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Cayley Fuzzy Digraph Structure Induced by Groups

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Abstract: In this paper we introduce a class of Cayley fuzzy digraph structure induced by groups. Further many graph properties are expressed in terms of algebraic properties.

Key Words: Fuzzy graph, Cayley digraph structure, vertex transitive graph.

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§1. Introduction

Digraph Structure Let V be a non-empty set and $S_1, S_2, \cdots, S_k$ are relations on V which are mutually disjoint, then $G' = (V, S_1, S_2, \cdots, S_n)$ is a *digraph structure*. In addition, if $S_1, S_2, \cdots, S_k$ are symmetric and irreflexive, then $G' = (V, S_1, S_2, \cdots, S_k)$ is a *graph structure*, see [2] for details.

Let G be a group and $S_1, S_2, \cdots, S_n$ be mutually disjoint subsets of G. Then the *Cayley digraph structure* of G with respect to $S_1, S_2, \cdots, S_n$ is defined as the graph structure $X = (G; E_1, E_2, \cdots, E_n)$, where $E_i = \{(x, y) : x^{-1}y \in S_i\}$ [1]. In case, a digraph structure with only one connection set is the usual Cayley digraph. So a Cayley digraph structure is a generalization of the Cayley digraph.

Fuzzy Digraph Structure Let $G' = (V, S_1, S_2, \cdots, S_k)$ be a graph (digraph) structure and $\mu, \rho_1, \rho_2, \cdots, \rho_k$ be fuzzy subsets of $V, S_1, S_2, \cdots, S_k$ respectively such that $\rho_i(x, y) \leq \mu(x) \wedge \mu(y)$, for all $x, y \in V$ and $i = 1, 2, \cdots, k$. Then $G = (\mu, \rho_1, \rho_2, \cdots, \rho_k)$ is a *fuzzy graph (digraph) structure* of G' [8], such that $\rho_i(x, y) \leq \mu(x) \wedge \mu(y)$, for all $x, y \in V$ and $i = 1, 2, \ldots, n$. Then $G = (\mu, \rho_1, \rho_2, \cdots, \rho_n)$ is a *fuzzy digraph structure* of G'.

Let V be a non-empty set, μ be fuzzy subset of V and $R_1, R_2, \ldots, R_n$ be mutually disjoint fuzzy relations on μ. Then $G = (\mu, R_1, R_2, \cdots, R_n)$ is a fuzzy digraph structure on V. In case $\mu = \chi_V$, where χ_V is the characteristic function on V, then the fuzzy digraph structure $(\mu, R_1, R_2, \cdots, R_n)$ is simply denoted by $G = (V; R_1, R_2, \cdots, R_n)$.

A fuzzy digraph structure $G = (V; R_1, R_2, \cdots, R_n)$ is called (i) trivial if $R_i \equiv 0$ for all i, (ii) reflexive if for all $x \in V, R_i(x, x) = 1$ for some i, (iii) symmetric if $R_i = R_i^{-1}$ for all i, (iv) transitive if for every i and j, $R_i \wedge R_j \leq R_k$ for some k, (v) a Hasse diagram if for every positive integer $m \geq 2$ and for every $x_1, x_2, \cdots, x_m$ of V with $R_i(x_j, x_{j+1}) > 0$ for all $j = 0, 1, 2, \cdots, m-1$, implies $R_i(x_0, x_m) = 0$ for all i, and (vi) complete if for any

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$x, y \in V, R_i(x, y) > 0$, for some $i = 1, 2, \cdots, n$. A walk of length k in a digraph structure is an alternating sequence $W = x_0, e_0, x_1, \cdots, e_k, x_k$, where $e_j = (x_j, x_{j+1})$ and $R_i(e_j) > 0$ for some i. A walk W is called a path if all the vertices are distinct. We use notation $x_0, x_1, x_2, \cdots, x_k$ for the walk W. A walk is called a circuit if its first and last vertices are the same, but no other vertex is repeated. A weak path is a sequence $x_1, x_2, \cdots, x_m$ of distinct vertices of V such that for $j = 1, 2, \cdots, m-1$, $R_i \vee R_i^{-1}(x_j, x_{j+1}) > 0$ for some $i = 1, 2, \cdots, n$. Distance between two vertices x and y in G is the length of the shortest path from x to y and is denoted by $d(x, y)$. Diameter of the fuzzy digraph structure G, denoted by $d(G)$, is defined by $d(G) = max_{x,y \in G} d(x, y)$. A fuzzy digraph structure $G = (V; R_1, R_2, \cdots, R_n)$ is called (i) connected (strongly connected) if y is connected to x for all $x, y \in V$, and (ii) weakly connected if any two vertices can be joined by a weak path, that is, the fuzzy digraph structure $G' = (V; R_1 \vee R_1^{-1}, R_2 \vee R_2^{-1}, \cdots, R_n \vee R_n^{-1})$ is connected. A weakly connected fuzzy digraph structure $G = (V; R_1, R_2, \cdots, R_n)$ with out any circuits is called a tree.

The present work is a generalisation of the work in [6] in which Madhavan Namboothiri N.M. et al. introduced a class of Cayley fuzzy graphs induced by groups.

§2. Cayley Fuzzy Digraph Structure

Definition 2.1 *Let V be a group and $\nu_1, \nu_2, \cdots, \nu_n$ be mutually disjoint fuzzy subsets of V. Then, Cayley Fuzzy Digraph Structure of V with respect to $\nu_1, \nu_2, \cdots, \nu_n$ is defined as $(V; R_1, R_2, \cdots, R_n)$ where $R_i(x, y) = \nu_i(x^{-1}y)$ and is denoted by $CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$. The subsets $\nu_1, \nu_2, \cdots, \nu_n$ are called connection fuzzy subsets of $CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$. In case, a Cayley fuzzy digraph structure with only one connection set is usual Cayley fuzzy graph.*

Theorem 2.2 *$G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ is vertex-transitive.*

Proof Let a and b be any two arbitrary elements in G. Define $\psi : V \to V$ by $\psi(x) = ba^{-1}x$ for all $x \in V$. Clearly, ψ is a bijection onto itself. Furthermore, we have, for each $x, y \in V$,

$$\begin{aligned} R_i(\psi(x), \psi(y)) &= R_i(ba^{-1}x, ba^{-1}y) \\ &= \nu_i((ba^{-1}x)^{-1}(ba^{-1}y)) \\ &= \nu_i(x^{-1}y) = R_i(x, y). \end{aligned}$$

Hence, the proof is complete. □

Theorem 2.3 *Cayley fuzzy digraph structures are regular.*

Proof Let $G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ be a cayley fuzzy digraph structure. Let $u, v \in V$. Since Cayley fuzzy digraph structures are vertex transitive, there exist an automorphism say, f on G such that, $f(u) = v$ and $R_i(f(x), f(y)) = R_i(x, y)$ for any $x, y \in V$ and $i = 1, 2, \cdots, n$.

Then the in-degree of u,

$$\begin{aligned} ind(u) &= \sum_{x\in V}\sum_{i=1}^{n} R_i(x,u) = \sum_{x\in V}\sum_{i=1}^{n} R_i(f(x),f(u)) \\ &= \sum_{x\in V}\sum_{i=1}^{n} R_i(f(x),v) = \sum_{f(x)\in V}\sum_{i=1}^{n} R_i(f(x),v) \\ &= \sum_{y\in V}\sum_{i=1}^{n} R_i(y,v) = ind(v). \end{aligned}$$

Similarly, we can prove that $outd(u) = outd(v)$. Therefore, G is in-regular and out-regular. Now to prove that G is regular we just need to show that $ind(1) = outd(1)$.

$$\begin{aligned} ind(1) &= \sum_{x\in V}\sum_{i=1}^{n} R_i(x,1) = \sum_{x\in V}\sum_{i=1}^{n} \nu_i(x^{-1}) \\ &= \sum_{x\in V}\sum_{i=1}^{n} \nu_i(x) = \sum_{x\in V}\sum_{i=1}^{n} R_i(1,x) = outd(1). \end{aligned}$$

Therefore, G is regular. □

Theorem 2.4 *$G = CayF_D(V;\nu_1,\nu_2,\cdots,\nu_n)$ is a trivial graph if and only if $\nu_i \equiv 0$ for all i.*

Proof By definition, G is trivial if and only if $R_i \equiv 0$ for all i. This implies that $\nu_i \equiv 0$ for all i. □

Theorem 2.5 *$G = CayF_D(V;\nu_1,\nu_2,\cdots,\nu_n)$ is reflexive if and only if $\nu_i(1) = 1$ for some i.*

Proof Assume that $G = CayF_D(V;\nu_1,\nu_2,\cdots,\nu_n)$ is reflexive. Then for every $x \in V$, $R_i(x,x) = 1$ for some i. This implies that $\nu_i(x^{-1}x) = \nu_i(1) = 1$ for some i.

Conversely, let $\nu_i(1) = 1$ for some i, say $i = k$. This implies that for each $x \in V$, $R_k(x,x) = \nu_k(x^{-1}x) = \nu_k(1) = 1$. That is G is reflexive. □

Theorem 2.6 *$G = CayF_D(V;\nu_1,\nu_2,\cdots,\nu_n)$ is symmetric if and only if $\nu_i(x) = \nu_i(x^{-1})$ for all $x \in V$, $i = 1,2,\cdots,n$.*

Proof Suppose that G is symmetric. Then for any $x \in V$,

$$\nu_i(x) = \nu(x^{-1}x^2) = R_i(x,x^2) = R_i{}^{-1}(x,x^2) = R_i(x^2,x) = \nu_i(x^{-1}x^{-1}x) = \nu_i(x^{-1}).$$

Therefore, $\nu_i(x) = \nu_i(x^{-1})$.

Conversely, suppose that $\nu_i(x) = \nu_i(x^{-1})$ for all $x \in V$. Then for any $x,y \in V$, $R_i(x,y) = \nu_i(x^{-1}y) = \nu_i((x^{-1}y)^{-1}) = \nu_i(y^{-1}x) = R_i(y,x)$. This implies that, R is symmetric. Hence the proof is complete. □

Theorem 2.7 *$G = CayF_D(V;\nu_1,\nu_2,\cdots,\nu_n)$ is transitive if and only if for every i,j and for* *any* $x, y \in V, \nu_i(x) \wedge \nu_j(y) \leq \nu_k(xy)$ *for some* k.

Proof First assume that G is transitive. That is, for every i, j, $R_i \circ R_j \leq R_k$ for some k. For $x, y \in V$,

$$\begin{aligned}\nu_i(x) \wedge \nu_j(y) &\leq \vee\{\nu_i(z) \wedge \nu_j(z^{-1}(xy)) : z \in V\}\\ &= \vee\{R_i(1, z) \wedge R_j(z, xy) : z \in V\}\\ &= R_i \circ R_j(1, xy)\\ &\leq R_k(1, xy) = \nu_k(xy).\end{aligned}$$

That is, $\nu_i(x) \wedge \nu_j(y) \leq \nu_k(xy)$ for some k.

Now let for any $x, y \in V$ and i, j, $\nu_i(x) \wedge \nu_j(y) \leq \nu_k(xy)$ for some k. Then,

$$\begin{aligned}(R_i \circ R_j)(x, y) &= \vee\{R_i(x, z) \wedge R_j(z, y) : z \in V\}\\ &= \vee\{\nu_i(x^{-1}z) \wedge \nu_j(z^{-1}y) : z \in V\}\\ &\leq \vee\{\nu_k((x^{-1}z)(z^{-1}y)) : z \in V\}\\ &= \nu_k(x^{-1}y) = R_k(x, y).\end{aligned}$$

Thus, $R_i \circ R_j \leq R_k$ for some k. This completes the proof. □

Theorem 2.8 *$G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ is complete if and only if $\cup{\nu_i}_\circ{}^+ = V$.*

Proof First assume that G is complete. That is $\cup {R_i}_\circ^+ = V \times V$. Clearly, $\cup {\nu_i}_\circ^+ \subseteq V$. Now let $x \in V$. Then $(1, x) \in {R_i}_\circ{}^+$ for some i. That is, $R_i(1, x) \geq 0$, which implies, $\nu_i(x) \geq 0$. Thus, $x \in \cup{\nu_i}_\circ{}^+$. Therefore, $V \subseteq \cup{\nu_i}_\circ{}^+$. That is, $\cup{\nu_i}_\circ{}^+ = V$.

Conversely, assume $\cup{\nu_i}_\circ{}^+ = V$. Let $(x, y) \in V \times V$. Then $x, y \in V \Rightarrow x^{-1}y \in V \Rightarrow x^{-1}y \in \cup{\nu_i}_\circ{}^+ \Rightarrow x^{-1}y \in {\nu_i}_\circ{}^+$ for some i. Then, $\nu_i(x^{-1}y) \geq 0$. That is, $R_i(x, y) \geq 0$ which implies $(x, y) \in {R_i}_\circ{}^+$. Hence, $V \times V \subseteq \cup{R_i}_\circ{}^+$. Therefore,

$$\bigcup {R_i}_\circ{}^+ = V \times V.$$

This completes the proof. □

Let A_k be the set of all elements $x \in V$ of the form $x = x_1x_2 \cdots x_k$, where $x_j \in {\nu_i}_0^+$ for some $i = 1, 2, \cdots, n$. Then $[\vartheta]$ is defined as $[\vartheta] = \bigcup_{k=1}^{n} A_k$. Let B_k be the set of all elements $y \in V$ of the form $y = y_1y_2 \cdots y_k$, where $y_j \in (\nu_i \wedge \nu_i^{-1})_0^+$ for some $i = 1, 2, \cdots, n$. Then $[[\vartheta]]$ is defined as $[[\vartheta]] = \bigcup_{k=1}^{n} B_k$.

Theorem 2.9 *$G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ is connected if and only if $V = [\vartheta]$.*

Proof First assume that $G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ is connected. Clearly, $[\vartheta] \subseteq V$. Now let $x \in V$. Then there exists a path from 1 *to* x say, $(1, y_1, y_2, \cdots, y_k = x)$. Then, for some i, $R_{i_1}(1, y_1) > 0$, that is, $y_1 \in \nu^+_{i_1 0}$. Also, $y_{j-1}^{-1} y_j \in \nu^+_{i_j 0}$,for $j = 2, 3, \cdots, k$. This implies that $x \in A_k$, since, $x = (1.y_1)(y_1^{-1} y_2)(y_2^{-1} y_3) \cdots (y_{k-1}^{-1} y_k)$. Therefore, $x \in \bigcup_{k=1}^{n} A_k = [\vartheta]$. Hence, $V = [\vartheta]$.

Conversely, assume that $V = [\vartheta]$. Let $x, y \in V$. Then $z = x^{-1}y \in V$, implies, $z \in [\vartheta] = \bigcup_{k=1}^{n} A_k$. Then $z = z_1 z_2 \cdots z_k$. Then $1, z_1, z_1z_2, \cdots, z_1z_2 \cdots z_k = z$ is a path from 1 to z. Then $x, xz_1, xz_1z_2, \cdots, xz_1z_2 \cdots z_k = xz = y$ is a path from x to y, implies G is connected. This completes the proof. □

Theorem 2.10 *$G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ is weakly connected if and only if $V = [[\vartheta]]$.*

Proof Assume G be weakly connected. Clearly, $[[\vartheta]] \subseteq V$. Let $x \in V$. Then there exist a weak path say, $1, x_1, x_2, \cdots, x_k = x$ from 1 to x. Then, $1x_1 \in (\nu_{i_1} \vee \nu_{i_1}^{-1})^+_0$, $x_1^{-1}x_2 \in (\nu_{i_2} \vee \nu_{i_2}^{-1})^+_0, \cdots,$ $x_{k-1}^{-1}x_k \in (\nu_{i_k} \vee \nu_{i_k})^+_0$, which clearly implies that

$$x \in \bigcup_{k} B_k = [[\vartheta]].$$

Hence, $V \in [[\vartheta]]$.

Conversely, assume that $V = [[\vartheta]]$. Let $x, y \in V$, implies $z = x^{-1}y \in V$. Therefore, $z \in [[\vartheta]]$. Then there exist elements $z_j \in (\nu_{i_j} \vee \nu_{i_j}^{-1})^+_0$, $j = 1, 2, \cdots, k$, such that $z = z_1 z_2 \cdots z_k$, for some $k \in \{1, 2, \cdots, n\}$. Then $1, z_1, z_1z_2, \cdots, z_1z_2 \cdots z_k = z$ is a weak path from 1 to z and hence $x, xz_1, xz_1z_2, \cdots, xz_1z_2 \cdots z_k = xz = y$ is a weak path from x to y. Therefore, G is weakly connected. This completes the proof. □

Theorem 2.11 *$G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ is partially ordered if and only if*

(i) $\nu_i(1) = 1$ for some i;

(ii) for every i, j and for any $x, y \in V, \nu_i(x) \wedge \nu_j(y) \leq \nu_k(xy)$ for some k;

(iii) $\{x : \nu(x) = \nu(x^{-1})\} = \{1\}$ for all $i = 1, 2, \cdots, n$.

Theorem 2.12 *$G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ is quasi-ordered if and only if*

(i) $\nu_i(1) = 1$ for some i;

(ii) for every i, j and for any $x, y \in V, \nu_i(x) \wedge \nu_j(y) \leq \nu_k(xy)$ for some k.

Theorem 2.13 *$G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ is a Hasse diagram if and only if G is connected and $\nu_k(x_1x_2 \cdots x_m) = 0$, $k = 1, 2, \cdots, n$, for any collection $x_1, x_2, \cdots, x_m$ of vertices in V with $m \geq 2$ and $\nu_{i_j}(x_j) > 0$ for $j = 1, 2, \cdots, m$.*

Proof Suppose G is a Hasse diagram. Since $\nu_{i_j}(x_j) > 0$ for $j = 1, 2, \cdots, m$, $(1, x_1, x_1x_2, \cdots, x_1x_2 \cdots x_m)$ is a path from 1 to $x_1x_2 \cdots x_m$. Now since G is a Hasse diagram, $R_k(1, x_1x_2 \cdots x_m) = 0$ for all k. Therefore $\nu_k(x_1x_2 \cdots x_m) = 0$ for all $k = 1, 2, \cdots, n$.

Conversely suppose, G is connected and $\nu_k(x_1x_2 \cdots x_m) = 0$, $k = 1, 2, \cdots, n$, for any collection $x_1, x_2, \cdots, x_m$ of vertices in V with $m \geq 2$ and $\nu_{i_j}(x_j) > 0$ for $j = 1, 2, \cdots, m$. Let

$(x_0, x_1, \cdots, x_m)$ be a path in G from x_1 to x_m, $m \geq 2$. Then $R_{i_1}(x_0, x_1) > 0$, $R_{i_2}(x_1, x_2) > 0$, $\cdots, R_{i_m}(x_{m-1}, x_m) > 0$ which implies, $\nu_{i_1}(x_0^{-1}x_1) > 0$, $\nu_{i_2}(x_1^{-1}x_2) > 0$, $\cdots, \nu_{i_m}(x_{m-1}^{-1}x_m) > 0$. Thus, by assumption, $\nu_k(x_0^{-1}x_1x_1^{-1}x_2 \cdots x_{m-1}^{-1}\, x_m) = \nu_k(x_0^{-1}x_m) = 0$. Therefore, $R_k(x_0, x_m) = 0$ for all $k = 1, 2, \cdots, n$. Hence, G is a Hasse diagram. This completes the proof. □

Theorem 2.14 *For $k = 1, 2, \cdots, n$, let A_k be the set of all products of the form $\nu_{i_1}\nu_{i_2} \cdots \nu_{i_k} = \{x_1x_2 \cdots x_k : x_j \in \nu_{i_j}{}_0^+, j = 1, 2, \cdots, k\}$. If $G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ has finite diameter, then the diameter of G is the least positive integer m such that*

$$G = \bigcup_{A \in A_m} A.$$

Theorem 2.15 *$G = CayF_D(V; \nu_1, \nu_2, \cdots, \nu_n)$ is a tree if and only if $V = [[\vartheta]]$ and $1 \notin [\vartheta]$.*

Definition 2.16([6]) *Let $(S, *)$ be a semigroup. Let A be a fuzzy subset of S. Then A is said to be fuzzy sub-semigroup of S if for all $a, b \in S$, $A(ab) \geq A(a) \wedge A(b)$.*

Definition 2.17 *Let $(S, *)$ be a semigroup and let $\nu_1, \nu_2, \cdots, \nu_n$ be mutually disjoint fuzzy subsets of S. The fuzzy sub-semigroup generated by $\nu_1, \nu_2, \cdots, \nu_n$ is the smallest fuzzy sub-semigroup of S which contains $\nu_1, \nu_2, \cdots, \nu_n$. Let us denote it by $\langle \nu_{(123\cdots n)} \rangle$.*

Theorem 2.18 *Let $(S, *)$ be a semigroup and let $\nu_1, \nu_2, \cdots, \nu_n$ be mutually disjoint fuzzy subsets of S. Then the fuzzy subset $\langle \nu_{(123\cdots n)} \rangle$ is precisely given by $\langle \nu_{(123\cdots n)} \rangle(x) = \vee\big\{\nu_{j_1}(x_1) \wedge \nu_{j_2}(x_2) \wedge \cdots \nu_{jk}(x_k) : x = x_1x_2 \cdots x_k$ with a finite positive integer $k, x_i \in S$ and $\nu_{j_i}(x_i) > 0$ for some $j_i = 1, 2, \cdots, n\big\}$ for any $x \in S$.*

Proof Let ν' be the fuzzy subset of V defined by $\nu'(x) = \vee\big\{\nu_{j_1}(x_1) \wedge \nu_{j_2}(x_2) \wedge \cdots \wedge \nu_{j_m}(x_m) : x = x_1x_2x_3 \cdots x_m,\ xj_i \in \nu_{j_{i_\circ}}^+, m \in \{1, 2, 3, \cdots, n\}\big\}$ for any $x \in V$. If $y \in V$, by definition of ν', it is clear that $\nu'(y) \geq \nu_{j_k}(y)$ where $j_k \in \{1, 2, \cdots, n\}$ and $\nu_{j_k}(y) \geq 0$. Thus, we have $\nu_{j_k} \leq \nu'$ for all j_i. This implies that ν' contains $\nu_1, \nu_2, \cdots, \nu_n$. Let $x, y \in V$. If $\nu_{j_i}(x) = 0$ or $\nu_{j_i}(y) = 0$, then $\nu_{j_i}(x) \wedge \nu_{j_i}(y) = 0$. Then, $\nu'(xy) \geq \nu_{j_i}(x) \wedge \nu_{j_i}(y)$. Again, if $\nu_{j_i}(x) \neq 0$ and $\nu_{j_i}(y) \neq 0$, then by definition of ν', we have $\nu'(xy) \geq \nu_{j_i}(x) \wedge \nu_{j_i}(y)$. Hence ν' is a fuzzy sub semigroup of V containing $\nu_i, i \in \{1, 2, \cdots, n\}$. Now let A be any fuzzy sub semigroup of V containing $\nu_i, i \in \{1, 2, \cdots, n\}$. Then, for any $x \in V$ with $x = x_1x_2x_3 \ldots x_m,\ x_i \in \nu_{j_{i_\circ}}^+$, for $i = 1, 2, \cdots, n, m \in \{1, 2, 3, \cdots, n\}$ we have $A(x) \geq A(x_1) \wedge A(x_2) \wedge \cdots \wedge A(x_m) \geq \nu_{j_1}(x_1) \wedge \nu_{j_2}(x_2) \wedge \cdots \wedge \nu_{j_m}(x_m)$, which implies that $A(x) \geq \vee\{\nu_{j_1}(x_1) \wedge \nu_{j_2}(x_2) \wedge \cdots \wedge \nu_{j_m}(x_m) : x = x_1x_2x_3 \cdots x_m,\ x_{j_i} \in \nu_{j_{i_\circ}}^+, m \in \{1, 2, 3, \cdots, n\}$ for $j_i \in \{1, 2, \ldots, n\}$ for any $x \in V$. Therefore, $A(x) \geq \nu'(x)$ for all $x \in V$. Thus, $\nu' = \langle \nu_{(123\cdots n)} \rangle$. That is, $\langle \nu_{(123\cdots n)} \rangle(x) = \vee\big\{\nu_{j_1}(x_1) \wedge \nu_{j_2}(x_2) \wedge \cdots \wedge \nu_{j_m}(x_m) : x = x_1x_2x_3 \cdots x_m,\ x_{j_i} \in \nu_{j_{i_\circ}}^+, m \in \{1, 2, 3, \cdots, n\}\big\}$ for any $x \in V$. □

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