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Hyperfloorplans and Superhyperfloorplans – Definitions, Properties and Perspectives (Revisit)

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Abstract: Floorplans are geometric arrangements of modules within defined boundaries, adhering to constraints such as area and aspect ratio [1-3]. They are well known for applications in VLSI design and have been the subject of extensive algorithmic research. Hyperstructures extend the concept of the powerset into advanced mathematical models, while superhyperstructures further generalize these models via n-th iterated powersets, enabling multi-layered hierarchical abstractions [4-5]. Floorplans have a wide range of applications, and because hyperstructures and superhyperstructures can represent hierarchical structures in the real world, they are recognized as highly important research areas. However, the fusion of these concepts has only just begun to be explored. Therefore, with the aim of contributing to the dissemination of knowledge, in this paper, we revisit floorplans and examine the notions of *hyperfloorplan* and *superhyperfloorplan* as defined in [6]. We hope that our analysis provides valuable insights and aids in the broader understanding of floorplan theory and its applications.

Key Words: Hyperfloorplan, superhyperfloorplan, hyperStructure, superHyperStructure, powerset.

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§1. Preliminaries

This section introduces the fundamental concepts and definitions that underpin the discussions in this paper. Throughout, all sets are assumed to be finite. Furthermore, any integer n used in the context of superhyperstructures and related constructs is taken to be a non-negative integer. For detailed information on the operations associated with each concept, the reader is referred to the relevant literature as appropriate.

1.1. Hyperstructures and Superhyperstructures

This subsection presents the formal foundations of *hyperstructures* and their higher-order generalization, *superhyperstructures*, which serve as powerful mathematical tools for modeling multi-tiered relational systems. Mathematical concepts such as networks, graphs, topology, and alge-

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bra, as well as various real-world models, are defined on a daily basis; however, in some cases, these frameworks cannot adequately represent hierarchical structures. To address this limitation, the notions of hyperstructure and superhyperstructure have been introduced in recent years.

A *hyperstructure* is defined over the powerset of a base set, allowing operations not on individual elements but on subsets of the underlying domain. This generalization offers a flexible and expressive framework for capturing interactions and dependencies within complex systems [7-12].

Building on this concept, a *superhyperstructure* extends the idea further by employing the n-th iterated powerset of a set. This construction facilitates the representation of deeply nested or hierarchical relationships, where elements can themselves be subsets of subsets, and so forth. Such structures are particularly useful in applications involving recursive abstraction or multilayer decision architectures [4,13 - 16] Closely related frameworks include the theory of *superhypergraphs* [17 - 20].

We now proceed to provide precise mathematical definitions of these two foundational concepts.

Definition 1.1(Set,[21]) *A set is a collection of distinct, well-defined objects, referred to as elements. For any object x, it can be determined whether x is an element of a given set. If x belongs to a set A, this is denoted as $x \in A$. Sets are often represented using curly braces.*

Definition 1.2(Base set,[22]) *A base set is a primary set S from which more complex structures, such as powersets and hyperstructures, are derived. It is formally expressed as:*

$$S = \{x \mid x \text{ is an element in the defined domain}\}.$$

The elements of advanced structures, such as $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$, are drawn from this base set S.

Definition 1.3(Power set,[23, 24]) *The powerset of a set S, denoted as $\mathcal{P}(S)$, is the set containing all subsets of S, including both the empty set and S itself. Formally, it is defined as:*

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 1.4(n-th Powerset, cf. [14,25,26]) *The n-th powerset of a set H, denoted by $\mathcal{P}_n(H)$, is constructed iteratively. Starting from the basic powerset, it is defined as:*

$$\mathcal{P}_1(H) = \mathcal{P}(H), \quad \mathcal{P}_{n+1}(H) = \mathcal{P}(\mathcal{P}_n(H)), \quad \text{for } n \geq 1.$$

Similarly, the n-th non-empty powerset, denoted by $P_n^*(H)$, is defined iteratively as:

$$\mathcal{P}_1^*(H) = \mathcal{P}^*(H), \quad \mathcal{P}_{n+1}^*(H) = \mathcal{P}^*(\mathcal{P}_n^*(H)).$$

Here, $\mathcal{P}^*(H)$ represents the powerset of H excluding the empty set.

Example 1.5(Real-world example of an n-th powerset) Consider a company H with three employees:

$$H = \{\text{Satoshi}, \text{Yuko}, \text{Kenji}\}.$$

The first powerset $\mathcal{P}_1(H)$ lists all possible groups of employees, such as $\{\text{Satoshi}, \text{Yuko}\}$ or $\{\text{Kenji}\}$.

The second powerset $\mathcal{P}_2(H)$ then treats each element of $\mathcal{P}_1(H)$ (i.e., each possible group of employees) as a single unit and forms all possible collections of such groups. For example, one element of $\mathcal{P}_2(H)$ could be:

$$\big\{ \{\text{Satoshi}\},\ \{\text{Yuko}, \text{Kenji}\} \big\},$$

which represents a scenario where the company organizes two independent task forces: one consisting solely of Satoshi, and another consisting of Yuko and Kenji.

In practical terms, the n-th powerset models higher-order organizational structures, such as teams of teams, or committees formed from existing working groups, thereby capturing multiple hierarchical levels of arrangement.

To establish a formal foundation for the concepts of Hyperstructures and Superhyperstructures, we present the following definitions and propositions.

Definition 1.6(Classical structure, cf. [14,26]) *A classical structure is a mathematical framework defined on a non-empty set H, equipped with one or more* Classical Operations *that satisfy specified classical axioms. Specifically, a classical operation is a function of the form*

$$\#_0 : H^m \to H,$$

where $m \geq 1$ is a positive integer, and H^m denotes the m-fold Cartesian product of H. Common examples include addition and multiplication in algebraic structures such as groups, rings, and fields.

Definition 1.7(Hyperstructure, cf. [14,26]) *A* Hyperstructure *extends the notion of a Classical Structure by operating on the powerset of a base set. Formally, it is defined as:*

$$\mathcal{H} = (\mathcal{P}(S), \circ),$$

where S is the base set, $\mathcal{P}(S)$ is the powerset of S, and $\circ$ is an operation defined on subsets of $\mathcal{P}(S)$. Hyperstructures allow for generalized operations that can apply to collections of elements rather than single elements.

Example 1.8(Real-world example of a hyperstructure) Let $S = \{\text{Satoshi}, \text{Yuko}, \text{Kenji}\}$ represent the set of individual project members in a company. The powerset $\mathcal{P}(S)$ consists of all possible project teams, such as $\{\text{Satoshi}, \text{Yuko}\}$ or $\{\text{Kenji}\}$.

Define an operation $\circ$ on $\mathcal{P}(S)$ where, given two subsets of S (teams), the result of $\circ$ is the set of all possible joint committees that can be formed by taking at least one member from each team. For example:

$$\{\text{Satoshi}, \text{Yuko}\} \circ \{\text{Kenji}\} = \big\{\{\text{Satoshi}, \text{Kenji}\}, \ \{\text{Yuko}, \text{Kenji}\}\big\}.$$

This construction forms a hyperstructure $\mathcal{H} = (\mathcal{P}(S), \circ)$, where the operation acts on *subsets* (teams) instead of on individual people. In practical terms, this models scenarios such as cross-team collaboration in organizations, where operations combine groups rather than single members.

Definition 1.9(n-Superhyperstructure, cf.[14,26]) *An* n-Superhyperstructure *further generalizes a Hyperstructure by incorporating the n-th powerset of a base set. It is formally described as:*

$$\mathcal{SH}_n = (\mathcal{P}_n(S), \circ),$$

where S is the base set, $\mathcal{P}_n(S)$ is the n-th powerset of S, and $\circ$ represents an operation defined on elements of $\mathcal{P}_n(S)$. This iterative framework allows for increasingly hierarchical and complex representations of relationships within the base set.

Example 1.10(2-Superhyperstructure over $S = \{a, b\}$) Let

$$S = \{a, b\}.$$

Then the first powerset is

$$\mathcal{P}_1(S) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\},$$

and the second powerset is

$$\mathcal{P}_2(S) = \mathcal{P}\big(\mathcal{P}_1(S)\big) = \big\{ X \mid X \subseteq \mathcal{P}_1(S)\big\},$$

which has $2^4 = 16$ elements. Define a hyperoperation

$$\circ_2 \ : \ \mathcal{P}_2(S) \times \mathcal{P}_2(S) \ \longrightarrow \ \mathcal{P}\big(\mathcal{P}_2(S)\big)$$

by

$$X \circ_2 Y \ = \ \big\{ Z \in \mathcal{P}_2(S) \ \big| \ X \cup Y \ \subseteq \ Z\big\}.$$

Then, $\big(\mathcal{P}_2(S), \ \circ_2\big)$ is a concrete 2-superhyperstructure.

Concrete computation. Choose

$$X = \{\{a\}, \{a, b\}\}, \qquad Y = \{\{b\}\}.$$

Then

$$X \cup Y = \{\{a\}, \{b\}, \{a, b\}\},$$

and

$$X \circ_2 Y = \big\{ Z \subseteq \mathcal{P}_1(S) \ \big| \ \{a\}, \{b\}, \{a, b\} \in Z\big\}.$$

The two minimal members of $X \circ_2 Y$ are

$$Z_1 = \{\{a\}, \{b\}, \{a, b\}\}, \quad Z_2 = Z_1 \cup \{\emptyset\}.$$

All other elements of $X \circ_2 Y$ are obtained by adding any of the remaining subsets of $\mathcal{P}_1(S)$ to Z_1.

1.2.Floorplan

Floorplans are geometric arrangements of modules within defined boundaries, adhering to constraints such as area and aspect ratio [3,27,28]. The definition of a general floorplan is provided below.

Definition 1.11(Floorplan, [3,27,28]) *A floorplan is a geometric arrangement of a given set of rectangular modules within a bounding rectangle, satisfying specific constraints related to module dimensions, aspect ratios, and interconnections. It is formally defined as follows:*

1. Modules. The floorplan consists of m rectangular modules $\{M_1, M_2, \dots, M_m\}$, where each module M_i is characterized by

- *Area. $A_i > 0$, the total area of the module;*
- *Aspect ratio bounds. l_i and u_i, the lower and upper bounds for the height-to-width ratio $\frac{h_i}{w_i}$, such that*

$$w_i \cdot h_i = A_i, \quad l_i \leq \frac{h_i}{w_i} \leq u_i;$$

- *A module is rigid if $l_i = u_i$, and flexible otherwise;*
- *A module may have a fixed orientation (dimensions w_i, h_i are fixed) or a free orientation (dimensions can be interchanged).*

2. Bounding Rectangle. The modules are arranged within a bounding rectangle R with dimensions W (width) and H (height), such that

$$p \leq \frac{H}{W} \leq q, \quad \text{where } p, q > 0$$

3. Partitioning The rectangle R is partitioned into m non-overlapping rectangular regions $\{r_1, r_2, \cdots, r_m\}$, each corresponding to a module M_i. Each region r_i satisfies:

$$x_i \cdot y_i \geq A_i, \quad l_i \leq \frac{y_i}{x_i} \leq u_i$$

where x_i and y_i are the width and height of r_i, respectively.

4. Objective Function. The quality of a floorplan is measured using the following objective function:

$$\text{Score} = \lambda \cdot (W \cdot H) + \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} \cdot d_{ij}$$

where

- *$W \cdot H$: Total area of the bounding rectangle R;*

- c_{ij}*: Connection cost between modules* M_i *and* M_j ($c_{ij} \geq 0$);
- d_{ij}*: Manhattan distance between the centers of* r_i *and* r_j;
- $\lambda > 0$*: User-defined weight balancing the importance of area and wirelength.*

5. Slicing Floorplans. A slicing floorplan is a recursive partitioning of R *using horizontal and vertical cuts, represented as*

- *Slicing Tree. A binary tree where internal nodes represent cuts and leaves represent modules;*
- *Polish Expression. A postfix expression encoding the slicing structure.*

For slicing floorplans, the bounding rectangle R *is recursively divided into smaller regions* $\{r_1, r_2, \cdots, r_m\}$ *using slicing operators* $+$ *(horizontal cut) and* $\times$ *(vertical cut).*

6. Feasibility. A floorplan is feasible if all regions r_i *satisfy:*

$$x_i \cdot y_i = A_i, \quad l_i \leq \frac{y_i}{x_i} \leq u_i$$

and no two regions overlap.

Example 1.12(3-module slicing floorplan) Let $m = 3$ and consider modules M_1, M_2, M_3 with parameters:

$$\begin{aligned} A_1 &= 10, \quad l_1 = 0.8,\ u_1 = 1.2,\ \text{free orientation},\\ A_2 &= 20, \quad l_2 = u_2 = 1.0,\ \text{rigid, fixed orientation},\\ A_3 &= 15, \quad l_3 = 0.5,\ u_3 = 2.0,\ \text{free orientation}. \end{aligned}$$

Choose module region dimensions (x_i, y_i) satisfying $x_i y_i = A_i$ and $l_i \leqslant y_i/x_i \leqslant u_i$:

$$(x_1, y_1) = (\sqrt{10}, \sqrt{10}), \quad (x_2, y_2) = (\sqrt{20}, \sqrt{20}), \quad (x_3, y_3) = (3, 5).$$

a slicing tree with a vertical root cut ($\times$) separating

$$\underbrace{\left(M_1 \ + \ M_3\right)}_{\substack{\text{horizontal}\\ \text{cut}}} \quad \text{and} \quad M_2,$$

whose Polish expression is

$$M_1\ M_3\ +\ M_2\ \times.$$

Then,

$$\begin{aligned} &W = \underbrace{\max\{x_1, x_3\}}_{=\sqrt{10}} + x_2 = \sqrt{10} + \sqrt{20} \approx 3.162 + 4.472 = 7.634,\\ &H = \max\{\, y_1 + y_3,\ y_2\} = \max\{\sqrt{10} + 5,\ \sqrt{20}\} \approx \max\{3.162 + 5,\ 4.472\} = 8.162,\\ &\frac{H}{W} \approx \frac{8.162}{7.634} \approx 1.069 \quad (p = 0.5,\ q = 2). \end{aligned}$$

Place all bottom-aligned, so region centers are

$$\begin{aligned} c_1 &= (\tfrac{\sqrt{10}}{2},\, 5+\tfrac{\sqrt{10}}{2}) \approx (1.581, 6.581), \\ c_3 &= (\tfrac{3}{2},\, \tfrac{5}{2}) \approx (1.5, 2.5), \\ c_2 &= \big(\sqrt{10}+\tfrac{\sqrt{20}}{2},\, \tfrac{\sqrt{20}}{2}\big) \approx (5.398, 2.236). \end{aligned}$$

Let connection costs $c_{12} = 2$, $c_{13} = 1$, $c_{23} = 3$. Then, the Manhattan distances

$$d_{12} \approx 8.162, \quad d_{13} \approx 4.162, \quad d_{23} \approx 4.162.$$

For weight $\lambda = 1$, the score is

$$\begin{aligned} \text{Score} &= (W \cdot H) + \sum_{1\leqslant i<j\leqslant 3} c_{ij}\, d_{ij} \\ &\approx 7.634 \cdot 8.162 + (2 \cdot 8.162 + 1 \cdot 4.162 + 3 \cdot 4.162) \\ &\approx 62.33 + 33.00 = 95.33. \end{aligned}$$

All regions satisfy $x_i y_i = A_i$ and $l_i \leqslant y_i/x_i \leqslant u_i$, and there is no overlap. Hence this floorplan is feasible.

§2. Review: Hyperfloorplan

Let us now build a *hyperfloorplan* starting from the set of modules S. We first consider the powerset $\mathcal{P}(S)$. Elements of $\mathcal{P}(S)$ are all possible subsets of modules. Our overarching goal is to capture geometric *feasibility* in a hyperoperation (cf.[6]).

Definition 2.1(Hyperfloorplan, cf.[6]) *Let $S = \{M_1, \cdots, M_m\}$ be a finite set of rectangular modules, and let $\mathcal{P}(S)$ be its powerset. We say that a subset $X \subseteq S$ admits a* feasible floorplan *if there exists a classical slicing—or non-slicing—arrangement of the modules in X within some bounding rectangle satisfying all area, aspect-ratio, non-overlap, and connectivity constraints.*

Define a hyperoperation

$$\circ \;:\; \mathcal{P}(S) \times \mathcal{P}(S) \;\longrightarrow\; \mathcal{P}\big(\mathcal{P}(S)\big)$$

by

$$A \circ B \;=\; \big\{ X \subseteq S \mid A \cup B \subseteq X \text{ and } X \text{ admits a feasible floorplan} \big\}.$$

Then the hyperfloorplan *of S is the hyperstructure*

$$\mathcal{HF}(S) \;=\; \big(\mathcal{P}(S), \circ\big).$$

Example 2.2(Hyperfloorplan of four modules) Let

$$S = \{M_1, M_2, M_3, M_4\}$$

with the following module parameters

Module	A_i	(l_i, u_i)	Orientation	Chosen (w_i, y_i)
M_1	8	[0.5, 2.0]	free	$(2,\ 4),\quad 2\cdot 4 = 8,\ 4/2 = 2.0$
M_2	12	[1.0, 1.0]	rigid	$(\sqrt{12},\ \sqrt{12}),\ \sqrt{12}^2 = 12,\ 1.0$
M_3	6	[0.75, 1.5]	free	$(2,\ 3),\quad 2\cdot 3 = 6,\ 3/2 = 1.5$
M_4	10	[0.8, 1.25]	free	$(\sqrt{10},\ \sqrt{10}),\ \sqrt{10}^2 = 10,\ 1.0$

Define two level-1 subsets by

$$A = \{M_1,\ M_2\}, \qquad B = \{M_2,\ M_3\}.$$

By Definition 2.1,

$$A \circ B = \big\{\, X \subseteq S \mid \{M_1, M_2, M_3\} \subseteq X \text{ and } X \text{ admits a feasible floorplan} \big\}.$$

We find two minimal supersets

- $X_1 = \{M_1, M_2, M_3\}$. A feasible slicing-floorplan

– First, slice M_1 and M_2 horizontally, $\begin{cases} W_0 &= \max\{w_1, w_2\} = \max\{2, \sqrt{12}\} \approx 3.464, \\ H_0 &= y_1 + y_2 = 4 + \sqrt{12} \approx 7.464. \end{cases}$

– Then, slice the block $\{M_1, M_2\}$ vertically with M_3,

$$\begin{cases} W &= W_0 + w_3 \approx 3.464 + 2 = 5.464, \\ H &= \max\{H_0,\ y_3\} = \max\{7.464,\ 3\} = 7.464. \end{cases}$$

– All aspect ratios and non-overlap conditions hold, so X_1 is feasible.

- $X_2 = \{M_1, M_2, M_3, M_4\}$, A feasible non-slicing floorplan.

– Place M_1, M_2 in bottom row side by side: $\begin{cases} W_b &= w_1 + w_2 = 2 + \sqrt{12} \approx 5.464, \\ H_b &= \max\{y_1, y_2\} = \max\{4, \sqrt{12}\} \approx 4. \end{cases}$

– Place M_3, M_4 in top row side by side $\begin{cases} W_t &= w_3 + w_4 = 2 + \sqrt{10} \approx 5.162, \\ H_t &= \max\{y_3, y_4\} = \max\{3, \sqrt{10}\} \approx 3.162. \end{cases}$ –
Overall bounding rectangle $\begin{cases} W &= \max\{W_b, W_t\} = 5.464, \\ H &= H_b + H_t \approx 4 + 3.162 = 7.162. \end{cases}$ – All regions satisfy area and aspect-ratio constraints, so X_2 is feasible.

Therefore,

$$A \circ B \ =\ \{\, X_1,\ X_2\}.$$

Proposition 2.3(Well-definedness) *For all $A, B \subseteq S$, the set $A \circ B$ is nonempty if and only if $A \cup B$ itself admits a feasible floorplan.*

Proof By definition, $A \circ B$ consists exactly of those supersets $X \supseteq A \cup B$ that admit a feasible floorplan. In particular, $X = A \cup B$ belongs to $A \circ B$ if and only if $A \cup B$ admits some feasible floorplan. Hence $A \circ B \neq \emptyset$ precisely when $A \cup B$ is floorplannable. □

Theorem 2.4(Hyperfloorplan generalizes classical floorplan) *Let $S = \{M_1, \ldots, M_m\}$. Then,*

(a) Every classical slicing floorplan of S can be realized by an iterated hyperproduct chain in $\mathcal{HF}(S)$;

(b) Conversely, every finite chain of hyperproducts whose last element is S corresponds to a classical slicing floorplan of S.

Particularly, the set of all slicing floorplans of S is in bijection with the set of all hyperproduct expressions in $\mathcal{HF}(S)$ evaluating to S.

Proof We prove parts (a) and (b) by induction on the number of internal nodes in a slicing tree.

(a) *From slicing tree to hyperproduct chain.* Let T be a binary slicing tree whose leaves, in left-to-right order, are the singleton sets $\{M_{i_1}\}, \cdots, \{M_{i_m}\}$ and whose internal nodes are labeled "×" (vertical cut) or "+" (horizontal cut). We construct a sequence of subsets

$$X_1,\, X_2,\, \cdots,\, X_{m-1}$$

and corresponding pairs (A_k, B_k) such that

$$X_k \;=\; A_k \;\circ\; B_k,$$

and at the end $X_{m-1} = S$.

• *Base Case.* If T has a single internal node whose children are leaves $\{M_i\}$ and $\{M_j\}$, then these two modules are sliced together to form a feasible two-module floorplan in $X_1 = \{M_i, M_j\}$. By Definition 2.1, $\{M_i\} \circ \{M_j\}$ contains X_1.

• *Inductive Step.* Suppose for a slicing tree with k internal nodes we have constructed a chain

$$X_1 = A_1 \circ B_1,\ X_2 = A_2 \circ B_2,\ \cdots,\ X_k = A_k \circ B_k$$

with X_k equal to the subset of modules in the subtree rooted at the k-th internal node. Now attach one more cut combining X_k with a leaf $\{M_\ell\}$ or with the result of another subtree Y_k. By feasibility of the slicing, $X_k \cup \{M_\ell\}$ (or $X_k \cup Y_k$) admits a feasible floorplan; hence by Definition 2.1

$$X_{k+1} \;=\; X_k \;\circ\; \{M_\ell\} \quad \big(\text{or } X_k \circ Y_k\big) \;\ni\; X_k \cup \{M_\ell\}\,.$$

This yields the extended chain.

After $m - 1$ steps we obtain $X_{m-1} = S$, realizing the full slicing floorplan.

(b) *From hyperproduct chain to slicing tree.* Conversely, let

$$X_1 = A_1 \circ B_1,\ X_2 = A_2 \circ B_2,\ \cdots,\ X_\ell = A_\ell \circ B_\ell$$

be any finite chain in $\mathcal{HF}(S)$ with $X_\ell = S$. By Proposition 2.3, each X_k admits a feasible floorplan for $A_k \cup B_k$. We form a binary tree whose root combines the two subsets A_ℓ and B_ℓ by the slicing cut used in that floorplan (vertical or horizontal), and whose children are either leaves (if A_ℓ or B_ℓ is a singleton) or the roots of subtrees constructed recursively from the partial chain ending at X_{k-1}. Since each step merges exactly two previously disjoint subsets, the result is a slicing tree describing a valid classical floorplan of S.

Thus every slicing floorplan corresponds exactly to one hyperproduct chain in $\mathcal{HF}(S)$, proving the bijection and hence the desired generalization. □

Proposition 2.5(Commutativity) *For all $A, B \subseteq S$,*

$$A \circ B \;=\; B \circ A.$$

Proof By Definition 2.1,

$$A \circ B = \{\, X \subseteq S \mid A \cup B \subseteq X,\ X \text{ feasible}\},$$

and

$$B \circ A = \{\, X \subseteq S \mid B \cup A \subseteq X,\ X \text{ feasible}\}.$$

Since $A \cup B = B \cup A$, the two sets coincide. □

Proposition 2.6(Identity element) *The empty set $\emptyset$ acts as a neutral element: for any $A \subseteq S$,*

$$\emptyset \circ A \;=\; A \circ \emptyset \;=\; \{\, X \subseteq S \mid A \subseteq X,\ X \textit{ feasible}\}.$$

Particularly, if A itself admits a feasible floorplan, then A is the minimal element of $\emptyset \circ A$.

Proof Immediate from Definition 2.1, since $\emptyset \cup A = A$ and a superset X must satisfy $A \subseteq X$ and feasibility. □

Proposition 2.7(Idempotence) *For any $A \subseteq S$ that admits a feasible floorplan,*

$$A \in A \circ A.$$

Proof By Definition 2.1,

$$A \circ A = \{\, X \subseteq S \mid A \cup A \subseteq X,\ X \text{ feasible}\} = \{\, X \subseteq S \mid A \subseteq X,\ X \text{ feasible}\}.$$

Since $A \subseteq A$ and A is feasible, $A \in A \circ A$. □

Proposition 2.8(Antitone property) *If $A \subseteq A' \subseteq S$ and $B \subseteq B' \subseteq S$, then*

$$A' \circ B' \ \subseteq\ A \circ B.$$

Proof Suppose $X \in A' \circ B'$. Then $A' \cup B' \subseteq X$ and X admits a feasible floorplan. Since $A \cup B \subseteq A' \cup B'$, it follows that $A \cup B \subseteq X$. Hence $X \in A \circ B$. □

Theorem 2.9(Hyper-Associativity) *For all $A, B, C \subseteq S$, define the triple hyperproduct by*

$$A \circ B \circ C := \bigcup_{X \in A \circ B} \big(X \circ C\big) = \bigcup_{Y \in B \circ C} \big(A \circ Y\big).$$

Then,

$$A \circ B \circ C = \{\, Z \subseteq S \mid A \cup B \cup C \subseteq Z,\ Z \textit{ feasible}\}.$$

Proof By definition,

$$\bigcup_{X \in A \circ B} (X \circ C) = \bigcup_{\substack{X \subseteq S \\ A \cup B \subseteq X}} \{\, Z \subseteq S \mid X \cup C \subseteq Z,\ Z \text{ feasible}\}.$$

Since $A \cup B \cup C \subseteq X \cup C \subseteq Z$, the union ranges exactly over all feasible Z containing $A \cup B \cup C$. The same argument applies to $\bigcup_{Y \in B \circ C} (A \circ Y)$, establishing the equality. □

§3. Review: n-Superhyperfloorplan

An n-superhyperfloorplan organizes hierarchical sets of modules using iterated powersets and feasibility constraints across n abstraction levels for multi-layer layout (cf.[6]).

Definition 3.1(n-Superhyperfloorplan, (cf.[6]) *Let $S = \{M_1, \cdots, M_m\}$ be a finite set of atomic modules, and define recursively the k-th powerset*

$$\mathcal{P}_1(S) = \mathcal{P}(S), \qquad \mathcal{P}_{k+1}(S) = \mathcal{P}\big(\mathcal{P}_k(S)\big).$$

We now introduce a feasibility predicate F_k on subsets of $\mathcal{P}_k(S)$

- *$F_1(X)$ holds for $X \subseteq \mathcal{P}_1(S)$ if and only if there exists a classical (slicing or non-slicing) floorplan of the modules in X within some bounding rectangle, satisfying all area, aspect-ratio, non-overlap, and interconnection constraints.*
- *For $k > 1$, $F_k(X)$ holds for $X \subseteq \mathcal{P}_k(S)$ if and only if*

(i) There exists a bounding rectangle R and an arrangement of the "supermodules' ' X inside R satisfying the usual floorplanning constraints at level k.

(ii) Every element $Y \in X$, viewed as a subset $Y \subseteq \mathcal{P}_{k-1}(S)$, satisfies $F_{k-1}(Y)$.

For a fixed $n \geqslant 1$, define the hyperoperation

$$\circ_n : \underbrace{\mathcal{P}_n(S) \times \cdots \times \mathcal{P}_n(S)}_{m\ times} \longrightarrow \mathcal{P}\big(\mathcal{P}_n(S)\big)$$

by

$$A_1 \circ_n A_2 \circ_n \cdots \circ_n A_m = \Big\{ X \in \mathcal{P}_n(S) \;\Big|\; \bigcup_{i=1}^{m} A_i \subseteq X \ and\ F_n(X) \Big\}.$$

The n-superhyperfloorplan *of S is the hyperstructure*

$$\mathcal{SHF}_n = \big(\mathcal{P}_n(S), \circ_n\big).$$

Example 3.2(2-Superhyperfloorplan of $S = \{M_1, M_2, M_3\}$) Let

$$S = \{M_1, M_2, M_3\},$$

with atomic module parameters

Module	A_i	(l_i, u_i)	Chosen (w_i, h_i)
M_1	8	$[0.5, 2.0]$	$(2, 4),\ 2 \cdot 4 = 8,\ 4/2 = 2.0$
M_2	12	$[1.0, 1.0]$	$(\sqrt{12}, \sqrt{12}),\ \sqrt{12}^2 = 12,\ 1.0$
M_3	6	$[0.75, 1.5]$	$(2, 3),\ 2 \cdot 3 = 6,\ 3/2 = 1.5$

We have $\mathcal{P}_1(S) = \mathcal{P}(S)$ and $\mathcal{P}_2(S) = \mathcal{P}(\mathcal{P}_1(S))$. Choose two level-2 elements

$$A = \big\{\{M_1\}, \{M_2, M_3\}\big\}, \quad B = \big\{\{M_2\}, \{M_1, M_3\}\big\} \subseteq \mathcal{P}_2(S).$$

By Definition 3.1,

$$A \circ_2 B = \Big\{ X \in \mathcal{P}_2(S) \;\Big|\; \{\{M_1\}, \{M_2\}, \{M_1, M_3\}, \{M_2, M_3\}\} \subseteq X \text{ and } F_2(X) \Big\}.$$

We exhibit two minimal supersets X_1 and X_2

$$X_1 = \big\{\{M_1\}, \{M_2\}, \{M_1, M_3\}, \{M_2, M_3\}\big\},$$
$$X_2 = X_1 \cup \big\{\{M_1, M_2, M_3\}\big\}.$$

Step 1. Verify F_1 for each "supermodule" $Y \in X_1 \cup X_2$.

- $Y_1 = \{M_1\}$, a trivial bounding box (2×4).
- $Y_2 = \{M_2\}$, a bounding box $\big(\sqrt{12} \times \sqrt{12}\big) \approx (3.464 \times 3.464)$.
- $Y_3 = \{M_1, M_3\}$, choose vertical slice

$$W_3 = 2 + 2 = 4, \quad H_3 = \max\{4, 3\} = 4.$$

• $Y_4 = \{M_2, M_3\}$, choose vertical slice:

$$W_4 = \sqrt{12} + 2 \approx 5.464, \quad H_4 = \max\{\sqrt{12}, 3\} = \sqrt{12} \approx 3.464.$$

• $Y_5 = \{M_1, M_2, M_3\}$. In this case, first horizontal slice of M_1, M_2: $W_a = \max\{2, \sqrt{12}\} \approx 3.464$, $H_a = 4 + \sqrt{12} \approx 7.464$; then vertical slice with M_3: $W_5 = W_a + 2 \approx 5.464$, $H_5 = \max\{7.464, 3\} = 7.464$.

Thus. $F_1(Y)$ holds for all $Y \in X_1 \cup X_2$.

Step 2. Verify F_2 (arrangement of supermodules).

(a) For X_1: four supermodules $Y_1, \cdots, Y_4$. Arrange in two rows

$$\begin{aligned} &\text{Bottom row: } Y_1\,(2\times 4),\ Y_2\,(3.464\times 3.464) && \Rightarrow W_b = 2 + 3.464 = 5.464, \quad H_b = 4,\\ &\text{Top row: } Y_3\,(4\times 4),\ Y_4\,(5.464\times 3.464) && \Rightarrow W_t = 4 + 5.464 = 9.464, \quad H_t = 4. \end{aligned}$$

Overall bounding box

$$W = \max\{5.464, 9.464\} = 9.464, \quad H = H_b + H_t = 4 + 4 = 8,$$

so $F_2(X_1)$ holds.

(b) For X_2: five supermodules $Y_1, \ldots, Y_5$. Arrange in two rows

$$\begin{aligned} &\text{Bottom row: } Y_1(2\times 4),\ Y_2(3.464\times 3.464),\ Y_3(4\times 4) \Rightarrow W_b = 2 + 3.464 + 4 = 9.464,\ H_b = 4,\\ &\text{Top row: } Y_4(5.464\times 3.464),\ Y_5(5.464\times 7.464) \Rightarrow W_t = 5.464 + 5.464 = 10.928,\ H_t = 7.464. \end{aligned}$$

Overall bounding box

$$W = \max\{9.464, 10.928\} = 10.928, \quad H = H_b + H_t = 4 + 7.464 = 11.464,$$

so $F_2(X_2)$ holds as well.

Hence

$$A \circ_2 B = \{ X_1,\ X_2\},$$

demonstrating a concrete 2-superhyperfloorplan of S.

Example 3.3(3-Superhyperfloorplan of $S = \{M_1, M_2\}$) Let

$$S = \{M_1, M_2\},$$

with module parameters

Module	A_i	Chosen dimensions (w_i, h_i)
M_1	4	$(2, 2)$
M_2	9	$(3, 3)$

so that $w_i h_i = A_i$ and all aspect ratios are 1. We have

$$\mathcal{P}_1(S) = \{\emptyset, \{M_1\}, \{M_2\}, \{M_1, M_2\}\},$$

$$\mathcal{P}_2(S) = \mathcal{P}(\mathcal{P}_1(S)), \quad \mathcal{P}_3(S) = \mathcal{P}(\mathcal{P}_2(S)).$$

Define three level-2 "supermodules" in $\mathcal{P}_2(S)$ by

$$U_0 = \emptyset, \quad U_1 = \{\{M_1\}\}, \quad U_2 = \{\{M_2\}\}, \quad U_3 = \{\{M_1\}, \{M_2\}\}.$$

Then, each U_j satisfies $F_2(U_j)$ since

- For U_1 and U_2, the single submodule is trivially placed in its own bounding box.
- For U_3, place $\{M_1\}$ and $\{M_2\}$ side-by-side: $W = 2 + 3 = 5,\ H = \max\{2, 3\} = 3$.
- For $U_0 = \emptyset$, no modules $\rightarrow$ trivial feasibility.

Choose two level-3 elements in $\mathcal{P}_3(S)$:

$$A = \{U_3\}, \qquad B = \{U_1,\ U_2\}.$$

Then, by Definition 3.1,

$$A \circ_3 B = \Big\{ X \in \mathcal{P}_3(S) \ \Big|\ \{U_1, U_2, U_3\} \subseteq X \text{ and } F_3(X) \Big\}.$$

We take the two minimal supersets:

$$X_1 = \{U_1, U_2, U_3\}, \quad X_2 = X_1 \cup \{U_0\}.$$

Step 1. Check F_2 for each U_j. As above, all four satisfy F_2.

Step 2. Verify $F_3(X_k)$ by arranging the level-2 supermodules U_j in a bounding rectangle, using their level-2 bounding dimensions

$$\dim(U_1) = (2, 2), \quad \dim(U_2) = (3, 3), \quad \dim(U_3) = (5, 3), \quad \dim(U_0) = (0, 0).$$

(a) For X_1: place U_1, U_2 in top row, U_3 in bottom row

$$W_{\text{top}} = 2 + 3 = 5,\ H_{\text{top}} = 3, \quad W_{\text{bot}} = 5,\ H_{\text{bot}} = 3,$$

so overall $W = \max\{5, 5\} = 5,\ H = 3 + 3 = 6$.

(b) For X_2: include the trivial U_0 (zero-area) alongside

$$\text{Same layout gives } W = 5,\ H = 6,$$

so $F_3(X_2)$ holds as well.

Therefore, $A \circ_3 B = \{X_1,\ X_2\}$ exhibiting a concrete 3-superhyperfloorplan of S.

Theorem 3.4(n-Superhyperfloorplan Generalizes Hyperfloorplan) *Let* $\mathcal{HF} = (\mathcal{P}_1(S), \circ_1)$ *be* *the hyperfloorplan of S. Then,*

(a) $\mathcal{SHF}_1 = (\mathcal{P}_1(S), \circ_1)$ coincides exactly with $\mathcal{HF}$.

(b) For each $n > 1$, the inclusion $\iota\colon \mathcal{P}_1(S) \hookrightarrow \mathcal{P}_n(S)$, $A \mapsto A$, induces an embedding of hyperstructures $\iota\colon \mathcal{HF} \hookrightarrow \mathcal{SHF}_n$ preserving the hyperoperation:

$$\iota\big(A \circ_1 B\big) \;=\; \iota(A) \circ_n \iota(B), \quad \forall\, A, B \subseteq S.$$

Thus every classical hyperfloorplan is recovered as the special case $n = 1$, and $\mathcal{SHF}_n$ strictly generalizes $\mathcal{HF}$.

Proof (a) By definition $\mathcal{P}_1(S) = \mathcal{P}(S)$ and F_1 is exactly the classical feasibility predicate. Hence $\circ_1$ agrees with the hyperfloorplan operation of the Definition, so $\mathcal{SHF}_1 = \mathcal{HF}$.

(b) For $n > 1$, observe that the inclusion map $\iota\colon A \mapsto A$ identifies each level-1 subset with itself inside $\mathcal{P}_n(S)$. If $F_1(X)$ holds for $X \subseteq \mathcal{P}_1(S)$, then for the same X regarded in $\mathcal{P}_n(S)$, condition (ii) of F_n is automatic (each $Y \in X$ is an element of $\mathcal{P}_1(S) \subseteq \mathcal{P}_{n-1}(S)$ and satisfies F_1), and condition (i) matches the original floorplanning feasibility. Therefore,

$$X \in A \circ_1 B \quad\Longleftrightarrow\quad X \in \iota(A) \circ_n \iota(B),$$

showing that ι preserves the hyperoperation and is injective. Hence $\mathcal{HF}$ embeds as a sub-hyperstructure of $\mathcal{SHF}_n$. □

Proposition 3.5(Commutativity) *For any $A_1, \ldots, A_m \subseteq \mathcal{P}_n(S)$ and any permutation σ of $\{1, \cdots, m\}$,*

$$A_1 \circ_n A_2 \circ_n \cdots \circ_n A_m \;=\; A_{\sigma(1)} \circ_n A_{\sigma(2)} \circ_n \cdots \circ_n A_{\sigma(m)}.$$

Proof By Definition 3.1,

$$A_1 \circ_n \cdots \circ_n A_m = \big\{\, X \in \mathcal{P}_n(S) \mid U := \bigcup_{i=1}^{m} A_i \subseteq X,\; F_n(X) \big\}.$$

Since set-union is commutative and $F_n(X)$ depends only on X, the right-hand side is unchanged by permuting the A_i. □

Proposition 3.6(Monotonicity) *If $A_i \subseteq B_i \subseteq \mathcal{P}_n(S)$ for all $i = 1, \ldots, m$, then*

$$B_1 \circ_n B_2 \circ_n \cdots \circ_n B_m \;\subseteq\; A_1 \circ_n A_2 \circ_n \cdots \circ_n A_m.$$

Proof Let

$$X \in B_1 \circ_n \cdots \circ_n B_m.$$

Then, $\bigcup_i B_i \subseteq X$ and $F_n(X)$. Since each $A_i \subseteq B_i$, we have $\bigcup_i A_i \subseteq \bigcup_i B_i \subseteq X$. Hence $X \in A_1 \circ_n \cdots \circ_n A_m$. □

Theorem 3.7(Hyper-Associativity) *For any $A, B, C \subseteq \mathcal{P}_n(S)$, define*

$$A \circ_n B \circ_n C := \bigcup_{X \in A \circ_n B} \bigl(X \circ_n C\bigr) \;=\; \bigcup_{Y \in B \circ_n C} \bigl(A \circ_n Y\bigr).$$

Then,

$$A \circ_n B \circ_n C = \bigl\{\, Z \in \mathcal{P}_n(S) \mid A \cup B \cup C \subseteq Z,\ F_n(Z) \bigr\}.$$

Proof By Definition 3.1,

$$A \circ_n B = \{\, X \mid A \cup B \subseteq X,\ F_n(X)\}.$$

Hence,

$$\bigcup_{X \in A \circ_n B} (X \circ_n C) = \bigcup_{\substack{X \subseteq \mathcal{P}_n(S) \\ A \cup B \subseteq X \\ F_n(X)}} \{\, Z \mid X \cup C \subseteq Z,\ F_n(Z)\}.$$

But $A \cup B \cup C \subseteq X \cup C \subseteq Z$ and $F_n(Z)$ is independent of how the union is parenthesized. The symmetric argument applies to $\bigcup_{Y \in B \circ_n C}(A \circ_n Y)$, giving the claimed equality. □

Theorem 3.8(Embedding of Lower Levels) *For each k with $1 \leqslant k < n$, the inclusion map $\iota_k : \mathcal{P}_k(S) \hookrightarrow \mathcal{P}_{k+1}(S)$, $X \mapsto \{X\}$, induces an embedding of hyperstructures*

$$\iota_k : \mathcal{SHF}_k = \bigl(\mathcal{P}_k(S), \circ_k\bigr) \;\longrightarrow\; \mathcal{SHF}_{k+1} = \bigl(\mathcal{P}_{k+1}(S), \circ_{k+1}\bigr).$$

Proof Let $A, B \subseteq \mathcal{P}_k(S)$. Under ι_k, they become singletons $\{A\}, \{B\} \subseteq \mathcal{P}_{k+1}(S)$. By Definition 3.1,

$$\iota_k(A) \circ_{k+1} \iota_k(B) = \{\, X \subseteq \mathcal{P}_{k+1}(S) \mid \{A, B\} \subseteq X,\ F_{k+1}(X)\}.$$

Since $F_{k+1}(X)$ requires each element of X to satisfy F_k, particularly, A, B must each satisfy F_k. Thus $\{A, B\} \subseteq X$, $F_{k+1}(X)$ holds exactly when $A \cup B \subseteq X' \subseteq \mathcal{P}_k(S)$ with $F_k(X')$, identifying X' with $X \setminus \{\dots\}$. One checks directly that this correspondence preserves distinctness and the hyperoperation $\circ_k$. Thus, ι_k is an injective homomorphism of hyperstructures. This completes the proof. □

Theorem 3.9(Reduction to Hyperfloorplan) *If $n = 1$, $\mathcal{SHF}_1 = (\mathcal{P}_1(S), \circ_1)$ coincides with the classical hyperfloorplan $\mathcal{HF}(S)$ of Definition* 2.1.

Proof For $n = 1$, we have $\mathcal{P}_1(S) = \mathcal{P}(S)$ and the feasibility predicate F_1 is exactly “admitting a classical floorplan”. Therefore,$\circ_1$ matches the operation $\circ$ of Definition 2.1, yielding

$$\mathcal{SHF}_1 = \mathcal{HF}(S).$$

□

§3. Conclusion

In this paper, we revisited floorplans and examined the notions of *hyperfloorplan* and *superhyperfloorplan* as defined in [6]. In the future, we expect further investigations into algorithms related to these concepts, as well as studies on their extended frameworks incorporating fuzzy sets [29,30], intuitionistic fuzzy sets [31,32], neutrosophic sets [31,32], and plithogenic sets [35 – 37].

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