A Note on Fixed Point Theorem in
Complex Valued Intuitionistic Fuzzy Metric Space

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Abstract: We show several common fixed point theorems for contraction condition satisfying certain requirements in complex valued intuitionistic fuzzy metric spaces in this study.

Key Words: Common fixed point, intuitionistic fuzzy set, complex valued, continuous $t$-norm.


§1. Introduction

In 1965, Zadeh [12] proposed the concept of fuzzy sets. Fuzzy set theory is a useful tool for describing situations involving imprecise or ambiguous data. Fuzzy sets deal with situations like these by assigning a degree of belonging to a set to each object. Since then, it has become a burgeoning field of study in engineering, medicine, social science, graph theory, metric space theory, and complex analysis, among other fields. Kramosil and Michalek [6] introduced fuzzy metric spaces in a variety of ways in 1975. With the help of continuous $t$-norms, George and Veermani [4] improved the concept of fuzzy metric spaces in 1994.

Buckley [3] was the one who originally established the concept of fuzzy complex numbers and fuzzy complex analysis. 1987. Some authors were influenced by Buckley’s work. Re-examination of fuzzy complex numbers continues. The year was 2002, and Fuzzy sets were extended to complicated fuzzy sets by Ramot et al. [8], as though it were a blanket statement Ramot et al. claim that $A$ membership function defines a sophisticated fuzzy set. function with a range that extends beyond $[0, 1]$the complicated plane’s unit circle Singh was born in the year 2016. The concept of complex valued fuzzy was introduced by et al.[10]. Using complex valued continuous to create metric spaces $t$-norm as well as the concept of convergent convergence. in a complex valued fuzzy sequence, Cauchy sequence in complex valued fuzzy metric spaces. By introducing the concept of non-membership grade to fuzzy set theory, Atanassov [1] created a stir in 1983.In this paper, we generalise the results of Jeyaraman, Shakila [13]

In the complex valued intuitionistic fuzzy metric spaces, this work gives some common fixed point theorems for pairs of occasionally weakly compatible mappings satisfying various requirements.

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§2. Preliminaries

**Definition 2.1** A binary operation $*$ : $r_s(\cos \theta + i \sin \theta) \times r_s(\cos \theta + i \sin \theta) \to r_s(\cos \theta + i \sin \theta)$, where $r_s \in [0, 1]$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$, is called complex valued continuous t-norm if it satisfies the followings:

1. $*$ is associative and commutative;
2. $*$ is continuous;
3. $a \ast e^{\theta} = a, \forall a \in r_s(\cos \theta + i \sin \theta)$;
4. $a \ast b \leq c \ast d$ whenever $a \leq c$ and $b \leq d, \forall a, b, c, d \in r_s(\cos \theta + i \sin \theta)$.

**Definition 2.2** A binary operation $r_s(\cos \theta + i \sin \theta) \times r_s(\cos \theta + i \sin \theta) \to r_s(\cos \theta + i \sin \theta)$, where $r_s \in [0, 1]$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$, is called complex valued continuous t-co norm if it satisfies the followings:

1. is associative and commutative;
2. is continuous;
3. $a \circ 0 = a, \forall a \in r_s(\cos \theta + i \sin \theta)$;
4. $a \circ b \leq c \circ d$ whenever $a \leq c$ and $b \leq d, \forall a, b, c, d \in r_s(\cos \theta + i \sin \theta)$.

**Example 2.3** The following are examples for complex valued continuous t-norm.

1. $a \ast b = \min\{a, b\}, \forall a, b \in r_s(\cos \theta + i \sin \theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$;
2. $a \ast b = \max(a + b - (\cos \theta + i \sin \theta), 0)$, for all $a, b \in r_s(\cos \theta + i \sin \theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$.

**Example 2.4** The following are examples for complex valued continuous t-conorm.

1. $a \circ b = \max\{a, b\}, \forall a, b \in r_s(\cos \theta + i \sin \theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right];$
2. $a \circ b = (a + b, 1)$, for all $a, b \in r_s(\cos \theta + i \sin \theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$.

**Definition 2.5** The 5-triplet $(X, M, N, *, \circ)$ is said to be complex valued intuitionistic fuzzy metric space if $X$ is an arbitrary non empty set, $*$ is a complex valued continuous t-norm, $\circ$ is a complex valued continuous t-conorm and $M, N : X \times X \times (0, \infty) \to r_s(\cos \theta + i \sin \theta)$ are complex valued fuzzy sets, where $r_s \in [0, 1], r_s(\cos \theta + i \sin \theta)$ are complex valued fuzzy sets, where $r_s \in [0, 1]$ and $\theta \in \left[0, \frac{\pi}{2}\right]$, satisfying the following conditions:

For all $x, y, z \in X; t, s \in (0, \infty); r_s \in [0, 1]$ and $\theta \in \left[0, \frac{\pi}{2}\right]$,

(cf1) $M(a, b, p) + M(a, b, p) \leq M(a, b, p)$;
(cf2) $M(a, b, p) > 0$;
(cf3) $M(a, b, p) = (\cos \theta + i \sin \theta)$, for all $p \in (0, \infty)$ if and only if $a = b$;
(cf4) $M(a, b, p) = M(b, a, p)$;
(cf5) $M(a, b, p + s) \geq M(a, c, p) * M(c, b, s)$;
(cf6) $M(a, b, p) : (0, \infty) \to r_s(\cos \theta + i \sin \theta)$ is continuous;
(cf7) $N(a, b, p) < (\cos \theta + i \sin \theta)$;
(cf8) $N(a, b, p) = 0$, for all $p \in (0, \infty)$ if and only if $a = b$;
(cf9) $N(a, b, p) = N(b, a, p)$;
(cf10) $N(a, b, p + s) \leq N(a, c, p) \circ N(c, b, s)$.
The pair \((M, N)\) is called a complex valued intuitionistic fuzzy metric space. The functions \(M(a, b, p)\) and \(N(a, b, p)\) denotes the degree of nearness and non-nearness between \(a\) and \(b\) with respect to \(t\). It is noted that if we take \(\theta = 0\), then complex valued intuitionistic fuzzy metric simply goes to real valued intuitionistic fuzzy metric.

§3. Main Results

**Theorem 3.1** Let \((X, M, N, *, \circ)\) be a complex valued intuitionistic fuzzy metric space with \(\lim_{t \to \infty} M(a, b, p) = (\cos \theta + i \sin \theta)\) and \(\lim_{t \to \infty} N(a, b, p) = 0\), for all \(a, b \in X, p > 0\) and let \(A\) and \(B\) be self mappings on \(X\). If there exists \(d \in (0,1)\) such that

\[
M(Aa, Bb, dp) \geq M(a, b, p), \quad N(Aa, Bb, dp) \leq N(a, b, p) \quad \text{for all} \quad a, b \in X \text{ and } p > 0, \quad (3.1)
\]

then \(A\) and \(B\) have a unique common fixed point in \(X\).

**Proof** Let \(a_0 \in X\) be an arbitrary point and we define the sequence \(\{a_n\}\) by \(a_{2n+1} = Aa_{2n}\) and \(a_{2n+2} = B a_{2n+1}; n = 0, 1, 2, \cdots\). Now, for \(d \in (0,1)\) and for all \(p > 0\), then from (3.1) we have

\[
M(a_{2n+1}, a_{2n+2}, dp) = M(Aa_{2n}, B a_{2n+1}, dp) \geq M(a_{2n}, a_{2n+1}, p)
\]

\[
M(a_{2n}, a_{2n+1}, dp) = M(Aa_{2n-1}, B a_{2n}, dp) \geq M(a_{2n-1}, a_{2n}, p), \quad \text{and}
\]

\[
N(a_{2n+1}, a_{2n+2}, dp) = N(Aa_{2n}, B a_{2n+1}, dp) \leq N(a_{2n}, a_{2n+1}, p)
\]

\[
N(a_{2n}, a_{2n+1}, dp) = N(Aa_{2n-1}, B a_{2n}, dp) \leq N(a_{2n-1}, a_{2n}, p).
\]

In general, we have

\[
M(a_{n+1}, a_{n+2}, dp) \geq M(a_n, a_{n+1}, p), \quad N(a_{n+1}, a_{n+2}, dp) \leq N(a_n, a_{n+1}, p)
\]

for for all \(p > 0\) and \(d \in (0,1); n = 0, 1, 2, \cdots\) but \(\{a_n\}\) be a sequence in a complex valued intuitionistic fuzzy metric space \((X, M, N, *, \circ)\), with \(\lim_{p \to \infty} M(a, b, p) = \cos \theta + i \sin \theta\) and \(\lim_{p \to \infty} N(a, b, p) = 0, \forall a, b \in X\). If \(\lim_{p \to 0} N(a, b, p) = 0\), there exists \(d \in (0,1)\) such that \(M(a_{n+1}, a_{n+2}, dp) \geq M(a_n, a_{n+1}, p)\) and \(N(a_{n+1}, a_{n+2}, dp) \leq (a_{n}, a_{n+1}p), \) for all \(p > 0\), then \(\{a_n\}\) is a cauchy sequence in \(X\). Since \(X\) is Complete then there exist \(V \in X\) such that \(a_n \to v\) as \(n \to \infty\) and \(\{a_{2n}\}\) and \(\{a_{2n+1}\}\) are subsequences of the same point \(v \in X\), i.e.

\[
a_{2n} \to v, a_{2n+1} \to v, \text{as } n \to \infty.
\]
Now from (3.1) we have,

\[ M(Av, v, dp) = M(Au, v, \frac{dp}{2}) \]

\[ \geq M(Au, a_{2n+2}, \frac{dp}{2}) \ast M(a_{2n+2}, v, \frac{dp}{2}) \]

\[ = M(Au, Ba_{2n+1}, \frac{dp}{2}) \ast M(a_{2n+2}, v, \frac{dp}{2}) \]

\[ \geq M(v, a_{2n+1}, \frac{p}{2}) \ast M(a_{2n+2}, v, \frac{dp}{2}) \]

\[ N(Av, v, dp) = N(Av, v, \frac{dp}{2}) \]

\[ \leq N(Av, a_{2n+2}, \frac{dp}{2}) \circ N(a_{2n+2}, v, \frac{dp}{2}) \]

\[ = N(Av, Ba_{2n+1}, \frac{dp}{2}) \circ N(a_{2n+2}, v, \frac{dp}{2}) \]

\[ \leq N(v, a_{2n+1}, \frac{p}{2}) \circ (a_{2n+2}, v, \frac{dp}{2}) \]

On taking limit \( n \to \infty \),

\[ M(Av, v, dp) \geq (\cos \theta + i \sin \theta) \ast (\cos \theta + i \sin \theta) \]

\[ = \cos \theta + i \sin \theta \]

\[ N(Av, v, dp) \leq 0 \circ 0 = 0. \]

So \( Av = v \) again, and

\[ M(Av, v, dp) = M(v, Bv, \frac{dp}{2}) \]

\[ \geq M(v, a_{2n+1}, \frac{dp}{2}) \ast M(a_{2n+1}, Bv, \frac{dp}{2}) \]

\[ = M(v, a_{2n+1}, \frac{dp}{2}) \ast M(Aa_{2n}, Bv, \frac{dp}{2}) \]

\[ \geq M(v, a_{2n+1}, \frac{p}{2}) \ast M(a_{2n}, v, \frac{p}{2}) \]

\[ N(Av, v, dp) = N(v, Bv, \frac{dp}{2}) \]

\[ \leq N(v, a_{2n+1}, \frac{dp}{2}) \circ N(a_{2n+1}, Bv, \frac{dp}{2}) \]

\[ = N(v, a_{2n+1}, \frac{dp}{2}) \circ N(Aa_{2n}, Bv, \frac{dp}{2}) \]

\[ \leq N(v, a_{2n+1}, \frac{p}{2}) \circ N(a_{2n}, v, \frac{p}{2}) \]
On taking limit $n \to \infty$,

$$M(Av, v, dp) \geq (\cos \theta + i \sin \theta) * (\cos \theta + i \sin \theta) \leq \cos \theta + i \sin \theta$$

$$N(Av, v, dp) \leq 0 \circ 0 = 0.$$

So $Bv = v$, and $Av = Bv = v$. Hence $v$ is a common fixed point of $A$ and $B$. For uniqueness let $c$ be any another fixed point of $A$ and $B$. Now from (3.1),

$$M(v, c, dp) = M(Av, Bc, dp) \geq M(v, c, p), \quad N(v, c, dp) = N(Av, Bc, dp) \leq N(v, c, p),$$

we know that when $(X, M, N, *, \circ)$ be a complex valued intuitionistic fuzzy metric space such that $\lim_{p \to \infty} M(a, b, p) = \cos \theta + i \sin \theta$ and $\lim_{p \to \infty} N(a, b, p) = 0, \forall a, b \in X$. If $M(a, b, dp) \geq M(a, b, p)$ and $N(a, b, dp) \leq N(a, b, p)$ for some $0 < d < 1$, for all $a, b \in X, p \in (0, \infty)$, then $a = b$. Hence $v = c$. □

References