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Neighborhood Total 2-Domination Number and Connectivity of Graphs

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Abstract: A subset S of V is called a dominating set in a graph G if every vertex in $V-S$ is adjacent to at least one vertex in S. A set $S \subseteq V$ is called the neighborhood total 2-dominating set (nt2d-set) of a graph G if every vertex in $V-S$ is adjacent to at least two vertices in S and the induced subgraph $\langle N(S)\rangle$ has no isolated vertices. The minimum cardinality of an nt2d-set of G is called the neighborhood total 2-domination number of G and is denoted by $\gamma_{2nt}(G)$. The connectivity $\kappa(G)$ of G is the minimum number of vertices whose removal results in a disconnected or trivial graph. In this paper we find an upper bound for the sum of the neighborhood total 2-domination number and connectivity of a graph and characterize the corresponding extremal graphs.

Key Words: Domination number, Smarandachely k-domination number, neighborhood total 2-domination number, connectivity.

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§1. Introduction

The graph $G=(V,E)$ we mean a finite, undirected, connected graph with neither loops nor multiple edges and with out isolated vertices. The order and size of G are denoted by n and m respectively. The degree of a vertex u in G is the number of edges incident with u and is denoted by deg$u$.The minimum and maximum degree of a graph G is denoted by $\delta(G)$ and $\Delta(G)$, respectively. For graph theoretic terminology we refer to Chartrand and Lesniak [1] and Haynes et.al [2,3].

Let $v \in V$. The open neighborhood and closed neighborhood of v are denoted by $N(v)$ and $N[v]=N(v)\cup\{v\}$. If $S \subseteq V$ then $N(S)=\bigcup_{v\in S} N(v)$ for all $v \in S$ and $N[S]=N(S)\cup S$. If $S \subseteq V$ and $u \in S$ then the private neighbor set of u with respect to S is defined by $pn[u,S]=\{v : N[v]\cap S=\{u\}\}$. $H(m_1,m_2,\cdots,m_n)$ denotes the graph obtained from the graph H by attaching m_i pendant edges to the vertex $v_i \in V(H), 1 \le i \le n$. The graph $K_2(m_1,m_2)$ is called bistar and it is also denoted by $B(m_1,m_2)$. $H(P_{m_1},P_{m_2},\cdots,P_{m_n})$ is

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the graph obtained from the graph H by attaching an end vertex of P_{m_i} to the vertex v_i in $H, 1 \leq i \leq n$.

A subset S of V is called a dominating set of G if every vertex in $V - S$ is adjacent to at least one vertex in S. Generally, a set of vertices S in a graph G is said to be a *Smarandachely k-dominating set* if each vertex of G is dominated by at least k vertices of S. Clearly, a dominating set is nothing else but a Smarandachely 1-dominating set of G. The *Smarandachely k-domination number* $\gamma_k(G)$ of G is the minimum cardinality of a Smarandachely k-dominating set of G. Particularly, if $k = 1$, such a number is called the *domination number* of G and denoted by $\gamma(G)$. C.Sivagnanam [5] introduced the concept of neighborhood total 2-domination in graphs. A set $S \subseteq V$ is called a neighborhood total 2-dominating set (nt2d-set) of a graph G if every vertex in $V - S$ is adjacent to at least two vertices in S, i.e., S is a Smarandachely 2-dominating set and the induced subgraph $\langle N(S) \rangle$ has no isolated vertices. The minimum cardinality of an nt2d-set of G is called the neighborhood total 2-domination number of G and is denoted by $\gamma_{2nt}(G)$.The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph.

Several authors have studied the problem of obtaining an upper bound for the sum of a dominating parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. J.Paulraj Joseph and S.Arumugam [4] proved that $\gamma(G) + \kappa(G) \leq n$ and characterized the corresponding extremal graphs. In this paper, we obtain a sharp upper bound for the sum of the neighborhood total 2-domination number and connectivity of a graph and characterize the corresponding extremal graphs.We use the following theorems.

Theorem 1.1 ([5]) *Let G be a connected graph on $n \geq 2$ vertices. Then $\gamma_{2nt}(G) \leq n$ and equality holds if and only if G is a star.*

Theorem 1.2 ([1]) *For any graph G, $\kappa(G) \leq \delta(G)$.*

§2. Main Results

Theorem 2.1 *For any graph G, $\gamma_{2nt}(G) + \kappa(G) \leq 2n - 1$ and equality holds if and only if G is isomorphic to K_2.*

Proof $\gamma_{2nt}(G) + \kappa(G) \leq n + \delta \leq n + n - 1 = 2n - 1$. Let $\gamma_{2nt}(G) + \kappa(G) = 2n - 1$. Then $\gamma_{2nt}(G) = n$ and $\kappa(G) = n - 1$ which gives G is a star as well as a complete graph on n vertices. Hence G is isomorphic to K_2. The converse is obvious. □

Theorem 2.2 *For any graph G, $\gamma_{2nt}(G) + \kappa(G) = 2n - 2$ if and only if G is isomorphic to either K_3 or $K_{1,2}$.*

Proof Let $\gamma_{2nt} + \kappa(G) = 2n - 2$. Then there are two cases to consider: (1) $\gamma_{2nt}(G) = n$ and $\kappa(G) = n - 2$ and (2) $\gamma_{2nt}(G) = n - 1$ and $\kappa(G) = n - 1$.

Case 1. $\gamma_{2nt}(G) = n$ and $\kappa(G) = n - 2$.

Then G is a star and hence $\kappa(G) = 1$ which gives $n = 3$. Thus G is isomorphic to $K_{1,2}$.

Case 2. $\gamma_{2nt}(G) = n - 1$ and $\kappa(G) = n - 1$.

Then G is a complete graph. This gives $\gamma_{2nt}(G) = 2$. Then $n = 3$ and hence G is isomorphic to K_3. The converse is obvious. □

Theorem 2.3 *For any graph G, $\gamma_{2nt}(G) + \kappa(G) = 2n - 3$ if and only if G is isomorphic to C_4 or $K_{1,3}$ or K_4.*

Proof Let $\gamma_{2nt}(G)+\kappa(G) = 2n-3$. Then there are three cases to consider: (1) $\gamma_{2nt}(G) = n$ and $\kappa(G) = n-3$; (2) $\gamma_{2nt}(G) = n-1$ and $\kappa(G) = n-2$; (3) $\gamma_{2nt}(G) = n-2$ and $\kappa(G) = n-1$.

Case 1. $\gamma_{2nt}(G) = n$ and $\kappa(G) = n - 3$.

Then G is a star and hence $\kappa(G) = 1$ which gives $n = 4$. Thus G is isomorphic to $K_{1,3}$.

Case 2. $\gamma_{2nt}(G) = n - 1$ and $\kappa(G) = n - 2$.

Then $n - 2 \leq \delta(G)$. If $\delta = n - 1$ then G is a complete graph which is a contradiction. Hence $\delta(G) = n - 2$. Then G is isomorphic to $K_n - Y$ where Y is a matching in K_n. Then $\gamma_{2nt}(G) \leq 3$. If $\gamma_{2nt}(G) = 3$ then $n = 4$ and hence G is isomorphic to C_4. If $\gamma_{2nt}(G) = 2$ then $n = 3$ and hence G is isomorphic to $K_{1,2}$ which is a contradiction.

Case 3. $\gamma_{2nt}(G) = n - 2$ and $\kappa(G) = n - 1$.

Then G is a complete graph on n vertices. Since $\gamma_{2nt} = 2$ we have $n = 4$. Hence G is isomorphic to K_4. The converse is obvious. □

Theorem 2.4 *For any graph G, $\gamma_{2nt}(G) + \kappa(G) = 2n - 4$ if and only if G is isomorphic to P_4 or K_5 or $K_4 - e$ or $K_{1,4}$ or $K_3(1, 0, 0)$.*

Proof Let $\gamma_{2nt}(G)+\kappa(G) = 2n-4$. Then there are four cases to consider: (1) $\gamma_{2nt}(G) = n$ and $\kappa(G) = n-4$; (2) $\gamma_{2nt}(G) = n-1$ and $\kappa(G) = n-3$; (3) $\gamma_{2nt}(G) = n-2$ and $\kappa(G) = n-2$; (4) $\gamma_{2nt}(G) = n - 3$ and $\kappa(G) = n - 1$.

Case 1. $\gamma_{2nt}(G) = n$ and $\kappa(G) = n - 4$.

Then G is a star and hence $\kappa(G) = 1$ which gives $n = 5$. Thus G is isomorphic to $K_{1,4}$.

Case 2. $\gamma_{2nt}(G) = n - 1$ and $\kappa(G) = n - 3$.

Then $n - 3 \leq \delta$. If $\delta = n - 1$ then G is a complete graph which is a contradiction. If $\delta = n - 2$ then G is isomorphic to $K_n - Y$ where Y is matching in K_n. Then $\gamma_{2nt}(G) = 2$ or 3. If $\gamma_{2nt}(G) = 3$ then $n = 4$. Hence G is either $K_4 - e$ or C_4. For these two graphs $\kappa(G) = 2 \neq n - 3$ which is a contradiction. If $\gamma_{2nt}(G) = 2$ then $n = 3$ which is a contradiction to $\kappa(G) = n - 3$. Hence $\delta = n - 3$. Let $X = \{v_1, v_2, \cdots, v_{n-3}\}$ be the vertex cut of G and let $V - X = \{x_1, x_2, x_3\}$.

Subcase 2.1 $\langle V - X\rangle = \overline{K_3}$.

Then every vertex of $V - X$ is adjacent to all the vertices in X. If $|X| = 1$ then G is a star which is a contradiction. If $|X| = 2$ then G is isomorphic to either $K_{2,3}$ or the graph obtained from $K_{2,3}$ by joining the vertices of degree 3 by an edge. But for these two graphs $\gamma_{2nt} \leq 3$ which is a contradiction.Hence $|X| \geq 3$. Then $\{v_1, v_2, x_1, x_2\}$ is a nt2d-set of G. Hence $\gamma_{2nt}(G) \leq 4$. Then $n = 5$ which is a contradiction.

Subcase 2.2 $\langle V - X\rangle = K_1 \cup K_2$.

Let $x_1x_2 \in E(G)$. Then x_3 is adjacent to all the vertices in X and x_1, x_2 are not adjacent to at most one vertex in X. If degx_1 or degx_2 is $n-2$ then $\{x_1, x_2, x_3\}$ is a nt2d-set of G and hence $\gamma_{2nt} \leq 3$. Then $n \leq 4$ which gives $n = 4$. Hence G is isomorphic to P_4 or $K_3(1, 0, 0)$.

Suppose degx_1 = deg$x_2 = n-3$. If $N(x_1) = N(x_2)$ then there is a vertex $v_1 \in X$ such that v_1 is not adjacent to both x_1 and x_2. Then v_1 is adjacent to all the vertices in X. It is clear that $\{v_1, x_1, x_2, x_3\}$ is a nt2d-set of G. Hence $\gamma_{2nt} \leq 4$. Thus $n \leq 5$ which gives $n = 5$. Then G is isomorphic to the graph obtained from two copies of C_3 by merging one vertex of a copy of C_3 to a vertex of another copy of C_3. For this graph $\kappa(G) = 1$ which is a contradiction.If $N(x_1) \neq N(x_2)$ then there are at least two vertices v_1 and v_2 such that v_1 is not adjacent to x_1 but adjacent to x_2 and v_2 is not adjacent to x_2 but adjacent to x_1. Then $\{x_1, x_2, x_3\}$ is a nt2d-set of G and hence $n \leq 4$ which is a contradiction.

Case 3. $\gamma_{2nt}(G) = n-2$ and $\kappa(G) = n-2$.

Then $n-2 \leq \delta(G)$. If $\delta = n-1$ then G is a complete graph which is a contradiction. Hence $\delta(G) = n-2$. Then G is isomorphic to $K_n - Y$where Y is a matching in K_n. Then $\gamma_{2nt}(G) \leq 3$. If $\gamma_{2nt}(G) = 3$ then $n = 5$. But $\gamma_{2nt}(K_5 - Y) = 2 \neq n-2$ which is a contradiction. If $\gamma_{2nt}(G) = 2$ then $n = 4$. Hence G is isomorphic to $K_4 - e$.

Case 4. $\gamma_{2nt}(G) = n-3$ and $\kappa(G) = n-1$.

Then G is a complete graph on n vertices. Since $\gamma_{2nt}(G) = n-3$ we have $n = 5$. Hence G is isomorphic to K_5. The converse is obvious. □

Notation 2.5 We use the following notations in this paper:

(i) G^* is a graph obtained from $C_5 + e$ by joining two non adjacent vertices one has degree two and another has degree three by an edge.

(ii) H^* is a graph obtained from K_4 by subdividing an edge once.

(iii) The set $A = \{G^*, H^*, P_5, C_5, C_5 + e, K_6, K_{1,5}, K_{2,3}, K_{3,3}, K_3(2, 0), B(2, 1), K_5 - Y$, where Y is a matching in $K_5\}$.

Theorem 2.6 *For any connected graph G, $\gamma_{2nt}(G) + \kappa(G) = 2n-5$ if and only if $G \in A$.*

Proof Let $\gamma_{2nt}(G) + \kappa(G) = 2n-5$. Then there are five cases to consider: (1) $\gamma_{2nt}(G) = n$ and $\kappa(G) = n-5$; (2) $\gamma_{2nt}(G) = n-1$ and $\kappa(G) = n-4$; (3) $\gamma_{2nt}(G) = n-2$ and $\kappa(G) = n-3$; (4) $\gamma_{2nt}(G) = n-3$ and $\kappa(G) = n-2$; (5) $\gamma_{2nt}(G) = n-4$ and $\kappa(G) = n-1$.

Case 1. $\gamma_{2nt}(G) = n$ and $\kappa(G) = n-5$.

Then G is a star and hence $\kappa(G) = 1$ which gives $n = 6$. Thus G is isomorphic to $K_{1,5}$.

Case 2. $\gamma_{2nt}(G) = n-1$ and $\kappa(G) = n-4$.

Then $n-4 \leq \delta(G)$. If $\delta(G) = n-1$ then G is a complete graph which is a contradiction. If $\delta(G) = n-2$ then G is isomorphic to $K_n - Y$ where Y is a matching in K_n. Then $\gamma_{2nt}(G) \leq 3$ and hence $n \leq 4$ which is a contradiction to $\kappa(G) = n-4$. Suppose $\delta(G) = n-3$. Let $X = \{v_1, v_2, \cdots, v_{n-4}\}$ be the vertex cut of G and let $V - X = \{x_1, x_2, x_3, x_4\}$. If $\langle V - X\rangle$ contains an isolated vertex then $\delta(G) \leq n-4$ which is a contradiction. Hence $\langle V - S\rangle$ is isomorphic to $K_2 \cup K_2$. Also every vertex of $V - X$ is adjacent to all the vertices of X. Then $\gamma_{2nt}(G) = 3$. Hence $n = 4$ which is a contradiction. Thus $\delta(G) = n-4$.

Subcase 2.1 $\langle V - X\rangle = \overline{K_4.}$

Then every vertex of $V - X$ is adjacent to all the vertices in X. If $|X| = 1$ then G is a star which is a contradiction. If $|X| = 2$ then G is isomorphic to either $K_{2,4}$ or the graph obtained from $K_{2,4}$ by joining the vertices of degree 4 by an edge. But for these two graphs $\gamma_{2nt} \leq 3$ which is a contradiction. Hence $|X| \geq 3$. Then $\{v_1, v_2, x_1, x_2\}$ is a nt2d-set of G. Hence $\gamma_{2nt} \leq 4$. Then $n \leq 5$ which is a contradiction.

Subcase 2.2 $\langle V - X\rangle = P_3 \cup K_1$.

Let x_1 be the isolated vertex and (x_2, x_3, x_4) be a path in $\langle V - X\rangle$. Then x_1 is adjacent to all the vertices in X and x_2, x_4 are not adjacent to at most one vertex in X. Let $v_1 \in X - N(x_2)$. If $N(x_3) \cap X = \emptyset$ then $\delta(G) = 2$ and hence $n = 6$ then $\{x_1, x_2, x_3, x_4\}$ or $\{x_1, x_2, x_3, v_2\}$ is a nt2d-set of G which is a contradiction to $\gamma_{2nt}(G) = n-1$. If $N(x_3) \cap X \neq \emptyset$ then $\{x_1, x_2, x_4, v_1\}$ is a nt2d-set of G and hence $\gamma_{2nt} \leq 4$. Thus $n = 5$. Then G is isomorphic to $B(2,1)$ or $K_3(1,1,0)$ or the graph G_1 where G_1 is obtained from $K_4 - e$ by attaching a pendant edge to the vertex of degree 3. But $\gamma_{2nt}(G_1) = \gamma_{2nt}(K_3(1,1,0)) = 3 \neq n-1$ which is a contradiction. Hence G is isomorphic to $B(2,1)$.

Suppose $N(x_2) = N(x_4) = X$. Then $\{x_1, x_2, x_4, v_1\}$ is a nt2d-set of G. Hence $\gamma_{2nt}(G) \leq 4$ which gives $n = 5$. Then G is isomorphic to either G_1 or $C_4(1,0,0,0)$. But $\gamma_{2nt}(G_1) = \gamma_{2nt}(C_4(1,0,0,0)) = 3 \neq n-1$ which is a contradiction.

Subcase 2.3 $\langle V - X\rangle = K_3 \cup K_1$.

Let x_1 be the isolated vertex in $\langle V - X\rangle$ and $\langle\{x_2, x_3, x_4\}\rangle$ be the complete graph. Then x_1 is adjacent to all the vertices in X and x_2, x_3, x_4 are not adjacent to at most two vertices in X. Then $\{x_1, x_2, x_3, v_1, v_2\}$ where $v_1, v_2 \in X - [N(x_2) \cup N(x_3)]$ is a nt2d-set of G and hence $n = 5$ or 6. Suppose $n = 5$. Then G is isomorphic to $K_4(1,0,0,0)$ or $K_3(P_3, P_1, P_1)$ or the graph obtained from $K_4 - e$ by attaching a pendant egde to the vertex of degree two. For these graphs $\gamma_{2nt}(G) + \kappa(G) \neq 2n-5$. Suppose $n = 6$. Then $\{x_1, x_2, x_3, v_1\}$ or $\{x_1, x_2, x_3, v_2\}$ or $\{x_2, x_3, v_1, v_2\}$ is a nt2d-set of G which is a contradiction to $\gamma_{2nt} = n-1$.

Subcase 2.4 $\langle V - X\rangle = K_2 \cup \overline{K_2.}$

Let $x_1x_2 \in E(G)$ and $x_3x_4 \in E(\overline{G})$. Then each $x_i, i = 1$ or 2 is non adjacent to at most one vertex in X and each $x_j, j = 3$ or 4 is adjacent to all the vertices in X. Then $\{x_1, x_3, x_4, v_1\}$ where $v_1 \in N(x_2) \cap X$ is a nt2d-set of G and hence $n = 5$. Then G is isomorphic to $B(2,1)$ or $K_3(2,0)$.

Subcase 2.5. $\langle V - X\rangle = K_2 \cup K_2$.

Let $x_1x_2, x_3x_4 \in E(G)$. Since $\delta(G) = n-4$ each x_i is non adjacent to at most one vertex in X. Then at most one vertex say $v_1 \in X$ such that $|N(v_1) \cap (V-X)| = 1$. If all $v_i \in X$ such that $|N(v_i) \cap (V-X)| \geq 2$ then $\{x_1, x_2, x_3, x_4\}$ is a nt2d-set of G and hence $n = 5$. Then $X = \{v_1\}$. If $|N(v_1) \cap (V-X)| = 2$ then G is isomorphic to P_5. If $|N(v_1) \cap (V-X)| \geq 3$ then $\gamma_{2nt}(G) = 3 \neq n-1$ which is a contradiction. Suppose $|N(v_1) \cap (V-X)| = 1$ and $|N(v_i) \cap (V-X)| \geq 2$ for $i \neq 1$ then $\{x_1, x_2, x_3, x_4, v_1\}$ is a nt2d-set of G and hence $n = 6$. For this graph $\gamma_{2nt}(G) + \kappa(G) \neq 2n-5$.

Case 3. $\gamma_{2nt}(G) = n-2$ and $\kappa(G) = n-3$.

Then $n-3 \leq \delta$. If $\delta = n-1$ then G is a complete graph which is a contradiction. If $\delta = n-2$ then G is isomorphic to $K_n - Y$ where Y is any matching in K_n. Then $\gamma_{2nt}(G) = 2$ or 3. If $\gamma_{2nt}(G) = 3$ then $n = 5$ which gives a contradiction. If $\gamma_{2nt}(G) = 2$ then $n = 4$. Hence G is either $K_4 - e$ or C_4. For these two graphs $\kappa(G) = 2 \neq n-3$ which is a contradiction. Hence $\delta = n-3$. Let $X = \{v_1, v_2, ..., v_{n-3}\}$ be the vertex cut of G and let $V - X = \{x_1, x_2, x_3\}$.

Subcase 3.1 $\langle V-X \rangle = \overline{K_3}$

Then every vertex of $V-X$ is adjacent to all the vertices in X. If $|X| = 1$ then G is a star which is a contradiction. If $|X| = 2$ then G is isomorphic to either $K_{2,3}$ or the graph H_1 which is obtained from $K_{2,3}$ by joining the vertices of degree three by an edge. But $\gamma_{2nt}(H_1) = 2$. Hence G is isomorphic to $K_{2,3}$. Suppose $|X| \geq 3$. Then $\{v_1, v_2, x_1, x_2\}$ is a nt2d-set of G. Hence $\gamma_{2nt}(G) \leq 4$. Then $n = 6$. Thus G is isomorphic to $K_{3,3}$ or $K_{3,3} + e$ or $P_3 + \overline{K_3}$. But $\gamma_{2nt}(K_{3,3} + e) = \gamma_{2nt}(P_3 + \overline{K_3}) = 3$ which is a contradiction. Hence G is isomorphic to $K_{3,3}$.

Subcase 3.2 $\langle V-X \rangle = K_1 \cup K_2$.

Let $x_1x_2 \in E(G)$. Then x_3 is adjacent to all the vertices in X and x_1, x_2 are not adjacent to at most one vertex in X. If degx_1 or degx_2 is $n-2$ then $\{x_1, x_2, x_3\}$ is a nc2d-set of G and hence $\gamma_{2nt}(G) \leq 3$. Then $n \leq 5$. If $n = 4$ then G is isomorphic to P_4 or $K_3(1,0,0)$. But for these graphs $\gamma_{2nt} \neq n-2$. Suppose $n = 5$. Let $X = \{v_1, v_2\}$ and let $v_1v_2 \in E(G)$. If deg$x_1 = 3$ and deg$x_2 = 2$ then G is isomorphic to G^*. If deg$x_1 = 3$ and deg$x_2 = 3$ then G is isomorphic to a graph H_2 which is obtained from $K_4 \cup K_1$ by joining any vertices of K_4 to the vertex of K_1 by the edges. But $\gamma_{2nt}(H_2) = 2 \neq n-2$ which is a contradiction. If $v_1v_2 \notin E(G)$ then G is isomorphic to $C_5 + e$ or H^*. Suppose deg$x_1 =$ deg$x_2 = n-3$. If $N(x_1) = N(x_2)$ then there is a vertex $v_1 \in X$ such that v_1 is not adjacent to both x_1 and x_2. Then v_1 is adjacent to all the vertices in X. If $|X| \geq 4$ then $\{x_1, x_2, x_3, v_1\}$ is a nt2d-set of G and hence $n \leq 6$ which is a contradiction. If $|X| = 3$ then $\{v_1, v_2, v_3\}$ is a nt2d-set of G and hence $n \leq 5$ which is a contradiction. If $N(x_1) \neq N(x_2)$ then two vertices say v_1 and v_2 such that v_1 is not adjacent to x_1 but adjacent to x_2 and v_2 is not adjacent to x_2 but adjacent to x_1. Then $\{x_1, x_2, x_3\}$ is a nt2d-set of G and hence $n \leq 5$. Then G is isomorphic to C_5 or $C_5 + e$.

Case 4. $\gamma_{2nt}(G) = n-3$ and $\kappa(G) = n-2$.

Then $n-2 \leq \delta(G)$. If $\delta = n-1$ then G is a complete graph which is a contradiction. Hence $\delta(G) = n-2$. Then G is isomorphic to $K_n - Y$ where Y is a matching in K_n. Then $\gamma_{2nt}(G) \leq 3$. If $\gamma_{2nt}(G) = 3$ then $n = 6$. But $\gamma_{2nt}(K_6 - Y) = 2 \neq n-3$ which is a contradiction. If $\gamma_{2nt}(G) = 2$ then $n = 5$. Hence G is isomorphic to $K_5 - Y$ where Y is any matching in K_5.

Case 5. $\gamma_{2nt}(G) = n - 4$ and $\kappa(G) = n - 1$.

Then G is a complete graph. Since $\gamma_{2nt}(G) = n - 4$ we have $n = 6$. Hence G is isomorphic to K_6. The converse is obvious. □

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