vertices u_{4r-3}, u_{4r-2}, u_{4r-1}, u_{4r} receive the labels 0, 0, 2, 3. Now we assign the labels 0, 1, 2, 3 respectively to the vertices x_1, x_2, x_3, x_4. We now assign the labels 0, 1, 2, 3 to the vertices x_5, x_6, x_7, x_8 respectively. Next we assign the labels 0, 1, 2, 3 respectively to the vertices $x_9, x_{10}, x_{11}, x_{12}$. Continuing like this until reach the vertices x_{4r}. Clearly the vertices x_{4r-3}, x_{4r-2}, x_{4r-1}, x_{4r} receive the labels 0, 1, 2, 3.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, where $r \in N$.

As in Case 1 assign the label to the vertices u_i, x_i $(1 \leq i \leq 4r)$. Finally, we assign the labels 0, 3 to the vertices u_{4r+1}, x_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, where $r \in N$. Label the vertices u_i, x_i $(1 \leq i \leq 4r)$ as in Case 1. Now we assign the labels 3, 0, 0, 2 to the vertices u_{4r+1}, u_{4r+2}, x_{4r+1},x_{4r+2}.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, where $r \in N$. In this case assign the label for the vertices u_i, x_i $(1 \leq i \leq 4r)$ as in Case 1. We now assign the labels 3, 0, 0, 0, 2, 3 to the vertices u_{4r+1}, u_{4r+2}, u_{4r+3}, x_{4r+1},x_{4r+2}, x_{4r+3}.

This vertex labeling f is a 4-total mean cordial labeling of $F_n \odot K_1$ follows from the Table 2.

Order of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$5r + 1$	$5r + 1$	$5r$	$5r + 1$
$n = 4r + 1$	$5r + 2$	$5r + 2$	$5r + 2$	$5r + 1$
$n = 4r + 2$	$5r + 3$	$5r + 3$	$5r + 3$	$5r + 3$
$n = 4r + 3$	$5r + 5$	$5r + 4$	$5r + 4$	$5r + 4$

Table 2.

Case 5. $n = 2, 3$.

A 4-total mean cordial labeling is given in Table 3.

n	u	v	u_1	u_2	u_3	v_1	v_2	v_3
2	1	3	0	0		3	3	
3	1	3	0	0	0	3	3	3

Table 3.

This completes the proof. □

Theorem 4.10 *A graph $F_n \odot \overline{K_3}$ is 4-total mean cordial for all values of $n \geq 2$.*

Proof Let P_n be the path $u_1u_2 \ldots u_n$. Let $V\left(F_n \odot \overline{K_3}\right) = \{u, x, y, z\} \cup V(P_n) \cup \{x_i, y_i, z_i : 1 \leq i \leq n\}$ and $E\left(F_n \odot \overline{K_3}\right) = \{uu_i : 1 \leq i \leq n\} \cup E(P_n) \cup \{u_ix_i, u_iy_i, u_iz_i : 1 \leq i \leq n\}$.

Clearly,

$$\left|V\left(F_n \odot \overline{K_3}\right)\right| + \left|E(F_n) \odot \overline{K_3}\right| = 9n + 6.$$

Assign the labels 2, 0, 2, 3 to the vertices u, x, y, z respectively.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, where $r \in N$. Label the vertices u_i, x_i $(1 \leq i \leq n)$ as in Case 1 of Theorem 4.2. Assign the labels 0, 1, 2, 3 respectively to the vertices y_1, y_2, y_3, y_4. Now we assign the labels 0, 1, 2, 3 to the vertices y_5, y_6, y_7, y_8 respectively. We now assign the labels 0, 1, 2, 3 respectively to the vertices y_9, y_{10}, y_{11}, y_{12}. Proceeding like this until reach the vertex y_{4r}. Obviously the vertices y_{4r-3}, y_{4r-2}, y_{4r-1}, y_{4r} receive the labels 0, 1, 2, 3. Next we assign the labels 0, 1, 2, 3 respectively to the vertices z_1, z_2, z_3, z_4. We now assign the labels 0, 1, 2, 3 to the vertices z_5, z_6, z_7, z_8 respectively. Now we assign the labels 0, 1, 2, 3 respectively to the vertices z_9, z_{10}, z_{11}, x_{12}. Continuing like this until reach the vertices z_{4r}. Clearly, the vertices z_{4r-3}, z_{4r-2}, z_{4r-1}, z_{4r} receive the labels 0, 1, 2, 3.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, where $r \in N$. As in Case 1 assign the label to the vertices u_i, x_i, y_i, z_i $(1 \leq i \leq 4r)$. Finally we assign the labels 2, 0, 0, 0 to the vertices u_{4r+1}, x_{4r+1}, y_{4r+1}, z_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, where $r \in N$. Label the vertices u_i, x_i, y_i, z_i $(1 \leq i \leq 4r)$ as in Case 1. Now we assign the labels 3, 0, 2, 0, 1, 0, 1, 1 to the vertices u_{4r+1}, u_{4r+2}, x_{4r+1},x_{4r+2}, y_{4r+1}, y_{4r+2}, z_{4r+1},z_{4r+2}.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r + 3$, where $r \in N$. In this case assign the label for the vertices u_i, x_i $(1 \leq i \leq 4r + 2)$ as in Case 3. We now assign the labels 2, 0, 0, 3 to the vertices u_{4r+3}, x_{4r+3}, y_{4r+3}, z_{4r+3}.

This labeling f is a 4-total mean cordial labeling of $F_n \odot K_1$ follows from the Table 4.

Order of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$5r+1$	$5r+1$	$5r$	$5r+1$
$n = 4r+1$	$5r+2$	$5r+2$	$5r+2$	$5r+1$
$n = 4r+2$	$5r+3$	$5r+3$	$5r+3$	$5r+3$
$n = 4r+3$	$5r+5$	$5r+4$	$5r+4$	$5r+4$

Table 4.

Case 5. $n = 2, 3$.

A 4-total mean cordial labeling is given in Table 5.

n	u	x	y	z	u_1	u_2	u_3	x_1	x_2	x_3	y_1	y_2	y_3	z_1	z_2	z_3
2	2	0	2	3	3	0		2	0		1	0		1	1	
3	2	0	2	3	3	0	2	3	0	1	2	0	1	2	1	3

Table 5.

This completes the proof. □

References

[1] I.Cahit, Cordial Graphs: A weaker version of Graceful and Harmonious graphs, *Ars Combin.*, 23(1987) 201-207.

[2] J.A.Gallian, A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 19 (2016) #Ds6.

[3] F.Harary, *Graph Theory*, Addision wesley, New Delhi (1969).

[4] R.Ponraj, S.Subbulakshmi, S.Somasundaram, k-total mean cordial graphs, *J.Math.Comput. Sci.*, 10(2020), No.5,1697-1711.

[5] R.Ponraj, S.Subbulakshmi, S.Somasundaram, 4-total mean cordial graphs derived from paths, *J.Appl and Pure Math.*, Vol 2(2020), 319-329.

[6] R.Ponraj, S.Subbulakshmi, S.Somasundaram, 4-total mean cordial labeling in subdivision graphs, *Journal of Algorithms and Computation*, 52(2020),1-11.

[7] R.Ponraj, S.Subbulakshmi, S.Somasundaram, Some 4-total mean cordial graphs derived from wheel, *J. Math. Comput. Sci.*, 11(2021), 467-476.

[8] R.Ponraj, S.Subbulakshmi, S.Somasundaram, 4-total mean cordial graphs derived from star and bistar, *J. Math. Comput. Sci.*, 11(2021), 467-476.

[9] U.M.Prajapati, N.B.Patel, Edge product cordial labeling of some cycle related graphs, *Open J. Discrete Math.*, 6(2016), 268-278.

[10] U.M.Prajapati, P.D.Shah, Some edge product cordial graphs in the context of duplication of some graphs elements, *Open J. Discrete Math.*, 6(2016), 248-258.

[11] U.M.Prajapati, K.P.Shah, Discussion on prime cordial labeling, *Pramana Research Journal.*, Vol 9(2019), 1090-1105.

[12] M.Seoud, S.Salman, On difference cordial graphs, *Mathematica Aeterna.*, Vol 5(2015), 105-124.

[13] M.Seoud, S.Salman, Some results and examples on difference cordial graphs, *Turkish Journal of Mathematics*, 40(2016), 417-427.