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Average Lower Domination Number for Some Middle Graphs

Derya Dogan

(Department of Mathematics, Faculty of Science, Ege University, 35100, Bornova-IZMIR, TURKEY)

E-mail: derya.dogan@ege.edu.tr

Abstract: Communication network is modeled as a simple, undirected, connected and unweighted graph G. Many graph theoretical parameters can be used to describe the stability and reliability of communication networks. If we consider a graph as modeling a network, the average domination number of a graph is one of the parameters for graph vulnerability. In this paper, we consider the average domination number for middle graphs of some well-known graphs, in particular we consider the middle graphs because these graphs are between total graphs and line graphs. In real life problems, every edge corresponds cost, so middle graphs make sense on this situation.

Key Words: Domination, Average lower domination number, middle graphs.

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§1. Introduction

The *line graph* $L(G)$ of a graph G is the graph whose vertex set is the edge set $\mathrm{E}(G)$ of G, with two vertices of $L(G)$ being adjacent if and only if the corresponding edges in G have a vertex in common. The *middle graph* $M(G)$ is the graph obtained from G inserting a new vertex into every edge of G and by joining edges those pairs of these new vertices which lie on adjacent edges of G. Another important graph is the total graph. The *total graph* $T(G)$ is the graph whose vertex set is the union of the vertex set $V(G)$ and the edge set $E(G)$ of G, with two vertices of $T(G)$ being adjacent if and only if the corresponding elements of G are adjacent or incident. There have been lots of research on various properties of line graphs, middle graphs and total graphs of graphs.

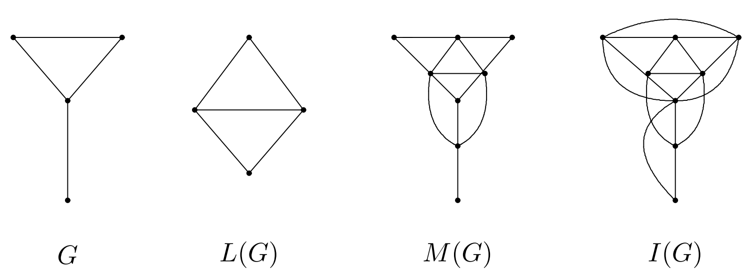

Fig.1

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The stability of communication network is composed of processing nodes and communication links, is the prime importance of network designers. Graph theoretical parameters can be used to describe the stability and reliability of communication networks. If we consider a graph as modeling a network, the domination number of a graph is one of the parameters for graph vulnerability.

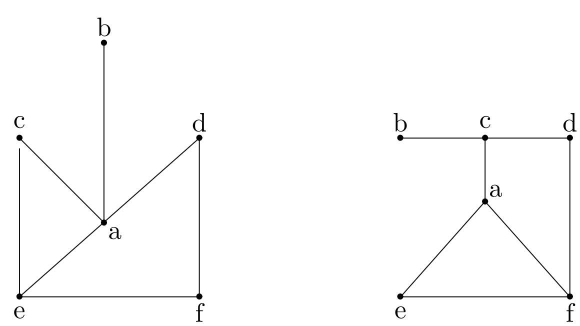
In this paper, we considered average domination number of a graph instead of domination number of a graph. Let's see what difference between these two parameters is and why we consider average domination number instead of domination number.

A vertex v in a graph G said to dominate itself and each of its neighbors, that is, v dominates the vertices in its closed neighborhood $N[v]$. A set S of vertices of G is a dominating set of G if every vertex of G is dominated by at least one vertex of S. Equivalently, a set S of vertices of G is dominating set if every vertex in $V(G) - S$ is adjacent to at least one vertex in S. The minimum cardinality among the dominating sets of G is called the domination number of G and is denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is then referred to as a minimum dominating set.

Let $G = (V, E)$ be a graph, the domination number $\gamma_v(G)$ of G relative to v is the minimum cardinality of a dominating set in G that contains v. The average domination number of G, $\gamma_{av}(G)$, can be written as:

$$\frac{1}{|V(G)|} \sum_{v \epsilon V(G)} \gamma_v(G).$$

Let H and G be a graphs which are have same order and size as below:



Graph G Graph H

Fig.2

It can easily seen that $\kappa(G) = \kappa(H) = 1$, $\beta(G) = \beta(H) = 3$, $\alpha(G) = \alpha(H) = 3$ and also $\gamma(G) = \gamma(H) = 2$. In this case, *how can we decide which graph is more reliable and how to find the average domination number of these graphs?*

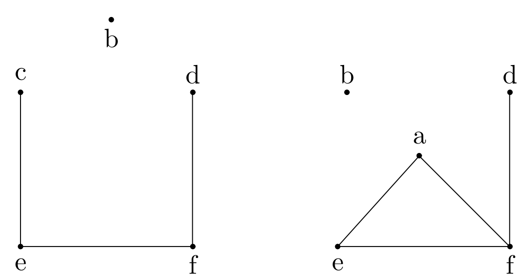
v		γ_v
a	$\{a, f\}$	2
b	$\{b, e, d\}$	3
c	$\{c, a, f\}$	3
d	$\{d, a\}$	2
e	$\{e, a\}$	2
f	$\{f, a\}$	2

$\gamma_{av}(G) = 14/6 = 2.33$

v		γ_v
a	$\{a, c\}$	2
b	$\{b, f\}$	2
c	$\{c, a, \}$	2
d	$\{d, a, b\}$	3
e	$\{e, c\}$	2
f	$\{f, b\}$	2

$\gamma_{av}(H) = 13/6 = 2.16$

So, $\gamma_{av}(H) < \gamma_{av}(G)$. Then we can say graph H is more reliable than graph G. If we consider the graphs $G-\{a\}$ and $H-\{c\}$, then we can see each graph has one isolated vertex and $G-\{a\}$ has P_4 but $H-\{c\}$ contains cycle in it. This means $H-\{c\}$ is more reliable than $G-\{a\}$.



Graph $G-\{a\}$ Graph $H-\{c\}$

Fig.3

Henning introduced average domination and independent domination numbers, studied trees for which these two parameters are equal. Our goal is to study the average domination number for the middle graphs of some well-known graphs.

§2. Average Domination Numbers for Middle Graphs of Some Well-Known Graphs

Theorem 2.1 *Let $M(K_{1,n})$ be the middle graph of $K_{1,n}$. Then*

$$\gamma_{av}(M(K_{1,n})) = \frac{2n^2+n+1}{2n+1}.$$

Proof The number of vertices of the graph $K_{1,n}$ and $M(K_{1,n})$ are $n+1$ and $2n+1$, respectively. Let say $M(K_{1,n}) = G$. We consider the vertex-set of graph $G = V_1(G) \cup V_2(G) \cup V_3(G)$ where,

$V_1(G)$: The set contains center vertex which has degree n.

$V_2(G)$: The set contains n vertices whose degree is 1.

$V_3(G)$: The set contains n new vertices with degree of $(n+1)$ which are obtained by definition of middle graph.

Then it is easily calculated that the average domination number of graph G. We consider three cases.

Case 1 Let v be the vertex of the $V_1(G)$. The vertex v is the center vertex which have the degree n in $K_{1,n}$. v dominates n vertices which are new vertices of $M(K_{1,n})$. In order to dominate vertices of $V_2(G)$, we have to put these vertices to dominating set including vertex v. Consequently, for the center vertex v, $\gamma_v(G) = n+1$.

Case 2 Let $v \in V_2(G)$, then degree of v is 1. Since the graph G have n vertices as v, then the cardinality of dominating set including vertex v is n, i.e., $\gamma_v(G) = n$.

Case 3 Let $v \in V_3(G)$. It is similar to that of Case 2. Thus $\gamma_v(G) = n$.

Consequently, we get that

$$\sum_{v\epsilon V(G)} \gamma_v(G) = \sum_{v\epsilon V_1(G)} \gamma_v(G) + \sum_{v\epsilon V_2(G)} \gamma_v(G) + \sum_{v\epsilon V_3(G)} \gamma_v(G)$$

$$\begin{aligned}\gamma_{av}(G) &= \frac{1}{|V(G)|}\Big(\sum_{v\epsilon V_1(G)} \gamma_v(G) + \sum_{v\epsilon V_2(G)} \gamma_v(G) + \sum_{v\epsilon V_3(G)} \gamma_v(G)\Big) \\ &= \frac{1}{2n+1}\left((n+1) + n^2 + n^2\right) = \frac{2n^2+n+1}{2n+1}.\end{aligned}$$ □

Observation 1 *The domination number of $M(P_n)$ is $\left\lceil \frac{n}{2} \right\rceil$.*

Proof We have to take $\left\lceil \frac{n}{2} \right\rceil$ vertices which have degree 4 in our dominating set to dominate n vertices each of which have degree 2. It is easy to see that $\gamma(M(P_n)) = \left\lceil \frac{n}{2} \right\rceil$. □

Observation 2 *If $n = 2k$, then the dominating set is unique.*

Proof The proof is clear. □

Theorem 2.2 *Let $M(P_n)$ be the middle graph of P_n. Then*

$$\gamma_{av}(M(P_n)) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil + \dfrac{\left\lfloor \frac{2n-1}{4} \right\rfloor}{2n-1}, & n \text{ is odd}; \\ \\ \dfrac{n^2+n-1}{2n-1}, & n \text{ is even}. \end{cases}$$

Proof Notice that $|V(P_n)| = n$ and $|V(M(P_n))| = 2n-1$. Let $M(P_n) = G$ and $v_1, v_2, v_3, \cdots, v_{2n-1}$ be the vertices of G. We need to consider two cases which are even and odd order of P_n.

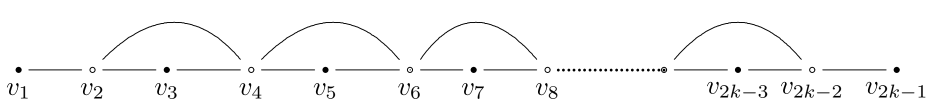


Fig.4

Case 1 If n is odd, then by Observation 1 $\gamma(M(P_n)) = \left\lceil \frac{n}{2} \right\rceil$ and we can find several dominating set that gives us domination number of these vertices (The set is not unique). For the rest of vertices. The cardinality of domination sets which are including v_i such that $i \equiv 3(mod\,4)$ is $\left\lceil \frac{n}{2} \right\rceil + 1$ and the number of these vertices is $\left\lfloor \frac{2n-1}{4} \right\rfloor$ in $M(P_n)$, where $k \geq 1$. For the rest of vertices, domination number is $\left\lceil \frac{n}{2} \right\rceil$. Then,

$$\begin{aligned}\gamma_{av}(M(P_n)) &= \frac{\left((2n-1) - \left\lfloor \frac{2n-1}{4} \right\rfloor\right)\left(\left\lceil \frac{n}{2} \right\rceil\right) + \left(\left\lfloor \frac{2n-1}{4} \right\rfloor\right)\left(\left\lceil \frac{n}{2} \right\rceil + 1\right)}{2n-1} \\ &= \left\lceil \frac{n}{2} \right\rceil + \frac{\left\lfloor \frac{2n-1}{4} \right\rfloor}{2n-1}.\end{aligned}$$

Case 2 If n is even, then by Observation 2 dominating set is unique and $\gamma(M(P_n)) = \frac{n}{2}$. It is clear that for the number of $\frac{n}{2}$ vertices we have the same average domination number. For the rest of vertices the dominating number increases 1 unit, so $\gamma_v = \frac{n}{2} + 1$ for the number of $\frac{3n-2}{2}$ vertices. Then,

$$\gamma_{av}(M(P_n)) = \frac{\frac{n}{2} \times \frac{n}{2} + \frac{3n-2}{2}\left(\frac{n}{2}+1\right)}{2n-1} = \frac{n^2+n-1}{2n-1}.$$

So,

$$\gamma_{av}(M(P_n)) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil + \frac{\left\lfloor \frac{2n-1}{4} \right\rfloor}{2n-1}, & n \text{ is odd,} \\ \frac{n^2+n-1}{2n-1}, & n \text{ is even.} \end{cases}$$

□

Theorem 2.3 *Let $M(C_n)$ be the middle graph of C_n. Then,*

$$\gamma_{av}(M(C_n)) = \frac{n+1}{2}.$$

Proof Notice that $|V(C_n)| = n$, $|V(M(C_n))| = 2n$. Let $M(C_n) = G$. We consider two cases dependent on the parity of $|G|$ following.

Case 1 n is odd.

In this case, we consider two cases, i.e.,$v \in V(C_n)$ and $v \in (V(M(C_n)) \setminus V(C_n)$. First, let $v \in V(C_n)$. Then we want to find γ_v, v dominates 2 new vertices in $M(C_n)$ and itself. The remain vertices which are not dominated gives us P_{n-1}, the domination number of P_{n-1} is $\gamma(M(P_{n-1})) = \frac{n-1}{2}$ by Observation 2. So, $\gamma_v = (1 + \frac{n-1}{2})$ and also we have n vertices as v. If $v \in (V(M(C_n)) \setminus V(C_n)$. Then we want to find γ_v, v dominates 2 new vertices in $M(C_n)$, 2 vertices in C_n and itself. The remain vertices which are not dominated gives us P_{n-2}, the domination number of P_{n-2} is $\gamma(M(P_{n-2})) = \left\lceil \frac{n-2}{2} \right\rceil$ by Observation 1. So, $\gamma_v = (1 + \left\lceil \frac{n-2}{2} \right\rceil)$ and also we have n vertices as v.

By these two subcases,

$$\begin{aligned}\gamma_{av}(M(C_n)) &= \frac{\left(1+(\frac{n-1}{2})\right)n+\left(\left\lceil\frac{n-2}{2}\right\rceil+1\right)n}{2n}\\ &= \left\lceil\frac{n}{2}\right\rceil=\frac{n+1}{2},\end{aligned}$$

where n is odd.

Case 2 n is even.

In this case, we also consider two cases,i.e., $v \in V(C_n)$ and $v \in (V(M(C_n)) \setminus V(C_n)$. First, let $v \in V(C_n)$. Then we want to find γ_v, v dominates 2 new vertices in $M(C_n)$ and itself. The remain vertices which are not dominated gives us P_{n-1}, the domination number of P_{n-1} is $\gamma(M(P_{n-1})) = \left\lceil\frac{n-1}{2}\right\rceil$ by Observation 1. So, $\gamma_v = (1+\left\lceil\frac{n-1}{2}\right\rceil)$ and also we have n vertices as v.

Now if $v \in (V(M(C_n)) \setminus V(C_n)$. Then we want to find γ_v, v dominates 2 new vertices in $M(C_n)$, 2 vertices in C_n and itself. The remain vertices which are not dominated gives us P_{n-2}, the domination number of P_{n-2} is $\gamma(M(P_{n-2})) = \frac{n-2}{2}$ by Observation 1. So, $\gamma_v = (1+\frac{n-2}{2})$ and also we have n vertices as v.

By these two subcases,

$$\gamma_{av}(M(C_n)) = \frac{(1+\left\lceil\frac{n-1}{2}\right\rceil)n+(1+\frac{n-2}{2})n}{2n}=\frac{n+1}{2},$$

where n is even. So,

$$\gamma_{av}(M(C_n)) = \frac{n+1}{2}. \qquad \square$$

Theorem 2.4 *Let $M(W_{1,n})$ be the middle graph of $W_{1,n}$. Then,*

$$\gamma_{av}(M(W_{1,n})) = \begin{cases} \dfrac{\left\lceil\frac{n}{2}\right\rceil(3n+3)}{3n+1}, & n \text{ is odd,}\\[2ex] \dfrac{n+2}{2}, & n \text{ is even.}\end{cases}$$

Proof The numbers of vertices of the graph $W_{1,n}$ and $M(W_{1,n})$ are $n+1$ and $3n+1$, respectively. Let $M(W_{1,n}) = G$. We consider the vertex-set of graph $G = V_1(G) \cup V_2(G) \cup V_3(G) \cup V_4(G)$ where,

$V_1(G)$: The set contains center vertex with degree of n of the graph G.

$V_2(G)$: The set contains n vertices whose degree is 3.

$V_3(G)$: The set contains n vertices whose degree is 6.

$V_4(G)$: The set contains n new vertices with degree of $n+3$ which are obtained by definition of middle graph.

Then it is easily calculated that the average domination number of graph G. Now, we have cases and also subcases.

Case 1 Let n be odd.

(i) Let $v \in V_1(G)$. The vertex v dominates vertices of $V_2(G)$. In order to dominate rest of vertices we have to put $\left\lceil \frac{n}{2} \right\rceil$ vertices in our dominating set which is $\gamma(M(C_n))$ by the proof of Case 1 for C_n. Consequently, $\gamma_v(G) = \left\lceil \frac{n+2}{2} \right\rceil$.

(ii) Let $v \in V_2(G)$. The vertex v dominates 3 vertices which are new vertices in $M(W_{1,n})$ and itself. In order to dominate K_{n+1} that is obtain from vertex of $V_1(G)$ and new vertices of $M(W_{1,n})$ which are obtain from incident edges of $V_1(G)$ in $W_{1,n}$, we need one vertex. Then to dominate the rest of vertices we have to put $\left\lceil \frac{n-2}{2} \right\rceil = \left\lceil \frac{n}{2} \right\rceil - 1$ vertices in our dominating set which is the $\gamma(M(P_{n-2}))$ by Observation 1. Hence,

$$\gamma_v(G) = \left\lceil \frac{n}{2} \right\rceil + 1.$$

(iii) Let $v \in V_3(G)$. The vertex v dominates 4 vertices which are new vertices in $M(W_{1,n})$, 2 of them outside and the other 2 inside of wheel, 2 vertices of $W_{1,n}$ and itself. In order to dominate K_{n+1}, we need one vertex which is belong to K_{n+1}, say u. u dominates 3 other vertices which are not element of $V(K_{n+1})$. So, for the rest of vertices we have to put $\frac{n-3}{2}$ vertices in our dominating set which is the $\gamma(M(P_{n-3}))$ by Observation 2. Hence,$\gamma_v(G) = 1 + 1 + \frac{n-3}{2} = \frac{n+1}{2} = \left\lceil \frac{n}{2} \right\rceil$.

(iv) Let $v \in V_4(G)$. The vertex v dominates K_{n+1}, 3 additional vertices(which are in outside of wheel) and itself. For other vertices, we have to put $\frac{n-1}{2}$ vertices in our dominating set which is the $\gamma(M(P_{n-1}))$ by Observation 2. Hence, $\gamma_v(G) = \frac{n-1}{2} + 1 = \frac{n+1}{2} = \left\lceil \frac{n}{2} \right\rceil$.

By these four subcases, we know that

$$\begin{aligned}
\sum_{v \epsilon V(G)} \gamma_v(G) &= \sum_{v \epsilon V_1(G)} \gamma_v(G) + \sum_{v \epsilon V_2(G)} \gamma_v(G) + \sum_{v \epsilon V_3(G)} \gamma_v(G) + \sum_{v \epsilon V_4(G)} \gamma_v(G) \\
\gamma_{av}(G) &= \frac{1}{|V(G)|}\Big(\sum_{v \epsilon V_1(G)} \gamma_v(G) + \sum_{v \epsilon V_2(G)} \gamma_v(G) + \sum_{v \epsilon V_3(G)} \gamma_v(G) + \sum_{v \epsilon V_4(G)} \gamma_v(G)\Big) \\
&= \frac{\left\lceil \frac{n+2}{2} \right\rceil + \left(\left\lceil \frac{n}{2} \right\rceil + 1 \right) n + \left\lceil \frac{n}{2} \right\rceil n + \left\lceil \frac{n}{2} \right\rceil n}{3n+1} = \frac{(3n+3)\left\lceil \frac{n}{2} \right\rceil}{3n+1}.
\end{aligned}$$

Case 2 Let n be even.

(i) Let $v \in V_1(G)$. The vertex v dominates vertices of $V_2(G)$. In order to dominate rest of vertices we have to put $\frac{n}{2}$ vertices in our dominating set which is the $\gamma(M(C_n))$ by the proof of Case 2 for C_n. Consequently, $\gamma_v(G) = \frac{n}{2} + 1$.

(ii) Let $v \in V_2(G)$. The vertex v dominates 3 vertices which are new vertices in $M(W_{1,n})$ and itself. In order to dominate K_{n+1}, we need to put one vertex in our dominating set which is belong to K_{n+1} and also this vertex dominates to more outside vertices. Then to dominate the rest of vertices we have to put $\frac{n-2}{2}$ vertices in our dominating set which is the $\gamma(M(P_{n-2}))$ by Observation 2. Hence,$\gamma_v(G) = \frac{n+2}{2}$.

(iii) Let $v \in V_3(G)$. The vertex v dominates 4 vertices which are new vertices in $M(W_{1,n})$, 2 of them outside and the other 2 inside of wheel, 2 vertices of $W_{1,n}$ and itself. In order to dominate K_{n+1}, we need one vertex which is belong to K_{n+1}, say u. u dominates 3 other vertices which are not element of $V(K_{n+1})$. So, for the rest of vertices we have to put $\left\lceil \frac{n-3}{2} \right\rceil$ vertices in our dominating set which is the $\gamma(M(P_{n-3}))$ by Observation 1. Hence,

$$\gamma_v(G) = \left\lceil \frac{n-3}{2} \right\rceil + 1 + 1 = \frac{n}{2} + 1.$$

(iv) Let $v \in V_4(G)$. The vertex v dominates K_{n+1}, 3 additional vertices(which are in outside of wheel) and itself. So, for the rest of vertices we have to put $\left\lceil \frac{n-1}{2} \right\rceil$ vertices in our dominating set which is the $\gamma(M(P_{n-1}))$ by Observation 1. Hence,$\gamma_v(G) = \left\lceil \frac{n-1}{2} \right\rceil + 1 = \frac{n}{2} + 1$.

By these four subcases;

$$\begin{aligned}
\sum_{v \epsilon V(G)} \gamma_v(G) &= \sum_{v \epsilon V_1(G)} \gamma_v(G) + \sum_{v \epsilon V_2(G)} \gamma_v(G) + \sum_{v \epsilon V_3(G)} \gamma_v(G) + \sum_{v \epsilon V_4(G)} \gamma_v(G) \\
\gamma_{av}(G) &= \frac{1}{|V(G)|}\Big(\sum_{v \epsilon V_1(G)} \gamma_v(G) + \sum_{v \epsilon V_2(G)} \gamma_v(G) + \sum_{v \epsilon V_3(G)} \gamma_v(G) + \sum_{v \epsilon V_4(G)} \gamma_v(G)\Big) \\
&= \frac{\left(\frac{n}{2}+1\right) + \left(\frac{n+2}{2}\right) n + \left(\frac{n}{2}+1\right) n + \left(\frac{n}{2}+1\right) n}{3n+1} = \frac{n}{2} + 1.
\end{aligned}$$

□

Theorem 2.5 *Let $M(K_n)$ be the middle graph of K_n. Then*

$$\gamma_{av}(M(K_n)) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil, & n \text{ is odd}; \\ \frac{n^2+n+4}{2n+2}, & n \text{ is even}. \end{cases}$$

Proof The number of vertices of the graph K_n and $M(K_n)$ are n and $\frac{n^2+n}{2}$, respectively. Let $M(K_n) = G$. We consider the vertex-set of graph $G = V_1(G) \cup V_2(G)$, where

$V_1(G)$: The set of vertices of K_n,
$V_2(G)$: The set of vertices of G which are not in V_1.

We need to consider for even and odd n and also each case will have two subcases, $v \in V_1(G)$ and $v \in V_2(G)$.

Case 1 n is odd.

(i) Let $v \in V_1(G)$. In order to find $\gamma_v(G)$, we have to put v in domination set, then still we have $n-1$ vertices in $V_1(G)$ that are not dominated, since each vertices of $V_2(G)$

dominate to two vertices of $V_1(G)$, we need to add $\frac{n}{2}$ vertices of $V_2(G)$ to domination set. Then, $\gamma_v(G) = \frac{n}{2} + 1 = \left\lceil \frac{n}{2} \right\rceil$ for all $v \in V_1(G)$.

(ii) Let $v \in V_2(G)$. In order to find $\gamma_v(G)$, we have to put v in domination set, then we dominated two vertices of $V_1(G)$ and also $2n-4$ vertices of $V_2(G)$ which are adjacent to v. We have to put $\frac{n-2}{2}$ vertices of $V_2(G)$ in domination set to dominate $n-2$ vertices of $V_1(G)$ which are not dominated. Thus, $\gamma_v(G) = \frac{n-2}{2} + 1 = \left\lceil \frac{n}{2} \right\rceil$ for all $v \in V_2(G)$.

By (i) and (ii), we get that

$$\begin{aligned} \sum_{v\epsilon V(G)} \gamma_v(G) &= \sum_{v\epsilon V_1(G)} \gamma_v(G) + \sum_{v\epsilon V_2(G)} \gamma_v(G) \\ &= \frac{1}{|V(G)|} \left(\sum_{v\epsilon V_1(G)} \gamma_v(G) + \sum_{v\epsilon V_2(G)} \gamma_v(G) \right) \\ &= \frac{1}{\frac{n^2+n}{2}} \left(n \left\lceil \frac{n}{2} \right\rceil + \left(\frac{n^2-n}{2} \right) \left\lceil \frac{n}{2} \right\rceil \right) = \left\lceil \frac{n}{2} \right\rceil. \end{aligned}$$

Case 2 n is even.

(i) Let $v \in V_1(G)$. The proof is similar to that of Case 1. In order to find $\gamma_v(G)$, we have to put v in domination set, then still we have $n-1$ vertices in $V_1(G)$ that are not dominated, since each vertices of $V_2(G)$ dominate to two vertices of $V_1(G)$, we need to add $\frac{n-1}{2}$ vertices of $V_2(G)$ to domination set. Since n is even, then $\left\lceil \frac{n-1}{2} \right\rceil$. Then, $\gamma_v(G) = \frac{n}{2} + 1 = \left\lceil \frac{n}{2} \right\rceil$ for all $v \in V_1(G)$.

(ii) Let $v \in V_2(G)$. In order to dominate n vertices of $V_1(G)$, we have to take $\frac{n}{2}$ vertices of $V_2(G)$. Thus, $\gamma_v(G) = \frac{n}{2}$ for all $v \in V_2(G)$.

By (i) and (ii),

$$\begin{aligned} \sum_{v\epsilon V(G)} \gamma_v(G) &= \sum_{v\epsilon V_1(G)} \gamma_v(G) + \sum_{v\epsilon V_2(G)} \gamma_v(G) \\ &= \frac{1}{|V(G)|} \left(\sum_{v\epsilon V_1(G)} \gamma_v(G) + \sum_{v\epsilon V_2(G)} \gamma_v(G) \right) \\ &= \frac{1}{\frac{n^2+n}{2}} \left(n \left(\frac{n}{2} + 1\right) + \left(\frac{n^2-n}{2} \right) \frac{n}{2} \right) = \frac{n^2+n+4}{2n+2}. \end{aligned}$$

By Cases 1 and 2, the average domination number of $M(K_n)$ is,

$$\gamma_{av}(M(K_n)) = \begin{cases} \left\lceil \dfrac{n}{2} \right\rceil, & \text{n is odd;} \\ \\ \dfrac{n^2+n+4}{2n+2}, & n \text{ is even.} \end{cases} \qquad \square$$

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