International J.Math. Combin. Vol.3(2014), 70-88

Necessary Condition for Cubic Planar 3-Connected Graph to be Non-Hamiltonian with Proof of Barnette's Conjecture

Mushtaq Ahmad Shah

(Department of Mathematics, Vivekananda Global University (Formerly VIT Jaipur))

E-mail: shahmushtaq81@gmail.com

Abstract: A conjecture of Barnette states that, every three connected cubic bipartite planar graph is Hamiltonian. This problem has remained open since its formulation. This paper has a threefold purpose. The first is to provide survey of literature surrounding the conjecture. The second is to give the necessary condition for cubic planar three connected graph to be non-Hamiltonian and finally, we shall prove near about 50 year Barnett's conjecture. For the proof of different results using to prove the results we illustrate most of the results by using counter examples.

Key Words: Cubic graph, hamiltonian cycle, planar graph, bipartite graph, faces, subgraphs, degree of graph.

AMS(2010): 05C25

§1. Introduction

It is not an easy task to prove the Barnette's conjecture by direct method because it is very difficult process to prove or disprove it by direct method. In this paper, we use alternative method to prove the conjecture. It must be noted that if any one property of the Barnette's graph is deleted graph is non Hamiltonian. A planar graph is an undirected graph that can be embedded into the Euclidean plane without any crossings. A planar graph is called polyhedral if and only if it is three vertex connected, that is, if there do not exists two vertices the removal of which would disconnect the rest of the graph. A graph is bipartite if its vertices can be colored with two different colors such that each edge has one end point of each color. A graph is cubic if each vertex is the end point of exactly three edges. And a graph is Hamiltonian if there exists a cycle that pass exactly once through each of its vertices. Self-loops and parallel edges are not allowed in these graphs. Barnett's conjecture states that every cubic polyhedral graph is Hamiltonian. P.G.Tait in (1884) conjectured that every cubic polyhedral graph is Hamiltonian; this came to be known as Tait's conjecture. It was disproved by W.T. Tutte (1946), who constructs a counter example with 46 vertices; other researchers later found even smaller counterexamples, however, none of these counterexamples is bipartite. Tutte himself

[1]Received November 13, 2013, Accepted August 27, 2014.

conjectured that every cubic 3-connected bipartite graph is Hamiltonian but this was shown to be false by discovery of a counterexample, the Horton graph [16] .David W. Barnett (1969) proposed a weakened combination of Tait's and Tutte's conjecture, stating that every cubic bipartite polyhedral graph is Hamiltonian this conjecture first announced in [12] and later in [3]. In [10], Tutte proved that all planar 4-connected graphs are Hamiltonian, and in [9] Thomassen extended this result by showing that every planar 4-connected graph is Hamiltonian connected, that is for any pair of vertices, there is a Hamiltonian path with those vertices as endpoints.

§2. Supports for the Conjecture

In [5] Holton confirmed through a combination of clever analysis and computer search that all Barnett graphs with up to and including 64 vertices are Hamiltonian. In an announcement [14,11], McKay used computer search to extend this result to 84 vertices this implies that if Barnett conjecture is indeed false than a minimal counterexample must contain at least 86 vertices, and is therefore considerable larger than the minimal counterexample to Tait and Tutte conjecture. This is not all we know about a possible counterexample; another interesting result is that of Fowler, who in an unpublished manuscript [15] provided a list of subgraphs that cannot appear in any minimal counterexample to Barnett's conjecture.

Goody in [2] consider proper subsets of the Barnett graphs and proved the following.

Theorem 2.1 *Every Barnett graph which has faces consisting exclusively of quadrilaterals, and hexagons is Hamiltonian, and further more in all such graphs, any edge that is common to both a quadrilateral and a hexagon is a part of some Hamiltonian cycle.*

Theorem 2.2 *Every Barnett graph which has faces consisting of 7 quadrilaterals, 1 octagon and any number of hexagons is Hamiltonian, and any edge that is common to both a quadrilateral and an octagon is a part of some Hamiltonian cycle.*

In [6] Jensen and Toft reported that Barnett conjecture is equivalent to following.

Theorem 2.3 *Barnett conjecture is true if and only if for every Barnett graph G, it is possible to partition its vertices in to two subsets so that each induced an acyclic subgraph of G. (This theorem is not correct)*

Theorem 2.4([8]) *The edges of any bipartite graph G can be colored with $\delta(G)$ colors, where $\delta(G)$ is the minimum degree of vertices in G.*

Theorem 2.5([4]) *Barnett conjecture holds if and only if any arbitrary edge in a Barnett graph is a part of some Hamiltonian cycle.*

Theorem 2.6([13]) *Barnett conjecture holds if and only if for any arbitrary face in a Barnett graph there is a Hamiltonian cycle which passes through any two arbitrary edges on that face.*

Theorem 2.7([7]) *Barnett conjecture holds if and only if for any arbitrary face in a Barnett graph and for any arbitrary edges e_1 and e_2 on that face there is a Hamiltonian cycle which* *passes through e_1 and avoids e_2.*

It is difficult to say whether any of the technique described above will aid in settling Barnett conjecture. Certainly many of them seems to be useful and worth extending. One strategy is to keep chipping away at it; if Barnett conjecture is true then Godey's result can be extended to show that successively large and large subsets of Barnett graphs are Hamiltonian.

The Grinberg's Theorem [1] is not useful to find the counter example to Barnett's conjecture because all faces in Barnett graphs have even number of sides.

§3. New Results Supporting the Conjecture

Definition 3.1 *Any closed subgraph H of cubic planar three connected graph G is called complete cubic planar 3-connected subgraph H^C if all possible edges in that subgraph H are drawn then it also becomes cubic planar 3-connected graph. Thus we say H^C is cubic planar 3-connected graph.*

We illustrate by counter example following.

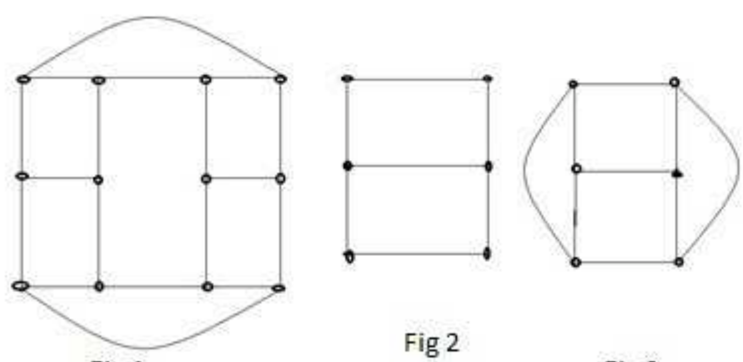


Fig 1 Fig 2 Fig 3

Let G be any cubic planar three connected graph as shown in Fig.1 we take its subgraph H shown in Fig.2 then we draw all possible edges in the subgraph as shown in Fig.3 the subgraph graph becomes complete cubic planar three connected H^C subgraph.

Definition 3.2 *Any closed subgraph H of cubic planar 3-connected graph G is called complete planar $n-1$ cubic 3-connected subgraph and is denoted by H^{C+} if all possible edges in that subgraph H are drawn then it becomes planar $n-1$ cubic 3-connected graph. i.e. Only one vertex has degree two and remaining graph is cubic planar 3-connected.*

Illustrate by counter example.

Let G be any cubic planar three connected graph as shown in Fig.4 H be its subgraph as shown in Fig.5 we draw all possible edges in the subgraph as shown in Fig.6 but still there exist a vertex having degree two only thus we say the subgraph H^{C+} be its complete planar $n-1$ cubic three connected graph.

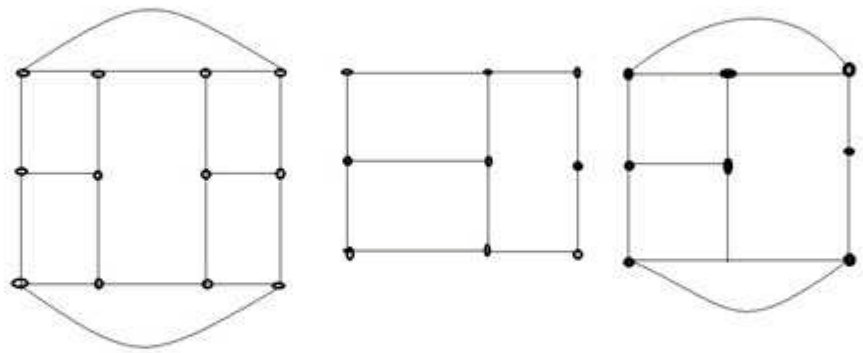


Fig. 4 Fig. 5 Fig. 6

Remark 3.1 A vertex can not have degree one in closed cubic planar three connected subgraphs, then it should be pendent vertex which is not possible in closed graphs so the degree of remaining vertices is two and degree cannot be more than three because it is the subgraph of cubic planar three connected graph so only possibility is that degree of remaining vertex is two.

Definition 3.3 *A closed subgraph H of cubic planar 3-connected graph is called complete planar $n-r$ cubic and 3-connected if all possible edges in that subgraph H are draw then it becomes cubic planar 3-connected, but it is still planar $n-r$ cubic and 3-connected, i,e its r vertices have degree two and remaining all vertices are cubic and three connected. It can be represented by H^{Cr+}.*

Lemma 3.1 *A planar bipartite 3-connected and $n-3$ cubic is non hamiltonian. In other words a planar graph which is bipartite 3-connected and $n-3$ cubic, i.e., only three of its vertices are of degree four and remaining graph is cubic then such a graph is non hamiltonian.* (Only encircle vertices is of degree four and rest of the graph is cubic and 3-connected)

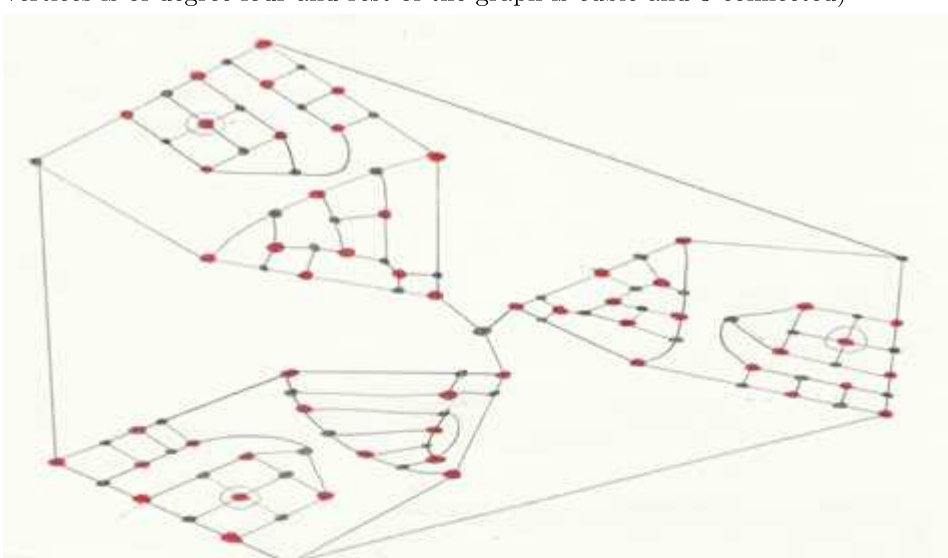


Fig. 7

We shall prove this result by counter example. The main aim behind the result is to prove that if a single property is deleted in cubic planar three connected bipartite graphs then it is non-Hamiltonian. This graph can be divided in to three closed subgraphs and an isolated vertex such that these closed subgraphs are H^{C+} sub graphs. Later we use this result in the main theorem.

Lemma 3.2 A cubic planar bipartite 2-connected graph is non-hamiltonian. It can be seen in this example. (It is not possible for me to give number of counter examples even though we can construct number of such examples) Fig.8 below is the cubic planar bipartite 2-connected graph but non-hamiltonian.

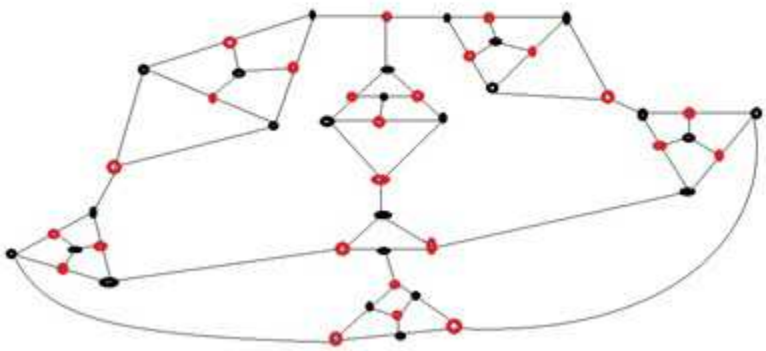


Fig.8

Remark 3.2 In every cubic planar three connected bipartite graph if any one of the property is deleted then the graph is non Hamiltonian.

Remark 3.3 Let $[\cdot]$ denotes the greatest integer function.

(1) If a and b are any two positive integers then $[a+b]=[a]+[b]$;

(2) If a is any positive integer and b is any positive real number then $[a+b]=[a]+[b]$.

Remark 3.4 Let G be any graph and if G is cubic planar three connected, we know that every cubic planar three connected graph, the Degree of each vertex is exactly equal to three. Thus the sum of all the degree of the Graph is $3n$ that is

$$\sum_{i=1}^{n} d_i = 3n.$$

Since each edge contributes two to the degrees thus the number of edges in the graph is

$$E = \frac{\sum\limits_{i=1}^{n} d_i}{2} = \frac{3n}{2},$$

where n is the number of vertices of the graph. Thus we conclude that if number of nodes is n number of edges is $\frac{3n}{2}$ and if number of edges is $\frac{3n}{2}$ the number of nodes is $\frac{2}{3}E = \frac{2}{3} \times \frac{3n}{2} = n$. Thus we conclude that in any cubic planar three connected graph edges and nodes are connected by certain relation.

The number of edges of any cubic planar three connected graph is always divisible by three if we take any planar cubic three connected graph and number of edges is not divisible by three then given graph is not H^C it is planar $n-1$ cubic and three connected, i.e., it contain a vertex of degree two only such a graph is denoted by H^{C+}. There does not exist any two vertices of degree two because we can draw an edge between them.

Lemma 3.3 *The number of regions in every cubic three connected planar graph and every cubic planar bipartite three connected graph of n vertices is* $\frac{n+4}{2}$*, where n is the number of vertices of the graph.*

Proof Since in every cubic planar three connected graph and every cubic planar three connected bipartite graph the degree of each vertices is exactly equal to three as graph is cubic. Thus the sum of all the degree of the Graph is $3n$ that is

$$\sum_{i=1}^{n} d_i = 3n.$$

Since each edge contributes two to the degrees thus the number of edges in these graph is

$$E = \frac{\sum\limits_{i=1}^{n} d_i}{2} = \frac{3n}{2},$$

where n is the number vertices of the graph. Thus we conclude that if number of vertices is n number of edges is $\frac{3n}{2}$ and if number of edges is $\frac{3n}{2}$ the number of vertices is $\frac{2}{3}e = \frac{2}{3} \times \frac{3n}{2} = n$, i.e., the number of vertices and edges are connected by certain relation. We know by Euler's theorem on planar graphs the number of regions is equal to

$$r = e - v + 2.$$

Since we have a graph of n vertices as we know it is cubic planar three connected or cubic planar three connected bipartite graph the number of edges in such graph's is $\frac{3n}{2}$ as shown above, now substitute these values in equation (i) we get

$$r = e - n + 2 = \frac{3n}{2} - n + 2 = \frac{n+4}{2}.$$

That proves the result. □

Thus from the above result we conclude that in every cubic planar three connected and every cubic planar bipartite three connected graph it is true that

$$e - v + 2 = \frac{n+4}{2}$$

The above result is not true for other planar graphs as we can take a counter example of three connected bipartite planar graph known as Herschel graph which contain 11 vertices and 18 edges. Contain 9 regions does not satisfy the above result.

Note 3.1 In every cubic planar three connected graph G and every cubic planar bipartite three connected graph G^+

1. The order of graphs G and G^+ is even;

2. The number of regions in both the graphs G and G^+ are equal to $\frac{n+4}{2}$, where n is the total number of vertices (See lemma 3);

3. The edges and vertices in both the graphs are connected by certain relation i.e

$$E = \frac{3n}{2} \text{ and } V = \frac{2E}{3}.$$

4. In G odd cycles are allowed but in G^+ it is bipartite thus odd cycles are not allowed.

§4. Necessary Condition for a Cubic Planar 3-Connected Graph to be Non-hamiltonian

Theorem A *A cubic planar 3-connected graph is non-hamiltonian if the graph is divided into three closed subgraphs of any order and an arbitrary isolated vertex such that these three closed subgraphs are planar $n-1$ cubic three connected I.e. they are H^{C+}subgraphs in other words a planar 3-connected graph is non-hamiltonian if these three subgraphs are such that*

$$\left[\frac{3n}{2}\right] \not\equiv 0(mod3),$$

where $[\cdot]$ denotes the greatest integer function and $\frac{3n}{2}$ is the number of edges in these subgraphs (Remark 3.4 above).

Proof Let G be any cubic planar 3-connected graph of order n number of edges is $\frac{3n}{2}$. Let us suppose that all the three closed subgraphs of G are complete closed planar cubic 3-connected, i.e. H^C subgraphs then

$$\left[\frac{3n}{2}\right] \equiv 0(mod3)$$

Since odd cycles are allowed so we can take any closed subgraph of any order in such a way that these closed subgraphs are necessarily H^{C+} first of all we shall take order of all closed subgraphs is odd if these closed subgraphs are H^{C+} then we have to stop the process of searching as such subgraphs exist but if such closed subgraphs are not H^{C+} then we try for different orders.

Let order of closed subgraph be odd, i.e., n is odd say $n = 2m+1$ or $n = 2m-1$ and

$$\left[\frac{3n}{2}\right] \equiv 0(mod3), \quad \left[\frac{3(2m+1)}{2}\right] \equiv 0(mod3), \quad \left[\frac{6m+3}{2}\right] \equiv 0(mod3).$$

Since in graphs the number of vertices and edges represent positive integers so

$$\begin{aligned}\left[\frac{6m}{2}\right] + \left[\frac{3}{2}\right] \equiv 0(mod3) \quad &\Rightarrow \quad 3m+1 \equiv 0(mod3)\\ &\Rightarrow \quad 3/3m+1 \text{ and } 3/-3m\\ &\Rightarrow \quad 3/3m+1-3m \Rightarrow 3/1,\end{aligned}$$

which is contradiction similarly if $n = 2m-1$. We get $3/3m-1-3m$ which gives $3/-1$. This again gives contradiction.

Thus we conclude that

$$\left[\frac{3n}{2}\right] \not\cong 0(mod3)$$

(Since such subgraphs exists we shell stop our search). Thus there exists one vertex in all the three closed subgraphs having degree 2 only (the degree cannot be one are more than three discussed above remark 4) that is these three subgraphs are H^{C+} subgraphs. If two vertices are of degree two we can draw an edge between them and subgraph becomes H^C only that is these subgraphs are cubic planar three connected which is not possible. When graph satisfy these conditions we first of all delete all those edge which we have added in the subgraphs then we join these sub graphs together with the arbitrary isolated vertex (it must be noted that such graphs does not contain only one arbitrary vertex it may contain more than one arbitrary vertex) with those vertices of the subgraphs having degree 2 in such a way that graph becomes cubic planar three connected. since odd cycles are allowed when we start from any arbitrary vertex it is not possible to travel all the vertices once and reaches back at the stating vertex because an arbitrary vertex can be traveled only at once so we can travel at most two of these H^{C+} subgraphs which we have joined to make the graph cubic planar three connected thus the graph so obtained is non-Hamiltonian.

Now we shall illustrate the result with following graphs and prove that this condition is satisfied by these graphs. All these graphs are cubic planar three connected and non Hamiltian satisfy the above conditions Fig.9 to 26 below.

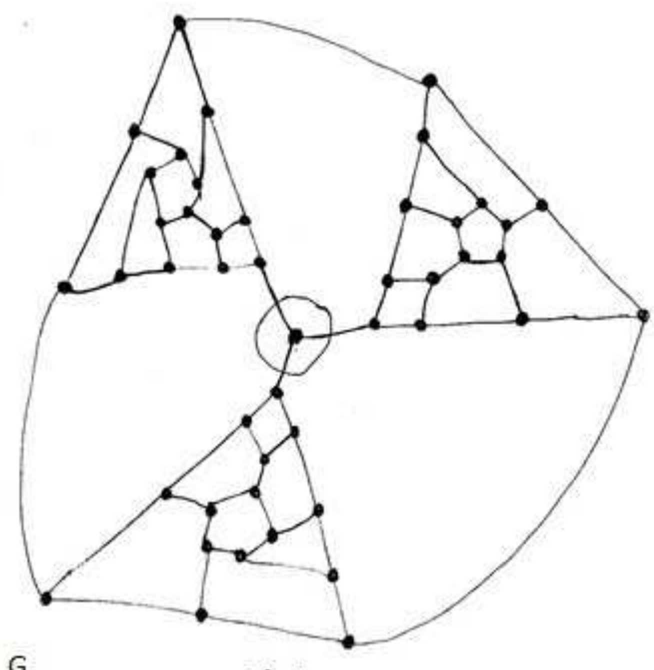
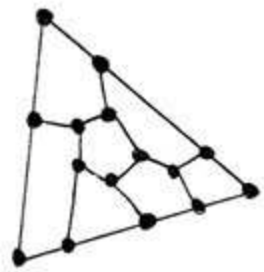


Fig.9

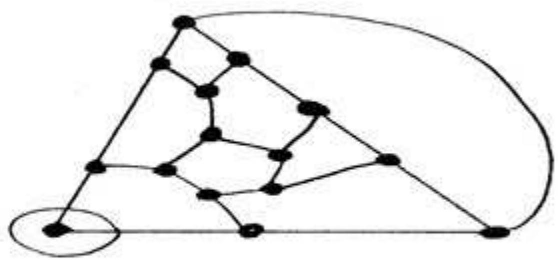
1) First of all let us take Tuttle graph, in which we take an encircle vertex as an isolated vertex and divide the remaining graph in three closed subgraphs not necessary of same order we shall show that these closed subgraphs are H^{C+} subgraphs.

Let H be its subgraph shown below



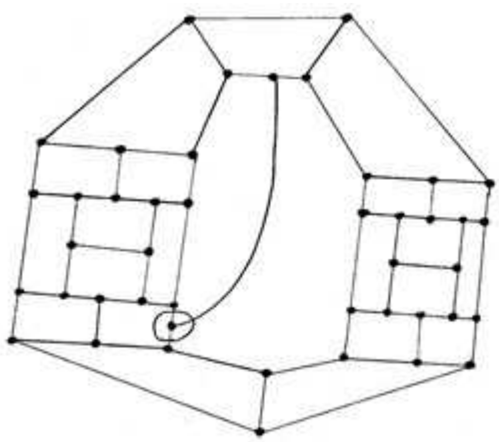
H **Fig. 10**

Again if we draw all possible edges in this closed subgraph the subgraph becomes planar $n-1$ cubic and three connected, i.e., H^{C+} subgraph as shown below and a vertex having degree two only has been shown by encircling the vertex.



H^{C+} **Fig. 11**

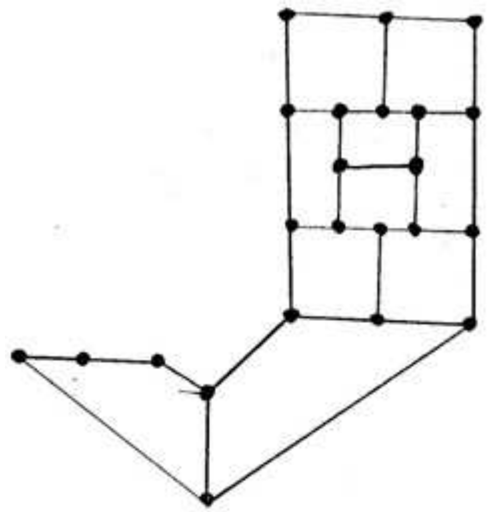
Since all the three closed subgraphs of this graph are of same order so other two subgraphs have same property as discussed above.



G **Fig. 12**

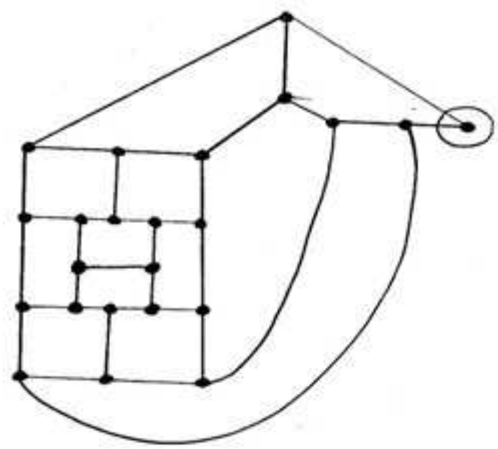
2) Now let us take younger graph of 44 vertices which is cubic planar three connected non-Hamiltonian. And isolated vertex is shown by encircle it, and remaining graph is divided into three closed subgraphs not necessarily of same order all these closed subgraphs are H^{C+}.

Let H_1 be its one closed subgraph as shown



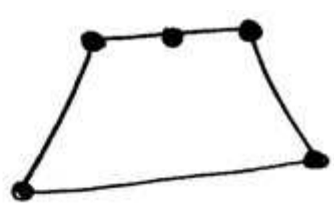
H_1 Fig.13

If we draw all possible edges in this subgraph it becomes planar $n-1$ cubic three connected as shown below H^{C+} only encircle vertex is of degree two.



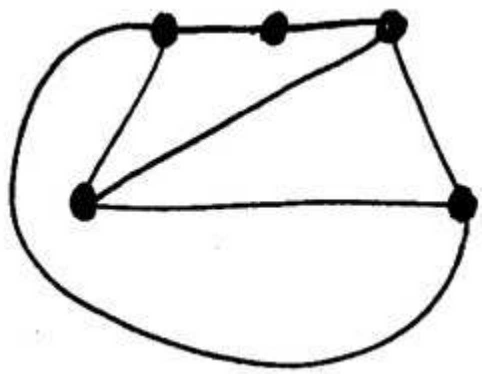
H^{C+} Fig. 14

Let another subgraph H_2 of the graph is given below.



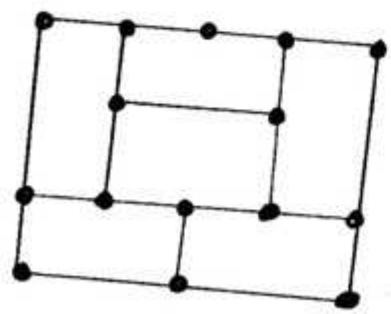
H_2 Fig. 15

If we draw all possible edges in the subgraph it also becomes H^{C+} subgraph as shown below.



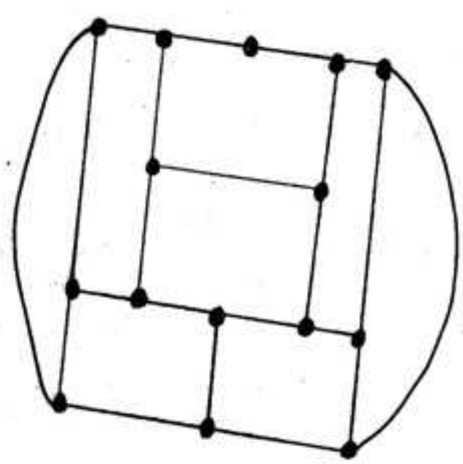
H^{C+} Fig. 16

Let another subgraph H_3 of a graph is given as



H3 Fig. 17

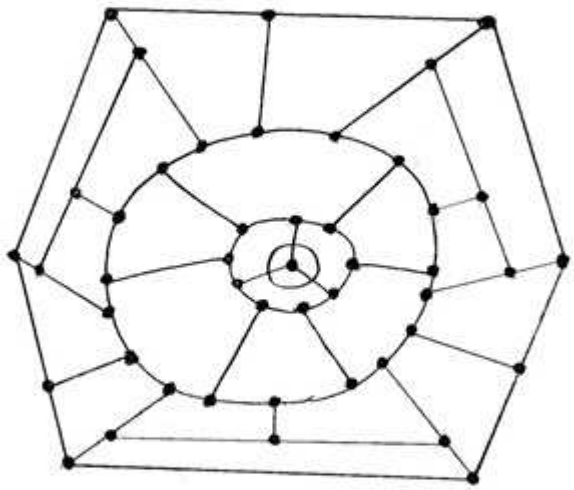
If we draw all possible edges in the subgraph it becomes H^{C+} subgraph as shown below



H^{C+} Fig. 18

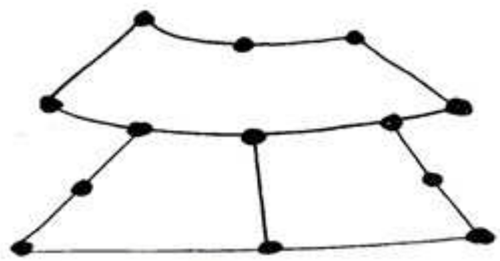
3) Now let us take another example of cubic planar three connected non Hamiltonian graph known as Grin berg graph 0f 46 vertices as shown below in which encircle vertex is an arbitrary

vertex.



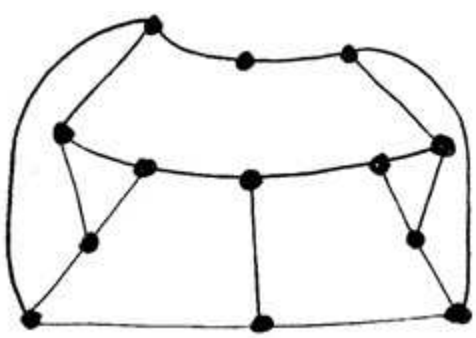
G Fig. 19

4) Let us take its closed subgraph H_1 as shown below



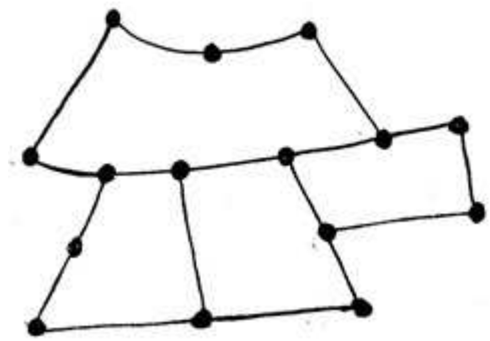
H_1 Fig. 20

Now if we draw all possible edges in the subgraph it becomes H^{C+} subgraph as shown below



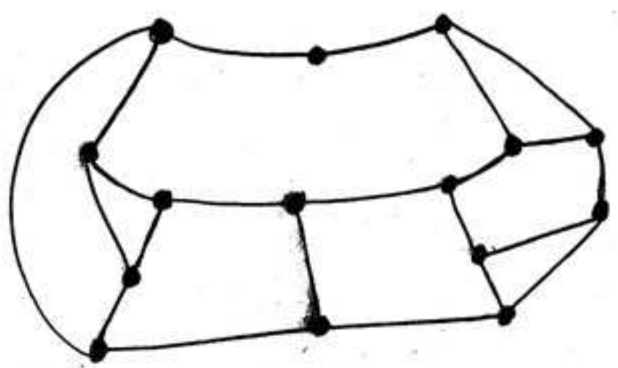
H^{C+} Fig.21

Let us take another subgraph H_2 of the graph given below



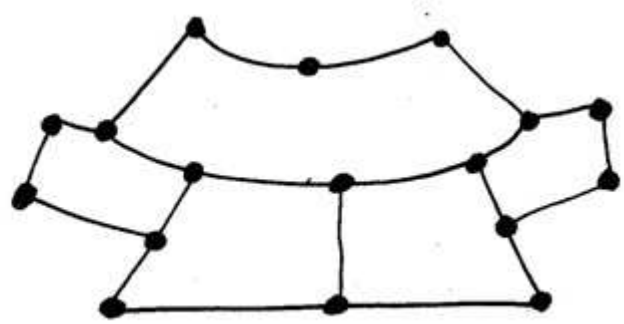
H_2 Fig. 22

If we draw all possible edges in the graph it becomes planar $n-1$ cubic and three connected as shown below



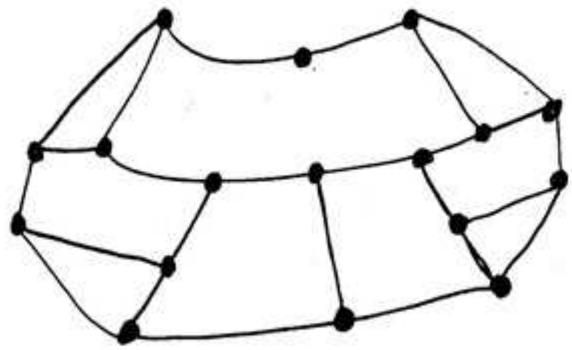
H^{C+} Fig. 23

Now again if we take another closed subgraph H_3 as below



H_3 Fig. 24

If we again draw all possible edges in the closed subgraph it becomes H^{C+} subgraph as shown below



H^{C+} Fig. 25

Now all other planar cubic three connected non Hamiltonian graphs satisfy this condition these graphs are shown in Fig.26 − 1 to Fig.26 − 3.

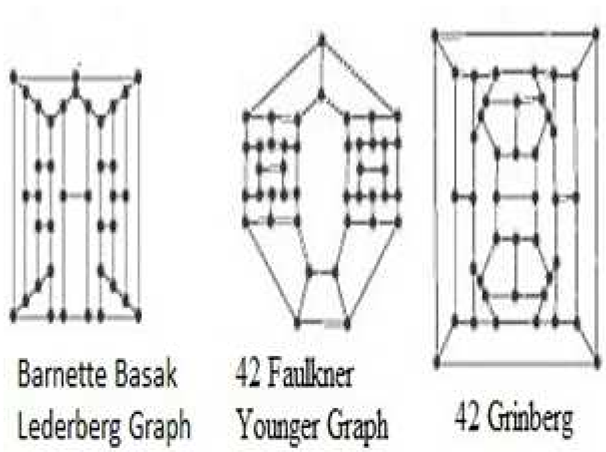


Fig.26-1

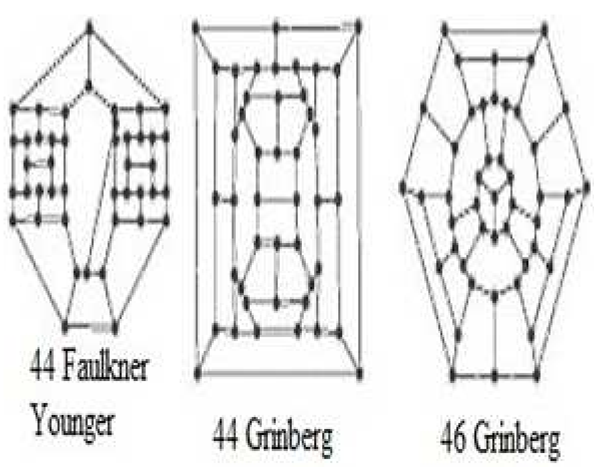


Fig.26-2

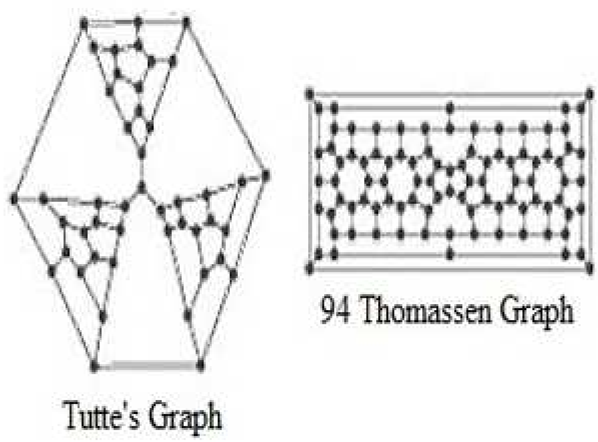
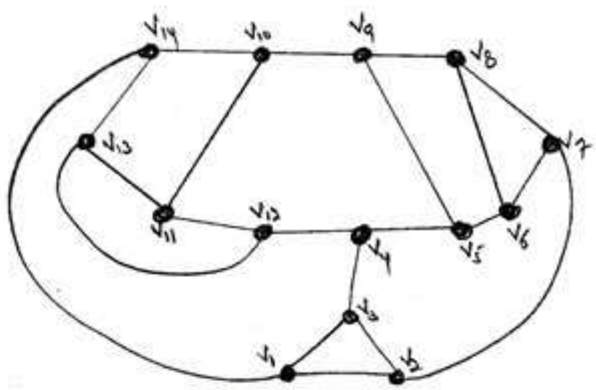


Fig.26-3

Note 4.1 One of the most important thing regarding the cubic planar three connected non-Hamiltonian graphs which was proved by professors Linfan Mao and Yanpei Liu in 2001 [17] there exists infinite three connected non-Hamiltonian cubic maps on every surface (orient able or non-orient able) not only the above graphs but also these infinite graphs satisfy the condition which we have proved above.

Now we shall show that above result is sharp. We use counter example to prove this sharpness. Below example Fig.27 is a graph which is cubic planar three connected contain Hamiltonian cycle start from $V_1, V_2, V_3, V_4, \cdots, V_{14}, V_1$. In this graph if we take V_4 as arbitrary vertex all the three closed subgraphs are not H^{C+} as shown below, it is not necessary we take V_4 as arbitrary vertex we can take any vertex as arbitrary vertex in such a way that remaining graph is divided into three closed subgraphs of any order but all such closed subgraphs are H^{C+} which is not possible in this graph.



G Fig. 27

Thus we conclude that all cubic planar three connected non Hamiltonian graphs can be divided in to three closed subgraphs of any order and an isolated vertex satisfying the property that all the three closed subgraphs are H^{C+}, But if graph is cubic planar three connected and Hamiltonian it is not necessary that all the three closed subgraphs satisfy H^{C+} property as shown below, thus the condition which we use to prove the theorem is sharp. In other words every cubic planar three connected graph which is Hamiltonian and can be divided into three closed subgraphs of any order and an isolated vertex all the three subgraphs may are may not be H^{C+} subgraphs, but if graph is non Hamiltonian all such closed subgraphs are H^{C+} (There are other examples as well but it is not possible to draw all in this paper).

Take a closed subgraph H and its H^{C+} subgraph

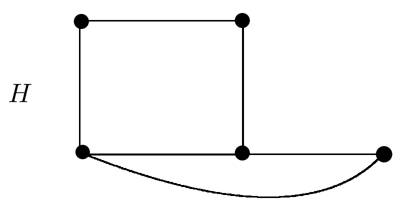


Fig.28

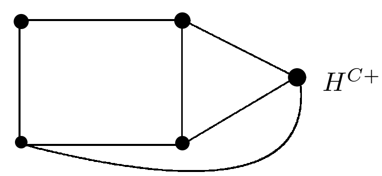


Fig.29

Take another closed subgraph H and its H^{C+} subgraph

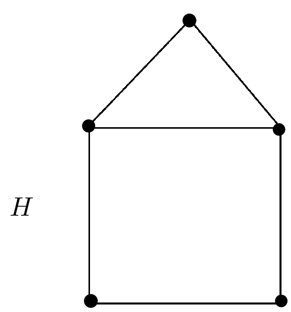


Fig.30

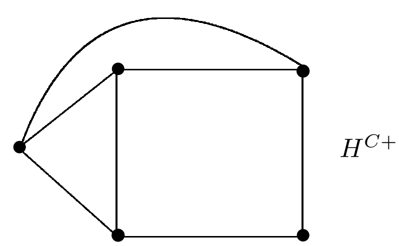


Fig.31

And finally take a closed subgraph H of order three so it is not H^{C+} because we cannot draw any more edge in this subgraph (these edges are parallel edges). □

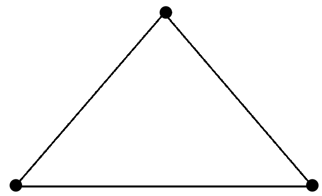


Fig.32

Remark 4.1 It has been discussed above that number of regions in cubic planar three connected graphs and cubic planar three connected bipartite graphs are $\frac{n+4}{2}$, thus it is necessary that every cubic planar three connected bipartite graph is non Hamiltonian if it has at least one closed subgraph which is H^{C+} also in lemma 1. we have given a counter example of $n-3$ cubic planar three connected bipartite non Hamiltonian graph satisfy H^{C+} property.

Theorem B *Every cubic planar bipartite three connected graph is Hamiltonian (Barnett's conjecture).*

Proof Since every bipartite graph is two colorable and thus without odd cycles so it contains only even cycles and number of vertices is also even, we cannot take any closed subgraph of odd order because it is not connected as odd cycles are not allowed, so every closed subgraph of cubic planar bipartite three connected graph is of even length. Thus in this type of graph n is always even, such graphs are non Hamiltonian only if there exist at least one subgraph H of any order which is planar $n-1$ cubic and three connected, i.e., H^{C+} subgraph, Then conjecture is not true because counter example can be constructed to disprove it if such a condition is satisfied. (See theorem A above) and Fig.7 of Lemma 1.

Let it is true that such graphs have at least one closed subgraph of H^{C+} then it must satisfy the following condition

$$\left[\frac{3n}{2}\right] \ncong 0(mod3)$$

Since n is necessarily even i.e. order of every subgraph is even because graph is bipartite and odd cycles are not allowed. Let $n = 2m$. Then

$$\begin{aligned} \frac{3n}{2} &= 3 \times \frac{(2m)}{2} = 3m \\ \frac{3n}{2} &\equiv 0(mod3) \\ 3m &\equiv 0(mod3) \\ &\Rightarrow 3/3m \text{ and} 3/-3m \\ &\Rightarrow 3/3m-3m \Rightarrow 3/0, \end{aligned}$$

which contradicts the given statement that

$$\left[\frac{3n}{2}\right] \not\equiv 0(mod3).$$

Thus we conclude that there does not exist any subgraph H of cubic planar bipartite three connected graph G which is planar $n-1$ cubic and three connected, i.e., which is H^{C+} thus there does not exists any counter example which proves that Barnett's conjecture does not hold, thus every cubic planar bipartite three connected graph is Hamiltonian that proves the conjecture. □

The above theorem can be verified by Lemma 3.1 above the non Hamiltonian graph G of Lemma 3.1 can be divided into an arbitrary vertex and three closed subgraphs H^{C+} even though graph is bipartite(without odd cycles) three connected planar $n-3$ cubic, only three vertices are of degree four and contain only even cycles.

Below is the graph which is cubic planar three connected contains Hamiltonian cycle. This cannot be divided into an arbitrary vertex and three closed subgraphs H^{C+}.

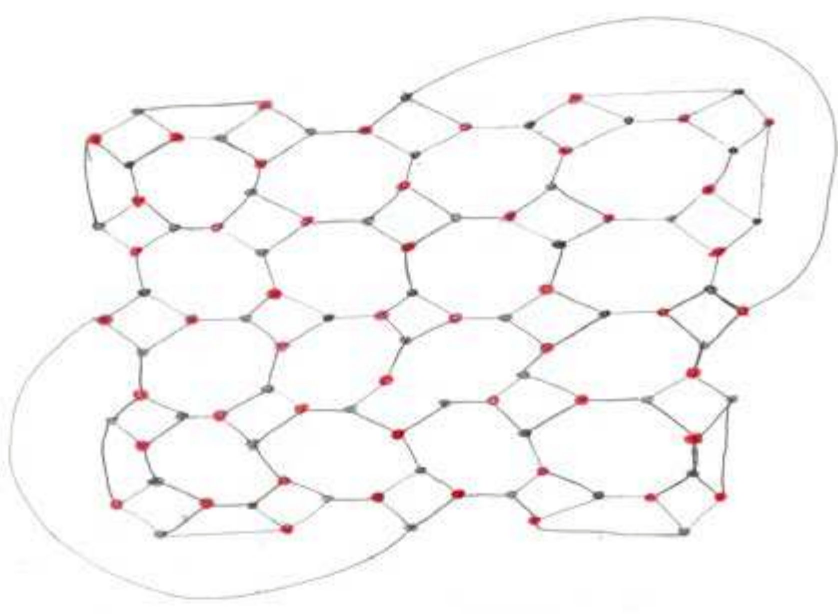


Fig. 33

References

[1] J.A.Bonday and U.S.R.Murty, *Graph Theory with Applications*, Macmillan, London, 1976.

[2] P.R.Goodey, Hamiltonian circuit in Polytopes with even sided faces, *Israel Journal of Mathematics*, 22: 52-56, 1975.

[3] B.Grunbaum, *Polytopes, Graphs and Complexes*, *Bull. Amer. Math. Soc.*, 76:1131-1201, 1970.

[4] A.Hertel, *Hamiltonian Cycle in Sparse Graphs*, Master's thesis, university of Toronto, 2004.

[5] B.D.McKay, D.A. Holton, B. Manvel, Hamiltonian cycles in cubic 3- connected bipartite planar graphs, *Journal of Combinatorial Theory*, series B, 38:279-297, 1985.

[6] T.R.Jensen and B.Toft, *Graph Coloring Problem*, J.Wiley and Sons New York, 1995.

[7] A.K.Kalmans, *Konstruktsii Kubicheskih Dvudolnyh 3-Svyaznyh Bez GamiltonovyhTsiklov,* Sb.TrVNiii Sistem, issled. 10:64-72, 1986.

[8] D.Conig.Uber, *Graphen und ihre Anwendung auf Determinantentheorie und Mengenlehre*, Math- ematische Annalen, 77: 453-465, 1916.

[9] C.Thomassen, A theorem on paths in planar graphs, *Journal of Graph Theory*, 7:169-176, 1983.

[10] W.T.Tutte, A theorem on planar graphs, *Trans. Amer. Math. Soc.*, 82:99-116, 1956.

[11] R.Aldred, G.Brinkmann and B.D McKay, Announcement, March 2000.

[12] D.Barnette, *Conjecture 5*, In W.T.Tutte ed. *Recent Progress in Combinatorics*, page 343 Academic press, New York, 1969.

[13] M.Braverman, Personal communication, 2004.

[14] T.Feder and C.Subi, *On Barnett's Conjecture*, Published on internet http//theory.stanford. edu/tomas/bar.ps, Submitted to *Journal of Graph Theory.*

[15] T.Fowler, Reducible Configurations for the Barnett's conjecture, Unpublished manuscript August 2001.

[16] Akiyama T., Nishizeki T. and Saiton N., NP-compleateness of the Hamiltonian cycle problem for bipartite graphs, *Journal of Information Processing*, 3(2): 73-76(1980), also cited by Hertel (2005).

[17] Linfan Mao and Yanpei Liu, An approach for constructing 3-connected non-Hamiltonian cubic maps on surfaces, *OR Transactions*, 4(2001), 1-7.