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4-Total Prime Cordiality of Certain Subdivided Graphs

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Abstract: Let G be a (p,q) graph. Let $f : V(G) \rightarrow \{1,2,\cdots,k\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $\gcd(f(u),f(v))$. f is called k-total prime cordial labeling of G if $|t_f(i) - t_f(j)| \leq 1$, $i,j \in \{1,2,\cdots,k\}$ where $t_f(x)$ denotes the total number of vertices and the edges labelled with x. A graph with a k-total prime cordial labeling is called k-total prime cordial graph. In this paper we investigate the 4-total prime cordial labeling for some subdivided graphs.

Key Words: Corona, ladder, triangular snake, k-total prime cordial labeling, Smarandache k-total prime cordial labeling.

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§1. Introduction

In this paper we consider simple, finite and undirected graphs only. The notion of k- total prime cordial labelling has been introduced in [4]. In [4–9], they investigate the k-total prime cordial labeling of some graphs and investigate the 4-total prime cordial labeling behaviour of path, cycle, star, bistar, ladder, triangular snake, friendship graph, comb, double comb, double triangular snake, flower graph, gear graph, Jelly fish, book, irregular triangular snake, prism, helm, dumbbell graph, sunflower graph, dragon, mobius ladder and subdivision of some graphs. In this paper we examine the 4-total prime cordial labeling of subdivision of some graphs like star, bistar, comb, double comb, ladder, triangular snake and double triangular snake. Terms are not defined here follows from [1], [3].

§2. Preliminary Results

Definition 2.1 *Let G_1, G_2 respectively be (p_1,q_1), (p_2,q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.*

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Definition 2.2 *If $e = uv$ is an edge of G then e is said to be subdivided when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G and is denoted by $S(G)$.*

§3. k-Total Prime Cordial Labeling

Definition 3.1 *Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \cdots, k\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-total prime cordial labeling of G if $|t_f(i) - t_f(j)| \le 1$, $i, j \in \{1, 2, \cdots, k\}$ where $t_f(x)$ denotes the total number of vertices and the edges labelled with x. A graph with a k-total prime cordial labeling is called k-total prime cordial graph. Generally, if there are integers $i, j \in \{1, 2, \cdots, k\}$ such that $|t_f(i) - t_f(j)| > 1$, f is called a Smarandache k-total prime cordial labeling and G a Smarandache k-total prime cordial labeling graph.*

Theorem 3.2 *The subdivision of comb $S(P_n \odot K_1)$ is 4-total prime cordial.*

Proof Let P_n be the path $u_1u_2 \cdots u_n$. Let x_i be the vertex which subdivide the edge u_iu_{i+1}. Let v_i be the vertex adjacent to u_i. Let w_i be the pendent vertices v_i. Clearly $|V(S(P_n \odot K_1))| + |E(S(P_n \odot K_1))| = 8n - 3$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$, $t \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_t$ and assign the label 3 to the vertices $u_{t+1}, u_{t+2}, \cdots, u_{2t}$. Next we assign the label 2 to the vertices $u_{2t+1}, u_{2t+2}, \cdots, u_{3t}$ and assign 1 to the vertices $u_{3t+1}, u_{3t+2}, \cdots, u_n$. Now we consider the vertices x_i $(1 \le i \le n-1)$. Assign the label 4 to the vertices $x_1, x_2, \cdots, x_t$ and assign the label 3 to the vertices $x_{t+1}, x_{t+2}, \ldots, x_{2t}$. Next we assign the label 2 to the vertices $x_{2t+1}, x_{2t+2}, \cdots, x_{3t}$. Finally we assign 1 to the vertices $x_{3t+1}, x_{3t+2}, \cdots, x_{n-1}$. Now we move to the vertices v_i, w_i $(1 \le i \le n)$. Assign the label 4 to the vertices $v_1, v_2, \ldots, v_t$ and $w_1, w_2, \cdots, w_t$. Then assign the label 3 to the vertices $v_{t+1}, v_{t+2}, \cdots, v_{2t}$ and $w_{t+1}, w_{t+2}, \cdots, w_{2t}$. Now we assign the label 2 to the vertices $v_{2t+1}, v_{2t+2}, \cdots, v_{3t}$ and $w_{2t+1}, w_{2t+1}, \cdots, w_{3t}$. Finally we assign the label 1 to the vertices $v_{3t+1}, v_{3t+2}, \cdots, v_n$ and $w_{3t+1}, w_{3t+2}, \cdots, w_n$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t + 1$, $t \in \mathbb{N}$. As in case 1, assign the label to the vertices u_i $(1 \le i \le n-2)$, x_i $(1 \le i \le n-2)$, v_i, w_i $(1 \le i \le n-1)$. Now we assign the labels 3, 4, 2, 3, 4 respectively to the vertices u_{n-1}, u_n, x_{n-1}, v_n and w_n.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4t + 2$, $t \in \mathbb{N}$. Assign the label to the vertices u_i, v_i, w_i $(1 \le i \le n-2)$ and x_i $(1 \le i \le n-3)$ by case 1. Next we assign the labels 4, 3, 4, 3, 2, 2, 4, 3 to the vertices u_{n-1}, u_n, x_{n-2}, x_{n-1}, v_{n-1}, v_n, w_{n-1} and w_n respectively.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4t+3$, $t \in \mathbb{N}$. As in Case 3, assign the label to the vertices u_i, v_i, w_i $(1 \leq i \leq n-2)$ and x_i $(1 \leq i \leq n-3)$. Next we assign the labels 2, 3, 4, 3, 3, 3, 2,1 respectively to the vertices u_{n-1}, u_n, x_{n-2}, x_{n-1}, v_{n-1}, v_n, w_{n-1} and w_n. □

Theorem 3.3 *The subdivision of double comb $S(P_n \odot 2K_1)$ is 4-total prime cordial.*

Proof Let P_n be the path $u_1u_2 \cdots u_n$. Let z_i be the vertex which subdivide the edge u_iu_{i+1}. Let v_i, w_i be the vertices adjacent to u_i. Let x_i, y_i be the pendent vertices adjacent to u_i,v_i respectively. Obviously $|V(S(P_n \odot 2K_1))| + |E(S(P_n \odot 2K_1))| = 12n - 3$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$, $t \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_t$ and assign the label 3 to the vertices $u_{t+1}, u_{t+2}, \cdots, u_{2t}$. Now we assign the label 2 to the vertices $u_{2t+1}, u_{2t+2}, \cdots, u_{3t}$. Next we assign 1 to the vertices $u_{3t+1}, u_{3t+2}, \cdots, u_{n-1}$. Finally we assign the label 3 to the vertex u_n. Now we consider the vertices z_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $z_1, z_2, \cdots, z_{t-1}$ and assign the label 3 to the vertices $z_{t+1}, z_{t+2}, \cdots, z_{2t-1}$. Next we assign the label 2 to the vertices $z_{2t+1}, z_{2t+2}, \cdots, z_{3t-1}$. Now we assign the label 1 to the vertices $z_t, z_{2t}, z_{3t}, z_{3t+1}, \cdots, z_{n-1}$. Assign the label to the vertices v_i, w_i, x_i, y_i $(1 \leq i \leq n-1)$. Finally we assign 2, 4, 4, 3 respectively to the vertices x_n, v_n, w_n and y_n.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t+1$, $t \in \mathbb{N}$. As in case 1, assign the label to the vertices u_i, v_i, x_i, w_i, y_i $(1 \leq i \leq n-1)$ and z_i $(1 \leq i \leq n-2)$. Now we assign the labels 2, 4, 4, 2, 3, 3 respectively to the vertices z_{n-1}, x_n, v_n, u_n, w_n and y_n.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4t+2$, $t \in \mathbb{N}$. Assign the label to the vertices u_i, v_i, x_i, w_i, y_i $(1 \leq i \leq n-1)$ and z_i $(1 \leq i \leq n-2)$ by case 2. Next we assign the label 1 to z_{n-1} and assign the labels 4, 4, 2, 3, 3 to the vertices x_n, v_n, u_n, w_n and y_n respectively.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4t+3$, $t \in \mathbb{N}$. As in Case 3, assign the label to the vertices u_i, v_i, x_i, w_i, y_i $(1 \leq i \leq n-1)$ and z_i $(1 \leq i \leq n-2)$. Next we assign the labels 2, 4, 4, 2, 3, 3 respectively to the vertices z_{n-1}, x_n, v_n, u_n, w_n and y_n. □

Theorem 3.4 *The subdivision of star $S(K_{1,n})$ is 4-total prime cordial.*

Proof Let u be the vertex of degree n and $u_1, u_2, \cdots, u_n$ be the vertices of degree 2. Let $v_1, v_2, \cdots, v_n$ be the pendent vertices.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$, $t \in \mathbb{N}$. Assign the label 4 to the vertex u. Next we now move to the vertices $u_1, u_2, \cdots, u_n$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_t$ and assign the label 2 to the vertices $u_{t+1}, u_{t+2}, \cdots, u_{2t}$. Next we assign the label 3 to the vertices $u_{2t+1}, u_{2t+2}, \cdots, u_{3t}$. Finally assign the label 1 to the non-labelled vertices of u_n. Now we consider the pendent vertices $v_1, v_2, \cdots, v_n$. Assign the label 4 to the vertices $v_1, v_2, \cdots, v_t$ and assign the label 2 to the vertices $v_{t+1}, v_{t+2}, \ldots, v_{2t}$. Finally assign 3 to the non-labelled vertices of v_n.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t+1$, $t \in \mathbb{N}$. In this case, assign the label to the vertices u, u_i $(1 \leq i \leq n)$ and v_i $(1 \leq i \leq n-2)$ by in case 1. Next assign the labels 2 and 4 to the vertices v_{n-1} and v_n respectively.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4t+2$, $t \in \mathbb{N}$. As in Case 1, assign the label to the vertices u, u_i $(1 \leq i \leq n-1)$ and v_i $(1 \leq i \leq n-2)$. Next assign the labels 2, 1 and 4 to the vertices respectively u_n, v_{n-1} and v_n.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4t+3$, $t \in \mathbb{N}$. Assign the label to the vertices u, u_i $(1 \leq i \leq n-2)$ and v_i $(1 \leq i \leq n-2)$ as in case 1. Finally assign the labels 3, 2, 4 and 4 to the vertices u_{n-1}, u_n, v_{n-1} and v_n respectively. □

Theorem 3.5 *The subdivision of bistar $S(B_{n,n})$ is 4-total prime cordial.*

Proof Let u, v be the vertices of degree n and w be the vertex of degree 2 adjacent to both u and v. Let u_i be the vertex of degree 2 adjacent to u and v_i be the vertex of degree 2 adjacent to v. Let x_i and y_i $(1 \leq i \leq n)$ be the pendent vertex adjacent to u_i and v_i respectively.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$, $t \in \mathbb{N}$. Assign the labels 4, 2 and 3 to the vertex u, w and v respectively. Next we move to the vertices $u_1, u_2, \cdots, u_n$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_{2t}$ and assign the label 2 to the vertices $u_{2t+1}, u_{2t+2}, \cdots, u_{4t}$. Now we consider the pendant vertices of u_n. Assign the label 4 to the vertices $x_1, x_2, \cdots, x_{2t}$ and assign the label 2 to the vertices $x_{2t+1}, x_{2t+2}, \cdots, x_{4t}$. Now we consider the vertices $v_1, v_2, \cdots, v_n$. Assign the label 3 to the vertices $v_1, v_2, \cdots, v_{2t}$ and assign the label 1 to the vertices $v_{2t+1}, v_{2t+2}, \cdots, v_{4t}$. Finally we move to the pendant vertices of v_n. Assign the label 3 to the vertices $y_1, y_2, \cdots, y_{2t}$ and assign the label 1 to the vertices $y_{2t+1}, y_{2t+2}, \cdots, y_{4t}$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t+1$, $t \in \mathbb{N}$. In this case assign the label to the vertices u, v, w, u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-1)$, x_i $(1 \leq i \leq n-1)$ and y_i $(1 \leq i \leq n-1)$ as in case 1. Next assign the labels 4, 2, 3 and 1 respectively to the vertices u_n, x_n, v_n and y_n.

Case 3. $n \equiv 2, 3 \pmod 4$.

Let $n = 4t+1$ and $n = 4t+2$ $t \in \mathbb{N}$. The proof is similar to that of Case 2. □

Theorem 3.6 *The subdivision of triangular snake $S(T_n)$ is 4-total prime cordial.*

Proof Let P_n be the path $u_1 u_2 \cdots u_n$. Let w_i be the vertex adjacent to u_i and u_{i+1}. Let

v_i be the vertices which subdivide the edge u_iu_{i+1} and x_i,y_i be the vertex which subdivided u_iw_i and $u_{i+1}w_i$ respectively. It is easy to verify that $|V(S(T_n))| + |E(S(T_n))| = 11n - 10$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$, $t \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_t$ and assign the label 2 to the vertices $u_{t+1}, u_{t+2}, \cdots, u_{2t}$. Next we assign the label 3 to the vertices $u_{2t+1}, u_{2t+2}, \cdots, u_{3t}$ then we assign the label 1 to the vertices $u_{3t+1}, u_{3t+2}, \cdots, u_{n-1}$. Finally, we assign 3 to the vertex u_n. Assign the label 4 to the vertices $v_1, v_2, \cdots, v_t$ and assign the label 2 to the vertices $v_{t+1}, v_{t+2}, \cdots, v_{2t-1}$ and assign the label 3 to the vertices $v_{2t}, v_{2t+1}, \cdots, v_{3t-1}$ then we assign the label 1 to the vertices $v_{3t}, v_{3t+1}, \cdots, v_{n-1}$. Assign the label to the vertices x_i $(1 \leq i \leq n-1)$ as in v_i $(1 \leq i \leq n-1)$. Now relabel the vertex x_{2t} by 2. Assign the label 4 to the vertices $y_1, y_2, \cdots, y_{t-1}$ and assign the label 2 to the vertices $y_t, y_{t+1}, \cdots, y_{2t-1}$ and assign the label 3 to the vertices $y_{2t}, y_{2t+1}, \cdots, y_{3t-1}$ then we assign the label 1 to the vertices $y_{3t}, y_{3t+1}, \cdots, y_{n-2}$. Finally we assign the label 2 to the vertices y_{n-1}. Now we consider the vertices w_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $w_1, w_2, \cdots, w_t$ and assign the label 2 to the vertices $w_{t+1}, w_{t+2}, \cdots, w_{2t-1}$ and assign the label 3 to the vertices $w_{2t}, w_{2t+1}, \cdots, w_{3t-1}$ then we assign the label 1 to the vertices $w_{3t}, w_{3t+1}, \cdots, w_{n-2}$. Finally we assign 4 to the vertex w_{n-1}.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t + 1$, $t \in \mathbb{N}$. As in Case 1, assign the label to the vertices u_i $(1 \leq i \leq n-1)$, v_i, x_i, y_i, w_i $(1 \leq i \leq n-2)$. Next we assign the labels 3, 3, 2, 4, 4 to the vertices v_{n-1}, u_n, x_{n-1}, y_{n-1} and w_{n-1} respectively.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4t + 2$, $t \in \mathbb{N}$. Assign the label to the vertices u_i $(1 \leq i \leq n-3)$, v_i, x_i, w_i $(1 \leq i \leq n-3)$ and y_i $(1 \leq i \leq n-4)$ as in Case 2. Now we assign the labels 4, 3, 3 respectively to the vertices u_{n-2}, u_{n-1} and u_n. Next we assign the labels to the vertices 4, 3, 2, 2, 2, 1 to the vertices v_{n-2}, v_{n-1}, x_{n-2}, x_{n-1}, w_{n-2}, and w_{n-1} respectively. Finally we assign the labels 4, 2, 3 respectively to the vertices y_{n-3}, y_{n-2} and y_{n-1}.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4t + 3$, $t \in \mathbb{N}$. Assign the label to the vertices u_i, x_i, y_i, w_i $(1 \leq i \leq n-3)$ and v_i $(1 \leq i \leq n-4)$ as in Case 3. Now we assign the labels 4, 3, 3, 3, 1, 1, 3, 4, 3 respectively to the vertices u_{n-2}, u_{n-1}, u_n, x_{n-2}, x_{n-1}, y_{n-2}, y_{n-1}, w_{n-2} and w_{n-2}. Finally we assign the labels 2, 4, 3 to the vertices v_{n-3}, v_{n-2} and v_{n-1} respectively. □

Theorem 3.7 *The subdivision of ladder $S(L_n)$ is 4-total prime cordial.*

Proof Let $V(L_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\}$. Let y_i, w_i and x_i be the vertices which subdivide the edges u_iu_{i+1}, u_iv_i and v_iv_{i+1} respectively. Clearly $|V(S(L_n))| + |E(S(L_n))| = 11n - 6$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4t$, $t \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_t$ and $v_1, v_2, \cdots, v_t$.

Assign the label 2 to the vertices $u_{t+1}, u_{t+2}, \cdots, u_{2t}$ and $v_{t+1}, v_{t+2}, \cdots, v_{2t}$. Next we assign the label 3 to the vertices u_{2t+1}, u_{2t+2},..., u_{3t} and $v_{2t+1}, v_{2t+2}, \cdots, v_{3t}$ then we assign the label 1 to the vertices $u_{3t+1}, u_{3t+2}, \cdots, u_{n-1}$ and $v_{3t+1}, v_{3t+2}, \cdots, v_{n-1}$. Finally, we assign the labels 4 and 3 to the vertices u_n and v_n respectively. Next we consider the vertices x_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $x_1, x_2, \cdots, x_t$ and assign the label 2 to the vertices $x_{t+1}, x_{t+2}, \cdots, x_{2t}$. Now we assign the label 3 to the vertices $x_{2t+1}, x_{2t+2}, \cdots, x_{3t}$. Finally we assign the label 1 to the vertices $x_{3t+1}, x_{3t+2}, \cdots, x_n$. Now we consider the vertices y_i, w_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $y_1, y_2, \cdots, y_t$ and $w_1, w_2, \cdots, w_t$. Assign the label 2 to the vertices $y_{t+1}, y_{t+2}, \cdots, y_{2t-1}$ and $w_{t+1}, w_{t+2}, \cdots, w_{2t-1}$. Next we assign the label 3 to the vertices $y_{2t}, y_{2t+1}, \cdots, y_{3t-1}$ and $w_{2t}, w_{2t+1}, \cdots, w_{3t-1}$ then we assign the label 1 to the vertices $y_{3t}, y_{3t+1}, \cdots, y_{n-2}$ and w_{3t}, $w_{3t+1}, \cdots$, w_{n-2}, w_{n-1}. Finally, we assign the labels 2 vertex y_{n-1}.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4t+1$, $t \in \mathbb{N}$. As in Case 1, assign the label to the vertices u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-1)$, x_i $(1 \leq i \leq n)$, y_i $(1 \leq i \leq n-2)$ and w_i $(1 \leq i \leq n-2)$. Finally we assign the labels 2, 2, 4, 3 respectively to the vertices u_n, v_n, y_{n-1} and w_{n-1}.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4t+2$, $t \in \mathbb{N}$. As in Case 2, assign the label to the vertices u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-1)$, x_i $(1 \leq i \leq n-1)$, y_i $(1 \leq i \leq n-2)$ and w_i $(1 \leq i \leq n-2)$. Finally we assign the labels 4, 3, 2, 4, 3 to the vertices u_n, v_n, x_n, y_{n-1} and w_{n-1} respectively.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4t+3$, $t \in \mathbb{N}$. As in Case 3, assign the label to the vertices u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-1)$, x_i $(1 \leq i \leq n-1)$, y_i $(1 \leq i \leq n-2)$ and w_i $(1 \leq i \leq n-2)$. Finally we assign the labels 3, 4, 2, 3, 4 respectively to the vertices u_n, v_n, x_n, y_{n-1} and w_{n-1}. □

Theorem 3.8 *The subdivision of double triangular snake $S(DT_n)$ is 4-total prime cordial.*

Proof Let P_n be the path $u_1u_2\cdots u_n$. Let v_i, w_i be the vertex adjacent to u_iu_{i+1}. Let x_i, y_i, z_i, s_i and r_i be the vertex which subdivide the edges u_iu_{i+1}, u_iv_i, v_iu_{i+1}, u_iw_i and w_iu_{i+1} respectively. Clearly $|V(S(DT_n))| + |E(S(DT_n))| = 18n - 17$.

Case 1. $n \equiv 0 \pmod 4$, $n \geq 8$.

Let $n = 4t$, $t \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_t$ and assign the label 2 to the vertices $u_{t+1}, u_{t+2}, \cdots, u_{2t}$. Next we assign the label 3 to the vertices $u_{2t+1}, u_{2t+2}, \cdots, u_{3t}$ then we assign the label 1 to the vertices $u_{3t+1}, u_{3t+2}, \cdots, u_{n-1}$. Finally, we assign the label 3 to the vertex u_n. Now we consider the vertices v_i,w_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $v_1, v_2, \cdots, v_t$ and $w_1, w_2, \cdots, w_t$. Assign the label 2 to the vertices $v_{t+1}, v_{t+2}, \cdots, v_{2t-1}$ and $w_{t+1}, w_{t+2}, \cdots, w_{2t-1}$. Next we assign the label 3 to the vertices $v_{2t}, v_{2t+1}, \cdots, v_{3t-1}$ and $w_{2t}, w_{2t+1}, \cdots, w_{3t-1}$ then we assign the label 1 to the vertices $v_{3t}, y_{3t+1}, \cdots, v_{n-3}$ and $w_{3t}, w_{3t+1}, \cdots, w_{n-3}$. Finally we assign the labels 2, 4, 2, 4 respectively to the vertices v_{n-2}, v_{n-1}, w_{n-2} and w_{n-1}. Next we move to the vertices x_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $x_1, x_2, \cdots, x_t$ and assign the label 2 to the vertices $x_{t+1}, u_{t+2}, \cdots, x_{2t-1}$. Next we assign the label 3 to the vertices $x_{2t}, x_{2t+1}, \cdots, x_{3t-1}$ then we assign the label 1 to the vertices $x_{3t}, x_{3t+1}, \cdots, x_{n-2}$. Finally, we assign the label 3 to the vertex x_{n-1}. Now we consider the vertices y_i,s_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $y_1, y_2, \cdots, y_t$ and $s_1, s_2, \cdots, s_t$. Assign the label 2 to the vertices $y_{t+1}, y_{t+2}, \cdots, y_{2t}$ and $s_{t+1}, s_{t+2}, \cdots, s_{2t}$. Next we assign the label 3 to the vertices $y_{2t+1}, y_{2t+2}, \cdots, y_{3t-1}$ and $s_{2t+1}, s_{2t+2}, \cdots, s_{3t-1}$. Finally we assign the label 1 to the vertices $y_{3t}, y_{3t+1}, \cdots, y_{n-1}$ and $s_{3t}, s_{3t+1}, \cdots, s_{n-1}$. Next we move to the vertices z_i, r_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $z_1, z_2, \cdots, z_{t-1}$ and $r_1, r_2, \cdots, r_{t-1}$. Assign the label 2 to the vertices $z_t, z_{t+1}, \cdots, z_{2t-1}$ and $r_t, r_{t+1}, \cdots, r_{2t-1}$. Next we assign the label 3 to the vertices $z_{2t}, z_{2t+1}, \cdots, z_{3t-1}$ and $r_{2t}, r_{2t+1}, \cdots, r_{3t-1}$. Finally we assign the label 1 to the vertices $z_{3t}, z_{3t+1}, \cdots, z_{n-1}$ and $r_{3t}, r_{3t+1}, \cdots, r_{n-1}$. Clearly $t_f(1) = t_f(2) = t_f(3) = 18t-4$ and $t_f(4) = 18t-5$.

Case 2. $n \equiv 1 \pmod 4$, $n \geq 9$.

Let $n = 4t+1$, $t \in \mathbb{N}$. As in Case 1, assign the label to the vertices u_i $(1 \leq i \leq n-1)$, v_i, w_i, x_i, y_i, z_i, s_i, r_i $(1 \leq i \leq n-2)$. Finally we assign the labels 4, 4, 2, 3, 1, 4, 3, 2 respectively to the vertices u_n, v_{n-1}, w_{n-1}, x_{n-1}, y_{n-1}, z_{n-1}, s_{n-1} and r_{n-1}. Obviously $t_f(1) = 18t+1$ and $t_f(2) = t_f(3) = t_f(4) = 18t$.

Case 3. $n \equiv 2 \pmod 4$, $n \geq 10$.

Let $n = 4t+2$, $t \in \mathbb{N}$. Assign the label to the vertices u_i $(1 \leq i \leq n-2)$, v_i, w_i, x_i, y_i, z_i $(1 \leq i \leq n-3)$, s_i, r_i $(1 \leq i \leq n-2)$ by in case 1. Finally we assign the labels 2, 4, 3, 4, 3, 3, 1, 2, 3, 4, 2, 4, 1, 4 to the vertices u_{n-1}, u_n, v_{n-2}, v_{n-1}, w_{n-2}, w_{n-1}, x_{n-2}, x_{n-1}, y_{n-2}, y_{n-1}, z_{n-2}, z_{n-1}, s_{n-1} and r_{n-1} respectively. It is easy to verify that $t_f(1) = t_f(2) = t_f(3) = 18t+5$ and $t_f(4) = 18t+4$.

Case 4. $n \equiv 3 \pmod 4$, $n \geq 11$.

Let $n = 4t+3$, $t \in \mathbb{N}$. As in Case 3, assign the label to the vertices u_i $(1 \leq i \leq n-1)$, v_i, w_i, x_i, y_i, z_i, s_i, r_i $(1 \leq i \leq n-2)$. Finally we assign the labels 3, 3, 2, 2, 4, 3, 4, 1 respectively to the vertices u_n, v_{n-1}, w_{n-1}, x_{n-1}, y_{n-1}, z_{n-1}, s_{n-1} and r_{n-1}. Clearly $t_f(1) = t_f(2) = t_f(4) = 18t+9$ and $t_f(3) = 18t+10$.

Case 5. $t = 2, 3, 4, 5, 6, 7$.

A 4-total prime cordial labeling is given in Table 1.

n	2	3	4	5	6	7
u_1	3	4	4	4	4	4
u_2	4	2	2	4	4	4
u_3		3	3	2	2	2
u_4			3	3	3	2
u_5				1	1	3

u_6					2	1
u_7						1
v_1	4	4	4	4	4	4
v_2		3	3	2	2	4
v_3			4	3	3	2
v_4				3	3	3
v_5					4	3
v_6						1
w_1	2	4	4	4	4	4
w_2		1	2	2	2	4
w_3			1	3	3	2
w_4				1	1	3
w_5					3	3
w_6						1
x_1	1	2	4	4	4	4
x_2		3	2	2	2	2
x_3			3	3	3	2
x_4				3	3	3
x_5					4	3
x_6						1
y_1	3	4	4	4	4	4
y_2		3	3	2	2	4
y_3			1	3	3	2
y_4				1	1	3
y_5					3	3
y_6						1
z_1	4	2	2	4	4	4
z_2		3	3	2	2	2
z_3			4	3	3	2
z_4				1	1	3
z_5					4	1
z_6						1
s_1	3	4	4	4	4	4
s_2		1	2	2	2	4

s_3			1	3	3	2
s_4				1	1	3
s_5					3	3
s_6						1
r_1	2	2	2	4	4	4
r_2		1	3	2	2	2
r_3			1	3	3	2
r_4				1	1	3
r_5					2	3
r_6						1

Table 1

This completes the proof. □

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