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Neighbourly and Highly Irregular Neutrosophic Fuzzy Graphs

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Abstract: In this paper, the concepts of neighbourly irregular neutrosophic fuzzy graphs, highly irregular neutrosophic fuzzy graphs, neighbourly totally irregular and highly totally irregular neutrosophic fuzzy graphs are introduced. Also, we proved some theorems and results of these graphs.

Key Words: Neighbourly irregular neutrosophic fuzzy graphs, highly irregular neutrosophic fuzzy graphs, neighbourly totally irregular and highly totally irregular neutrosophic fuzzy graphs.

AMS(2010): 05C12, 03E72, 05C72.

§1. Introduction

F.Smarandache [13] introduced notion of neutrosophic set which is useful for dealing real life problems having imprecise, indeterminacy and inconsistent data. They are generalization of the theory of fuzzy sets, intuitionistics fuzzy set, interval valued fuzzy set, and interval valued intuitionistic fuzzy sets.

N. Shah and Hussain [11, 14] introduced the notion of soft neutrosophic graphs. N. Shah [12] introduces the notion of neutrosophic graphs and different operations like union, intersection and complement in his work. A neutrosophic set is characterized by a truth membership degree (t), an indeterminacy membership degree (i), falsity membership degree (f) independently, which are with in the real standard or non standard unit interval $]^{-}0, 1^{+}[$.

N. R. Santhi Maheswari and C. Sekar [7] introduced the notion of Neighbourly irregular graphs and semi neighbourly irregular graphs [8] on m - neighbourly irregular Fuzzy graphs, on neighbourly edge irregular fuzzy graphs [9]. N. R. Santhi Maheswari, R. Muneeswari and S. Ravi Narayanan [10] introduced the notion of 2 - highly irregular fuzzy graphs.

Divya and Dr. J. Malarvizhi [1] introduced the notion of neutrosophic fuzzy graph and few fundamental operation on neutrosophic fuzzy graph. This idea motivate us to introduce regular and irregular neutrosophic fuzzy graphs.

[1]Received November 19, 2020, Accepted March 5, 2021.

§2. Preliminaries

In this section, we recall the notions related to Neutrosophic set, fuzzy graph, Neutrosophic fuzzy set and neturosophic fuzzy graph.

Definition 2.1([13]) *Let X be a space of points with generic elements in X denoted by x. A neutrosophic set $A(NSA)$ is an object having the form*

$$A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\},$$

where the functions $T, I, F \to]^{-}0, 1^{+}[$ define respectively a truth membership function, an indeterminacy membership function and a falsity membership function of the element $x \in X$ to the set A with the condition

$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}.$$

The functions $T_A(x), I_A(x), F_A(x)$ are real standard or non standard subsets of $]^{-}0, 1^{+}[$.

Definition 2.2([5]) *A fuzzy graph is a pair of functions $G = (\sigma, \mu)$, where σ is a fuzzy subset of a non-empty set V and is a symmetric fuzzy relation of σ i.e $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(uv) \leq \sigma(u)\Lambda\sigma(v)$ for $\forall u, v \in V$ where uv denote the edge between u and v and $\sigma(u)\Lambda\sigma(v)$ denotes the minimum of $\sigma(u)$ and $\sigma(v)$, σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.*

Definition 2.3([1]) *Let X be a space of points with generic elements in X denoted by x. A neutrosophic fuzzy set $A(NFSA)$ is characterized by truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$.*

For each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0, 1]$. A $(NFSA)$ can be written as $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$.

Definition 2.4([1]) *Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be neutrosophic fuzzy sets on a set X. If $A = (T_A, I_A, F_A)$ is a neutrosophic fuzzy relation on a set X, then $A = (T_A, I_A, F_A)$ is called a neutrosophic fuzzy relation on $B = (T_B, I_B, F_B)$ if*

$$\begin{aligned} T_B(x, y) &\leq T_A(x).T_A(y), \\ I_B(x, y) &\leq I_A(x).I_A(y), \\ F_B(x, y) &\leq F_A(x).F_A(y) \end{aligned}$$

for all $x, y \in X$, where . means the ordinary multiplication.

Definition 2.5([1]) *A neutrosophic fuzzy graph $(NFgraph)$ with underlying set V is defined to be a pair $N_G = (A, B)$, where*

(i) the functions $T_A, I_A, F_A : V \to [0, 1]$ denote the degree of truth membership, degree of indeterminacy membership and the degree of falsity membership of the element $v_i \in V$ respectively and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$;

(ii) $E \subseteq V \times V$ *where the functions* $T_B, I_B, F_B : V \times V \to [0,1]$ *are defined by*

$$\begin{aligned} T_B(v_i, v_j) &\leq T_A(v_i).T_A(v_j), \\ I_B(v_i, v_j) &\leq I_A(v_i).I_A(v_j), \\ F_B(v_i, v_j) &\leq F_A(v_i).F_A(v_j) \end{aligned}$$

for all $v_i, v_j \in V$*, where . means ordinary multiplication denotes the degrees of truth membership, indeterminacy membership and falsity membership of the edge* $(v_i, v_j) \in E$ *respectively, where*

$$0 \leq T_B(x) + I_B(x) + F_B(x) \leq 3$$

for all $(v_i, v_j) \in E$ $(j = 1, 2, \cdots, n)$.

§3. Degree of Vertex in Neutrosophic Fuzzy Graphs

Throughout this paper, we denote $G = (V, E)$ a crisp graph, $N_G = (A, B)$ a neutrosophic fuzzy graph of graph G.

Definition 3.1 *Let* $N_G = (A, B)$ *be a neutrosophic fuzzy graph. The neighbourhood degree of a vertex* x *in* N_G *defined by*

$$d_{N_G}(x) = (deg_T(x), deg_I(x), deg_F(x)),$$

where

$$\begin{aligned} deg_T(x) &= \sum_{xy \in E} T_B(xy), \\ deg_I(x) &= \sum_{xy \in E} I_B(xy), \\ deg_F(x) &= \sum_{xy \in E} F_B(xy). \end{aligned}$$

Example 3.2 Let N_G be the neutrosophic fuzzy graph shown in Fig.1.

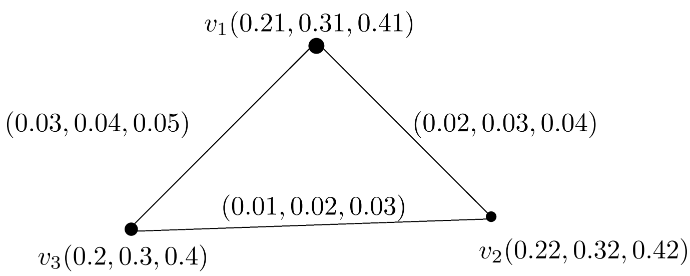

Fig.1

In this graph,

$$d_{N_G}(v_1) = (0.05, 0.07, 0.09),$$
$$d_{N_G}(v_2) = (0.03, 0.05, 0.07),$$
$$d_{N_G}(v_3) = (0.04, 0.06, 0.08).$$

Definition 3.3 *Let $N_G = (A, B)$ be a neutrosophic fuzzy graph. The closed neighbourhood degree of a vertex x in N_G defined by*

$$d_{N_G}[x] = (deg_T[x], deg_I[x], deg_F[x]),$$

where

$$\begin{aligned} deg_T(x) &= \sum_{xy\in E} T_B(xy) + T_A(x), \\ deg_I(x) &= \sum_{xy\in E} I_B(xy) + I_B(x), \\ deg_F(x) &= \sum_{xy\in E} F_B(xy) + F_B(x). \end{aligned}$$

Example 3.4 Consider the neutrosophic fuzzy graph N_G in Fig.2.

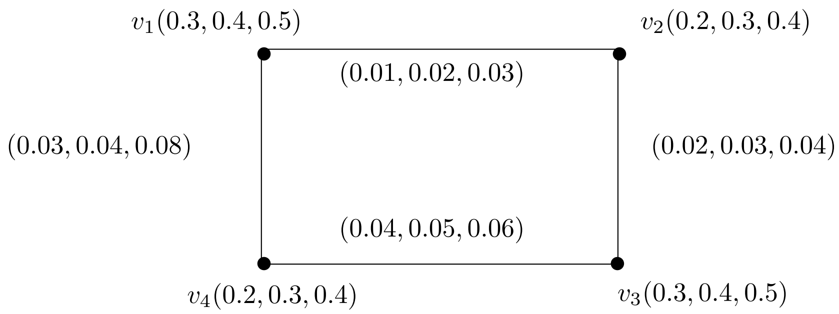

Fig.2

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.04, 0.06, 0.08),\ d_{N_G}(v_2) = (0.03, 0.05, 0.07), \\ d_{N_G}(v_3) &= (0.06, 0.08, 0.10),\ d_{N_G}(v_4) = (0.07, 0.09, 0.14), \\ d_{N_G}[v_1] &= (0.34, 0.46, 0.58),\ d_{N_G}[v_2] = (0.23, 0.35, 0.47), \\ d_{N_G}[v_3] &= (0.36, 0.48, 0.6),\ d_{N_G}[v_4] = (0.27, 0.39, 0.54). \end{aligned}$$

§4. Regular and Irregular Neutrosophic Fuzzy Graphs

Definition 4.1 *A neutrosophic fuzzy graph is called regular if all the vertices of N_G have the same open neighbourhood degree.*

Example 4.2 Consider the neutrosophic fuzzy graph N_G shown in Fig.3.

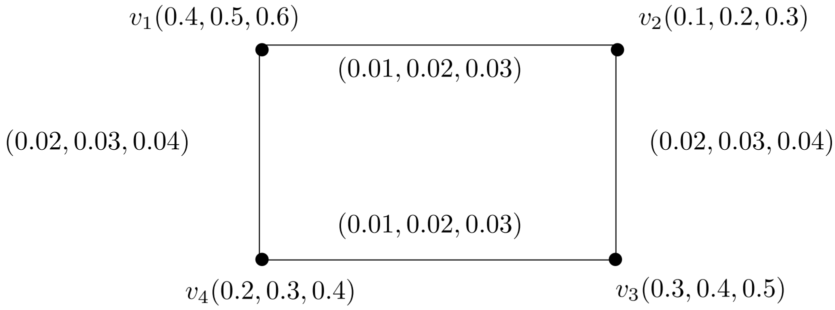

Fig. 3

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.03, 0.05, 0.07),\ d_{N_G}(v_2) = (0.03, 0.05, 0.07), \\ d_{N_G}(v_3) &= (0.03, 0.05, 0.07),\ d_{N_G}(v_4) = (0.03, 0.05, 0.07). \end{aligned}$$

Here all the vertices having same open neighbourhood degree. Hence this N_G is regular neutrosophic fuzzy graph.

Definition 4.3 *A neutrosophic fuzzy graph is said to be irregular if there is a vertex which is adjacent to vertices with distinct open neighbourhood degrees.*

Example 4.4 Let N_G be the neutrosophic fuzzy graph shown in Fig.4.

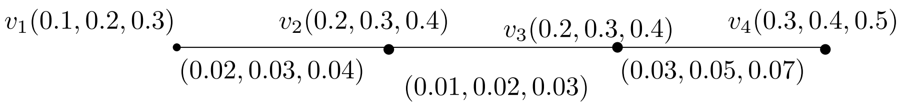

Fig.4

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.02, 0.03, 0.04),\ d_{N_G}(v_2) = (0.03, 0.05, 0.07), \\ d_{N_G}(v_3) &= (0.04, 0.07, 0.10),\ d_{N_G}(v_4) = (0.03, 0.05, 0.07). \end{aligned}$$

Here there is a vertex v_2 adjacent to the vertices v_1 and v_3 which are having distinct open neighbourhood degrees. Hence this graph is irregular neutrosophic fuzzy graph.

Definition 4.5 *A neutrosophic fuzzy graph N_G is called totally irregular neutrosophic fuzzy graphs if there is a vertex which is adjacent to the vertices with distinct closed neighbourhood degrees.*

Example 4.6 Let N_G be the neutrosophic fuzzy graph shown in Fig.5.

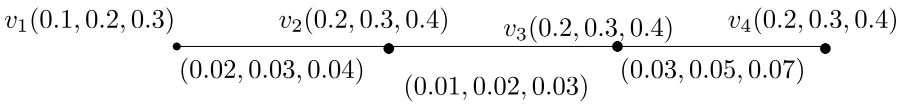

Fig.5

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.02, 0.03, 0.04),\ d_{N_G}(v_2) = (0.03, 0.05, 0.07), \\ d_{N_G}(v_3) &= (0.04, 0.07, 0.10),\ d_{N_G}(v_4) = (0.03, 0.05, 0.07), \\ d_{N_G}[v_1] &= (0.12, 0.23, 0.34),\ d_{N_G}[v_2] = (0.23, 0.35, 0.47), \\ d_{N_G}[v_3] &= (0.24, 0.37, 0.5),\ d_{N_G}[v_4] = (0.23, 0.35, 0.47). \end{aligned}$$

Here there is a vertex v_2 adjacent to the vertices v_1 and v_3 which are having distinct closed neighbourhood degrees. Hence this graph is totally irregular neutrosophic fuzzy graph.

§5. Neighbourly Irregular Neutrosophic Fuzzy Graphs

Definition 5.1 *Let N_G be a neutrosophic fuzzy graph. If every two adjacent vertices of N_G have distinct open neighbourhood degrees, then it is referred as a neighbourly irregular neutrosophic fuzzy graph.*

Example 5.2 Let N_G be the neutrosophic fuzzy graph shown in Fig.6.

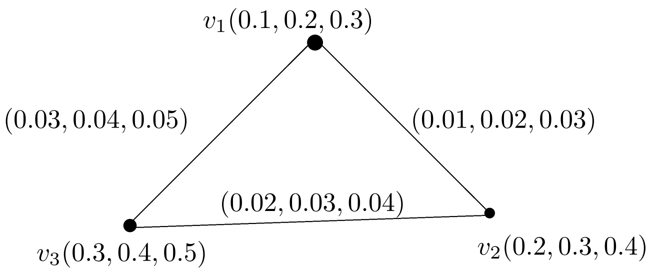

Fig.6

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.04, 0.06, 0.08),\ d_{N_G}(v_2) = (0.03, 0.05, 0.07), \\ d_{N_G}(v_3) &= (0.05, 0.07, 0.09). \end{aligned}$$

Here every two adjacent vertices having distinct open neighbourhood degrees. Hence this graph is neighbourly irregular neutrosophic fuzzy graph.

Definition 5.3 *A neutrosophic fuzzy graph is said to be neighbourly totally irregular neutrosophic fuzzy graph if every two adjacent vertices in N_G have distinct closed neighbourhood degrees.*

Example 5.3 Let N_G be the neutrosophic fuzzy graph shown in Fig.7.

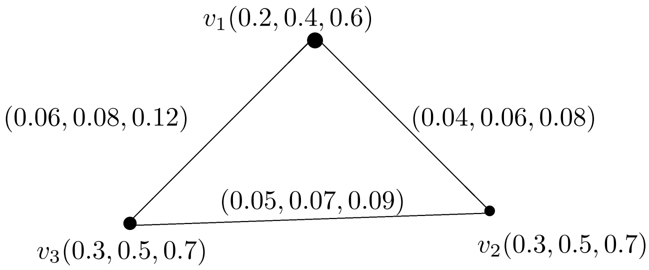

Fig.7

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.10, 0.14, 0.20),\ d_{N_G}(v_2) = (0.09, 0.13, 0.17), \\ d_{N_G}(v_3) &= (0.11, 0.15, 0.21),\ d_{N_G}[v_1] = (0.20, 0.44, 0.80), \\ d_{N_G}[v_2] &= (0.39, 0.63, 0.87),\ d_{N_G}[v_3] = (0.41, 0.65, 0.91). \end{aligned}$$

Here every two adjacent vertices having distinct closed neighbourhood degrees. Hence this graph is neighbourly totally irregular neutrosophic fuzzy graph.

Observation 5.5 Every neighbourly irregular neutrosophic fuzzy graphs need not be a neighbourly totally irregular neutrosophic fuzzy graph.

Example 5.6 Let N_G be the neutrosophic fuzzy graph shown in Fig.8.

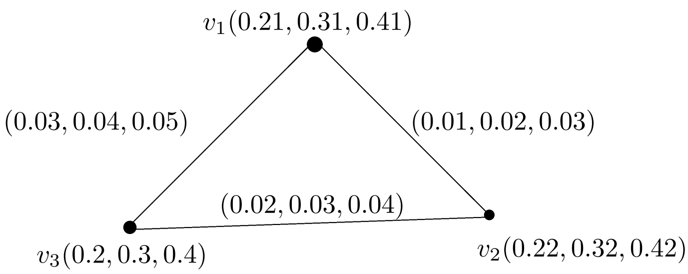

Fig.8

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.04, 0.06, 0.08),\ d_{N_G}(v_2) = (0.03, 0.05, 0.07), \\ d_{N_G}(v_3) &= (0.05, 0.07, 0.09),\ d_{N_G}[v_1] = (0.25, 0.37, 0.49), \\ d_{N_G}[v_2] &= (0.25, 0.37, 0.49),\ d_{N_G}[v_3] = (0.25, 0.37, 0.49). \end{aligned}$$

Here, every pair of adjacent vertices having distinct open neighbourhood degrees but every pair adjacent vertices having same closed neighbourhood degrees.

Hence N_G is neighbourly irregular neutrosophic fuzzy graph. But N_G is not neighbourly totally irregular neutrosophic fuzzy graph.

Observation 5.7 Every neighbourly totally irregular neutrosophic fuzzy graphs need not be a neighbourly irregular neutrosophic fuzzy graph.

Example 5.8 Consider the neutrosophic fuzzy graph N_G in Fig.9, we get that

$$\begin{aligned} d_{N_G}(v_1) &= (0.04, 0.12, 0.24),\ d_{N_G}(v_2) = (0.04, 0.12, 0.24), \\ d_{N_G}(v_3) &= (0.04, 0.12, 0.24),\ d_{N_G}(v_4) = (0.04, 0.12, 0.24), \\ d_{N_G}[v_1] &= (0.24, 0.42, 0.64),\ d_{N_G}[v_2] = (0.14, 0.22, 0.54), \\ d_{N_G}[v_3] &= (0.24, 0.42, 0.64),\ d_{N_G}[v_4] = (0.14, 0.22, 0.54). \end{aligned}$$

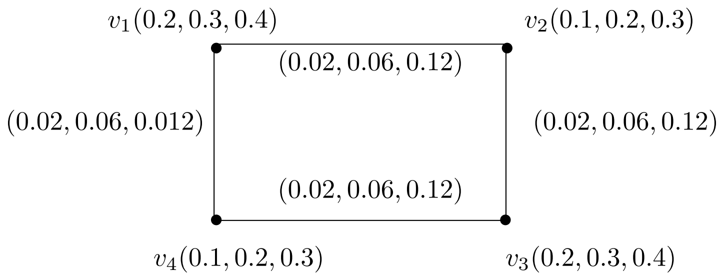

Fig.9

Here every pair of adjacent vertices having distinct closed neighbourhood degrees. But every pair adjacent vertices having same open neighbourhood degrees. Hence N_G is neighbourly totally irregular neutrosophic fuzzy graph. But N_G is not neighbourly irregular neutrosophic fuzzy graph.

§6. Highly Irregular Neutrosophic Fuzzy graphs

Definition 6.1 *Let N_G be a neutrosophic fuzzy graph. If every vertex in N_G is adjacent to vertices with distinct open neighbourhood degrees, then it is called as highly irregular neutrosophic fuzzy graph.*

Example 6.2 Consider the neutrosophic fuzzy graph N_G shown in Fig.10.

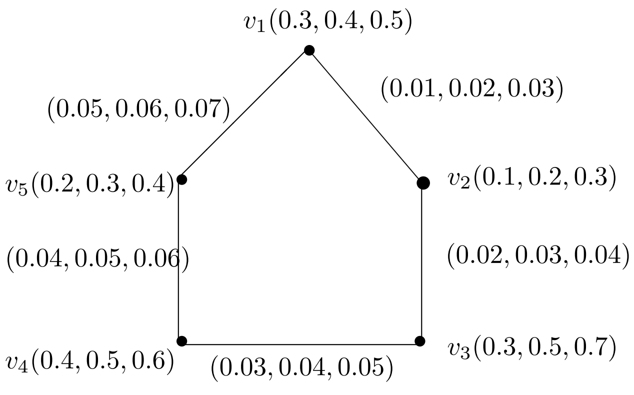

Fig.10

In this graph,

$$d_{N_G}(v_1) = (0.06, 0.08, 0.10),\ d_{N_G}(v_2) = (0.03, 0.05, 0.07),$$
$$d_{N_G}(v_3) = (0.05, 0.07, 0.09),\ d_{N_G}(v_4) = (0.07, 0.09, 0.11),\ d_{N_G}(v_4) = (0.09, 0.11, 0.13).$$

Here every vertex in N_G is adjacent to vertices with distinct open neighbourhood degrees. Hence this N_G is highly irregular neutrosophic fuzzy graph.

Definition 6.3 *Let N_G be a neutrosophic fuzzy graph. If every vertex in N_G is adjacent to vertices with distinct closed neighbourhood degrees, then it is called as highly totally irregular neutrosophic fuzzy graph.*

Example 6.4 Let N_G be the neutrosophic fuzzy graph shown in Fig.11.

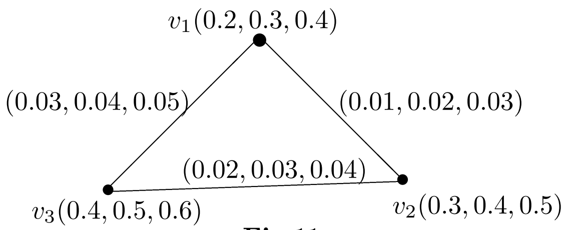

Fig.11

In this graph,

$$d_{N_G}(v_1) = (0.04, 0.06, 0.08),\ d_{N_G}(v_2) = (0.03, 0.05, 0.07),$$
$$d_{N_G}(v_3) = (0.05, 0.07, 0.09),\ d_{N_G}[v_1] = (0.24, 0.36, 0.48),$$
$$d_{N_G}[v_2] = (0.33, 0.45, 0.57),\ d_{N_G}[v_3] = (0.45, 0.57, 0.69).$$

Here every vertex in N_G is adjacent to vertices with distinct closed neighbourhood degrees. Hence this N_G is highly totally irregular neutrosophic fuzzy graph.

Observation 6.5 Every highly irregular neutrosophic fuzzy graphs need not be a highly totally irregular neutrosophic fuzzy graph.

Example 6.6 Consider the neutrosophic fuzzy graph N_G shown in Fig.12.

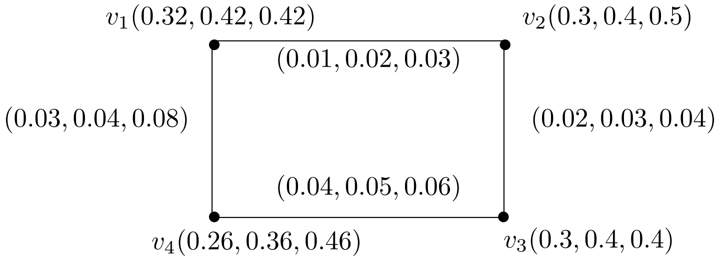

Fig. 12

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.04, 0.06, 0.08),\ d_{N_G}(v_2) = (0.03, 0.05, 0.07),\\ d_{N_G}(v_3) &= (0.06, 0.08, 0.10),\ d_{N_G}(v_4) = (0.07, 0.09, 0.11),\\ d_{N_G}[v_1] &= (0.36, 0.48, 0.5),\ d_{N_G}[v_2] = (0.33, 0.45, 0.57),\\ d_{N_G}[v_3] &= (0.36, 0.48, 0.5),\ d_{N_G}[v_4] = (0.33, 0.45, 0.57). \end{aligned}$$

Here, every vertex in N_G is adjacent to vertices with distinct open neighborhood degrees. But a vertex v_2 adjacent to the vertices v_1 and v_3 which are having same closed neighbourhood degrees. Hence N_G is highly irregular neutrosophic fuzzy graph. But N_G is not highly totally irregular neutrosophic fuzzy graph.

Observation 6.7 Every highly totally irregular neutrosophic fuzzy graph needs not be a highly irregular neutrosophic fuzzy graph.

Example 6.8 Consider the neutrosophic fuzzy graph N_G shown in Fig.13. We know that

$$\begin{aligned} d_{N_G}(v_1) &= (0.05, 0.07, 0.09),\ d_{N_G}(v_2) = (0.05, 0.07, 0.09),\\ d_{N_G}(v_3) &= (0.04, 0.06, 0.08),\ d_{N_G}(v_4) = (0.03, 0.05, 0.07),\\ d_{N_G}(v_5) &= (0.05, 0.07, 0.09),\ d_{N_G}[v_1] = (0.35, 0.47, 0.57),\\ d_{N_G}[v_2] &= (0.15, 0.27, 0.39),\ d_{N_G}[v_3] = (0.34, 0.36, 0.48),\\ d_{N_G}[v_4] &= (0.13, 0.25, 0.37),\ d_{N_G}[v_5] = (0.25, 0.37, 0.49). \end{aligned}$$

Here, every vertex in N_G is adjacent to the vertices with distinct closed neighbourhood degrees. But a vertex v_2 is adjacent to the vertices v_1 and v_5 which are having same neighbourhood degrees. Hence N_G is highly totally irregular neutrosophic fuzzy graph. But N_G is not highly irregular neutrosophic fuzzy graph.

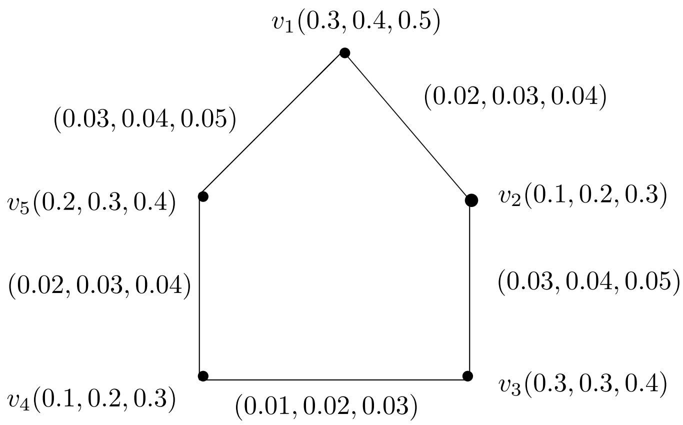

Fig.13

Theorem 6.9 *Let N_G be a neutrosophic fuzzy graph. Then N_G is highly irregular neutrosophic* *fuzzy graph and neighbourly irregular neutrosophic fuzzy graph iff the open neighbourhood degrees of all the vertices of N_G are distinct.*

Proof Let N_G be a neutrosophic fuzzy graph with n vertices $v_1, v_2, \cdots, v_n$. Suppose N_G is both highly and neighbourly irregular neutrosophic fuzzy graph. we shall show that the open neighbourhood degrees of all the vertices of N_G are distinct.

Let $d_{N_G}(v_i) = (deg_T(v_i), deg_I(v_i), deg_F(v_i)),\ i = 1, 2, \cdots, n$. Let the adjacent vertices of v_1 be $v_2, v_3, \cdots, v_n$ with open neighbourhood degrees

$$\begin{gathered}(deg_T(v_2), deg_I(v_2), deg_F(v_2)),\\ (deg_T(v_3), deg_I(v_3), deg_F(v_3)),\\ \cdots\cdots\cdots\cdots\cdots\cdots,\\ (deg_T(v_n), deg_I(v_n), deg_F(v_n)),\end{gathered}$$

respectively. Since N_G is highly irregular neutrosophic fuzzy graph, we have

$$\begin{gathered}deg_T(v_2) \neq deg_T(v_3) \neq \cdots \neq deg_T(v_n),\\ deg_I(v_2) \neq deg_I(v_3) \neq \cdots \neq deg_I(v_n),\\ deg_F(v_2) \neq deg_F(v_3) \neq \cdots \neq deg_F(v_n).\end{gathered}$$

Also since N_G is neighbourly irregular neutrosophic fuzzy graph, we have

$$\begin{gathered}deg_T(v_1) \neq deg_T(v_2) \neq deg_T(v_3) \neq \cdots \neq deg_T(v_n),\\ deg_I(v_1) \neq deg_I(v_2) \neq deg_I(v_3) \neq \cdots \neq deg_I(v_n),\\ deg_F(v_1) \neq deg_F(v_2) \neq deg_F(v_3) \neq \cdots \neq deg_F(v_n).\end{gathered}$$

Thus,

$$\begin{aligned}(deg_T(v_1), deg_I(v_1), deg_F(v_1)) &\neq (deg_T(v_2), deg_I(v_2), deg_F(v_2))\\ &\neq \cdots \neq (deg_T(v_n), deg_I(v_n), deg_F(v_n)).\end{aligned}$$

Hence the open neighbourhood degrees of all vertices are distinct.

Conversely, suppose we take the neighbourhood degrees of all vertices are distinct. To show that N_G is highly and neighbourly irregular neutrosophic fuzzy graphs. Let $d_{N_G}(v_i) = (deg_T(v_i), deg_I(v_i), deg_F(v_i)),\ i = 1, 2, \cdots, n$.

Given that

$$\begin{gathered}deg_T(v_1) \neq deg_T(v_2) \neq deg_T(v_3) \neq \cdots \neq deg_T(v_n),\\ deg_I(v_1) \neq deg_I(v_2) \neq deg_I(v_3) \neq \cdots \neq deg_I(v_n),\\ deg_F(v_1) \neq deg_F(v_2) \neq deg_F(v_3) \neq \cdots \neq deg_F(v_n).\end{gathered}$$

Therefore, any two adjacent vertices have distinct open neighbourhood degrees and also every vertex, which is adjacent to the vertices having distinct open neighbourhood degrees. Hence N_G is both highly and neighbourly irregular neutrosophic fuzzy graphs. □

Theorem 6.10 *Let N_G be neutrosophic fuzzy graph. If N_G is neighbourly irregular neutrosophic fuzzy graph and (T_A, I_A, F_A) is a constant function, then N_G is a neighbourly totally irregular neutrosophic fuzzy graph.*

Proof Let N_G be a neighbourly irregular neutrosophic fuzzy graph. Let $v_i, v_j \in V$, where v_i and v_j are adjacent vertices with distinct open neighbourhood degrees

$$(deg_T(v_1), deg_I(v_1), deg_F(v_1)) \text{ and } (deg_T(v_2), deg_I(v_2), deg_F(v_2))$$

respectively.

Suppose we assume that

$$(T_A(v_i), I_A(v_i), F_A(v_i)) = (T_A(v_j), I_A(v_j), F_A(v_j)) = (c_1, c_2, c_3),$$

where c_1, c_2, c_3 are constants and $c_1, c_2, c_3 \in [0, 1]$. Then,

$$\begin{aligned}
deg_T[v_i] &= deg_T(v_i) + T_A(v_i) = deg_T(v_i) + c_1,\\
deg_I[v_i] &= deg_I(v_i) + I_A(v_i) = deg_I(v_i) + c_2,\\
deg_F[v_i] &= deg_F(v_i) + F_A(v_i) = deg_F(v_i) + c_3,\\
deg_T[v_j] &= deg_T(v_j) + T_A(v_j) = deg_T(v_j) + c_1,\\
deg_I[v_j] &= deg_I(v_j) + I_A(v_j) = deg_I(v_j) + c_2,\\
deg_F[v_j] &= deg_F(v_j) + F_A(v_j) = deg_F(v_j) + c_3.
\end{aligned}$$

We need to show that $deg_T[v_i] \neq deg_T[v_j]$, $deg_I[v_i] \neq deg_I[v_j]$ and $deg_F[v_i] \neq deg_F[v_j]$. Suppose

$$\begin{aligned}
deg_T[v_i] = deg_T[v_j] &\implies deg_T(v_i) + c_1 = deg_T(v_j) + c_1\\
&\implies deg_T(v_i) - deg_T(v_j) = c_1 - c_1\\
&\implies deg_T(v_i) - deg_T(v_j) = 0\\
&\implies deg_T(v_i) = deg_T(v_j),
\end{aligned}$$

which is a contradiction to the fact that $deg_T(v_i) \neq deg_T(v_j)$. Therefore, $deg_T[v_i] \neq deg_T[v_j]$.

Similarly, $deg_I[v_i] \neq deg_I[v_j]$, $deg_F[v_i] \neq deg_F[v_j]$. Hence N_G is neighbourly totally irregular neutrosophic fuzzy graph. □

Observation 6.11 Every neighbourly irregular neutrosophic fuzzy graph need not be a highly irregular neutrosophic fuzzy graph.

Example 6.12 Consider the neutrosophic fuzzy graph N_G shown in Fig.14.

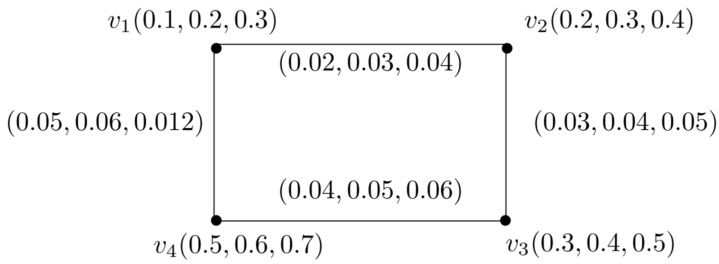

Fig. 14

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.07, 0.09, 0.1),\ d_{N_G}(v_2) = (0.05, 0.07, 0.09), \\ d_{N_G}(v_3) &= (0.07, 0.09, 0.11),\ d_{N_G}(v_4) = (0.09, 0.11, 0.17). \end{aligned}$$

Here, every two adjacent vertices having distinct open neighbourhood degrees. But a vertex v_2 adjacent to the vertices v_1 and v_3 which are having same open neighbourhood degree. Hence N_G is neighbourly irregular neutrosophic fuzzy graph. But N_G is not highly irregular neutrosophic fuzzy graph.

Observation 6.13 Every neighbourly totally irregular neutrosophic fuzzy graph need not be a highly totally irregular neutrosophic fuzzy graph.

Example 6.14 Consider the neutrosophic fuzzy graph N_G shown in Fig.15.

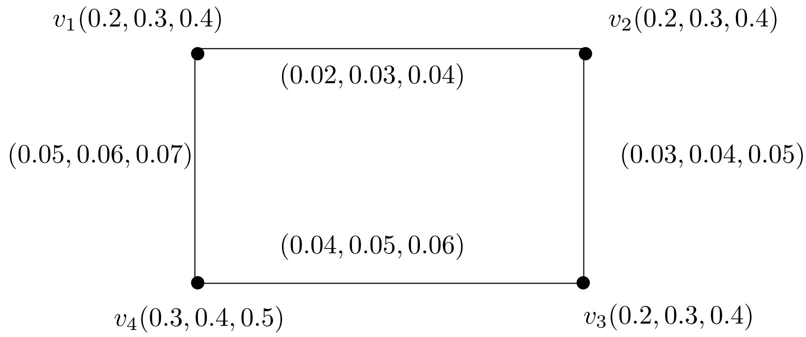

Fig. 15

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.07, 0.09, 0.11),\ d_{N_G}(v_2) = (0.05, 0.07, 0.09), \\ d_{N_G}(v_3) &= (0.07, 0.09, 0.11),\ d_{N_G}(v_4) = (0.09, 0.11, 0.13), \\ d_{N_G}[v_1] &= (0.27, 0.39, 0.51),\ d_{N_G}[v_2] = (0.25, 0.37, 0.49), \\ d_{N_G}[v_3] &= (0.27, 0.39, 0.51),\ d_{N_G}[v_4] = (0.39, 0.51, 0.63). \end{aligned}$$

Here every two adjacent vertices having distinct closed neighbourhood degrees but a vertex v_2 adjacent to the vertices v_1 and v_3 having same closed neighbourhood degree. Hence N_G is neighbourly totally irregular neutrosophic fuzzy graph. But N_G is not highly totally irregular neutrosophic fuzzy graph.

Observation 6.15 Every highly irregular neutrosophic fuzzy graph need not be a neighbourly irregular neutrosophic fuzzy graph.

Example 6.16 Consider the neutrosophic fuzzy graph N_G shown in Fig.16.

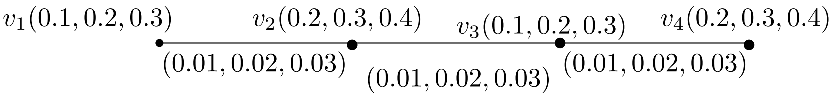

Fig.16

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.01, 0.02, 0.03),\ d_{N_G}(v_2) = (0.02, 0.04, 0.06), \\ d_{N_G}(v_3) &= (0.02, 0.04, 0.06),\ d_{N_G}(v_4) = (0.01, 0.02, 0.03). \end{aligned}$$

Here, every vertex is adjacent to the vertices with distinct degrees but the two adjacent vertices v_2 and v_3 have same degree. Hence N_G is highly irregular neutrosophic fuzzy graph. But N_G is not neighbourly irregular neutrosophic fuzzy graph.

Observation 6.17 Every highly totally irregular neutrosophic fuzzy graph need not be a neighbourly totally irregular neutrosophic fuzzy graph.

Example 6.18 Consider the neutrosophic fuzzy graph N_G shown in Fig.17.

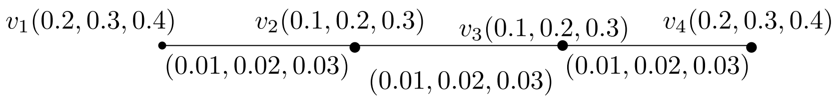

Fig.17

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.01, 0.02, 0.03),\ d_{N_G}(v_2) = (0.02, 0.04, 0.06), \\ d_{N_G}(v_3) &= (0.02, 0.04, 0.06),\ d_{N_G}(v_4) = (0.01, 0.02, 0.03), \\ d_{N_G}[v_1] &= (0.21, 0.32, 0.43),\ d_{N_G}[v_2] = (0.12, 0.24, 0.36), \\ d_{N_G}[v_3] &= (0.12, 0.24, 0.36),\ d_{N_G}[v_1] = (0.21, 0.32, 0.43). \end{aligned}$$

Here, every vertex is adjacent to the vertices with distinct closed neighborhood degrees but the two adjacent vertices v_2 and v_3 have same closed neighbourhood degree. Hence N_G is highly totally irregular neutrosophic fuzzy graph. But N_G is not neighbourly totally irregular neutrosophic fuzzy graph.

Theorem 6.19 *Let N_G be a neutrosophic fuzzy graph. If N_G is neighbourly totally irregular neutrosophic fuzzy graph and (T_A, I_A, F_A) is a constant function. Then N_G is neighbourly irregular neutrosophic fuzzy graph.*

Proof Let N_G be a neighbourly totally irregular neutrosophic fuzzy graph. Then by definition, the closed neighbourhood degree of every two adjacent vertices are distinct. Let

$v_i, v_j \in V$, where v_i, v_j are adjacent vertices with distinct closed neighbourhood degrees. To prove this N_G is neighbourly irregular neutrosophic fuzzy graph. Suppose we assume that

$$(T_A(v_i), I_A(v_i), F_A(v_i)) = (T_A(v_j), I_A(v_j), \quad F_A(v_j)) = (c_1, c_2, c_3),$$

where c_1, c_2, c_3 are constants and $c_1, c_2, c_3 \in [0, 1]$ and $deg[v_i] \neq deg[v_j]$. We show that $deg(v_i) \neq deg(v_j)$. Since $deg[v_i] \neq deg[v_j]$, we have

$$deg_T[v_i] \neq deg_T[v_j],\ deg_I[v_i] \neq deg_I[v_j],\ deg_F[v_i] \neq deg_F[v_j].$$

Now take

$$\begin{aligned} deg_T[v_i] \neq deg_T[v_j] &\implies deg_T(v_i) + c_1 \neq deg_T(v_j) + c_1 \\ &\implies deg_T(v_i) - deg_T(v_j) \neq c_1 - c_1 = 0 \\ &\implies deg_T(v_i) \neq deg_T(v_j). \end{aligned}$$

Similarly, $deg_I(v_i) \neq deg_I(v_j)$ and $deg_F(v_i) \neq deg_F(v_j)$. Therefore, the degree of $v_i, v_j \in V$ are distinct. This result is true for all pair of adjacent vertices in N_G. Hence N_G is neighbourly irregular neutrosophic fuzzy graph. □

Observation 6.20 Let N_G be a neutrosophic fuzzy graph, where N_G is a cycle with (T_B, I_B, F_B) is a constant function, Then N_G is neighbourly and highly regular neutrosophic fuzzy graph.

Example 6.21 Let N_G be the neutrosophic fuzzy graph shown in Fig.18. Then, we know that

$$\begin{aligned} d_{N_G}(v_1) &= (0.02, 0.04, 0.06),\ d_{N_G}(v_2) = (0.02, 0.04, 0.06), \\ d_{N_G}(v_3) &= (0.02, 0.04, 0.06). \end{aligned}$$

Here, every two adjacent vertices having same degree and every vertex adjacent to the vertices having same degree. Hence N_G is neighbourly and highly regular neutrosophic fuzzy graph.

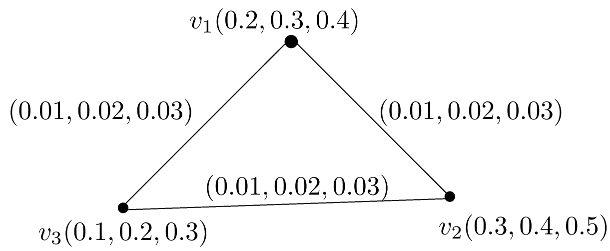

Fig.18

Observation 6.22 Let N_G be a neutrosophic fuzzy graph, where N_G is a cycle with (T_A, I_A, F_A) and (T_B, I_B, F_B) is a constant function, Then N_G is neighbourly and highly regular neutrosophic fuzzy graph.

Example 6.23 Consider the neutrosophic fuzzy graph N_G shown in Fig.19.

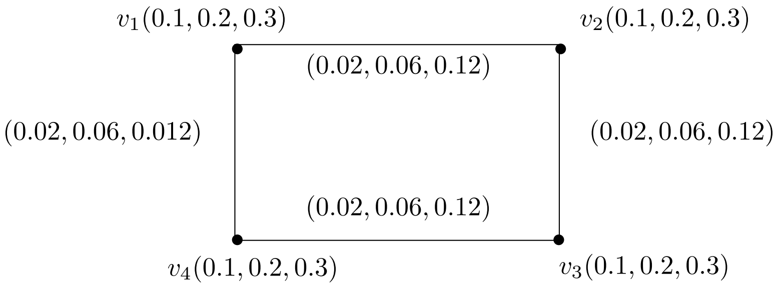

Fig. 19

In this graph,

$$\begin{aligned} d_{N_G}(v_1) &= (0.04, 0.12, 0.24),\ d_{N_G}(v_2) = (0.04, 0.12, 0.24), \\ d_{N_G}(v_3) &= (0.04, 0.12, 0.24),\ d_{N_G}(v_4) = (0.04, 0.12, 0.24), \\ d_{N_G}[v_1] &= (0.14, 0.22, 0.54),\ d_{N_G}[v_2] = (0.14, 0.22, 0.54), \\ d_{N_G}[v_3] &= (0.14, 0.22, 0.54),\ d_{N_G}[v_4] = (0.14, 0.22, 0.54). \end{aligned}$$

Here, every two adjacent vertices having same closed neighbourhood degree and every vertex adjacent to the vertices having same closed neighbourhood degree. Hence N_G is neighbourly and highly regular neutrosophic fuzzy graph.

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