*International J.Math. Combin. Vol.1-Vol.2(2025), 78-89*

# Modulo Two Square Mean Labeling of Some Path and Path Related Graphs

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**Abstract**: We introduce the new concept of modulo two square mean labeling. A graph is said to be modulo two square mean labeling, if there is a function $\phi$ from the vertex set of G to $\{1,2,3,\cdots n\}$, $\phi'$ from the edge set of G to $\{1\}$ where $\phi^{'}(uv) = \left\lceil \frac{f(u)^2+f(v)^2}{2} \right\rceil$ mod 2. In this paper we Prove that the modulo two square mean labeling of some path related graphs and $H-$graph with more than 3 vertices. Additionally, we provide a $C++$ program designed to determine the modulo two square mean labeling for the above mentioned graphs.

**Key Words**: Square sum labeling, mean labeling, root mean square labeling, Smarandachely mean labeling.

**AMS(2010)**: 05C78, 05C85.

## §1. Introduction

In this paper, we consider only simple, finite, undirected and non-trivial graph G = (V(G),E(G)) with the vertex set V(G) and the edge set E(G). Labeling of a graph G is an assignment integers to vertices or edges or both following certain rules. A useful survey on graph labeling by J.A.Gallian (2015) can be found in [1]. Labeled graph has its own applications in various fields such as engineering, technology, etc. A particular type of labeling becomes more interesting if there arises a number of problems that kindles the interest of the researchers. Prominent among the types of labeling is square sum labeling [2],[3], [4], [5]. In this paper we deal only finite, simple, connected and undirected graphs obtained through graph operations. Another labeling has been introduced by Somasundaram and Ponraj [6] the notion of mean labeling of graphs. A graph $G$ with $p$ vertices and $q$ edges is called a mean graph if there is an injective

[1]Received November 15, 2024. Accepted May 16, 2025

function $f$ from the vertices of $G$ to $\{0, 1, 2, \cdots, q\}$ such that when each edge $uv$ is labeled with

$$\begin{cases} \dfrac{f(u)+f(v)}{2}, & \text{if } f(u)+f(v) \text{ is even,} \\ \dfrac{f(u)+f(v)+1}{2}, & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

then the resulting edge labels are distinct. Generally, let $V' \subset V(G)$. If $G \backslash V'$ has a mean labeling $\phi$, then $\phi$ is called a Smarandachely mean labeling respect to $V'$ on $G$. The concept of root square mean labeling has been introduced by S. S. Sandhya, S. Somasundaram and S. Anusa in 2014. Meena. S and Mani. R investigated this labeling for some cycle related graphs.

## §2. Basic Definitions

We use the following definitions in the subsequent section to prove the main result.

**Definition** 2.1([4]) *A path is a trail in which all vertices are distinct.*

**Definition** 2.2 *The H-Graph of a path $P_n, n \geq 3$ is obtained from two copies of $v_1, v_2, v_3, \cdots v_n$ and $u_1, u_2, u_3, \cdots u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ by an edge if $n$ is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}+1}$ if $n$ is even.*

## §3. Main Results

**Theorem** 3.1 *The graph $P_n, n \geq 2$ is a modulo two square mean labeling.*

*Proof* Let $v(P_n) = \{v_i/1 \leq i \leq n\}$ be the vertex set and $E(P_n) = \{e_i = 1, 1 \leq i \leq n-1\}$is the edge set. The graph has n vertices and n-1 edges.

Let $f\colon v \to \{1, 2, \cdots n\}$ by defining the vertex labeling $f(v_i) = \{i \ for 1 \leq i \leq n\}$. Then the induced edge labels are $f(e_i) = \left\lceil \frac{f(v_i)^2 + f(v_{i+1})^2}{2} \right\rceil \bmod 2$, $1 \leq i \leq n-1$ for $e_i \in P_n$ if $n \geq 2$. Then for every $e \in E(P_n)$ is $f(e_i) = \{1\}$. Hence $f$ is a modulo two square mean labeling. □

**Illustration 3.1** A modulo two square mean labeling of $P_5$ is shown in Figure 1.

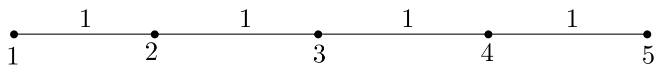


**Figure 1.** Modulo two square mean labeling of $P_5$

**Program 1**

```
# include <iostream>
# include <cmath>
int main()
{
int n, x[100],y[100],v[100],i;
```

```
std::cout << "Enter The Number of Vertices n = ";
std::cin>> n;
for(i = 1; i <= n; i++)
{
std::cout<< "\n The Path of the Vertices of v[" << i << "] = " << i << "\n";
x[i]= (i*i)+((i+1)*(i+1));
y[i]=ceil(float(x[i])/2);
}
for(i = 1;i <= n;i++)
{
std:: cout<< "\n" <<"The Square of the Edges of e(" << i << ") = " << y[i];
std:: cout<< "\t\t" <<" Edges of e(" << i << ") = " << y[i]%2;
std::cout<< "\n";}
return 0;
}
```

**Theorem** 3.2 *A graph $G$ obtained by attaching each vertex of $P_n$ to the central vertex of $K_{1,2}$ is a modulo two square mean labeling.*

*Proof* Let $G = P_n \otimes K_{1,2}$ be a graph obtained from a path $P_n$ with vertices $u_i$ and joining the vertices $v_i, w_i$ of $K_{1,2}$ for the vertices $u_i$ of $P_n$, $1 \le i \le n$ respectively.

Define $f\colon V(G) \to \{1, 2, 3, \cdots 3n\}$ to be

$$f(u_i) = 3i - 1 \;\; 1 \le i \le n,$$

$$f(v_i) = 3i - 2 \;\; 1 \le i \le n,$$

$$f(w_i) = 3i \;\; 1 \le i \le n.$$

by the definition of modulo two square mean labeling. The edges get labels

$$f(u_iu_{i+1}) = \left\lceil \frac{f(u_i)^2 + f(u_{i+1})^2}{2} \right\rceil \bmod 2,\ 1 \le i \le n-1,$$

$$f(u_iv_i) = \left\lceil \frac{f(u_i)^2 + f(v_i)^2}{2} \right\rceil \bmod 2,\ 1 \le i \le n,$$

$$f(u_iw_i) = \left\lceil \frac{f(u_i)^2 + f(w_i)^2}{2} \right\rceil \bmod 2,\ 1 \le i \le n.$$

Clearly, $f$ admits modulo two square mean labeling. □

**Illustration 3.2** A modulo two square mean labeling of $P_3 \otimes K_{1,2}$ is shown in Figure 2.

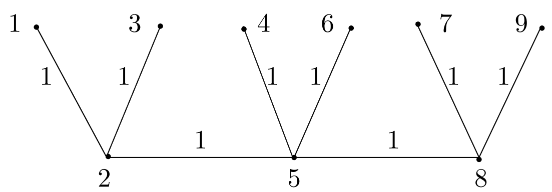


**Figure 2.** Modulo two square mean labeling of $P_3 \otimes K_{1,2}$

**Program 2**

```
# include <iostream>
# include <cmath>
int main()
{
int n, i,c=0; int v[100],u[100],w[100];
int e1[100],e2[100],e3[100],e4[100],e5[100],e6[100];
std::cout<< "Enter the Number of Vertices = ";
std::cin>>n;
for (i=1;i<=n;i++)
{
v[i]=3*i-2;
u[i]=3*i-1;
w[i]=3*i;
std::cout<< "\t The Vertices of v[" << i << "] = " << v[i];
std::cout<< "\t The Vertices of u[" << i << "] = " << u[i];
std::cout<<" \t The Vertices of w[" << i << "] = " << w[i];
std::cout<< "\n";
}
std::cout<< "\n";
for (i = 1; i <= n; i ++)
{
e3[i]=((u[i]*u[i])+(v[i]*v[i]));
e4[i]= ceil( float (e3[i])/2);
std::cout<<"\n Edges addition of e[" << i << "] = " << e3[i] << "\t\t";
std::cout<< "\t\t Edges of e[" << i << "] = " << e4[i]%2 << "\t";
}
for (i = 1; i <= n; i ++)
{
e5[i]=((u[i]*u[i])+(w[i]*w[i]));
e6[i]= ceil( float (e5[i])/2);
std::cout<< "\n Edges addition of e[" << i + n << "] = " << e5[i] << "\t\t";
std::cout<< "\t\t Edges of e[" << i + n << "] = " << e6[i]%2 << "\t";
}
for (i=1; i¡=n-1; i++)
{
e1[i]=((u[i]*u[i])+(u[i+1]*u[i+1]));
e2[i]= ceil( float (e1[i])/2);
std::cout<< "\n Edges addition of e[" << 2 * n + i << "] = " << e1[i] << "\t\t";
std::cout<< "\t\t Edges of e[" << 2 * n + i << "] = " << e2[i]%2 << "\t";
}
return 0;
```

```
}
```

**Theorem** 3.3 *The graph $K_{1,2} \otimes P_n$ is a modulo two square mean labeling.*

*Proof* Let $G = K_{1,2} \otimes P_n$ be a graph obtained by joining a pendant vertex of a path $P_n$ with a star $K_{1,2}$ and let $u$ be the central vertex of $K_{1,2}$. Let $u_1, u_2$ be the other vertices of $K_{1,2}$ and let $v_1, v_2, \cdots v_n$ be the vertices of $P_n$.

Define $f\colon V(G) \to \{1, 2, 3, \cdots n + 2\}$ as follows:

$f(u_1) = 1$ and $f(u_2) = 2$, $f(u = v_1) = 3$,

$f(v_{i+1}) = 3 + i$. and $1 \le i \le n - 1$

by the definition of modulo two square mean labeling. They are labeled as

$$f(u_1 v_1) = \left\lceil \frac{f(u_1)^2 + f(v_1)^2}{2} \right\rceil \bmod 2,$$

$$f(u_2 v_1) = \left\lceil \frac{f(u_2)^2 + f(v_1)^2}{2} \right\rceil \bmod 2,$$

$$f(v_i v_{i+1}) = \left\lceil \frac{f(v_i)^2 + f(v_{i+1})^2}{2} \right\rceil \bmod 2,\ 1 \le i \le n - 1.$$

Clearly, $G$ admits modulo two square mean labeling. □

**Illustration 3.3** A modulo two square mean labeling of $K_{1,2} \otimes P_3$ is shown in Figure 3.

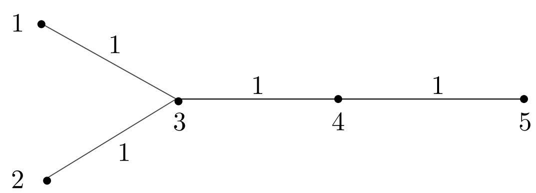


**Figure 3.** Modulo two square mean labeling of $K_{1,2} \otimes P_3$

**Program 3**

```
# include <iostream>
# include <cmath>
int main()
{
int n, i,j,c=0;
int v[100],u[100],w[100];
int e1[500],e2[500];
std::cout<< "Enter the Number of Vertices of path Graph = ";
std::cin>>n;
u[1]=1;
u[2]=2;
v[1]=3;
std::cout<< "\n The Vertices of u[" << 1 << "] = " << u[1];
std::cout<< "\n The Vertices of u[" << 2 << "] = " << u[2];
```

```
std::cout<< "\n\n The path Vertices of v[" << 1 << "] = " << v[1];
for (i = 1; i <= n − 1; i + +)
{
v[i+1]=3+i;
std::cout<< "\n The path Vertices of v[" << i + 1 << "] = " << v[i + 1];
}
std::cout<< "\n";
e1[1]=((u[1]*u[1])+(v[1]*v[1]));
e2[1]=ceil( float (e1[1])/2);
std::cout<< "\n Edges addition of e[" << 1 << "] = " << e1[1] << "\t";
std::cout<< "\t\t Edges of e[" << 1 << "] = " << e2[1]%2 << "\t";
e1[2]=((u[2]*u[2])+(v[1]*v[1]));
e2[2]= ceil( float (e1[2])/2);
std::cout<< "\t\t Edges addition of e[" << 2 << "] = " << e1[2] << "\t";
std::cout<< "\t\t Edges of e[" << 2 << "] = " << e2[2]%2 << "\t";
for(i = 1; i <= n − 1; i + +)
{
e1[i+2]=((v[i]*v[i])+(v[i+1]*v[i+1]));
e2[i+2]= ceil (float (e1[i+2])/2);
std::cout<< "\n Edges addition of e[" << i + 2 << "] = " << e1[i + 2] << "\t";
std::cout<< "\t\t Edges of e[" << i + 2 << "] = " << e2[i + 2]%2 << "\t";
}
return 0;
}
```

**Theorem** 3.4 *The graph $H_n$ with odd $n$ and $n \geq 3$, is a modulo two square mean labeling.*

*Proof* Let $H_n, n \geq 3$ be a H-Graph with vertex set $\{u_1, u_2, u_3, \cdots u_n, v_1, v_2, v_3, \cdots v_n\}$ and edge set $\{u_i u_{i+1}, v_i v_{i+1} \,|\, 1 \leq i \leq n-1\} \cup \{u_{\frac{n+1}{2}} \;\; v_{\frac{n+1}{2}} if\, n\, is\, odd\}$. Define $f : v(G) \to \{1, 2, 3, \cdots .2n\}$ by

$$f(u_i) = i \;, \quad 1 \leq i \leq n,$$
$$f(v_i) = n + i, \quad 1 \leq i \leq n$$

by defining the edge labels

$$f(e_i) = \left\lceil \frac{f(u_i)^2 + f(u_{i+1})^2}{2} \right\rceil \bmod 2 \text{ for } 1 \leq i \leq n-1,$$

$$f(e_{i+n-1}) = \left\lceil \frac{f(v_i)^2 + f(v_{i+1})^2}{2} \right\rceil \bmod 2 \quad \text{for } 1 \leq i \leq n-1,$$

$$f(e_{2n-1}) = \left\lceil \frac{f\left(\frac{u_{n+1}}{2}\right)^2 + f\left(\frac{v_{n+1}}{2}\right)^2}{2} \right\rceil \bmod 2.$$

Then, for every $e \in E(P_n)$ is $f(e_i) = 1$. Hence, the graph $H_n$ , $n \geq 3$ and $n$ is odd has a modulo two square mean labeling. □

**Illustration 3.4** A modulo two square mean labeling of $H_5$ is shown in Figure 4.

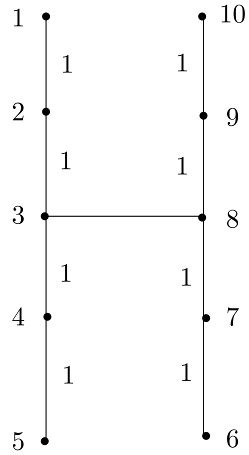


**Figure 4.** Modulo two square mean labeling of $H_5$

**Program 4**

```
# include <iostream>
# include <cmath>
int main()
{
int n, i,j,c=0; int v[100],u[100];
int e1[100],e2[100],e3[100],e4[100],e5[1],e6[1];
std::cout<< "Enter the Number of Vertices of H-Graph with odd Number of vertices = ";
std::cin>>n;
for (i = 1; i <= n; i++)
{
u[i]=i;
v[i]=n+i;
}
for (i = 1; i <= n; i++)
std::cout<< "\n The Vertices of u[" << i << "] = " << u[i];
std::cout<< "\n";
for (i = 1; i <= n; i++)
std::cout<< "\n The Vertices of v[" << i << "] = " << v[i];
std::cout<< "\n";
for(i = 1; i <= n - 1; i++)
{
e1[i]=((u[i]*u[i])+(u[i+1]*u[i+1]));
e2[i]= ceil( float (e1[i])/2);
std::cout<< "\n Edges addition of e[" << i << "] = " << e1[i] << "\t";
std::cout<< "\t\t Edges of e[" << i << "] = " << e2[i]%2 << "\t";
}
```

```
for(i = 1; i <= n − 1; i + +)
{
e3[i]=((v[i]*v[i])+(v[i+1]*v[i+1]));
e4[i]= ceil( float (e3[i])/2);
std::cout<< "\n Edges addition of e[" << n + i − 1 << "] = " << e3[i] << "\t";
std::cout<< "\t\t Edges of e[" << n + i − 1 << "] = " << e4[i]%2 << "\t";
}
e5[1]=((v[(n+1)/2]*v[(n+1)/2])+(u[(n+1)/2]*u[(n+1)/2]));
e6[1]= ceil( float (e5[1])/2);
std::cout<< "\n Middle Edges addition of e[" << n + i − 1 << "] = " << e5[1] << "\t";
std::cout<< "\t\t Edges of e[" << n + i − 1 << "] = " << e6[1]%2 << "\t";
return 0;
}
```

**Theorem** 3.5 *The graph $H_n \otimes K_{1,2}$ where $n$ is odd and $n \geq 3$ is a modulo two square mean labeling.*

*Proof* Let $G = H_n \otimes K_{1,2}$ where $H_n$ is a $H-$graph with vertices $u_1, u_2, u_3, \cdots u_n$ and $v_1, v_2, v_3, \cdots v_n$. for $1 \leq i \leq n$. Let $t_i, s_i$ be the vertices of $K_{1,2}$ attached at $u_i$, and $x_i, y_i$ be the vertices of $K_{1,2}$ joined at $v_i$. Define $f : V(G) \to \{1, 2, \cdots, 6n\}$, then the label to the vertices are as follows:

$f(t_i) = 3i - 2, 1 \leq i \leq n.$ ,

$f(s_i) = 3i, 1 \leq i \leq n.$ ,

$f(u_i) = 3i - 1, 1 \leq i \leq n.$, ,

$f(x_{i-n}) = 3i - 2, n + 1 \leq i \leq 2n.$ ,

$f(y_{i-n}) = 3i, n + 1 \leq i \leq 2n.$ ,

$f(v_{i-n}) = 3i - 1, n + 1 \leq i \leq 2n.$

by the definition of a modulo two square mean labeling. We define the edge labels are defined to be

$$f(u_i t_i) = \left\lceil \frac{f(u_i)^2 + f(t_i)^2}{2} \right\rceil mod\, 2, \qquad 1 \leq i \leq n,$$

$$f(u_i s_i) = \left\lceil \frac{f(u_i)^2 + f(s_i)^2}{2} \right\rceil mod\, 2, \qquad 1 \leq i \leq n,$$

$$f(v_i x_i) = \left\lceil \frac{f(v_i)^2 + f(x_i)^2}{2} \right\rceil mod\, 2, \qquad 1 \leq i \leq n,$$

$$f(v_i y_i) = \left\lceil \frac{f(v_i)^2 + f(y_i)^2}{2} \right\rceil mod\, 2, \qquad 1 \leq i \leq n,$$

$$f(u_i u_{i+1}) = \left\lceil \frac{f(u_i)^2 + f(u_{i+1})^2)}{2} \right\rceil mod\, 2, \quad 1 \leq i \leq n - 1,$$

$f(v_iv_{i+1}) = \left\lceil \frac{f(v_i)^2+f(v_{i+1})^2)}{2} \right\rceil mod\, 2, \quad 1 \leq i \leq n-1,$

$f(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}) = \left\lceil \frac{f(u_{\frac{n+1}{2}})^2+f(v_{\frac{n+1}{2}})^2}{2} \right\rceil mod\, 2,$ if $n$ is odd.

Hence, the edge label satisfying the a modulo two square mean labeling and $H_n \otimes K_{1,2}$ has a modulo two square mean labeling. □

**Illustration 3.5** A modulo two square mean labeling of $H_5 \otimes K_{1,2}$is shown in Figure 5.

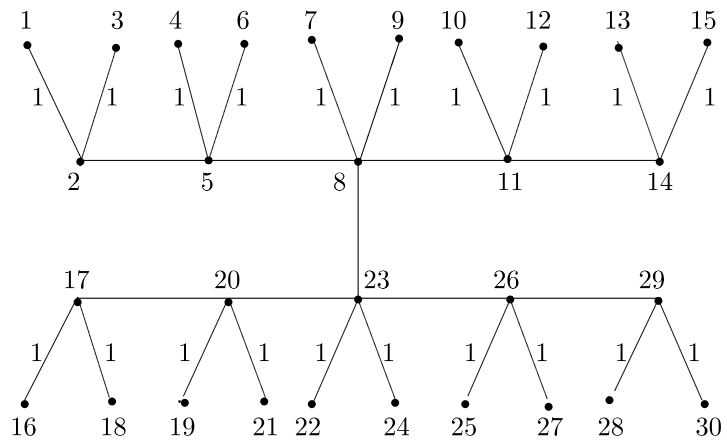


**Figure 5.** Modulo two square mean labeling of $H_5 \otimes K_{1,2}$

**Program 5**

```
# include <iostream>
# include <cmath>
int main()
{
int n, i,j,c=0; int v[100],u[100],w[100],t[100],s[100],x[100],y[100];
int e1[100],e2[100],e3[100],e4[100],e5[100],e6[100],e7[100],e8[100];
int e9[100],e10[100],e11[100],e12[100],e13[100],e14[100];
std::cout<< "Enter the Number of Vertices of H-Graph with odd Number of vertices = ";
std::cin>>n;
for (i = 1; i <= n; i++)
{
t[i]=3*i-2;
s[i]=3*i;
u[i]=3*i-1;
}
for (i = n + 1; i <= 2 * n; i++)
{
x[i-n]=3*i-2;
y[i-n]=3*i;
```

```
v[i-n]=3*i-1;
}
std::cout<< "\n";
for (i = 1; i <= n; i++)
{
std::cout<< "\t The Vertices of t[" << i << "] = " << t[i];
std::cout<< "\t The Vertices of u[" << i << "] = " << u[i];
std::cout<< "\t The Vertices of s[" << i << "] = " << s[i];
std::cout<< "\n";
}
std::cout<< "\n";
for (i = n + 1; i <= 2 * n; i++)
{
std::cout<< "\t The Vertices of x[" << i - n << "] = " << x[i - n];
std::cout<< "\t The Vertices of v[" << i - n << "] = " << v[i - n];
std::cout<< "\t The Vertices of y[" << i - n << "] = " << y[i - n];
std::cout<< "\n";
}
std::cout<< "\n";
for(i = 1; i <= n; i++)
{
e1[i]=((u[i]*u[i])+(t[i]*t[i]));
e2[i]= ceil(float (e1[i])/2);
std::cout<< "\n Edges addition of e[" << i << "] = " << e1[i] << "\t";
std::cout<< "\t Edges of e[" << i << "] = " << e2[i]%2 << "\t";
}
for(i = 1; i <= n; i++)
{
e3[i]=((u[i]*u[i])+(s[i]*s[i]));
e4[i]= ceil( float (e3[i])/2);
std::cout<< "\n Edges addition of e[" << n + i << "] = " << e3[i] << "\t";
std::cout<< "\t Edges of e[" << n + i << "] = " << e4[i]%2 << "\t";
}
for(i = 1; i <= n; i++)
{
e5[i]=((v[i]*v[i])+(x[i]*x[i]));
e6[i]= ceil( float (e5[i])/2);
std::cout<< "\n Edges addition of e[" << 2 * n + i << "] = " << e5[i] << "\t";
std::cout<< "\t Edges of e[" << 2 * n + i << "] = " << e6[i]%2 << "\t";
}
for (i = 1; i <= n; i++)
{
```

```
e7[i]=((v[i]*v[i])+(y[i]*y[i]));
e8[i]= ceil( float (e7[i])/2);
std::cout<< "\n Edges addition of e[" << 3*n+i << "] = " << e7[i] << "\t";
std::cout<< "\t Edges of e[" << 3*n+i << "] = " << e8[i]%2 << "\t";
}
for (i=1; i<=n-1; i++)
{
e9[i]=((u[i]*u[i])+(u[i+1]*u[i+1]));
e10[i]= ceil( float (e9[i])/2);
std::cout<< "\n Edges addition of e[" << 4*n+i << "] = " << e9[i] << "\t";
std::cout<< "\t Edges of e[" << 4*n+i << "] = " << e10[i]%2 << "\t";
}
for(i=1; i<=n-1; i++)
{
e11[i]=((v[i]*v[i])+(v[i+1]*v[i+1]));
e12[i]= ceil( float (e11[i])/2);
std::cout<< "\n Edges addition of e[" << 5*n-1+i << "] = " << e11[i] << "\t";
std::cout<< "\t Edges of e[" << 5*n+i-1 << "] = " << e12[i]%2 << "\t";
}
e13[1]=((v[(n+1)/2]*v[(n+1)/2])+(u[(n+1)/2]*u[(n+1)/2]));
e14[1]= ceil( float (e13[1])/2);
std::cout<< "\n Middle Edges addition of e[" << 6*n-1 << "] = " << e13[1] << "\t";
std::cout<< "\t Edges of e[" << 6*n-1 << "] = " << e14[1]%2 << "\t";
return 0;
}
```

## §4. Conclusion

In this Paper, we have introduced the new concept of modulo two square mean labeling of path, path related graphs and $H-$ class graphs. This new approach will be helpful to attack standard conjectures and unsolved open problems.

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