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D-Conformal Curvature Tensor in Generalized (κ, μ)-Space Forms

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Abstract: The object of the first two sections is to give brief history of generalized (κ, μ) space forms and some basic results related to such manifold. In the last section we have derived few results regarding D-conformal curvature tensor in generalized (κ, μ) space-forms.

Key Words: Generalized (κ, μ)-space form, D-conformal curvature tensor, η-Einstein manifold.

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§1. Introduction

In [1], Carriazo jointly with P. Alegre and D.E. Blair defined a generalized Sasakian space form as an almost contact metric manifold (M, ϕ, ξ, η, g) whose curvature tensor R is given by

$$\begin{aligned} R(X,Y)Z &= f_1\{g(Y,Z)X - g(X,Z)Y\} \\ &+ f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \end{aligned} \tag{1.1}$$

for any vector fields X, Y, Z on M.

In particular a Sasakian manifold $M(\phi, \xi, \eta, g)$ is said to be a Sasakian space form if all the $\phi-$sectional curvatures $K(X \wedge \phi X)$ are equal to a constant c, where $K(X \wedge \phi X)$ denotes the sectional curvature of the section spanned by the unit vector field X, orthogonal to ξ and ϕX. Later on many scientists R. Al-Ghefari, F. R. Alsomy [2],[5], M. H. Shahid have studied the CR-submanifolds of generalized Sasakian space forms. After them Ricci curvature of contact CR-submanifolds of such space were studied in [6].

In [2] authors studied contact metric and generalized Sasakian-space forms. In [7] and [8] authors studied locally ϕ-symmetric and η-recurrent Ricci tensor and also studied the projective curvature tensor respectively. Generalized Sasakian space form with few properties like conformally flat, locally symmetric were studied by Kim [9].

In recent paper [10], the authors (jointly with M. M. Tripathi) defined a generalized (κ, μ)-space form as an almost contact metric manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ whose curvature tensor is

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given as

$$R = f_1R_1 + f_2R_2 + f_3R_3 + f_4R_4 + f_5R_5 + f_6R_6, \tag{1.2}$$

where $f_1, f_2, f_3, f_4, f_5, f_6$ are differentiable functions on M, and $R_1, R_2, R_3, R_4, R_5, R_6$ are tensors defined as follows:

$$R_1(X,Y)Z = g(Y,Z)X - g(X,Z)Y, \tag{1.3}$$

$$R_2(X,Y)Z = g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z, \tag{1.4}$$

$$R_3(X,Y)Z = \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi, \tag{1.5}$$

$$R_4(X,Y)Z = g(Y,Z)hX - g(X,Z)hY + g(hY,Z)X - g(hX,Z)Y, \tag{1.6}$$

$$R_5(X,Y)Z = g(hY,Z)hX - g(hX,Z)hY + g(\phi hX,Z)\phi hY - g(\phi hY,Z)\phi hX, \tag{1.7}$$

$$R_6(X,Y)Z = \eta(X)\eta(Z)hY - \eta(Y)\eta(Z)hX + g(hX,Z)\eta(Y)\xi - g(hY,Z)\eta(X)\xi, \qquad (1.8),$$

where $2h = \pounds_\xi\phi$ and L is the usual Lie derivative. Usually, this manifold is denoted by $M(f_1, f_2, f_3, f_4, f_5, f_6)$. If $f_4 = f_5 = f_6 = 0$ then the manifold is the usual Sasakian space form.

Again, (κ, μ)-space forms are natural examples of generalized (κ, μ) space forms for constant functions ([10])

$$f_1 = \frac{c+3}{4},\ f_2 = \frac{c-1}{4},\ f_3 = \frac{c+3}{4},\ f_4 = 1,\ f_5 = \frac{1}{2},\ f_6 = 1-\mu. \tag{1.9}$$

In this paper we have established few conditions related to D-conformal curvature tensor.

§2. Preliminaries

An almost contact metric manifold is a $(2n+1)$-dimensional manifold endowed with an almost contact structure (ϕ, ξ, η) consisting of a tensor field ϕ of type $(1,1)$, a structure vector field ξ and 1-form η satisfying:

$$\phi^2 = -I + \eta \otimes \xi, \eta(\xi) = 1, \phi(\xi) = 0, \eta \circ \phi = 0, \tag{2.1}$$

for any vector field $X, Y \in \tilde{M}$ and a Riemannian metric g defined as

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y). \tag{2.2}$$

From above equation we can easily derive

$$g(X,\xi) = \eta(X). \tag{2.3}$$

The metric tensor satisfies the following properties:

$$g(\phi X, Y) = -g(X, \phi Y), \tag{2.4}$$

$$(\nabla_X \eta)Y = g(\nabla_X \xi, Y). \tag{2.5}$$

In a $(2n+1)$ dimensional generalized (κ,μ)-space form we obtain from (1.2)

$$\begin{aligned} R(X,Y)\xi &= f_1\{g(Y,\xi)X - g(X,\xi)Y\} \\ &+f_2\{g(X,\phi\xi)\phi Y - g(Y,\phi\xi)\phi X + 2g(X,\phi Y)\phi\xi\} \\ &+f_3\{\eta(X)\eta(\xi)Y - \eta(Y)\eta(\xi)X + g(X,\xi)\eta(Y)\xi - g(Y,\xi)\eta(X)\xi\} \\ &+f_4\{g(Y,\xi)hX - g(X,\xi)hY + g(hY,\xi)X - g(hX,\xi)Y\} \\ &+f_5 g(hY,\xi)hX - g(hX,\xi)hY + g(\phi hX,\xi)\phi hY - g(\phi hY,\xi)\phi hX \\ &+f_6\eta(X)\eta(\xi)hY - \eta(Y)\eta(\xi)hX + g(hX,\xi)\eta(Y)\xi - g(hY,\xi)\eta(X)\xi \end{aligned} \tag{2.6}$$

After some brief calculations we obtain from [4]

$$R(X,Y)\xi = (f_1 - f_3)\{\eta(X)Y - \eta(Y)X\} + (f_4 - f_6)\{\eta(Y)hX - \eta(X)hY\}. \tag{2.7}$$

Now putting $X = \xi,\ Y = X,\ Z = Y$ we get

$$R(\xi,X)Y = (f_1 - f_3)\{g(X,Y)\xi - \eta(Y)X\} + (f_4 - f_6)\{g(hX,Y) - \eta(Y)hX\}. \tag{2.8}$$

Again putting $Y = \xi$ in (2.7) we get

$$R(\xi,X)\xi = (f_1 - f_3)\{\eta(X)\xi - X\} - (f_4 - f_6)\{hX\}. \tag{2.9}$$

Applying η on both side of the equation (1.2) we calculate

$$\begin{aligned} \eta(R(X,Y)Z) &= (f_1 - f_3)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\} \\ &+(f_4 - f_6)\{g(hY,Z)\eta(X) - g(hX,Z)\eta(Y)\}. \end{aligned} \tag{2.10}$$

Putting $Z = \xi$ we can easily write

$$\eta(R(X,Y)\xi) = 0. \tag{2.11}$$

Applying η on both side of equation (2.7) we can get the following equations

$$\begin{aligned} \eta(R(\xi,X)Y) &= (f_1 - f_3)\{g(X,Y) - \eta(Y)\eta(X)\} \\ &+(f_4 - f_6)g(hX,Y), \end{aligned} \tag{2.12}$$

$$\begin{aligned} S(X,Y) &= \{2nf_1 + 3f_2 - f_3\}g(X,Y)\{(2n-1)f_4 - f_6\}g(hX,Y) \\ &-\{3f_2 + (2n-1)f_3\}\eta(X)\eta(Y). \end{aligned} \tag{2.13}$$

From (2.13) we obtain

$$S(X,\xi) = 2n(f_1 - f_3)\eta(X), \tag{2.14}$$

$$r = 2n\{(2n+1)f_1 + 3f_2 - 2f_3\}, \tag{2.15}$$

$$S(\phi X,\phi Y) = S(X,Y)-2n(f_1-f_3)\eta(X)\eta(Y) -\{(2n-1)f_4-f_6\}g(hX,Y), \tag{2.16}$$

$$QX = \{2nf_1+3f_2-f_3\}X+\{(2n-1)f_4-f_6\}hX-\{3f_2 +(2n-1)f_3\}\eta(X)\xi, \tag{2.17}$$

$$Q\xi = 2n(f_1-f_3)\xi. \tag{2.18}$$

From [12], D-conformal curvature tensor on a Riemannian manifold (M^{2n+1},g) is defined as

$$\begin{aligned} B(X,Y)Z &= R(X,Y)Z+\frac{1}{2(n-1)}\{S(X,Z)Y-S(Y,Z)X+g(X,Z)QY \\ &-g(Y,Z)QX\}-S(X,Z)\eta(Y)\xi+S(Y,Z)\eta(X)\xi-\eta(X)\eta(Z)QY \\ &-\eta(Y)\eta(Z)QX\}-\frac{k-2}{2(n-1)}\{g(X,Z)Y-g(Y,Z)X\} \\ &+\frac{k}{2(n-1)}\{g(X,Z)\eta(Y)\xi-g(Y,Z)\eta(X)\xi+\eta(X)\eta(Z)Y \\ &-\eta(Y)\eta(Z)X\}, \end{aligned} \tag{2.19}$$

where $k=\frac{r+4n}{2n-1}$, R is the curvature tensor, S is the Ricci tensor and r is the scalar curvature.

Now we give the definition of D-conformally flat generalized (κ,μ) space form following.

Definition 2.1 *A $(2n+1)$-dimensional generalized (κ,μ) space form $M(f_1,f_2,f_3,f_4,f_5,f_6)$ is said to be D-conformally flat if*

$$B(X,Y)Z=0. \tag{2.20}$$

We give the definition of $\xi-D$-conformally flat generalized (κ,μ) space form following.

Definition 2.2 *A $(2n+1)$-dimensional generalized (κ,μ) space form $M(f_1,f_2,f_3,f_4,f_5,f_6)$ is said to be $\xi-D$-conformally flat if*

$$B(X,Y)\xi=0. \tag{2.21}$$

Also we mention the following definition.

Definition 2.3 *A $(2n+1)$-dimensional generalized (κ,μ) space form $M(f_1,f_2,f_3,f_4,f_5,f_6)$ is said to be $\phi-D$-conformally flat if*

$$g(B(\phi X,\phi Y)\phi Z,\phi W)=0. \tag{2.22}$$

§3. Main Results

From Definition 2.1 we can draw the following theorem.

Theorem 3.1 *If a $(2n+1)$-dimensional generalized (κ,μ) space form $M(f_1,f_2,f_3,f_4,f_5,f_6)$ is D-conformally flat then $f_3=f_1+1$ and $f_4=f_6$.*

Proof Let us consider a $(2n+1)$-dimensional generalized (κ,μ) space form which satisfy the condition $B(X,Y)Z=0$ then from (2.19) we obtain on using Definition 2.1 and taking inner product with W we obtain

$$\begin{aligned}0 \;=\;& R(X,Y,Z,W)+\frac{1}{2(n-1)}\{S(X,Z)g(Y,W)-S(Y,Z)g(X,W)+g(X,Z)g(QY,W)\\&-g(Y,Z)g(QX,W)\}-S(X,Z)\eta(Y)\eta(W)+S(Y,Z)\eta(X)\eta(W)-\eta(X)\eta(Z)g(QY,W)\\&-\eta(Y)\eta(Z)g(QX,W)\}-\frac{k-2}{2(n-1)}\{g(X,Z)g(Y,W)-g(Y,Z)g(X,W)\}\\&+\frac{k}{2(n-1)}\{g(X,Z)\eta(Y)\eta(W)-g(Y,Z)\eta(X)\eta(W)+\eta(X)\eta(Z)g(Y,W)\\&-\eta(Y)\eta(Z)g(X,W)\}\end{aligned}\tag{3.1}$$

because of $R(X,Y,Z,W)=g(R(X,Y)Z,W))$.

Now setting $W=\xi$ in (3.1) and using (2.1) and (2.2), we have

$$\begin{aligned}0 \;=\;& \eta(R(X,Y)Z)+\frac{1}{2(n-1)}\{S(Y,\xi)g(X,Z)-S(X,\xi)g(Y,Z)-S(Y,\xi)\eta(X)\eta(Z)\\&+S(X,\xi)\eta(Y)\eta(Z)+2\{g(X,Z)\eta(Y)-g(Y,Z)\eta(X)\}.\end{aligned}\tag{3.2}$$

On using (2.10) and (2.14) we get on brief calculation

$$\begin{aligned}0 \;=\;& \frac{f_3-f_1-1}{n-1}\{g(Y,Z)\eta(X)-g(X,Z)\eta(Y)\}+(f_4-f_6)\{g(hY,Z)\eta(X)\\&-g(hX,Z)\eta(Y)\}.\end{aligned}\tag{3.3}$$

Since L.H.S. is equal to zero and

$$\{g(Y,Z)\eta(X)-g(X,Z)\eta(Y)\},\ \{g(hY,Z)\eta(X)-g(hX,Z)\eta(Y)\}\ \neq\ 0$$

we must have$f_3-f_1-1=0$ and $f_4-f_6=0$.

Hence

$$f_3=f_1+1=0,\quad f_4=f_6.\tag{3.4}$$

Therefore the above equation proves our theorem. □

Now on basis of the definition (2.2) we give our next theorem.

Corollary 3.1 *If a $(2n+1)$-dimensional generalized (κ,μ) space form $M(f_1,f_2,f_3,f_4,f_5,f_6)$ is said to be $\xi-D$-conformally flat then $f_3=f_1+1, f_4=f_6$.*

Proof Suppose the condition $B(X,Y)\xi=0$ holds in a $(2n+1)-$dimensional generalized

(κ, μ) space form. We have from (2.1), (2.2) in (2.19)

$$\begin{aligned} 0 &= R(X,Y)\xi + \frac{1}{2(n-1)}\{S(X,\xi)Y - S(Y,\xi)X - S(X,\xi)\eta(Y)\xi + S(Y,\xi)\eta(X)\xi\} \\ &+2\{\eta(X)Y - \eta(Y)X\}. \end{aligned} \tag{3.5}$$

Using (2.7) and (2.15) we calculate

$$0 = \frac{f_3 - f_1 - 1}{n-1}\{\eta(Y)X-)\eta(X)Y\} + (f_4 - f_6)\{\eta(Y)hX - \eta(X)hY\} \tag{3.6}$$

because of $\{\eta(Y)X-)\eta(X)Y\} \neq 0$ we must have $f_3 - f_1 - 1 = 0$ and $f_4 - f_6 = 0$. Hence we obtain our proof. □

From Definition 2.3 we can state our next theorem.

Theorem 3.2 *If a $(2n+1)$-dimensional generalized (κ, μ) space form $M(f_1, f_2, f_3, f_4, f_5, f_6)$ is $\phi - D$-conformally flat then Ricci tensor reduces to the form*

$$S(Y,Z) = \alpha g(Y,Z) + \beta\eta(Y)\eta(Z) + \gamma g(hY,Z) \tag{3.7}$$

under the condition $Tr.\phi = 0$, and $\mu = 0$, where α, β, γ are constants.

Proof Let $M(f_1, f_2, f_3, f_4, f_5, f_6)$ be a $(2n+1)$-dimensional generalized (κ, μ) space form. Suppose M satisfies $g(B(\phi X, \phi Y)\phi Z, \phi W) = 0$ then from (2.1), (2.19) we obtain

$$\begin{aligned} 0 &= g(R(\phi X, \phi Y)\phi Z, \phi W) \\ &+\frac{1}{2(n-1)}\{S(\phi X, \phi Z)g(\phi Y, \phi W) - S(\phi Y, \phi Z)g(\phi X, \phi W) \\ &+S(\phi Y, \phi W)g(\phi X, \phi Z) - S(\phi X, \phi W)g(\phi Y, \phi Z)\} \\ &-\frac{k-2}{2(n-1)}\{g(\phi X, \phi Z)g(\phi Y, \phi W) - g(\phi Y, \phi Z)g(\phi X, \phi W)\} \end{aligned} \tag{3.8}$$

In view of (2.22) and (3.8) and having few steps of calculations we get

$$\begin{aligned} 0 &= f_1\{g(Y,Z)g(X,W) - g(Y,Z)\eta(X)\eta(W) - g(X,W)\eta(Y)\eta(Z) \\ &-g(X,Z)g(Y,W) - g(Y,W)\eta(X)\eta(Z) + g(X,Z)\eta(Y)\eta(W)\} \\ &+f_2\{g(X,\phi Z)g(\phi Y,W) - g(\phi Y,Z)g(X,\phi W) + 2g(X,\phi Y)g(\phi Z,W)\} \\ &f_4\{-g(Y,Z)g(hX,W) + \eta(Y)\eta(Z)g(hX,W) + g(X,Z)g(hY,W) \\ &-\eta(X)\eta(Z)g(hY,W) - g(hY,Z)g(X,W) + \eta(X)\eta(W)g(hY,Z) + g(Y,W)g(hX,Z) \\ &-g(hX,Z)\eta(Y)\eta(W)\} \\ &+f_5\{g(hY,Z)g(hX,W) - g(hX,Z)g(hY,W) + g(hX,\phi Z)g(hY,\phi W) \\ &-g(hY,Z)g(hX,W)\} + \frac{1}{2(n-1)}\{S(X,Z)g(Y,W) - S(X,Z)\eta(Y)\eta(W) \end{aligned} \tag{3.9}$$

$$
\begin{aligned}
&-2n(f_1-f_3)g(Y,W)\eta(X)\eta(Z)-S(Y,Z)g(X,W)+S(Y,Z)\eta(X)\eta(W)\\
&+2n(f_1-f_3)g(X,W)\eta(Y)\eta(Z)+S(Y,W)g(X,Z)-S(Y,W)\eta(X)\eta(Z)\\
&-2n(f_1-f_3)g(X,Z)\eta(Y)\eta(W)-S(X,W)g(Y,Z)+S(X,W)\eta(Y)\eta(Z)\\
&+2n(f_1-f_3)g(Y,Z)\eta(X)\eta(W)\}+\frac{(2n-1)f_4-f_6}{2(n-1)}\{-g(hX,Z)g(Y,W)\\
&+g(hX,Z)\eta(Y)\eta(W)+g(hY,Z)g(X,W)\\
&-g(hY,Z)\eta(X)\eta(W)-g(hY,Z)g(X,Z)+g(hY,Z)\eta(X)\eta(Z)\\
&+g(hX,W)g(Y,Z)-g(hX,W)\eta(Y)\eta(Z)\}\\
&-\frac{k-2}{2(n-1)}\{g(X,Z)g(Y,W)-g(X,Z)\eta(Y)\eta(W)-g(Y,W)\eta(X)\eta(Z)\\
&-g(Y,Z)g(X,W)+g(Y,Z)\eta(X)\eta(W)+g(X,W)\eta(Y)\eta(Z)\}
\end{aligned}
$$

Let $\{e_i : i = 1, 2, \cdots, 2n+1\}$ be an orthonormal basis of the tangent space at any point of the manifold. Putting $X = W = e_i$ in (3.14) and taking summation over i, $1 \le i \le 2n+1$, we get

$$
\begin{aligned}
0 \;=\;& (2n-1)f_1\{g(Y,Z)-\eta(Y)\eta(Z)\}+f_2\{3g(\phi Y,\phi Z)-g(\phi Y,Z)Tr\phi\}\\
&-(2n-1)f_4 g(hY,Z)+\frac{1}{2(n-1)}[-2(n-1)S(Y,Z)-S(Z,\xi)\eta(Y)\\
&+\{2n(2n-1)(f_1-f_3)+r\}\eta(Y)\eta(Z)-S(Y,\xi)\eta(Z)\\
&+\{2n(2n-1)(f_1-f_3)-r\}g(Y,Z)]+\frac{r+2}{2(n-1)}[g(Y,Z)-\eta(Y)\eta(Z)]\\
&+0\frac{[(2n-1)f_4-f_6](2n-1)}{2(n-1)}g(hY,Z) \qquad (3.10)
\end{aligned}
$$

By using (2.3) and (2.14) we obtain

$$
\begin{aligned}
S(Y,Z) \;=\;& \frac{[(2n^2-2n+1)+3(n-1)f_2-nf_3+1]}{n-1}g(Y,Z)\\
&+\frac{[n(3-2n)f_3-f_1-3(n-1)f_2-1]}{n-1}\eta(Y)\eta(Z)\\
&+\frac{(f_4-f_6)(2n-1)}{2(n-1)}g(hY,Z). \qquad (3.11)
\end{aligned}
$$

Assuming

$$\alpha=\frac{[(2n^2-2n+1)+3(n-1)f_2-nf_3+1]}{n-1},$$

$$\beta=\frac{[n(3-2n)f_3-f_1-3(n-1)f_2-1]}{n-1},$$

$$\gamma=\frac{(f_4-f_6)(2n-1)}{2(n-1)}.$$

Hence we arrive at our proposed result. □

Theorem 3.3 *If a $(2n+1)$-dimensional generalized (κ,μ) space form $M(f_1,f_2,f_3,f_4,f_5,f_6)$*

satisfies the condition $B(\xi, X).S = 0$, then the scalar curvature is given by

$$r = 2n(f_1 - f_3)(2n+1) - 2n(f_4 - f_6)(f_1 - f_3 - 1)[(2n-1)f_4 - f_6]. \tag{3.12}$$

Proof Let $M(f_1, f_2, f_3, f_4, f_5, f_6)$ be a $(2n+1)$-dimensional generalized (κ, μ) space form. we suppose that M satisfies the condition $(B(\xi, X).S)(U, V) = 0$, where S is the Ricci tensor. Then we get

$$S(B(\xi, X)U, V) + S(U, B(\xi, X)V) = 0. \tag{3.13}$$

From equation (2.19) we can write after having few steps of calculations

$$\begin{aligned} B(\xi, Y)Z &= (f_1 - f_3 - \frac{1}{n-1})\{g(Y, Z)\xi - \eta(Z)Y\} + (f_4 - f_6)\{g(hY, Z)\xi - \eta(Z)hY\} \\ &+ \frac{1}{2(n-1)}\{2n(f_1 - f_3)[Y - \eta(Y)\xi]\eta(Z) \\ &- [g(Y, Z) - \eta(Y)\eta(Z)]Q\xi\}. \end{aligned} \tag{3.14}$$

Similarly replacing Y with X and Z with U we obtain

$$\begin{aligned} B(\xi, X)U &= (f_1 - f_3 - \frac{1}{n-1})\{g(X, U)\xi - \eta(U)X\} + (f_4 - f_6)\{g(hX, U)\xi - \eta(U)hX\} \\ &+ \frac{1}{2(n-1)}\{2n(f_1 - f_3)[X - \eta(X)\xi]\eta(U) \\ &- [g(X, U) - \eta(X)\eta(U)]Q\xi\}. \end{aligned} \tag{3.15}$$

Putting equations (3.14), (3.15) in (3.13) and replacing V with ξ we infer

$$\begin{aligned} 0 &= (f_1 - f_3 - \frac{1}{n-1})\{2n(f_1 - f_3)g(X, U) - S(X, \xi)\eta(U)\} \\ &\quad \frac{n(f_1 - f_3)}{n-1}[\{S(X, \xi) - 2n(f_1 - f_3)\eta(X)\}\eta(U) - 2n(f_1 - f_3)\{g(X, U) - \eta(x)\eta(U)\}] \\ &\quad + (f_1 - f_3 - \frac{1}{n-1})\{2n(f_1 - f_3)\eta(X)\eta(U) - S(X, U)\} \\ &\quad + \frac{n(f_1 - f_3)}{n-1}[S(X, U) - 2n(f_1 - f_3)\{g(X, U) - \eta(x)\eta(U)] \\ &\quad + (f_4 - f_6)\{2n(f_1 - f_3)g(hX, U) - S(hX, U)\}. \end{aligned} \tag{3.16}$$

Using equations (2.1), (2.2), (2.12) we obtain

$$S(X, U) = 2n(f_1 - f_3)g(X, U) + \frac{1}{a}[(f_4 - f_6)\{2n(f_1 - f_3)g(hX, U) - S(hX, U)\}], \tag{3.17}$$

where $a = f_1 - f_3 - \frac{2}{n-1}$. Again taking the orthonormal frame field at any point of the manifold and contracting over X and U we get from above equation

$$r = 2n(f_1 - f_3)(2n+1) - 2n(f_4 - f_6)(f_1 - f_3 - 1)[(2n-1)f_4 - f_6]. \tag{3.18}$$

Hence we get the result. □

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