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Combinatorics – A Mathematical Approach for Holding on the Realty of Thing in the Universe

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Abstract: Usually, one holds a thing T on its appearance or characters and particularly, by mathematics. But *is the mathematical reality equal to the reality of thing T?* The answer is not certain because the recognition of human on thing T is only a local or conditional one, implied in the fable of blind men with an elephant, i.e., the sophist told the blind men that an elephant has all characteristics that they are talking about. Then, *what is the significance of this fable?* It lies essentially in the shape of an elephant and generally, the reality of a thing is a combinatorial one, i.e., combinatorics is priori to the recognition of human because all of us are similar to the blind in front of a thing. In this report, I discuss the *non-harmonious group* with *Smarandache multispace* inherited a topological graph G in first, generalize it to *G-flows* $\overrightarrow{G}^L$ or networks $\overrightarrow{N}$ with vector flows and then, *continuity flows* $\overrightarrow{G}^L$, i.e., mathematics over 1-dimensional topological graphs, which extends the classic mathematics over combinatorial structures $\overrightarrow{G}$. This report surveys how to establish such a system by viewing continuity flow $\overrightarrow{G}^L$ as a mathematical element for establishing the Banach flow space, Hilbert flow space over topological graphs $\overrightarrow{G}$ and then, how to apply it to generalize a few of important conclusions in functional analysis such as those of the inverse mapping theorem, closed graph theorem and the Hahn-Banach theorem for providing the recognition of human on the reality of things, including the subdivision of a matter M into elementary particles with a mathematical supporting, which forms a complex network on M in physics, and shows also the 12 meridians on human body in traditional Chinese medicine is an example of G-flows or generally, continuity flows with dynamic equations.

Key Words: Combinatorial notion, contradiction, non-solvable system of equations, non-harmonious group, Smarandachely denied axiom, Smarandache multispace, G-flow, continuity flow, mathematical combinatorics, 12 meridians on human body.

AMS(2010): 05C10, 05C21, 35A08, 46B25, 51D20, 51H20, 51P05.

§1. Introduction

Usually, one holding a thing T on its appearance or characters and particularly, by mathematical reality. Then, *what is the reality of a thing T?* In dictionary, the word "*reality*" is explained to

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be the state of things as they actually exist, including everything that is and has been, whether or not it is observable or comprehensible. *Can one really hold on the reality of thing T?* The answer is not certain unless the mathematical reality. Generally, a thing T is multilateral or complex one but the recognition of human on thing T has certain limitations, i.e., it is only the local rather than the whole. Then, *how to solve this problem and how to cross the gap from the local to the whole?* The answer is nothing else but the combinatorics.

1.1. Combination Prior to Reductionism. As we all known, the reduction on a matter T is subdivided it into the minimum recognizable elements so that humans can understand the reality of matter T. For example, to subdivide a matter $T \to$ molecule $\to$ atom $\to$ nucleus $\to$ proton and neutron $\to$ elementary particle consisting of quarks, leptons with interaction quanta including photons and other particles of mediated interactions ([35]), and a living $L \to$ biological macromolecule $\to$ cell or gene, such as those shown in Figure 1 on subdividing of matter with elementary particles.

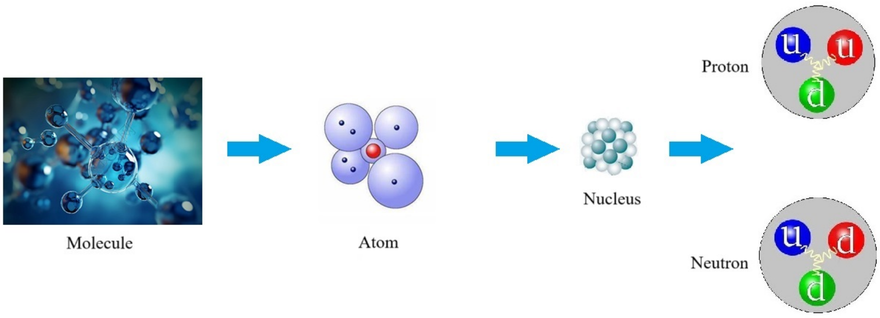

Figure 1

Actually, there is an assumption implied in reductionism without proof, i.e., the reality of matter T can be held if the behavior of elementary particles over a topological 1-dimensional structure is recognized locally by humans. For example, the models of proton, neutron are both over graphs K_3 by quarks in Figure 1. But, *is this assumption right and so, one can holds on the reality of matter T by reductionism?* The answer is not certain unless all elementary particles are in stationary or synchronization.

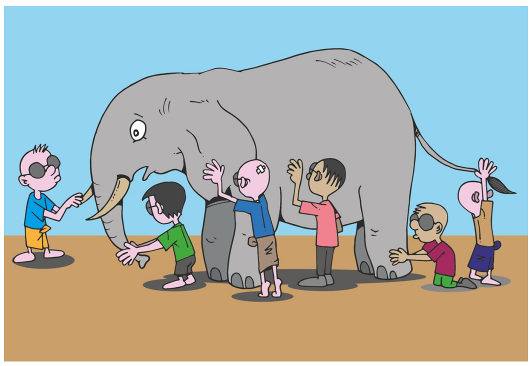

Figure 2

For holding the whole with the local, we all learned a famous fable of the *blind men with an elephant* in elementary school, which narrates that there are 6 blind men wanted to know an elephant looked like by feeling its body one by one, see Figure 2. Their recognitive process is like this, namely the 1st one touched the elephant's tooth and claimed "*an elephant is like a big, thick and smooth radish*", the 2nd one touched the elephant's trunk and claimed "*an elephant is like a tube*", the 3rd one touched the elephant's ear and claimed "*an elephant is like a big fan*", the 4th one touched the elephant's belly and claimed "*an elephant is clearly like a wall*", the 5th one touched the elephant's leg and claimed "*an elephant is clearly like a big pillar*" and finally, the 6th one touched the elephant's tail and claimed "*an *elephant is like a piece of grass rope"* . Each of them believed the perception of himself on the shape of elephant is right and insisting on his own opinion, kept an endless quarrelling with others. At this time, a sophist came forward and told them "*why you are thinking about the elephant's shape different is because each of you touches the different part of the elephant's body. Essentially, an elephant has all characteristics that you are talking about!"* This fable shows the limitation of recognition of blind man compared to the normal, namely the elephant shape in eyes of the blind men is very different from, even a bit ridiculous to that of the normal human. Then, *what is the elephant shape in eyes of the sophist by that of the blind men?* The answer is certainly nothing else but a union of characteristics recognized locally by the 6 blind men, namely the combination of all the local to form a whole such as those shown in Figure 3.

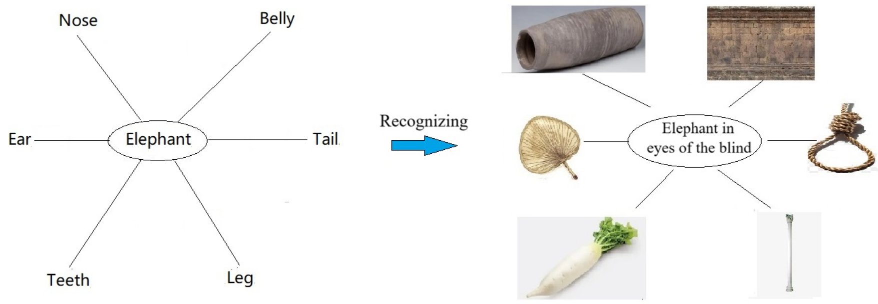

Figure 3

$$\textbf{An elephant} = \{\textit{4 big pillars}\}\bigcup\{\textit{1 gross rope}\}\bigcup\{1 \text{ tubes}\}$$
$$\bigcup\{\textit{2 big fans}\}\bigcup\{\textit{1 big wall}\}\bigcup\{\textit{2 big radishes}\} \quad (1.1)$$

with a combinatorial structure

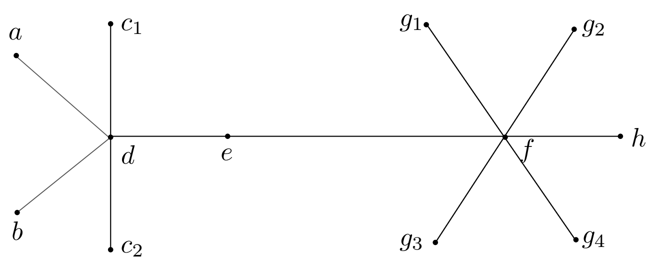

Figure 4

where a_1, a_2 = big radishes, b_1, b_2 = big fans, c= elephant's head, d = elephant's neck, e = big wall, g_1, g_2, g_3, g_4 = big pillars, f = grass rope. So, *what is the philosophical implication of the fable of blind men with an elephant?* Certainly, the elephant is existing in a 3-dimension $\mathbb{R}^3$. It is clear that the shape of an elephant can not be any combination of 2 big teeth (or big radishes), 1 trunk (or tube), 2 ears (or big fans), 1 belly (or wall), 4 legs (or big pillars) and 1 tail (or grass rope) but such a combination over the 1-dimensional topological graph G^L shown in Figure 4 in eyes of the sophist, inherited in the recognition of blind men by feeling the different parts on the elephant body.

Notice that the sophist told the blind men that "*an elephant has all characteristics that you are talking about*" is essentially a claim on the elephant shape, i.e., its shape is a union (1.1) of all characteristics of blind men hold on the elephant shape. Certainly, this is the recognitive way of human on thing T by reductionism. Generally, let the observable characteristics be $\chi_1, \chi_2, \cdots, \chi_n$ in reductionism of thing T and denote the mathematical reality of thing T by $T_{\mathcal{M}}$. So, one recognizes the mathematical reality of thing T by a union

$$T_{\mathcal{M}} = \bigcup_{1}^{n} \mathcal{R}(\chi_i) \tag{1.2}$$

of local recognitions $\mathcal{R}(\chi_i)$ of human, called a *Smarandache multispace* ([14],[36]), where $\mathcal{R}(\chi_i)$ is the mathematical reality on characteristic χ_i for integers $1 \leq i \leq n$. Now, *is any combination of characteristics of $\chi_1, \chi_2, \cdots, \chi_n$ necessarily the thing T?* The answer is certainly Not because one can not assert the characteristics $\chi_1, \chi_2, \cdots, \chi_n$ are complete, and the recognitive process of human on thing T is like the blind men on the elephant by feeling its partly body. Furthermore, T is existing in space, which inherits a topological 1-dimensional stricture G^L in the reductionism process and the combination is conclusively priori to the reductionism in recognition of thing T, which naturally leads to a complex network ([1]-[2]) or combinatorial fields ([10]-[12]) on thing T. For example, there are 3.6×10^{13} and 2.8×10^{13} cells respectively in a male or female body, which can be characterized by complex networks with 3.6×10^{13} or 2.8×10^{13} nodes, respectively.

1.2. Combinatorics Implied in Contradictory System. Usually, human quarrelling is because of the differences in recognition on one thing, which leads to contradiction, even by mathematics. For example, let $S_i, 1 \leq i \leq 6$ be the elephant shape of blind men in fable of the blind men with an elephant. They were quarrelling because their recognition are very different, i.e., $S_i \neq S_j$ if $1 \leq i \neq j \leq 6$. However, the contradiction arising is not due to the nature of elephant but the modeling of blind men, and the sophist told the blind men is its shape should be essentially a contradictory system holding with a *Smarandachely denied axiom* ([35],[36]), i.e., the axiom *the elephant shape S = a big radish* is simultaneously validated and invalided, or the axiom *the elephant shape S = the reality of elephant shape* only invalided but in six different shapes S_i,$1 \leq i \leq 6$ simultaneously, contradicting to a definite recognition on the elephant shape. Indeed, there is a fundamental question on the recognition of human, i.e., *is a contradictory system in mathematics worthless in recognition?* The answer is certainly Not because we can not asserted so, i.e., the contradiction is essentially caused by the modeling way of human, which violates the axiom adopted in the mathematical system. However, all things are harmonious in nature, namely the contradiction arising in a mathematical modeling only implies its inappropriate, not the objective of thing in nature. Particularly, the modeling of elements in a self-organized system such as those of biological population, cell system, gene, etc., i.e., all elements are self-motivated, not necessarily in stationary and synchronization are the case.

Figure 5

For example, let $A = \{C_1, C_2, C_3\}$ and $B = \{C'_1, C'_2, C'_3\}$ be two groups consisting of three Tom cats chasing three Jerry mice in Figure 5 along three straight lines on Euclidean plane $\mathbb{R}^2$

respectively, shown in Figure 6. Then, *how to modeling the running behavior of cats in groups A or B on plane $\mathbb{R}^2$?* For answering this question, a natural idea is to describe the running behavior of the two groups by moving orbits, i.e., lines on Euclidean plane $\mathbb{R}^2$, solve the two systems of linear equations and then, answer this question. In fact, the obits of three cats in groups A or B respectively form two systems of linear equations by cats running lines shown in Figure 6, i.e

$$(LES_3^N)\ \begin{cases} y &= 4 \\ y &= 2 \\ x+y &= 8 \end{cases} \qquad (LES_3^S)\ \begin{cases} x &= 3 \\ y &= 3 \\ x+y &= 6 \end{cases}$$

However, the system (LES_3^N) is non-solvable and the system (LES_3^S) has a solution $(3,3)$. Now, *can we conclude that the running behavior of cats in A are nothing unless an empty set $\emptyset$, and cats in B are all still at the point $(3,3)$ without moving?* Of course Not because all cats in group A and B are running on Euclidean plane $\mathbb{R}^2$.

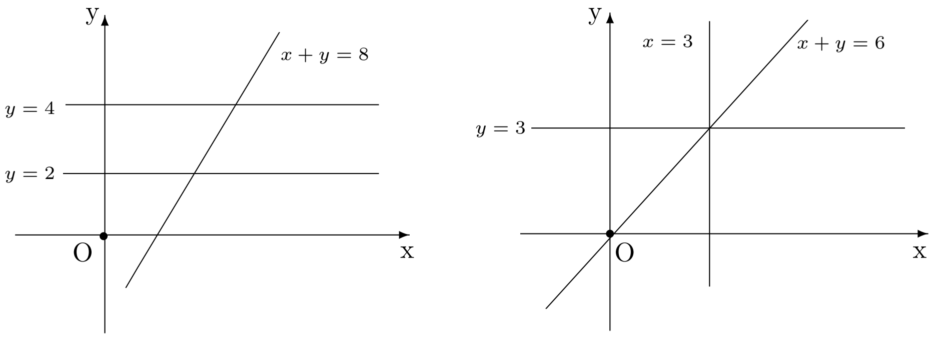

Figure 6

Then, *what is wrong with the modeling of running behavior of cats?* The answer is modeling by the solutions of systems (LES_3^N) and (LES_3^S) on running behavior of cats in groups A and B is inappropriate. Certainly, a running orbit of cat in groups A or B can be characterized by the solution of line equation, i.e., the straight line of cat in A or B on Euclidian plane $\mathbb{R}^2$ but not the solution of system of linear equations.

In this case, the orbits Orb(A) or Orb(B) should be a union of points of cats in groups A or B passing on plane $\mathbb{R}^2$, i.e.,

$$\begin{aligned} \mathrm{Orb}(A) &= \{(x,y):\ y=4\}\bigcup\{(x,y):\ y=2\}\bigcup\{(x,y):\ x+y=8\}, \\ \mathrm{Orb}(B) &= \{(x,y):\ x=3\}\bigcup\{(x,y):\ y=3\}\bigcup\{(x,y):\ x+y=6\}, \end{aligned}$$

where each of the orbits Orb(A) and Orb(B) is a Smarandache multispace, i.e., combinatorial one. For example, denote the points of a cat running on straight line $ax+by=c$ by the set $L_{a,b,c} = \{(x,y)|ax+by=c, a\neq 0 \text{ or } b\neq 0\}$ in Figure 6, the line intersections of (LES_3^N) and

(LES_3^S) for cats in groups A, B by

$$\begin{aligned} v_1 &= L_{0,1,4}\bigcap L_{1,1,8},\ v_2 = L_{0,1,2}\bigcap L_{1,1,8},\ L_{0,1,2}\bigcap L_{0,1,4} = \emptyset, \\ u_1 &= L_{1,0,3}\bigcap L_{1,1,6},\ u_2 = L_{0,1,3}\bigcap L_{1,0,3},\ u_3 = L_{0,1,3}\bigcap L_{1,1,6} \end{aligned}$$

and so, the running of cats in groups A, B can be characterized by combinatorial solutions of systems (LES_3^N) and (LES_3^S) respectively, i.e., labeled graphs P_3^L, C_3^L shown in Figure 7.

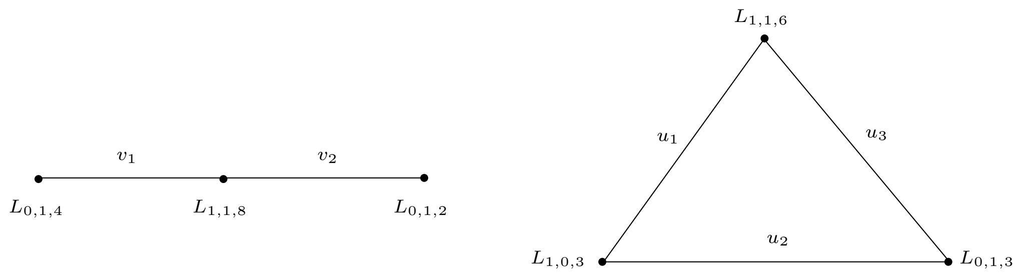

Figure 7

Generally, let $\widetilde{S}$ be a Smarandache multispace on n distinct spaces $S_1, S_2, \cdots, S_n$ for an integer $n \geq 1$. Define a labeled graph $G^L[\widetilde{S}]$ associated with $\widetilde{S}$ by

$$\begin{aligned} V\left(G^L\left[\widetilde{S}\right]\right) &= \{S_1, S_2, \cdots, S_n\} \\ E\left(G^L\left[\widetilde{S}\right]\right) &= \left\{(S_i, S_j)\,|S_i\bigcap S_j \neq \emptyset,\ 1 \leq i \neq j \leq n\right\} \end{aligned}$$

and labels on the vertex $\mathcal{S}_i$, edge $(\mathcal{S}_i, \mathcal{S}_j)$ for integers $1 \leq i \neq j \leq n$ respectively by

$$L: \ S_i \to L(S_i) \ = \ S_i \text{ and } L: \ (S_i, S_j) \to L(S_i, S_j) \ = \ S_i\bigcap S_j.$$

Certainly, a Smarandache multispace $\widetilde{S}$ is equivalent to the labeled graph $G^L[\widetilde{S}]$ by definition. However, $\widetilde{S}$ is a multiset implying the mathematical reality of thing but $G^L[\widetilde{S}]$ is sets over topological graph G which can contributes to establish a mathematics, i.e., *mathematical combinatorics* by viewing $G^L[\widetilde{S}]$ as a mathematical element. So, *could we really establish such a mathematics on elements* $G^L[\widetilde{S}]$*?* The answer is definite by combinatorics.

The main purpose of this report is to survey the establishing of mathematics on continuity flows $\overrightarrow{G}^L$ holding with vertex conservation law, show the importance of non-harmonious groups with G-solutions in recognition of the reality of thing T, generalize G-solutions to G-flows, continuity flows for constructing mathematics as those of Banach flow space, Hilbert flow space over topological graphs G and then, generalize a few of important conclusions in classic mathematics such as the inverse mapping theorem, closed graph theorem and the Hahn-Banach theorem in these flow spaces for providing the recognition of human on the reality of things, including the subdivision of a matter M into elementary particles with a mathematical supporting, and shows also the 12 meridians, Ren and Du meridians on human body in traditional Chinese medicine contributing an example of G-flows or generally, continuity flows by viewing the *Ying Qi* and

Wei Qi on meridians to be vital energy ([24]-[27], [33]).

All terminologies and notations not defined in this paper are standard such as those of algebra, complex systems, functional analysis, topology are referred to [3]-[5] and [34], topological graph is referred to [6]-[7], [15] and Smarandache multispace are referred to [14] and [36].

§2. Mathematical Reality

A mathematical reality on thing T is such an abstraction of T with a priori assumption that its evolution can be modeled consistent with the symbol behavior in a mathematical system. Usually, it is characterized by solvable equation in the classic. However, it should be not a solvable but non-solvable system, i.e., Smarandachely denied system or Smarandache multispace by Godel's incompleteness theorem on formal system, which concludes that *there exist always statements in a formal system S that can neither be proved nor disproved so long as S contains the Peano's axioms of arithmetic*, namely it can be only characterized by non-solvable system of solvable equations, i.e., *non-harmonious group* defined following.

Definition 2.1([29]) *A non-harmonious group $\mathcal{S}$ is such a system $\mathcal{S}$ consisting of elements P_i, $1 \leq i \leq m, m \geq 2$ with interactions that P_i is constrained on a system of equations*

$$(ES_m)\begin{cases} \mathscr{F}_{P_1}(\mathbf{x}, \mathbf{y}) = 0 \\ \mathscr{F}_{P_2}(\mathbf{x}, \mathbf{y}) = 0 \\ \cdots\cdots\cdots\cdots \\ \mathscr{F}_{P_m}(\mathbf{x}, \mathbf{y}) = 0 \end{cases} \tag{2.1}$$

at time t, where $\mathscr{F}_i(\mathbf{x}^0, \mathbf{y}^0) = 0$ and $\mathscr{F}_i$ satisfies the existence condition of implicit function theorem in a neighborhood U of point $(\mathbf{x}^0, \mathbf{y}^0)$ in Euclidean space $\mathbb{R}^n$ for integers $1 \leq i \leq m$.

Notice that each function of $\mathscr{F}_{v_1}, \mathscr{F}_{v_2}, \cdots, \mathscr{F}_{v_m}$ in equation (2.1) satisfying the condition of implicit function theorem. There must be a solution manifold $S_{\mathscr{F}_i} \subset \mathbb{R}^n$ with $\mathscr{F}_i : S_{\mathscr{F}_i} \to 0$ for integer $1 \leq i \leq m$. Then, the system (2.1) has no solution or has a solution is because of

$$\bigcap_{i=1}^{m} S_{\mathscr{F}_i} = \emptyset \quad \text{or} \quad \bigcap_{i=1}^{m} S_{\mathscr{F}_i} \neq \emptyset. \tag{2.2}$$

geometrically. So, *what is the meaning of system* (2.1) *has or has no solution?* The answer is that the solution shows the overlap state of elements $P_1, P_2, \cdots, P_m$ at time t, not the state of elements $P_1, P_2, \cdots, P_m$ because the behavior of element P_i is the solution manifold $S_{\mathscr{F}_i}$ for integer $1 \leq i \leq m$. Accordingly, the non-solvable case of system (2.1) indicates only that there is no overlap state in elements $P_1, P_2, \cdots, P_m$, not implies the state of P_i existing or not because its state is characterized by the solution manifold $S_{\mathscr{F}_i}$ for integers $1 \leq i \leq m$.

And so, *how to characterize the group behavior of $P_1, P_2, \cdots, P_m$?* the answer should be the union $\bigcup_{i=1}^{m} S_{\mathscr{F}_i}$ or Smarandache multispace on solution manifolds $S_{\mathscr{F}_i}, 1 \leq i \leq m$, not the intersection $\bigcap_{i=1}^{m} S_{\mathscr{F}_i}$ in classical mathematics for non-harmonious group $\mathcal{S}$. In other words, the solution of system (2.1) can be only applied to the recognition of thing T if all element states are the same in evolving, holds with $S_{\mathscr{F}_i} = S_{\mathscr{F}_j}, 1 \leq i, j \leq m$. It is worth noted that $\bigcup_{i=1}^{m} S_{\mathscr{F}_i} \subset \mathbb{R}^n$ is a union that characterizes the state of elements to some extent but still not completely the state of elements in group S. Then, *how should we characterize the state of group S in this case?* The answer is the G-*solution* of equations in system (2.1).

Definition 2.2([29]) *For any integer $m \geq 1$, the G-solution of system* (2.1) *on non-harmonious group S is a labeled graph G^L with vertex and edge sets defined by*

$$V\left(G^L\right) = \{S_{\mathscr{F}_i},\ 1 \leq i \leq m\},$$
$$E\left(G^L\right) = \left\{\ (S_{\mathscr{F}_i}, S_{\mathscr{F}_j}) \mid if\ S_{\mathscr{F}_i} \bigcap S_{\mathscr{F}_j} \neq \emptyset\ for\ integers\ 1 \leq i, j \leq m\ \right\}$$

and labels on vertices and edges of G by

$$L:\ S_{\mathscr{F}_i} \to S_{\mathscr{F}_i}, \quad (S_{\mathscr{F}_i}, S_{\mathscr{F}_j}) \to S_{\mathscr{F}_i} \bigcap S_{\mathscr{F}_j}, \quad 1 \leq i \neq j \leq m.$$

Such a G-solution $\bigcup_{i=1}^{m} S_{\mathscr{F}_i}$ is called a *combinatorial manifold* in geometry ([9]). Notice that the case of $\bigcap_{i=1}^{m} S_{\mathscr{F}_i} = \emptyset$ is meaninglessness in classical mathematics because it includes contradiction. However, it is mostly due to the overlap of element states rather than the non-existence of state of elements, namely it is more meaningful to study the G-solution of non-harmonious group S than that of classical one and get

Theorem 2.3([29]) *For any integer $m \geq 1$, a G-solution G^L of system* (2.1) *on a non-harmonious group S is always existing.*

Generally, we can apply G-solution to discuss respectively those of non-solvable systems of algebraic equations, ordinary differential equations and partial differential equations for characterizing the states of non-harmonious groups with stability of the system, see [11]-[18] for details. For example, let (LEq_m^1), $(LDES_m^1)$ be respectively a non-solvable system of linear equations and ordinary differential equations, i.e.,

$$(LEq_m^1)\ \ AX = B, \qquad (LDES_m^1)\begin{cases} \dot{X} = A_1X \\ \dot{X} = A_2X \\ \cdots\cdots\cdots \\ \dot{X} = A_mX \end{cases}$$

as examples, where matrixes $A = (a_{ij})_{m\times n}$, $A_k = \left(a_{ij}^k\right)_{m\times n}$, $X = (x_1, x_2, \cdots, x_n)^T$, $B = (b_1, b_2, \cdots, b_m)^T$ and $a_{ij}, a_{ij}^{[k]}, b_i$ are real numbers for integers $1 \leq i \leq m$, $1 \leq j \leq n$. For any integers $1 \leq i, j \leq m, i \neq j$, two linear equations

$$\begin{aligned} a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n &= b_i, \\ a_{j1}x_1 + a_{j2}x_2 + \cdots + a_{jn}x_n &= b_j \end{aligned}$$

are called *parallel* if there exists a constant c such that

$$c = \frac{a_{j1}}{a_{i1}} = \frac{a_{j2}}{a_{i2}} = \cdots = \frac{a_{jn}}{a_{in}} \neq \frac{b_j}{b_i}, \tag{2.3}$$

which is essentially the condition of parallel planes in Euclidean space $\mathbb{R}^n$. Now, let L_i be the ith linear equation in (LEq_m^1). We classify these equations $L_i,\ 1 \leq i \leq m$ to *parallel families*

$$\mathscr{C}_1, \mathscr{C}_2, \cdots, \mathscr{C}_s \tag{2.4}$$

with the maximal property, i.e., all linear equations in family $\mathscr{C}_i$ are parallel and there are no other equations parallel to lines in $\mathscr{C}_i$, $|\mathscr{C}_i| = n_i$ for integers $1 \leq i \leq s$. Then, the linear algebraic non-solvable system (LEq_m^1) can be easily characterized.

Theorem 2.4([16]) *Let (LEq_m^1) be a linear equation system for integers $m, n \geq 1$. Then*

$$G[LEq_m^1] \simeq K_{n_1, n_2, \cdots, n_s} \tag{2.5}$$

with $n_1 + n + 2 + \cdots + n_s = m$, where $\mathscr{C}_i$ is the parallel family with $n_i = |\mathscr{C}_i|$ for integers $1 \leq i \leq s$ in (LEq_m^1) and the system (LEq_m^1) is non-solvable if $s \geq 2$.

Particularly, if $n = 2$, let H be a planar graph with each edge of straight segment on $\mathbb{R}^2$. Define its *c-line graph* $L_C(H)$ by

$V(L_C(H)) = \{$straight lines $L = e_1 e_2 \cdots e_l, s \geq 1$ in $H\}$;

$E(L_C(H)) = \{(L_1, L_2)|$ if e_i^1 and e_j^2 are adjacent in H *for* $L_1 = e_1^1 e_2^1 \cdots e_l^1,\ L_2 = e_1^2 e_2^2 \cdots e_s^2,\ l, s \geq 1\}$.

For example, a planar graph H with its c-line graph $L_c(H)$ is shown in Figure 8.

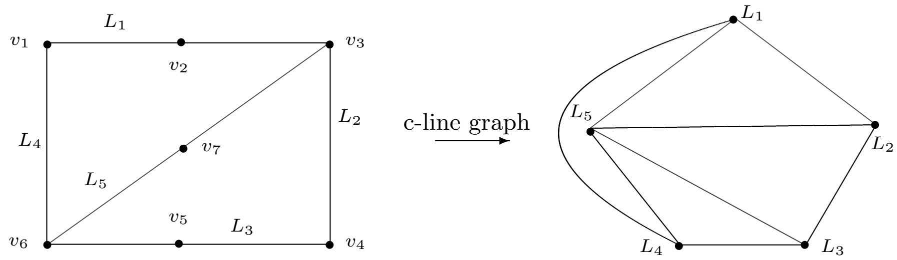

Figure 8

And so, the non-solvable system (LEq_m^1) of linear equations can also be characterized by c-line graph $L_c(H)$, i.e.,

Theorem 2.5([16]) *If $n = 2$, a linear equation system (LEq_m^1) is non-solvable if and only if*

$$G[LEq_m^1] \simeq L_C(H)), \tag{2.6}$$

where H is a planar graph of order $|H| \geq 2$ on $\mathbb{R}^2$ with each edge a straight segment

Notice that the solution of differential equation in system $(LDES_m^1)$ is a linear spaces spanned by its basic solutions. Thus, we can label each vertex of G-solution to get a *basis graph* of $(LDES_m^1)$ by its base.

Theorem 2.6([17]) *Let H be a basis graph. Then, there is a unique linear homogeneous* *differential equation system* $(LDES_m^1)$ *with G-solution H.*

For example, let $(LDES_6^1)$ be a system of homogeneous differential equations $(1)-(6)$. It is easily to know the bases of $(1)-(6)$ are respectively

$$\{e^t, e^{2t}\}, \{e^{2t}, e^{3t}\}, \{e^{3t}, e^{4t}\}, \{e^{4t}, e^{5t}\}, \{e^{5t}, e^{6t}\}, \{e^{6t}, e^t\}$$

with a basis graph $G^L[LDES_6^1]$ shown in Figure 9.

$$(LDES_6^1)\begin{cases} \ddot{x}-3\dot{x}+2x=0 & (1)\\ \ddot{x}-5\dot{x}+6x=0 & (2)\\ \ddot{x}-7\dot{x}+12x=0 & (3)\\ \ddot{x}-9\dot{x}+20x=0 & (4)\\ \ddot{x}-11\dot{x}+30x=0 & (5)\\ \ddot{x}-7\dot{x}+6x=0 & (6)\end{cases}$$

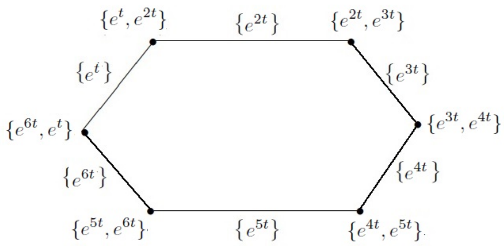

Figure 9

Furthermore, two linear spaces are isomorphic if and only if they have the same dimension. We can replace each basis by its dimension, obtain a basis graph $G^L[LDES_m^1]$ of systems $(LDES_m^1)$ labeled with integers, called an *integral graph* and then, classify systems $(LDES_m^1)$ of linear differential equations by integral graphs.

Theorem 2.7([17]) *Let* $(LDES_m^1)$, $(LDES_m^1)'$ *be two systems of linear homogeneous differential equation with integral graphs* H, H'*, respectively. Then* $(LDES_m^1) \overset{\mathcal{L}}{\simeq} (LDES_m^1)'$ *if and only if the integral graph* $H = H'$.

Similarly, let $(PDES_m^1)$ be a system of partial differential equations with

$$\begin{cases} F_1(x_1, x_2, \cdots, x_n, u, u_{x_1}, \cdots, u_{x_n}, u_{x_1x_2}, \cdots, u_{x_1x_n}, \cdots) = 0\\ F_2(x_1, x_2, \cdots, x_n, u, u_{x_1}, \cdots, u_{x_n}, u_{x_1x_2}, \cdots, u_{x_1x_n}, \cdots) = 0\\ F_m(x_1, x_2, \cdots, x_n, u, u_{x_1}, \cdots, u_{x_n}, u_{x_1x_2}, \cdots, u_{x_1x_n}, \cdots) = 0\end{cases}$$

on a function $u(x_1, \cdots, x_n, t)$. Then, its *symbol* is determined by

$$\begin{cases} F_1(x_1, x_2, \cdots, x_n, u, p_1, \cdots, p_n, p_1p_2, \cdots, p_1p_n, \cdots) = 0\\ F_2(x_1, x_2, \cdots, x_n, u, p_1, \cdots, p_n, p_1p_2, \cdots, p_1p_n, \cdots) = 0\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ F_m(x_1, x_2, \cdots, x_n, u, p_1, \cdots, p_n, p_1p_2, \cdots, p_1p_n, \cdots) = 0,\end{cases}$$

i.e., substitute $p_1^{\alpha_1}, p_2^{\alpha_2}, \cdots, p_n^{\alpha_n}$ into $(PDES_m^1)$ for the term $u_{x_1^{\alpha_1}x_2^{\alpha_2}\cdots x_n^{\alpha_n}}$, where $\alpha_i \geq 0$ for integers $1 \leq i \leq n$ and a non-solvable system $(PDES_m^1)$ is *algebraically contradictory* if its symbol is non-solvable. Otherwise, *differentially contradictory.* Then, we can characterize the G-solution of non-solvable systems of partial differential equations of first order, which form a non-harmonious groups and accordingly, to the reality of things.

Theorem 2.8([20]) *A Cauchy problem on systems*

$$\begin{cases} F_1(x_1, x_2, \cdots, x_n, u, p_1, p_2, \cdots, p_n) = 0 \\ F_1(x_1, x_2, \cdots, x_n, u, p_1, p_2, \cdots, p_n) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ F_m(x_1, x_2, \cdots, x_n, u, p_1, p_2, \cdots, p_n) = 0 \end{cases}$$

of partial differential equations of first order is non-solvable with initial values

$$\begin{cases} x_i|_{x_n=x_n^0} = x_i^0(s_1, s_2, \cdots, s_{n-1}) \\ u|_{x_n=x_n^0} = u_0(s_1, s_2, \cdots, s_{n-1}) \\ p_i|_{x_n=x_n^0} = p_i^0(s_1, s_2, \cdots, s_{n-1}), \quad i = 1, 2, \cdots, n \end{cases}$$

if and only if the system

$$F_k(x_1, x_2, \cdots, x_n, u, p_1, p_2, \cdots, p_n) = 0, \ 1 \leq k \leq m$$

is algebraically contradictory, in this case there must be an integer $k_0, \ 1 \leq k_0 \leq m$ *such that*

$$F_{k_0}(x_1^0, x_2^0, \cdots, x_{n-1}^0, x_n^0, u_0, p_1^0, p_2^0, \cdots, p_n^0) \neq 0 \qquad (2.7)$$

or it is differentially contradictory itself, i.e., there is an integer $j_0, \ 1 \leq j_0 \leq n-1$ *such that*

$$\frac{\partial u_0}{\partial s_{j_0}} - \sum_{i=0}^{n-1} p_i^0 \frac{\partial x_i^0}{\partial s_{j_0}} \neq 0. \qquad (2.8)$$

Theorem 2.9([20]) *A Cauchy problem on system* $(PDES_m^1)$ *of partial differential equations of first order with initial values* $x_i^{[k^0]}, u_0^{[k]}, p_i^{[k^0]}, \ 1 \leq i \leq n$ *for the* k*th equation in* $(PDES_m^1)$*,* $1 \leq k \leq m$ *such that*

$$\frac{\partial u_0^{[k]}}{\partial s_j} - \sum_{i=0}^{n} p_i^{[k^0]} \frac{\partial x_i^{[k^0]}}{\partial s_j} = 0 \qquad (2.9)$$

is uniquely G*-solvable, i.e.,* $G^L[PDES]$ *is uniquely determined.*

Now, *what is the role of* G*-solution on non-harmonious groups* S*?* Its role is on the global stability of non-harmonious group. Usually, a solution of system (ES_m) of differential equations is called stable or asymptotically stable if for all solutions $Y(t)$ of the differential equations (ES_m) with $|Y(0) - X(0)| < \delta(\varepsilon)$, exists $|Y(t) - X(t)| < \varepsilon$ for $\forall\varepsilon > 0$ and $t \geq 0$ or furthermore, $\lim_{t\to 0} |Y(t) - X(t)| = 0$. However, if $\bigcap_{i=1}^{m} S_{T_i} = \emptyset$ there are no solutions of (ES_m), the classical theory is failed to apply. By Theorem 2.3, any non-harmonious groups S has a G-solution $G^L[ES_m]$ of system (ES_m) whenever it is solvable or not, namely the G-solution $G^L[ES_m]$ can be used to characterize the stability of system (ES_m).

Let $G^L(t)$ be a G-solution of (ES_m) with initial values $G^L(t_0)$ and let $\omega : V\left(G^L[ES_m]\right) \to$

$\mathbb{R}$ be a functional. A system (ES_m) is said to be *ω-stable* if there exists a number $\delta(\varepsilon)$ for any number $\varepsilon > 0$ such that

$$\left\|\omega\left(G^{L_1(t)-L_2(t)}\right)\right\| < \varepsilon \tag{2.10}$$

or furthermore, *asymptotically ω-stable* if

$$\lim_{t\to\infty}\left\|\omega\left(G^{L_1(t)-L_2(t)}\right)\right\| = 0 \tag{2.11}$$

if the initial values hold with $\|L_1(t_0)(v) - L_2(t_0)(v)\| < \delta(\varepsilon)$ for $\forall v \in V\left(\overrightarrow{G}\right)$. In this case, if there is a Liapunov ω-function $L(\omega(t)) : \mathscr{O} \to \mathbb{R}, n \geq 1$ on $\overrightarrow{G}$ with open $\mathscr{O} \subset \mathbb{R}^n$ such that $L(\omega(t)) \geq 0$ with equality hold only if $(x_1, x_2, \cdots, x_n) = (0, 0, \cdots, 0)$ and $\dot{L}(\omega(t)) < 0$ if $t \geq t_0$. Denoted by $\mathbf{O}$ the zero G-solution of system (ES_m), i.e., all vertices and edges on $G^L[ES_m]$ are labeled by $\mathbf{0}$, we get a result on ω-stability of (ES_m) following.

Theorem 2.10([21]) *If there is a Liapunov ω-function $L(\omega(t)) : \mathscr{O} \to \mathbf{R}$ on $G^L[ES_m]$ of system (ES_m), then it is ω-stable, and furthermore, if $\dot{L}(\omega(t)) < 0$ for $G^L[ES_m] \neq \mathbf{O}$, then it is asymptotically ω-stable.*

For a linear system $(LDES_m^1)$ of differential equations, we can further introduce the sum-stable and prod-stable on $(LDES_m^1)$, i.e., a system $G^L[LDES_m^1]$ is *sum-stable* or *asymptotically sum-stable* if for all solutions $Y_v(t)$, $v \in V(G^L)$ of linear differential equations in $(LDES_m^1)$ with $|Y_v(0) - X_v(0)| < \delta_v$ exists

$$\left|\sum_{v\in V(H^L)} Y_v(t) - \sum_{v\in V(H^L)} X_v(t)\right| < \varepsilon \tag{2.12}$$

for all $t \geq 0$ or furthermore,

$$\lim_{t\to 0}\left|\sum_{v\in V(H^L)} Y_v(t) - \sum_{v\in V(H^L)} X_v(t)\right| = 0 \tag{2.13}$$

and *prod-stable* or *asymptotically prod-stable* if for all solutions $Y_v(t)$, $v \in V(G)$ of $(LDES_m^1)$ with $|Y_v(0) - X_v(0)| < \delta_v$ exists

$$\left|\prod_{v\in V(G)} Y_v(t) - \prod_{v\in V(G)} X_v(t)\right| < \varepsilon \tag{2.14}$$

for all $t \geq 0$, or furthermore,

$$\lim_{t\to 0}\left|\prod_{v\in V(G)} Y_v(t) - \prod_{v\in V(G)} X_v(t)\right| = 0. \tag{2.15}$$

We get criterions on sum-stable and prod-stable of the linear system $(LDES_m^1)$ following.

Theorem 2.11([17]) *A zero* **O***-solution of system* $(LDES_m^1)$ *of linear homogenous differential equation is asymptotically sum-stable if and only if* $\text{Re}\alpha_v < 0$ *for each* $\overline{\beta}_v(t)e^{\alpha_v t} \in \mathscr{B}_v$ *with vertex* $v \in G^L[LDES_m^1]$.

Theorem 2.12([17]) *A zero* **O***-solution of systems* $(LDES_m^1)$ *of linear homogenous differential equation is asymptotically prod-stable if and only if*

$$\sum_{v\in V(G)} \text{Re}\alpha_v \ < \ 0 \tag{2.16}$$

for each $\overline{\beta}_v(t)e^{\alpha_v t} \in \mathscr{B}_v$ *with vertex* $v \in G^L[LDES_m^1]$.

Similarly, we can also discuss non-solvable systems of differential equations by linearizing its non-linear differential parts, get criterions on the global stability of non-linear differential equations and then, apply to the stability of system (ES_m) of differential equations. For example, the stability of food web in biological systems. Notice that a food web is a complex network of interconnecting and overlapping food chains, i.e., "*eating or being eaten*" among various organisms within an ecosystem and it is more suitable characterized by labeled directed graphs $\overrightarrow{G}^L$ with by Kolmogorov model.

Theorem 2.13([22]) *A food web* $\overrightarrow{G}^L$ *with initial value* $\overrightarrow{G}^{L_0}$ *is globally stable or asymptotically stable if and only if there is an Eulerian multi-decomposition*

$$\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right)^{\widehat{L}} = \bigoplus_{i=1}^{s} \overrightarrow{H}_i^L \tag{2.17}$$

with solvable stable or asymptotically stable conservative equations on labeling Eulerian subgraphs $\overrightarrow{H}_i^L$ *for integers* $1 \leq i \leq s$, *where* $\overleftarrow{G}$ *is the digraph reversing orientation on every edge in* $\overrightarrow{G}$, $\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right)^{\widehat{L}}$ *is a labeled graph with labeling* $\widehat{L} : V(\overrightarrow{G}\bigcup\overleftarrow{G}) = L\left(V(\overrightarrow{G})\right)$ *and* $\widehat{L} : E\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right) \to L\left(E\left(\overrightarrow{G}\bigcup\overleftarrow{G}\right)\right)$ *by* $\widehat{L}:\ (u,v) \to \{0,(x,y),yf'\},\ (v,u) \to \{xf,(x,y),0\}$ *if* $L : (u,v) \to \{xf,(x,y),yf'\}$ *for* $\forall(u,v) \in E(\overrightarrow{G})$ *such as those shown in Figure 10.*

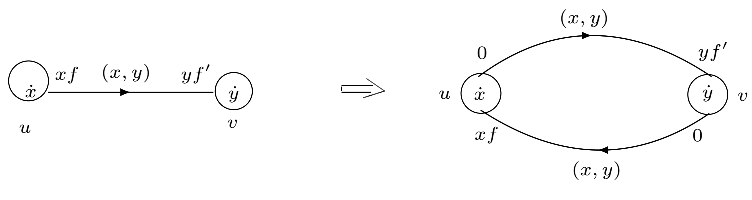

Figure 10

§3. Mathematical Combinatorics

A G-solution $G^L[ES_m]$ of system (ES_m) characterize the state of non-harmonious group $\mathcal{S}$ without direction on edges, i.e., not the action within itself. However, all things are in constantly moving and evolved by their internal action under the external with elements moving in thing, which is only in one-way. Whence, we generalize G-solution $G^L[ES_m]$ to a directed situation for modeling substance flow on the evolution of thing T, holding with conservation at each vertex, i.e., a generalization of network N to continuity flow, which is vectors in Banach space over a topological graph $\overrightarrow{G}$ with end-operator actions, i.e., *mathematical combinatorics*.

Definition 3.1([23]) *A continuity flow* $\left(\overrightarrow{G}; L, \mathscr{A}\right)$ *is an oriented topological graph* $\overrightarrow{G}^L$ *in space* $\mathscr{S}$ *associated with a mapping* $L : v \to L(v),\ (v,u) \to L(v,u)$, *2 end-operators* $A^+_{vu} \in \mathscr{A} : L(v,u) \to L^{A^+_{vu}}(v,u)$ *and* $A^+_{uv} \in \mathscr{A} : L(u,v) \to L^{A^+_{uv}}(u,v)$ *on a Banach space* $\mathscr{B}$ *over a field* $\mathscr{F}$ *such as those shown in Figure* 11 *with* $L(v,u) = -L(u,v)$, $A^+_{vu}(-L(v,u)) = -L^{A^+_{vu}}(v,u)$ *for* $\forall (v,u) \in E(\overrightarrow{G}^L)$ *and meanwhile, holding with the continuity equation*

$$\sum_{u \in N^-_G(v)} L^{A^+_{uv}}(u,v) - \sum_{u \in N^+_G(v)} L^{A^+_{uv}}(u,v) \ = \ L(v) \tag{3.1}$$

at any vertex $v \in V(\overrightarrow{G}^L)$ *of topological graph* $\overrightarrow{G}^L$, *where* $N^-_G(v), N^+_G(v)$ *are respectively the in-neighborhood and out-neighborhood of vertex* $v \in V(\overrightarrow{G}^L)$, *namely all vertices in* $N^-_G(v) \subset N_G(v)$ *or* $N^+_G(v) \subset N_G(v)$ *flow into or out of the vertex* v *and* $N^-_G(v) \cup N^+_G(v) = N_G(v)$.

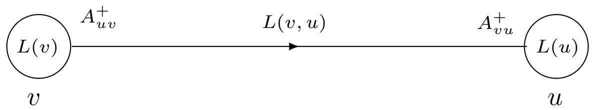

Figure 11

Notice that the continuity equations on vertices of $\overrightarrow{G}^L$ form a non-solvable system (ES_m) of equations. For example, let the $L : (v,u) \to L(v,u) \in \mathbb{R}^n \times \mathbb{R}^+$ with end-operators $A^+_{vu} = a_{vu}\dfrac{\partial}{\partial t}$ and $a_{vu} :\ \mathbb{R}^n \to \mathbb{R}$ for any edge $(v,u) \in E\left(\overrightarrow{G}\right)$ in Figure 12 following.

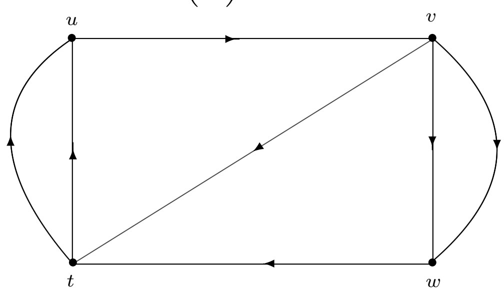

Figure 12

Then, the continuity equations on vertices of $\overrightarrow{G}^L$ are partial differential equations

$$\left\{\begin{array}{l}
a_{tu^1}\dfrac{\partial L(t,u)^1}{\partial t} + a_{tu^2}\dfrac{\partial L(t,u)^2}{\partial t} = a_{uv}\dfrac{\partial L(u,v)}{\partial t}\\
a_{uv}\dfrac{\partial L(u,v)}{\partial t} = a_{vw^1}\dfrac{\partial L(v,w)^1}{\partial t} + a_{vw^2}\dfrac{\partial L(v,w)^2}{\partial t} + a_{vt}\dfrac{\partial L(v,t)}{\partial t}\\
a_{vw^1}\dfrac{\partial L(v,w)^1}{\partial t} + a_{vw^2}\dfrac{\partial L(v,w)^2}{\partial t} = a_{wt}\dfrac{\partial L(w,t)}{\partial t}\\
a_{wt}\dfrac{\partial L(w,t)}{\partial t} + a_{vt}\dfrac{\partial L(v,t)}{\partial t} = a_{tu^1}\dfrac{\partial L(t,u)^1}{\partial t} + a_{tu^2}\dfrac{\partial L(t,u)^2}{\partial t}
\end{array}\right.$$

Usually, a continuity flow $(\overrightarrow{G}; L, \mathscr{A})$ is abbreviated to $\overrightarrow{G}^L$ for simplicity, which is a generalization of G-solution of non-harmonious group $\mathcal{S}$ and substance flow in physics, a more accurate model on the reality of thing T and includes most mathematical models on thing T. For example, if $L(v) = \dot{x}_v$, $v \in V(\overrightarrow{G})$, a continuity flow $\overrightarrow{G}^L$ is a *complex flow* ([23]); if x_v is a constant $\mathbf{v}_v$ dependent on v for $v \in V(\overrightarrow{G})$, a continuity flow $\overrightarrow{G}^L$ is an *action flow* ([21]); if $\mathscr{A} = \mathbb{Z}$ or $\mathbb{C}$, particularly, $\mathscr{A} = \mathbf{1}_{\mathscr{V}}$, a continuity flow $\overrightarrow{G}^L$ is $\overrightarrow{G}$*-flow* ([19]) and if $\mathscr{A} = \{\mathbf{1}_{\mathscr{B}}\}$ and $\mathscr{B}$ is the number field $\mathbb{Z}$ or $\mathbb{R}$, a continuity flow $\overrightarrow{G}^L$ is *complex network* ([4]), which was discussed extensively in complex science.

Now, *could we really establish mathematics on continuity flows $\overrightarrow{G}^L$ by view it as a mathematical element?* The answer is definite by considering $\overrightarrow{G}^L$ to be a family of vectors underlying a topological graph $\overrightarrow{G}$ with addition, multiplication and scalar multiplication for continuity flows $\overrightarrow{G}^L, \overrightarrow{G'}^{L'}$ and $\lambda \in \mathscr{F}$ defined by

$$G^L + G'^{L'} = (G \setminus G')^L \bigcup \left(G \bigcap G'\right)^{L+L'} \bigcup (G' \setminus G)^{L'}, \tag{3.2}$$

$$G^L \cdot G'^{L'} = (G \setminus G')^L \bigcup \left(G \bigcap G'\right)^{L\cdot L'} \bigcup (G' \setminus G)^{L'}, \tag{3.3}$$

$$\lambda \cdot G^L = G^{\lambda \cdot L}. \tag{3.4}$$

where, for any vertex $v \in V(G)$ and edge $(v,u) \in E(G)$, $L(v), L'(v), L(v,u), L'(v,u) \in \mathscr{B}$, $L + L' : v \to L(v) + L'(v)$, $(v,u) \to L(v,u) + L'(v,u)$, $L \cdot L' : v \to L(v) \cdot L'(v)$, $(v,u) \to L(v,u) \cdot L'(v,u)$, $\lambda \cdot L : v \to \lambda \cdot L(v)$, $(v,u) \to \lambda \cdot L(v,u)$, $L(v) \cdot L'(v)$ and $L(v,u) \cdot L'(v,u)$ denotes the *Hadamard product* of vectors in Banach space $\mathscr{B}$, namely

$$(x_1, x_2, \cdots, x_n) \cdot (y_1, y_2, \cdots, y_n) = (x_1y_1, x_2y_2, \cdots, x_ny_n). \tag{3.5}$$

such as those shown for addition and scalar multiplication in Figure 13.

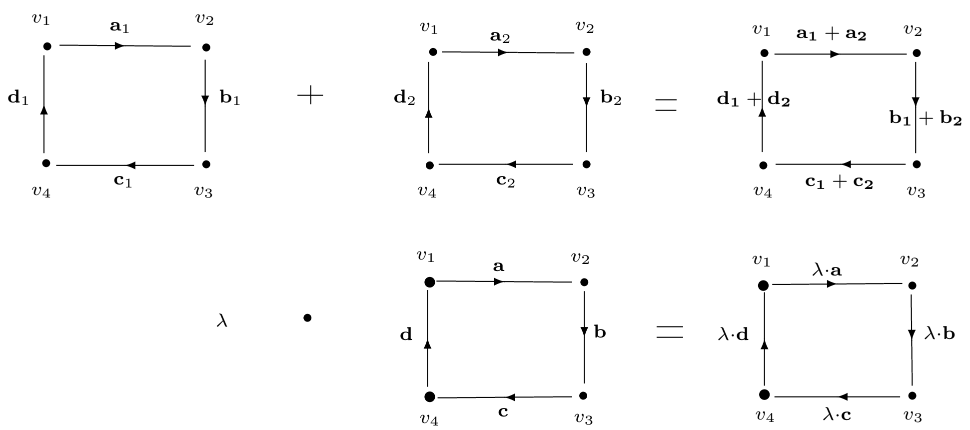

Figure 13

Notice that the addition "+", scalar multiplication "·" on vectors $\mathbf{a}, \mathbf{b}$ can be viewed as the operations on a particular continuity flow $\overrightarrow{G}^L$, i.e., path $\overrightarrow{P}^L_{n+1}$, which enables us to establish mathematics on continuity flows $\overrightarrow{G}^L$ on Banach space $\mathscr{B}$ over topological graphs $\overrightarrow{G}$.

3.1. Banach Flow Space

Let $\mathscr{G}$ be a graph family closed under the union operation of graph, $\mathscr{B}$ be a Banach space over field $\mathscr{F}$ and denoted by $\mathscr{G}_{\mathscr{B}}$ all continuity flows $\overrightarrow{G}^L$ with $\overrightarrow{G} \in \mathscr{G}$, $L : V(\overrightarrow{G}) \bigcup E(\overrightarrow{G}) \to \mathscr{B}$. Then, we have

Theorem 3.1([25],[32]) *If $\mathscr{G}$ is a closed family of graphs under the union operation and $\mathscr{B}$ a linear space $(\mathscr{B}; +, \cdot)$, then, all continuity flows $(\mathscr{G}_{\mathscr{B}}; +, \cdot)$ is a linear space, and furthermore, a commutative ring if $\mathscr{B}$ is a commutative ring $(\mathscr{B}; +, \cdot)$ over a field $\mathscr{F}$.*

Assume all end-operators are continuous linear operators in $\mathscr{A}$ and define the norm of a continuity flow $\overrightarrow{G}^L$ by

$$\left\| \overrightarrow{G}^L \right\| = \sum_{(v,u) \in E\left(\overrightarrow{G}\right)} \left\| L^{A^+_{vu}}(v,u) \right\|, \tag{3.6}$$

where $\|\cdot\|$ is the norm on Banach space $\mathscr{B}$. Then, we can verify the non-negative, homogeneity and the triangle inequality hold with $\mathscr{G}_{\mathscr{B}}$ and the non-negative, conjugacy and the linearity if $\mathscr{B}$ is further a Hilbert space, i.e.,

Theorem 3.2([25],[32]) *If $\mathscr{G}$ is a closed family of graphs under the union operation and $\mathscr{B}$ a Banach space $(\mathscr{B}; +, \cdot)$, then, $\mathscr{G}_{\mathscr{B}}$ with linear operators A^+_{vu}, A^+_{uv} for $\forall (v,u) \in E\left(\bigcup\limits_{G \in \mathscr{G}} \overrightarrow{G}\right)$ is a Banach space, and furthermore, $\mathscr{G}_{\mathscr{B}}$ is a Hilbert space if $\mathscr{B}$ is a Hilbert space.*

3.2. G-Isomorphic Operators on Banach Flow Space

Let $G_1^{L_1}, G_2^{L_2} \in \mathscr{G}_{\mathscr{B}}$ be continuity flows. Usually, a mapping $f : G_1^{L_1} \to G_2^{L_2}$ is said to be a *G-isomorphic operator* between continuity flows $G_1^{L_1}$, $G_2^{L_2}$ and the continuity flow $G_1^{L_1}$ is said to be G-isomorphic to $G_2^{L_2}$ if G_1, G_2 are isomorphic in graphs, i.e., there is an isomorphism $\varphi : G_1 \to G_2$ of graph and $L_2 = f \circ \varphi \circ L_1$ for $\forall (v,u) \in E(G_1)$, i.e., $G_1^{L_1}, G_2^{L_2}$ are isomorphic of labeled graphs. Furthermore, we conventionalize that $\widehat{G}^{\widehat{L}} = G^L$ for a topological graph $\widehat{G} \supset G$ if $\widehat{L}(x) = L(x)$ for $x \in V(G) \cup E(G)$ and $\widehat{L}(x) = \mathbf{0}$ for $x \notin V(G) \cup E(G)$, which reflects the essence of continuity flow. And by this convention, a $\widehat{G}$-isomorphism between continuity flows $G_1^{L_1}, G_2^{L_2}$ can be generally defined even if $G_1^{L_1}, G_2^{L_2}$ are non-isomorphic but with a supergraph $\widehat{G}$ as $\widehat{G} \supseteq G_1 \bigcup G_2$, a G-isomorphic operator can be generally defined by

Definition 3.3([29]) *A mapping $f : G_1^{L_1} \to G_2^{L_2}$ is a G-isomorphic operator between continuity flows $G_1^{L_1}$ and $G_2^{L_2}$ if*

(1) *there is an isomorphism $\varphi : \widehat{G} \to \widehat{G}$ with $\widehat{G} \supset G_1, G_2$ in graph;*

(2) *for $\forall (v,u) \in E(G_1)$ there is $L_2 = f \circ \varphi \circ L_1$ but for $\forall (v,u) \in E(G_2 \setminus G_1)$, $f : \mathbf{0} \to L_2(v,u)$ and for $\forall (v,u) \in E(G_1 \setminus G_2)$ and $\forall (v,u) \in E(\widehat{G} \setminus (G_1 \bigcup G_2))$, $f : L(v,u) \to \mathbf{0}$.*

Notice that a G-isomorphic operator $f : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}'}$ is naturally commutative with end-operators in $\mathscr{A}$ on edges of continuity flows $\overrightarrow{G}^L \in \mathscr{G}_{\mathscr{B}}$ by definition. Let $G_1^{L_1}$, $G_2^{L_2} \in \mathscr{G}_{\mathscr{B}}$ be two continuity flows with scalars $\lambda, \mu \in \mathscr{F}$. Then, a G-isomorphic operator $f : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}'}$ is *linear* if

$$f\left(\lambda G_1^{L_1} + \mu G_2^{L_2}\right) = \lambda f\left(G_1^{L_1}\right) + \mu f\left(G_2^{L_2}\right), \tag{3.7}$$

is *continuous* at continuity flow $G_0^{L_0}$ if for any number $\varepsilon > 0$ there always exists a real number $\delta(\varepsilon)$ such that

$$\left\|{G_1}^{L_1} - G_0^{L_0}\right\| < \delta(\varepsilon) \ \Rightarrow \ \left\|f\left(G^L\right) - f\left(G_0^{L_0}\right)\right\| < \varepsilon \tag{3.8}$$

and is *bounded* if there exists a constant $\xi \in [0, \infty)$ such that $\|f(G^L)\| \leq \xi\|G^L\|$ for any continuity flow $G^L \in \mathscr{G}_{\mathscr{B}}$. Furthermore, if

$$\left\|f\left(G_1^{L_1}\right) - f\left(G_2^{L_2}\right)\right\| \leq \xi\left\|G_1^{L_1} - G_2^{L_2}\right\|, \quad \xi \in [0, 1) \tag{3.9}$$

for two continuity flows $G_1^{L_1}, G_2^{L_2} \in \mathscr{G}_{\mathscr{B}}$ and a constant ξ, then f is a *contraction* on continuity flow space $\mathscr{G}_{\mathscr{B}}$. And then, we can generalize a few of well-known theorems in classical functional analysis to Banach flow space following.

Theorem 3.4(Fixed Flow Theorem, [25],[29],[32]) *For a continuous G-isomorphic contractor $f : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}'}$ there is only one continuity flow $G^L \in \mathscr{G}_{\mathscr{B}}$ such that $f\left(G^L\right) = G^L$.*

Theorem 3.5(Banach Inverse Theorem, [25],[29],[32]) *A G-isomorphic linear operator $f : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}}$ is continuous if and only if it is bounded and furthermore, if f is $1-1$ then the inverse operator f^{-1} of f is also a G-isomorphic continuous operator.*

For a G-isomorphic operator $f : \mathscr{G}_{\mathscr{B}} \to \mathscr{G}_{\mathscr{B}}$, its image Grap$f$ of is defined by

$$\text{Grap}f \ = \ \left\{\left(\overrightarrow{G}^L, f\left(\overrightarrow{G}^L\right)\right)\middle|\ \overrightarrow{G}^L \in \mathscr{G}_{\mathscr{B}}\right\} \tag{3.10}$$

and f is *closed* if the image Grap$f$ of f is closed.

Theorem 3.6(Closed Graph Theorem, [25],[29],[32]) *If $\mathbf{T} : \mathscr{G}_{\mathscr{B}_1} \to \mathscr{G}_{\mathscr{B}_2}$ is a closed linear operator with Banach spaces $\mathscr{B}_1$, $\mathscr{B}_2$, then $\mathbf{T}$ is continuous.*

Particularly, a G-isomorphic linear operator $f : \mathscr{G}_{\mathscr{B}} \to \mathbb{R}$ or $\mathbb{C}$ is called a *flow functional*, which can be applied to generalize the Hahn-Banach theorem to Banach flow space $\mathscr{G}_{\mathscr{B}}$.

Theorem 3.7(Hahn-Banach Theorem, [25],[29],[32]) *Let $\mathscr{H}_{\mathscr{B}}$ be a subspace of Banach flow space $\mathscr{G}_{\mathscr{B}}$ and let $F : \mathscr{H}_{\mathscr{B}} \to \mathbb{C}$ be a continuous linear flow functional on $\mathscr{H}_{\mathscr{B}}$. Then, there is a continuous linear flow functional $\widetilde{F} : \mathscr{G}_{\mathscr{B}} \to \mathbb{C}$ satisfies the conditions that if $\overrightarrow{G}^L \in \mathscr{H}_{\mathscr{B}}$ then $\widetilde{F}(\overrightarrow{G}^L) = F(\overrightarrow{G}^L)$ and $\|\widetilde{F}\| = \|F\|$.* Particularly, *if $\mathbf{O} \neq \overrightarrow{G}_0^{L_0} \in \mathscr{G}_{\mathscr{B}}$, there is a continuous linear flow functional F such that $\|F\| = 1$ and $\|F(\overrightarrow{G}_0^{L_0})\| = \|\overrightarrow{G}_0^{L_0}\|$.*

Then, *what is the important role of Hahn-Banach theorem in Banach flow space $\mathscr{G}_{\mathscr{B}}$?* Certainly, it can extend a flow functional from a small range to a large one. Furthermore, it convinces us that the subdividing of matter does not affect the validity of quantum hypothesises, i.e., a pure state in quantum mechanics is represented in terms of a normalized vector $|\psi\rangle$ in Hilbert space $\mathcal{H}$ with $\langle\psi|\psi\rangle = 1$, for an observable physical quantity a of quantum there exists a corresponding Hermitian operator H acting on $\mathcal{H}$ and the time evolving of state is governed by Schrödinger equation

$$i\hbar\frac{d\,|\psi\rangle}{dt} = H\,|\psi\rangle\,, \tag{3.11}$$

where $\hbar$ is the Planck's constant. Now, *can we conclude that the existence of Hermitian operator H in a quantum Q with quark structure $\overrightarrow{G}^L$ or a matter T such as the proton and neutron consist of quanta over a topological graph $\overrightarrow{G}^L$?* The answer is certainly Yes by a generalization of Fréchet-Riesz representation theorem in Banach space to Banach flow space $\mathscr{G}_{\mathscr{B}}$.

Theorem 3.8(Fréchet-Riesz Theorem, [25],[29],[32]) *Let $f : \mathscr{G}_{\mathscr{B}} \to \mathbb{C}$ be a continuous linear flow functional. For any continuous flow $G^L \in \mathscr{G}_{\mathscr{B}}$, there uniquely exists a continuous flow of $\widehat{G}^{\widehat{L}} \in \mathscr{G}_{\mathscr{B}}$ holding with $f(\overrightarrow{G}^L) = <\overrightarrow{G}^L, \widehat{G}^{\widehat{L}}>$.*

3.3. Integral and Differential on Continuity Flow

Let the isomorphism $\varphi = \mathrm{id}_G$, i.e., the identity mapping of topological graph $\overrightarrow{G}$ and let $\mathscr{B}$ be a Hilbert space and particularly, a function field on variable $\mathbf{x}$, Then, a G-isomorphic operator f is determined ([29]) by equation

$$L_2(v,u) \ = \ f \circ L_1(v,u), \quad \forall (v,u) \in E(\overrightarrow{G}) \tag{3.11}$$

which is equivalent to

$$f\left(\overrightarrow{G}^L[\mathbf{x}]\right) \ = \ \overrightarrow{G}^{f(L[\mathbf{x}])}. \tag{3.12}$$

Thereby, we can define the power

$$\overrightarrow{G}^{aL}[\mathbf{x}] = \overrightarrow{G}^{aL[\mathbf{x}]}, \quad a^{G^L[\mathbf{x}]} = \overrightarrow{G}^{a^{L[\mathbf{x}]}} \tag{3.13}$$

of continuity flow $\overrightarrow{G}^L[\mathbf{x}]$ for a number $a \in \mathbb{R}$ and respectively, the polynomial, sum and product of continuity flows by

$$\begin{aligned}
a_0 + a_1\overrightarrow{G}^L + a_2\overrightarrow{G}^{L^2} + \cdots + a_n\overrightarrow{G}^{L^n} \ &= \ \overrightarrow{G}^{a_0+a_1L+a_2L^2+\cdots+a_nL^n}, \\
a_1\overrightarrow{G}_1^{L_1} + a_2\overrightarrow{G}_2^{L_2} + \cdots + a_n\overrightarrow{G}_n^{L_n} \ &= \ \left(\bigcup_{i=1}^n \overrightarrow{G}_i\right)^{a_1L_1+a_2L_2+\cdots+a_nL_n}, \\
\left(a_1\overrightarrow{G}_1^{L_1}\right)\cdot\left(a_2\overrightarrow{G}_2^{L_2}\right)\cdots\left(a_n\overrightarrow{G}_n^{L_n}\right) \ &= \ \left(\bigcup_{i=1}^n \overrightarrow{G}_i\right)^{a_1L_1\cdot a_2L_2\cdot\cdots\cdot a_nL_n}.
\end{aligned}$$

Particularly, the 3 interesting exponential identities ([31]) for integer $n \geq 1$ following

$$e^x \ = \ 1 + \frac{x}{1!} + \frac{x^{2]}}{2!} + \cdots + \frac{x^n}{n!} + \cdots, \tag{3.14}$$

$$e^{tA} \ = \ \mathbf{I} + \frac{tA}{1!} + \frac{t^2A^2}{2!} + \cdots + \frac{t^nA^n}{n!} + \cdots, \tag{3.15}$$

$$e^{G^L[\mathbf{x}]} \ = \ \mathbf{I} + \frac{\overrightarrow{G}^L[\mathbf{x}]}{1!} + \frac{\overrightarrow{G}^{2L}[\mathbf{x}]}{2!} + \cdots + \frac{\overrightarrow{G}^{nL}[\mathbf{x}]}{n!} + \cdots, \tag{3.16}$$

where A is an $n \times n$ matrix, $G^L[\mathbf{x}]$ is a continuity flow that continuous in variable $\mathbf{x}$.

Furthermore, we can define the limitation of continuity flow sequence $\{\overrightarrow{G}_i^{L_i}[\mathbf{x}]\}_1^\infty$, differential and integral on continuity flow $\overrightarrow{G}^L[\mathbf{x}]$ by equality (3.12) and also, establish the calculus on $\mathscr{G}_{\mathscr{B}}$. For example, the fundamental theorem

$$\int_a^b f\frac{d}{dt}\left(\overrightarrow{G}^L[t]\right)dt = f\left(\overrightarrow{G}^L[t]\right)\Big|_{t=b} - f\left(\overrightarrow{G}^L[t]\right)\Big|_{t=a} \tag{3.17}$$

can be obtained similar to that of the calculus and introduce the variation on continuity flows $\overrightarrow{G}^L[\mathbf{x}]$, namely assume a G-isomorphic mapping $\mathscr{L}:(v,u)\in E(\overrightarrow{G})\to \mathscr{L}\left[\mathcal{L}(t,\mathbf{x}(t),\dot{\mathbf{x}}(t))(v,u)\right]$ is differentiable and respectively define the action $J[\overrightarrow{G}^{\mathscr{L}}[t]]$ and variation $\delta J[\overrightarrow{G}^{\mathscr{L}}[t]]$ on a continuity flow $\overrightarrow{G}^{\mathscr{L}}[t]$ by

$$J\left[\overrightarrow{G}^{\mathscr{L}}[t]\right] = \left|\int_{t_1}^{t_2}\overrightarrow{G}^{\mathscr{L}[\mathcal{L}(t,\mathbf{x}(t),\dot{\mathbf{x}}(t))]}dt\right|,\quad \delta J\left[\overrightarrow{G}^{\mathscr{L}}[t]\right] = \left|\delta\int_{t_1}^{t_2}\overrightarrow{G}^{\mathscr{L}[\mathcal{L}(t,\mathbf{q}(t),\dot{\mathbf{q}}(t))]}dt\right|, \tag{3.18}$$

where the variation $\delta:\mathscr{G}_{\mathscr{B}}\to\mathscr{G}_{\mathscr{B}}$ is a G-isomorphic operator. And so, the Euler-Lagrange equations

$$\frac{\partial\overrightarrow{G}^{\mathscr{L}}}{\partial q_i} - \frac{d}{dt}\frac{\partial\overrightarrow{G}^{\mathscr{L}}}{\partial\dot{q}_i} = \mathbf{O},\quad 1\le i\le n. \tag{3.19}$$

on continuity flow $\overrightarrow{G}^{\mathscr{L}}[t]$ can be established by the least action principle $\delta J[\overrightarrow{G}^{\mathscr{L}}[t]](v,u)=0$ for $\forall(v,u)\in E(\overrightarrow{G}^{\mathscr{L}}[t])$ with the properties of norm in Banach flow space $\mathscr{G}_{\mathscr{B}}$.

§4. **An Interesting Example**

Although we are all human but it is very hard to answer what a human is unless by behavioral characteristics. A more useful definition on human is by the pair $\{Y^-,Y^+\}$ of Yin and Yang with meridians running the vital energy on body in traditional Chinese medicine, including 12 meridians, i.e., the lung meridian of hand-Taiyin (LU), heart meridian of hand-Shaoyin (HT), pericardium meridian of hand-Jueyin (PC), the spleen meridian of foot-Taiyin (SP), kidney meridian of foot-Shaoyin (KI), liver meridian of foot-Jueyin (LR), large intestine meridian of hand-Yangming (LI), small intestine meridian of hand-Taiyang (SI), Sanjiao (triple burner) meridian of hand-Shaoyang (SJ), stomach meridian of foot-Yangming (ST), bladder meridian of foot Taiyang (BL), gallbladder meridian of foot-Shaoyang (GB), Ren meridian, Du meridian and 671 acupoints such as those shown in Figure 14. All of them connect the five organs and six bowels, communicating the up and down of vital energy of human running with a balanced pair $\{Y^-,Y^+\}$, called to be *Ying Qi* $\mathbf{\Psi}^-$ and *Wei Qi* $\mathbf{\Psi}^+$, i.e., an operating ruler for human body in traditional Chinese medicine, and there must be imbalance acupoints in one of the 12

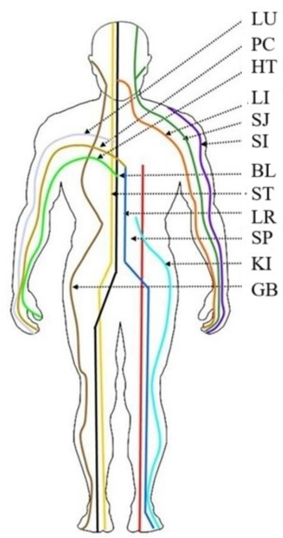

Figure 14

meridians for a patient. Then, the essence of doctor treating illness is by a natural law, i.e., *reducing the excess with supply the insufficient*, regulates on the meridians of human so that the restore balance of them by acupuncture or drugs.

Then, *how to modeling the running of a living body of human?* The answer is by the running of vital energy on 12 meridians with Ren and Du meridians in traditional Chinese medicine. Furthermore, *how to modeling the running of vital energy on 12 meridians with Ren and Du meridians?* The answer is nothing else but a G-flow $\overrightarrow{G}^L_{12}$ defined by ([24],[26])

$$V\left(\overrightarrow{G}_{12}\right) = \{\text{All acupoints } v \text{ on 12 meridians with Ren and Du meridians}\},$$

$$E\left(\overrightarrow{G}_{12}\right) = \{\text{All segments } (v,u) \text{ connecting adjacent points on 12 meridians with Ren and Du meridians with orientation of Ying Qi running}\}$$

such as those shown in Figure 15

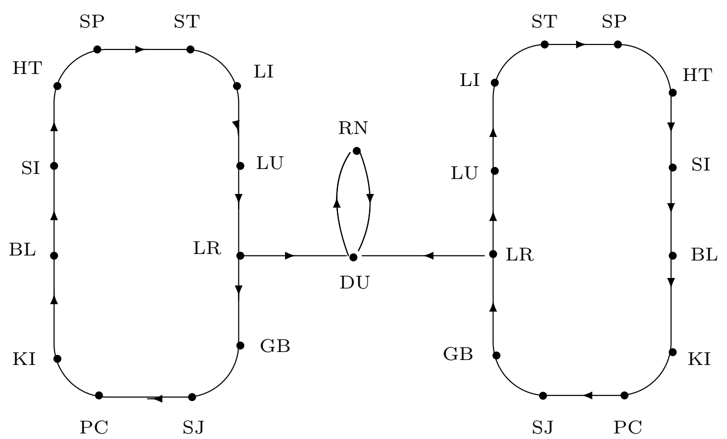

Figure 15

with a labeling $L:(v,u)\rightarrow\{\mathbf{\Psi}^-(v,u),\mathbf{\Psi}^+(v,u)\}$, $L:(v,u)\rightarrow\mathbf{\Psi}^-(v,u)$ or $L:(v,u)\rightarrow\mathbf{\Psi}^+(v,u)$ for edge $\forall(v,u)\in E(\overrightarrow{G}_{12})$. Notice that the *Ying Qi* $\mathbf{\Psi}^-$ and *Wei Qi* $\mathbf{\Psi}^+$ both are the vital energy with constant distributing c on each meridian of human body. Whence, they run in accordance with the law of conservation of energy and thereby, hold with continuity equation at each vertex of $\overrightarrow{G}^L_{12}$, i.e., $\overrightarrow{G}^L_{12}$ is a G-flow of continuity flow, which is characterized ([33]) by

$$\left\{\begin{array}{rcl} \dfrac{\partial\overrightarrow{G}^{\mathbf{\Psi}^-}_{12}}{\partial x_i}-\dfrac{d}{dt}\dfrac{\partial\overrightarrow{G}^{\mathbf{\Psi}^-}_{12}}{\partial\dot{x}_i} &=& \mathbf{O}\\ \dfrac{\partial\overrightarrow{G}^{\mathbf{\Psi}^+}_{12}}{\partial x_i}-\dfrac{d}{dt}\dfrac{\partial\overrightarrow{G}^{\mathbf{\Psi}^+}_{12}}{\partial\dot{x}_i} &=& \mathbf{O}\\ \overrightarrow{G}^{\mathbf{\Psi}^-}_{12}+\overrightarrow{G}^{\mathbf{\Psi}^+}_{12} &=& \overrightarrow{G}^{L_c}_{12}\end{array}\right. \quad (4.1)$$

for integers $1\le i\le n$ if we assume that $\mathbf{\Psi}^-$ and $\mathbf{\Psi}^+$ both are Lagrangian on the vital energy field of human, where the 1st and 2nd equations are Euler-Lagrange equations (3.19), the 3rd equation is the balance equation of *Ying Qi* and *Wei Qi* on meridians of human body, and the labeling $L_c : \forall(v,u) \in E(\overrightarrow{G}_{12}) \to \mathbf{c}_{vu}$ is constant. Notice that the system (4.1) of differential equations is essentially the theoretical foundation and leads to clinical techniques of traditional Chinese medicine, i.e., the amazing acupuncture and the prescription on compatibility of traditional Chinese medicines for a disease treatment.

§5. Conclusion

The reality of a thing T existing in universe should be a combinatorial one in the eyes of human because of the recognitive limitation and particularly, the mathematical reality. However, there are no mathematics applicable to reality of things unless the partial or conditional. Thus, a new mathematics should be established for the recognition of human on reality of thing by combinatorics, from the local to the whole. I introduce how to do such an objective in this report from non-harmonious groups to mathematical combinatorics, i.e., mathematics over topological graph, which is a natural way for recognizing the reality of thing T because thing T is not isolated but consisted of its elements, connected also with other things in unverse and also, the recognitive results by reductionism is nothing else but a complex network, we have to establish such a mathematics over topological graphs $\overrightarrow{G}$ inherited in thing T for crossing the recognitive gap from the local to the whole, including both of the macroscopic and microscopic such as the system of celestial body, particle moving or living evolution, the digital economy devolving of international or domestic trade, namely the evolution of all system, no matter it is harmonious or self-organized can be globally characterized by *mathematical combinatorics*.

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