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A Note on Fixed Point Theorem in Complex Valued Intuitionistic Fuzzy Metric Space

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Abstract: We show several common fixed point theorems for contraction condition satisfying certain requirements in complex valued intuitionistic fuzzy metric spaces in this study.

Key Words: Common fixed point, intuitionistic fuzzy set, complex valued, continuous t-norm.

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§1. Introduction

In 1965, Zadeh [12] proposed the concept of fuzzy sets. Fuzzy set theory is a useful tool for describing situations involving imprecise or ambiguous data. Fuzzy sets deal with situations like these by assigning a degree of belonging to a set to each object. Since then, it has become a burgeoning field of study in engineering, medicine, social science, graph theory, metric space theory, and complex analysis, among other fields. Kramosil and Michalek [6] introduced fuzzy metric spaces in a variety of ways in 1975. With the help of continuous t-norms, George and Veermani [4] improved the concept of fuzzy metric spaces in 1994.

Buckley [3] was the one who originally established the concept of fuzzy complex numbers and fuzzy complex analysis. 1987. Some authors were influenced by Buckley's work. Reexamination of fuzzy complex numbers continues. The year was 2002, and Fuzzy sets were extended to complicated fuzzy sets by Ramot et al. [8]. as though it were a blanket statement Ramot et al. claim that A membership function defines a sophisticated fuzzy set. function with a range that extends beyond [0, 1]the complicated plane's unit circle Singh was born in the year 2016. The concept of complex valued fuzzy was introduced by et al.[10]. Using complex valued continuous to create metric spaces t -norm as well as the concept of convergent convergence. in a complex valued fuzzy sequence, Cauchy sequence in complex valued fuzzy metric spaces.By introducing the concept of non-membership grade to fuzzy set theory, Atanassov [1] created a stir in 1983.In this paper, we generalise the results of Jeyaraman, Shakila [13]

In the complex valued intuitionistic fuzzy metric spaces, this work gives some common fixed point theorems for pairs of occasionally weakly compatible mappings satisfying various requirements.

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§2. Preliminaries

Definition 2.1 *A binary operation* $* : r_s(\cos\theta + i\sin\theta) \times r_s(\cos\theta + i\sin\theta) \to r_s(\cos\theta + i\sin\theta)$, *where* $r_s \in [0,1]$ *and a fix* $\theta \in \left[0, \frac{\pi}{2}\right]$, *is called complex valued continuous t-norm if it satisfies the followings:*

(1) $*$ *is associative and commutative;*
(2) $*$ *is continuous;*
(3) $a * e^{i\theta} = a, \forall a \in r_s(\cos\theta + i\sin\theta)$*;*
(4) $\mathrm{a} * \mathrm{b} \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d, \forall \mathrm{a,b,c,d} \in r_s(\cos\theta + i\sin\theta)$.

Definition 2.2 *A binary operation* $r_s(\cos\theta + i\sin\theta) \times r_s(\cos\theta + i\sin\theta) \to r_s(\cos\theta + i\sin\theta)$,*where* $r_s \in [0,1]$ *and a fix* $\theta \in \left[0, \frac{\pi}{2}\right]$, *is called complex valued continuous t-co norm if it satisfies the followings:*

(1) *is associative and commutative;*
(2) *is continuous;*
(3) $\mathrm{a} \diamond 0 = a, \forall a \in r_s(\cos\theta + i\sin\theta)$*;*
(4) $\mathrm{a} \diamond \mathrm{b} \leq \mathrm{c} \diamond \mathrm{d}$ *whenever* $a \leq c$ *and* $b \leq d, \forall \mathrm{a,b,c,d} \in r_s(\cos\theta + i\sin\theta\ )$*.*

Example 2.3 The following are examples for complex valued continuous t-norm.

(i) $a * b = \min\{a, b\}, \forall a, b \in r_s(\cos\theta + i\sin\theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$
(ii) $a * b = \max(a + b - (\cos\theta + i\sin\theta), 0)$, for all $a, b \in r_s(\cos\theta + i\sin\theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$.

Example 2.4 The following are examples for complex valued continuous t-conorm.

(i) $\mathrm{a} \diamond b = \max\{a, b\}, \forall a, b \in r_s(\cos\theta + i\sin\theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$;
(ii) $\mathrm{a} \diamond b = \min(a + b, 1)$, for all $a, b \in r_s(\cos\theta + i\sin\theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$.

Definition 2.5 *The 5-triplet* $(\ X, \mathrm{M}, \mathrm{N}, *, \diamond)$ *is said to be complex valued intuitionistic fuzzy metric space if* X *is an arbitrary non empty set,* $*$ *is a complex valued continuous* t*-norm,* $\diamond$ *is a complex valued continuous t-conorm and* $M, N : X \times X \times (0, \infty) \to r_s(\cos\theta + i\sin\theta)$ *are complex valued fuzzy sets, where* $r_s \in [0,1], r_s(\cos\theta + i\sin\theta)$ *are complex valued fuzzy sets, where* $r_s \in$ *and* $\theta \in \left[0, \frac{\pi}{2}\right]$, *satisfying the following conditions:*

For all $\mathrm{x,y,z} \in X; \mathrm{t,s} \in (0,\infty); r_s \in [0,1]$ *and* $\theta \in \left[0, \frac{\pi}{2}\right]$,
$(cf1)$ $M(a, b, p) + M(a, b, p) \leq (\cos\theta + i\sin\theta)$;
$(cf2)$ $M(a, b, p) > 0$;
$(cf3)$ $M(a, b, p) = (\cos\theta + i\sin\theta)$, for all $p \in (0, \infty)$ if and only if $\mathrm{a} = \mathrm{b}$;
$(cf4)$ $M(a, b, p) = M(b, a, p)$;
(cff) $M(a, b, p + s) \geq M(a, c, p) * M(c, b, s)$;
$(cf6)$ $M(a, b, p) : (0, \infty) \to r_s(\cos\theta + i\sin\theta)$ *is continuous;*
$(cf7)$ $N(a, b, p) < (\cos\theta + i\sin\theta)$;
$(cf8)$ $N(a, b, p) = 0$*, for all* $p \in (0, \infty)$ *if and only if* $\mathrm{a} = \mathrm{b}$*;*
$(cf9)$ $N(a, b, p) = N(b, a, p)$;
$(cf10)$ $N(a, b, p + s) \leq N(a, c, p) \diamond N(c, b, s)$;

$(cf11)$ *$N(a,b,p):(0,\infty)\to r_s(\cos\theta+i\sin\theta)$ is continuous.*

The pair (M,N) is called a complex valued intuitionistic fuzzy metric space. The functions $M(a,b,p)$ and $N(a,b,p)$ denotes the degree of nearness and non-nearness between a and b with respect to t. It is noted that if we take $\theta=0$, then complex valued intuitionistic fuzzy metric simply goes to real valued intuitionistic fuzzy metric.

§3. Main Results

Theorem 3.1 *Let (X, M, N, $*$, $\diamond$) be a complex valued intuitionistic fuzzy metric space with $\lim_{t\to\infty} M(a,b,p)=(\cos\theta+i\sin\theta)$ and $\lim_{t\to\infty} N(a,b,p)=0$, for all $a,b\in X, p>0$ and let A and B be self mappings on X. If there exists $d\in(0,1)$ such that*

$$M(Aa,Bb,dp)\geq M(a,b,p),\quad N(Aa,Bb,dp)\leq N(a,b,p)\ for\ all a,b\in X\ and dp>0, \tag{3.1}$$

then A and B have a unique common fixed point in X.

Proof Let $a_0\in X$ be an arbitrary point and we define the sequence $\{a_n\}$ by $a_{2n+1}=Aa_{2n}$ and $a_{2n+2}=Ba_{2n+1}; n=0,1,2,\cdots$. Now, for $d\in(0,1)$ and for all $p>0$, then from (3.1) we have

$$\begin{aligned} M\left(a_{2n+1},a_{2n+2},dp\right) &= M\left(Aa_{2n},Ba_{2n+1},dp\right)\\ &\geq M\left(a_{2n},a_{2n+1},p\right)\\ M\left(a_{2n},a_{2n+1},dp\right) &= M\left(Aa_{2n-1},Ba_{2n},dp\right)\\ &\geq M\left(a_{2n-1},a_{2n},p\right),\ \text{and}\\ N\left(a_{2n+1},a_{2n+2},dp\right) &= N\left(Aa_{2n},Ba_{2n+1},dp\right)\\ &\leq N\left(a_{2n},a_{2n+1},p\right),\\ N\left(a_{2n},a_{2n+1},dp\right) &= N\left(Aa_{2n-1},Ba_{2n},dp\right)\\ &\leq N\left(a_{2n-1},a_{2n},p\right). \end{aligned}$$

In general, we have

$$M\left(a_{n+1},a_{n+2},dp\right)\geq M\left(a_n,a_{n+1},p\right),\quad N\left(a_{n+1},a_{n+2},dp\right)\leq N\left(a_n,a_{n+1},p\right)$$

for for all $p>0$ and $d\in(0,1); n=0,1,2,\cdots$ but $\{a_n\}$ be a sequence in a complex valued intuitionistic fuzzy metric space (X, M, N, $*$, $\diamond$), with $\lim_{p\to\infty} M(a,b,p)=\cos\theta+\mathrm{i}\sin\theta$ and $\lim_{p\to\infty} N(a,b,p)=0, \forall a,b\in X$. If $\lim_{p\to 0} N(a,b,p)=0$, there exists $d\in(0,1)$ such that $M\left(a_{n+1},a_{n+2},dp\right)\geqslant M\left(a_n,a_{n+1},p\right)$ and $N\left(a_{n+1},a_{n+2},dp\right)\leqslant\left(a_n,a_{n+1}p\right)$, for all $p>0$, then $\{a_n\}$ is a cauchy sequence in X.Since X is Complete then there exist $V\in X$ such that $a_n\to v$ as $n\to\infty$ and $\{a_{2n}\}$ and $\langle a_{2n+1}\}$ are subsequences of the same point $v\in X$, i.e.

$$a_{2n}\to v, a_{2n+1}\to v, \text{as } n\to\infty.$$

Now from (3.1) we have,

$$\begin{aligned}
M(Av, v, dp) &= M\left(Av, v, \frac{dp}{2} + \frac{dp}{2}\right) \\
&\geqslant M\left(Au, a_{2n+2}\frac{dp}{2}\right) * M\left(a_{2n+2}, v, \frac{dp}{2}\right) \\
&= M\left(Au, Ba_{2n+1}, \frac{dp}{2}\right) * M\left(a_{2n+2}, v, \frac{dp}{2}\right) \\
&\geqslant M\left(v, a_{2n+1}, \frac{p}{2}\right) * M\left(a_{2n+2}, v, \frac{dp}{2}\right)
\end{aligned}$$

$$\begin{aligned}
N(Av, v, dp) &= N\left(Av, v, \frac{dp}{2} + \frac{dp}{2}\right) \\
&\leq N\left(Av, a_{2n+2}, \frac{dp}{2}\right) \diamond N\left(a_{2n+2}, v, \frac{dp}{2}\right) \\
&= N\left(Av, Ba_{2n+1}, \frac{dp}{2}\right) \diamond \mathrm{N}\left(q_{2n+2}, v, \frac{dp}{2}\right) \\
&\leq N\left(v, a_{2n+1}, \frac{p}{2}\right) \diamond \left(a_{2n+2}, v, \frac{dp}{2}\right)
\end{aligned}$$

On taking limit $n \to \infty$,

$$\begin{aligned}
M(Av, v, dp) &\geqslant (\cos\theta + i\sin\theta) * (\cos\theta + i\sin\theta) \\
&= \cos\theta + i\sin\theta
\end{aligned}$$

$$N(Av, v, dp) \quad \leqslant 0 \diamond 0 = 0.$$

So $Av = v$ again, and

$$\begin{aligned}
M(Av, v, dp) &= M\left(v,\ \mathrm{B}v, \frac{dp}{2} + \frac{dp}{2}\right) \\
&\geqslant M\left(v, a_{2n+1}, \frac{dp}{2}\right) * M\left(a_{2n+1},\ \mathrm{B}v, \frac{dp}{2}\right) \\
&= M\left(v, a_{2n+1}, \frac{dp}{2}\right) * M\left(Aa_{2n},\ \mathrm{B}v, \frac{dp}{2}\right) \\
&\geqslant M\left(v, a_{2n+1}, \frac{p}{2}\right) * M\left(a_{2n}, v, \frac{p}{2}\right) \\
N(Av, v, dp) &= N\left(v,\ \mathrm{B}v, \frac{dp}{2} + \frac{dp}{2}\right) \\
&\leq N\left(v, a_{2n+1}, \frac{dp}{2}\right) \diamond N\left(a_{2n+1},\ \mathrm{B}v, \frac{dp}{2}\right) \\
&= N\left(v, a_{2n+1}, \frac{dp}{2}\right) \diamond N\left(Aa_{2n},\ \mathrm{B}v, \frac{dp}{2}\right) \\
&\leq N\left(v, a_{2n+1}, \frac{p}{2}\right) \diamond N\left(a_{2n}, v, \frac{p}{2}\right)
\end{aligned}$$

On taking limit $n \to \infty$,

$$\begin{aligned} M(Av, v, dp) &\geqslant (\cos\theta + i\sin\theta) * (\cos\theta + i\sin\theta) \\ &= \cos\theta + i\sin\theta \end{aligned}$$

$$N(Av, v, dp) \quad \leqslant 0 \diamond 0 = 0.$$

So $Bv = v$, and $Av = Bv = v$. Hence v is a common fixed point of A and B. For uniqueness let c be any another fixed point of A and B. Now from (3.1),

$$M(v, c, dp) = M(Av, Bc, dp) \geq M(v, c, p), \quad N(v, c, dp) = N(Av, Bc, dp) \leq N(v, c, p),$$

we know that when $(X, \mathrm{M}, \mathrm{N}, *, \diamond)$ be a complex valued intuitionistic fuzzy metric space such that $\lim_{p\to\infty} M(a, b, p) = \cos\theta + \mathrm{i}\sin\theta$ and $\lim_{p\to\infty} N(a, b, p) = 0, \forall a, b \in X$. If $M(a, b, dp) \geq M(a, b, p)$ and $N(a, b, dp) \leq N(a, b, p)$ for some $0 < d < 1$, for all $a, b \in X, p \in (0, \infty)$, then $a = b$. Hence $v = c$. □

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