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# More Results on Vector Basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-Cordial Graphs

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**Abstract**: Let $G$ be a $(p,q)$ graph. Let $V$ be an inner product space with basis $S$. We denote the inner product of the vectors $x$ and $y$ by $\langle x,y\rangle$. Let $\phi : V(G) \to S$ be a function. For edge $uv$ assign the label $\langle \phi(u), \phi(v)\rangle$. Then $\phi$ is called a vector basis $S$-cordial labeling of $G$ if $|\phi_x - \phi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where $\phi_x$ denotes the number of vertices labeled with the vector $x$ and $\gamma_i$ denotes the number of edges labeled with the scalar $i$. A graph which admits a vector basis $S$-cordial labeling is called a vector basis $S$-cordial graph. In this paper, we investigate the existence of vector basis $\{$(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)$\}$-cordial labeling of octopus graph, $BC(n) \odot K_2$ and $BC(n) \odot mK_1$, where $mG$ denote the m copies of $G$.

**Key Words**: Vector basis $S$-cordial labeling, bicycle, complete graph, octopus graph, star graph, Smarandachely vector basis $S$-cordial labeling.

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## §1. Introduction

All graphs considered here are finite, simple and undirected graph. The order and size of a graph $G$ are denoted by $p$ and $q$, respectively. Terms not defined here are used in the sense of Harary [6] and Herstein [7]. The first research paper on graph theory was published by Leonhard Euler. Rosa [15] introduced the concept and notion of graph labeling. Since that time, various type of graph labeling method have been explored. Graph labeling finds application in various fields of science and technology. The concept of cordial labeling of graphs was introduced by Cahit [2]. Graph labeling is a dynamic field of study within graph theory that has primarily developed due to its various applications in mobile telecommunications systems, optimal circuit designs, graph decomposition problems, coding theory and communication networks. Topological cordial labeling of graph was introduced by Selestin Lina and Asha [16] and investigated the topological

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cordial labeling of some special graphs in [17]. Various graph labeling method of bicyclic related graphs were discussed in [8, 10, 11]. Meena and Gajalakshmi [9] have investigated the odd prime labeling of circular ladder related graphs. Some labeling methods of octopus related graphs were explored in [1, 3, 4].

A dynamic survey on different graph labeling problems with an extensive bibliography can be found in Gallian [5]. The concept of vector basis S-cordial labeling of graphs was introduced by Ponraj and Jeya [12], and examined the vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of behavior of the path, cycle, star, comb, complete graph, generalized friendship graph, tadpole graph and gear graph and thorn related graphs in [12, 13, 14]. In this paper, we investigate the existence of vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of octopus graph, $BC(n) \odot K_2$ and $BC(n) \odot mK_1$.

## §2. Preliminaries

In this section, we state a few definitions which are relevant for proving the main results.

**Definition** 2.1([1]) *An octopus graph $O_n$, $n \geq 2$ can be constructed by a fan graph $f_n$, $n \geq 2$ joining a star graph $K_{1,n}$ with sharing a common vertex.*

**Definition** 2.2([8]) *The bicyclic graph $BC(n)$ is obtained from two copies of cycles $C_n$ : $u_1u_2 \dots u_nu_1$ and $C_n{}'$ : $v_1v_2 \dots v_nv_1$ by identifying $u_1$ with $v_1$.*

**Definition** 2.3([5]) *The corona graph $G_1 \odot G_2$ is the graph obtained by taking one copy of $G_1$ and n copies of $G_2$ and joining $i^{th}$ vertex of $G_1$ with an edge to every vertex in the $i^{th}$ copy of $G_2$, where $G_1$ is graph of order $n$.*

In this paper, we consider the inner product space $R^n$ and the standard inner product $\langle x, y\rangle = x_1y_1 + x_2y_2 + \cdots + x_ny_n$ where $x = (x_1, x_2, \cdots, x_n), y = (y_1, y_2, \cdots, y_n)$ , $x_i, y_i \in R, 1 \leq i \leq n$.

## §3. Vector Basis S-Cordial Labeling

**Definition** 3.1 *Let $G$ be a $(p, q)$ graph. Let $V$ be an inner product space with basis $S$. We denote the inner product of the vectors $x$ and $y$ by $\langle x, y\rangle$. Let $\phi : V(G) \to S$ be a function. For edge $uv$ assign the label $\langle\phi(u), \phi(v)\rangle$. Then $\phi$ is called a vector basis S-cordial labeling of $G$ if $|\phi_x - \phi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where $\phi_x$ denotes the number of vertices labeled with the vector $x$ and $\gamma_i$ denotes the number of edges labeled with the scalar $i$. A graph $G$ which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph. Otherwise, if $|\phi_x - \phi_y| \leq 1$ or $|\gamma_i - \gamma_j| \leq 1$, $G$ is called a Smarandachely vector basis S-cordial graph.*

An example of a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of graph is shown in Figure 1.

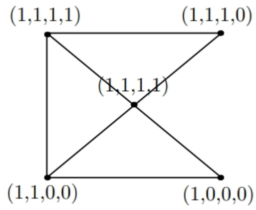


**Figure 1** A vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial graph

## §4. Main Results

In this section, we discuss the existence and non-existence of vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of $BC(n) \odot K_2$ and $BC(n) \odot mK_1$.

**Theorem** 4.1 *The octopus graph $O_n$ is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0, 0,0)}-cordial if and only if $n \neq 3$.*

*Proof* Let $V(O_n) = \{u, u_i, v_i \mid 1 \leq i \leq n\}$ and $E(O_n) = \{uu_i, uv_i \mid 1 \leq i \leq n\} \cup \{u_iu_{i+1} \mid 1 \leq i \leq n-1\}$ respectively be the vertex and edge sets of $O_n$. Clearly $p = |V(O_n)| = 2n+1$ and $q = |E(O_n)| = 3n-1$. We have consider the following five cases:

**Case 1.** $n = 3$

Consider the octopus graph $O_3$. Then $p = 7$ and $q = 8$. Clearly $\phi_{(1,1,1,1)} = 2$. This forces $\gamma_4 < 2$, which is a contradiction to the size of $O_3$.

**Case 2.** $n \equiv 0 \pmod 4$

Let $n = 4t$, $t > 0$. We get $p = 8t+1$ and $q = 12t-1$. Assign the vector (1,1,1,1) to the vertex $u$. Then assign the vector (1,1,1,1) to the first $t$ vertices $u_1, u_2, \ldots, u_t$ and to the $t$ vertices $v_1, v_2, \ldots, v_t$. Assign the vector (1,1,1,0) to the next $t$ vertices $u_{t+1}, u_{t+2}, \ldots, u_{2t}$ and to the next $t$ vertices $v_{t+1}, v_{t+2}, \ldots, v_{2t}$. Thereafter assign the vector (1,1,0,0) to the next $t$ vertices $u_{2t+1}, u_{2t+2}, \ldots, u_{3t}$ and to the next $t$ vertices $v_{2t+1}, v_{2t+2}, \ldots, v_{3t}$. Finally assign the vector (1,0,0,0) to the next $t$ vertices $u_{3t+1}, u_{3t+2}, \ldots, u_{4t}$ and to the next $t$ vertices $v_{3t+1}, v_{3t+2}, \ldots, v_{4t}$.

**Case 3.** $n \equiv 1 \pmod 4$

Let $n = 4t+1$, $t > 0$. We obtain $p = 8t+3$ and $q = 12t+2$. Assign the vector (1,1,1,1) to the vertex $u$. Then assign the vector (1,1,1,1) to the first $t+1$ vertices $u_1, u_2, \cdots, u_{t+1}$ and to the $t-1$ vertices $v_1, v_2, \cdots, v_{t-1}$. Assign the vector (1,1,1,0) to the next $t$ vertices $u_{t+2}, u_{t+3}, \cdots, u_{2t+1}$ and to the next $t+1$ vertices $v_t, v_{t+1}, \cdots, v_{2t}$. Thereafter assign the vector (1,1,0,0) to the next $t$ vertices $u_{2t+2}, u_{2t+3}, \ldots, u_{3t+1}$ and to the next $t+1$ vertices $v_{2t+1}, v_{2t+2}, \cdots, v_{3t+1}$. Finally assign the vector (1,0,0,0) to the next $t$ vertices $u_{3t+2}, u_{3t+3}, \ldots, u_{4t+1}$ and to the next $t$ vertices $v_{3t+2}, v_{3t+3}, \cdots, v_{4t+1}$.

**Case 4.** $n \equiv 2 \pmod 4$

Let $n = 4t+2$, $t > 0$. We obtain $p = 8t+5$ and $q = 12t+5$. Assign the vector (1,1,1,1) to the vertex $u$. Then assign the vector (1,1,1,1) to the first $t+1$ vertices $u_1, u_2, \cdots, u_{t+1}$ and to the $t$ vertices $v_1, v_2, \cdots, v_t$. Assign the vector (1,1,1,0) to the next $t+1$ vertices $u_{t+2}, u_{t+3}, \cdots, u_{2t+2}$ and to the next $t$ vertices $v_{t+1}, v_{t+2}, \cdots, v_{2t}$. Thereafter assign the vector (1,1,0,0) to the next $t$ vertices $u_{2t+3}, u_{2t+4}, \cdots, u_{3t+2}$ and to the next $t+1$ vertices $v_{2t+1}, v_{2t+2}, \ldots, v_{3t+1}$. Finally assign the vector (1,0,0,0) to the next $t$ vertices $u_{3t+3}, u_{3t+4}, \cdots, u_{4t+2}$ and to the next $t+1$ vertices $v_{3t+2}, v_{3t+3}, \cdots, v_{4t+2}$.

**Case 5.** $n \equiv 3 \pmod 4$

Let $n = 4t+2$, $t > 0$. We obtain $p = 8t+7$ and $q = 12t+8$. Assign the vector (1,1,1,1) to the vertex $u$. Then assign the vector (1,1,1,1) to the first $t+2$ vertices $u_1, u_2, \cdots, u_{t+2}$ and to the $t-1$ vertices $v_1, v_2, \cdots, v_{t-1}$. Assign the vector (1,1,1,0) to the next $t+1$ vertices $u_{t+3}, u_{t+4}, \cdots, u_{2t+3}$ and to the next $t$ vertices $v_t, v_{t+1}, \cdots, v_{2t-1}$. Thereafter assign the vector (1,1,0,0) to the next $t$ vertices $u_{2t+4}, u_{2t+5}, \cdots, u_{3t+3}$ and to the next $t+2$ vertices $v_{2t}, v_{2t+1}, \cdots, v_{3t+1}$. Finally assign the vector (1,0,0,0) to the next $t$ vertices $u_{3t+4}, u_{3t+5}, \ldots, u_{4t+3}$ and to the next $t+2$ vertices $v_{3t+2}, v_{3t+3}, \ldots, v_{4t+3}$.

Hence the vertex labeling $\phi$ is a vector basis {(1,1,1,1), (1,1,1,0),(1, 1,0,0),(1,0,0,0)}-cordial labeling of $O_n$ for all $n \geq 4$. □

**Example** 4.2 An illustration for the vector basis {(1,1,1,1),(1,1,1,0),(1, 1,0,0),(1,0,0,0)}-cordial labeling of $O_5$ for the case when $n \equiv 1 \pmod 4$ is shown in Figure 2.

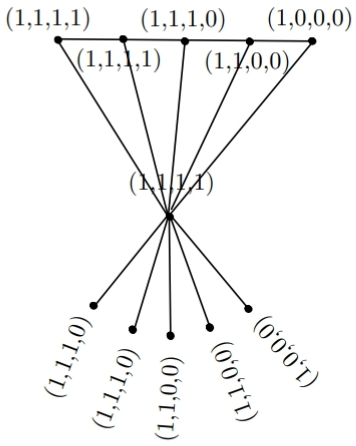


**Figure 2** A vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of $O_5$.

**Theorem** 4.3 *The corona product of bicyclic graph with $K_2$, $BC(n) \odot K_2$ is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial if and only if* $n \equiv 0, 2 \pmod 4$.

*Proof* Let $V(BC(n) \odot K_2) = \{u_i, u_{ij} \mid 1 \leq i \leq n \text{ and } 1 \leq j \leq 2\} \cup \{v_i, v_{ij} \mid 2 \leq i \leq n \text{ and } 1 \leq j \leq 2\}$ and $E(BC(n) \odot K_2) = \{u_i u_{i+1}, u_n u_1, u_1 v_2 \mid 1 \leq i \leq n-1\} \cup \{v_i v_{i+1}, v_n u_1 \mid 2 \leq i \leq n-1\} \cup \{u_i u_{ij}, u_{i1} u_{i2} \mid 1 \leq i \leq n \text{ and } 1 \leq j \leq 2\} \cup \{v_i v_{ij}, v_{i1} v_{i2} \mid 2 \leq i \leq n \text{ and } 1 \leq j \leq 2\}$ respectively be the vertex and edge sets of $BC(n) \odot K_2$. Clearly $p = |V(BC(n) \odot K_2)| = 3(2n-1)$ and $q = |E(BC(n) \odot K_2)| = 4(2n-1)+1$. Then assign the vectors to the vertices in the following order $u_1, u_2, \cdots, u_n, u_{11}, u_{12}, v_2, v_3, \cdots, v_n, u_{21}, u_{22}, \ldots, u_{n1}, u_{n2}, v_{11}, v_{12}, v_{21}, v_{22}, \ldots, v_{n1}, v_{n2}$. We consider the following four cases:

**Case 1.** $n \equiv 0 \pmod 4$

Let $n = 4t_1$, $t_1 > 0$. We get $p = 24t_1 - 3$. Then assign the vector (1,1,1,1) to the first $6t_1$ vertices and assign the vector (1,1,1,0) to the next $6t_1 - 1$ vertices. Thereafter assign the vector (1,1,0,0) to the next $6t_1 - 1$ vertices and assign the vector (1,0,0,0) to the next $6t_1 - 1$ vertices.

**Case 2.** $n \equiv 1 \pmod 4$

Let $n = 4t_1 + 1$, $t_1 > 0$. We get $p = 24t_1 + 3$ and $q = 32t_1 + 5$. From $6t_1 + 1$ vertices with vertex label (1,1,1,1), we can get $8t_1$ edges with edge label 4. We have to get $8t_1 + 1$ edges with edge label 4 from $6t_1 + 1$ vertices with vertex label (1,1,1,1) and so we are led to a contradiction.

**Case 3.** $n \equiv 2 \pmod 4$

Let $n = 4t_1 + 2$, $t_1 > 0$. We get $p = 24t_1 + 9$. Then assign the vector (1,1,1,1) to the first $6t_1 + 3$ vertices and assign the vector (1,1,1,0) to the next $6t_1 + 2$ vertices. Thereafter assign the vector (1,1,0,0) to the next $6t_1 + 2$ vertices and assign the vector (1,0,0,0) to the next $6t_1 + 2$ vertices.

**Case 4.** $n \equiv 3 \pmod 4$

Let $n = 4t_1 + 3$, $t_1 \geq 0$. We get $p = 24t_1 + 15$ and $q = 32t_1 + 21$. From $6t_1 + 4$ vertices with vertex label (1,1,1,1), we can get $8t_1 + 4$ edges with edge label 4. But we have to get $8t_1 + 5$ edges with edge label 4 from $6t_1 + 4$ vertices with vertex label (1,1,1,1) and so we are led to a contradiction.

Considering the findings in the above 4 cases, it emerges that a vector basis {(1,1,1,1),(1,1, 1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of $BC(n) \odot K_2$ exists for the cases of $n \equiv 0, 2 \pmod 4$ only. □

**Example** 4.4 An illustration for the vector basis {(1,1, 1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of $BC(4) \odot K_2$ for the case when $n \equiv 0 \pmod 4$ is shown in Figure 3.

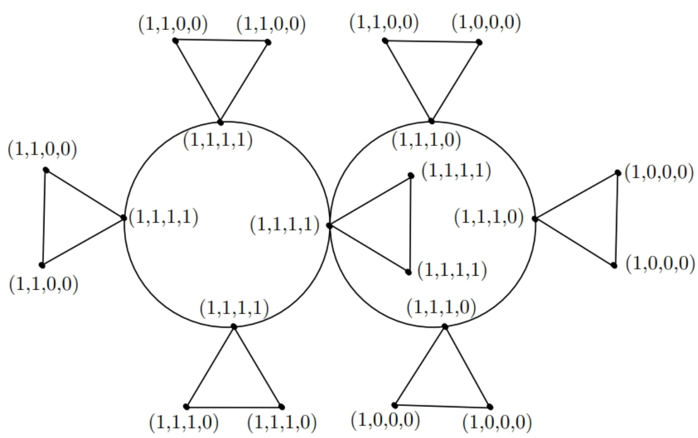


**Figure 3** A vector basis {(1,1,1,1),(1,1, 1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of $BC(4) \odot K_2$.

**Theorem** 4.5 *The corona product of bicyclic graph with $mK_1$, $BC(n)\odot mK_1$ is a vector basis $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial for all $n \geq 3$ and $m \geq 1$.*

*Proof* Let $V(BC(n)\odot mK_1) = \{u_i, u_{ij} \mid 1 \leq i \leq n \text{ and } 1 \leq j \leq m\} \cup \{v_i, v_{ij} \mid 2 \leq i \leq n \text{ and } 1 \leq j \leq m\}$ and $E(BC(n)\odot mK_1) = \{u_iu_{i+1}, u_nu_1 \mid 1 \leq i \leq n-1\} \cup \{v_iv_{i+1}, v_nu_1, u_1v_2 \mid 2 \leq i \leq n-1\} \cup \{u_iu_{ij} \mid 1 \leq i \leq n \text{ and } 1 \leq j \leq m\} \cup \{v_iv_{ij} \mid 2 \leq i \leq n \text{ and } 1 \leq j \leq m\}$ respectively be the vertex and edge sets of $BC(n)\odot mK_1$. Clearly $p = |V(BC(n)\odot mK_1)| = (2n-1)(m+1)$ and $q = |E(BC(n)\odot mK_1)| = (2n-1)(m+1)+1$. Then, assign the vectors to vertices in the following order $u_1, u_2, \cdots, u_n, v_2, v_3, \cdots, v_n, u_{11}, u_{12}, \cdots, u_{1m}, u_{21}, u_{22}, \cdots, u_{2m}, \cdots, u_{n1}, u_{n2}, \cdots, u_{nm}, v_{21}, v_{22}, \cdots, v_{2m}, \cdots, v_{n1}, v_{n2}, \cdots, v_{nm}$. We consider the following four cases:

**Case 1.** $n \equiv 0 \pmod 4$

Let $n = 4t_1$, $t_1 > 0$. We get $2n-1 = 8t_1 - 1$.

**Subcase 1.1** $m \equiv 0 \pmod 4$

Let $m = 4t_2$, $t_2 > 0$. We get $p = 4(8t_1t_2 + 2t_1 - t_2) - 1$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 - t_2 - 1$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 - t_2$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 - t_2$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 - t_2$ vertices.

**Subcase 1.2** $m \equiv 1 \pmod 4$

Let $m = 4t_2 + 1$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 4t_1 - t_2) - 2$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 - t_2 - 1$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1 - t_2 - 1$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1 - t_2$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 - t_2$ vertices.

**Subcase 1.3** $m \equiv 2 \pmod 4$

Let $m = 4t_2 + 2$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 6t_1 - t_2) - 3$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 - t_2 - 1$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 6t_1 - t_2 - 1$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 6t_1 - t_2 - 1$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 - t_2$ vertices.

**Subcase 1.4** $m \equiv 3 \pmod 4$

Let $m = 4t_2 + 3$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 8t_1 - t_2 - 1)$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 8t_1 - t_2 - 1$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 8t_1 - t_2 - 1$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 8t_1 - t_2 - 1$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 8t_1 - t_2 - 1$ vertices.

**Case 2.** $n \equiv 1 \pmod 4$

Let $n = 4t_1 + 1$, $t_1 > 0$. We get $2n-1 = 8t_1 + 1$.

**Subcase 2.1** $m \equiv 0 \pmod 4$

Let $m = 4t_2$, $t_2 > 0$. We get $p = 4(8t_1t_2 + 2t_1 + t_2) + 1$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 + t_2$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 + t_2$

vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 + t_2$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + t_2 + 1$ vertices.

**Subcase 2.2** $m \equiv 1 \pmod 4$

Let $m = 4t_2 + 1$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 4t_1 + t_2) + 2$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 + t_2$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1 + t_2$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1 + t_2 + 1$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 + t_2 + 1$ vertices.

**Subcase 2.3** $m \equiv 2 \pmod 4$

Let $m = 4t_2 + 2$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 6t_1 + t_2) + 3$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 + t_2$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 6t_1 + t_2 + 1$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 6t_1 + t_2 + 1$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 + t_2 + 1$ vertices.

**Subcase 2.4** $m \equiv 3 \pmod 4$

Let $m = 4t_2 + 3$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 8t_1 + t_2 + 1)$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 8t_1 + t_2 + 1$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 8t_1 + t_2 + 1$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 8t_1 + t_2 + 1$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 8t_1 + t_2 + 1$ vertices.

**Case 3.** $n \equiv 2 \pmod 4$

Let $n = 4t_1 + 2$, $t_1 > 0$. We get $2n - 1 = 8t_1 + 3$.

**Subcase 3.1** $m \equiv 0 \pmod 4$

Let $m = 4t_2$, $t_2 > 0$. We obtain $p = 4(8t_1t_2 + 2t_1 + 3t_2) + 3$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 + 3t_2$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 + 3t_2 + 1$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 + 3t_2 + 1$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + 3t_2 + 1$ vertices.

**Subcase 3.2** $m \equiv 1 \pmod 4$

Let $m = 4t_2 + 1$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 4t_1 + 3t_2 + 1) + 2$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 + 3t_2 + 1$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1 + 3t_2 + 1$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1 + 3t_2 + 2$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 + 3t_2 + 2$ vertices.

**Subcase 3.3** $m \equiv 2 \pmod 4$

Let $m = 4t_2 + 2$, $t_2 \geq 0$. We obtain $p = 4(8t_1t_2 + 6t_1 + 3t_2 + 2) + 1$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 + 3t_2 + 2$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 6t_1 + 3t_2 + 2$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2 + 6t_1 + 3t_2 + 2$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 + 3t_2 + 3$ vertices.

**Subcase 3.4** $m \equiv 3 \pmod 4$

Let $m = 4t_2 + 3$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 8t_1 + 3t_2 + 3)$. Then assign the vector

(1,1,1,1) to the first $8t_1t_2 + 8t_1 + 3t_2 + 3$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2+8t_1+3t_2+3$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2+8t_1+3t_2+3$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 8t_1 + 3t_2 + 3$ vertices.

**Case 4.** $n \equiv 3 \pmod 4$

Let $n = 4t_1 + 3$, $t_1 \geq 0$. We get $2n - 1 = 8t_1 + 5$.

**Subcase 4.1** $m \equiv 0 \pmod 4$

Let $m = 4t_2$, $t_2 > 0$. We obtain $p = 4(8t_1t_2 + 2t_1 + 5t_2 + 1) + 1$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 + 5t_2 + 1$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2+2t_1+5t_2+1$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2+2t_1+5t_2+1$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + 5t_2 + 2$ vertices.

**Subcase 4.2** $m \equiv 1 \pmod 4$

Let $m = 4t_2 + 1$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 4t_1 + 5t_2 + 2) + 2$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 + 5t_2 + 2$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2+4t_1+5t_2+2$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2+4t_1+5t_2+3$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 + 5t_2 + 3$ vertices.

**Subcase 4.3** $m \equiv 2 \pmod 4$

Let $m = 4t_2+2$, $t_2 \geq 0$. We obtain $p = 4(8t_1t_2+6t_1+5t_2+3)+3$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 + 5t_2 + 3$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2+6t_1+5t_2+4$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2+6t_1+5t_2+4$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 + 5t_2 + 4$ vertices.

**Subcase 4.4** $m \equiv 3 \pmod 4$

Let $m = 4t_2 + 3$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 8t_1 + 5t_2 + 5)$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 8t_1 + 5t_2 + 5$ vertices and assign the vector (1,1,1,0) to the next $8t_1t_2+8t_1+5t_2+5$ vertices. Thereafter assign the vector (1,1,0,0) to the next $8t_1t_2+8t_1+5t_2+5$ vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 8t_1 + 5t_2 + 5$ vertices.

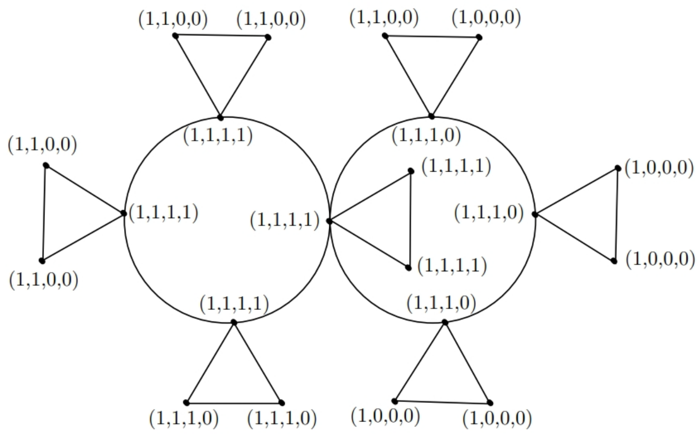


**Figure 4** A vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1, 0,0,0)}-cordial labeling of $BC(5) \odot 3K_1$.

For the above 4 cases, it emerges that a vector basis {(1,1,1,1), (1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of $BC(n) \odot mK_1$ exists for all $n \geq 3$ and $m \geq 1$. □

**Example** 4.6 An illustration for the vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of $BC(5) \odot 3K_1$ for the case when $n \equiv 1 \pmod 4$ and $m \equiv 3 \pmod 4$ is shown in Figure 4.

## §5. Conclusion

In this paper, we have investigated the existence of a vector basis {(1,1,1,1),(1,1,1,0),(1,1, 0,0),(1, 0,0,0)}-cordial labeling of octopus graph, $BC(n) \odot K_2$ and $BC(n) \odot mK_1$. The investigation of a vector basis {(1,1,1,1), (1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling behaviour of corona product of some more kind of graph families with $m$ copies of $K_1$ are the open problems for the future work.

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