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New Condition
Related with Higher Triple Centralizers in Semiprime Rings

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Abstract: The main objective of this work is shown that: if R is a 2-torsion free semiprime ring, consider that $T = (t_i)_{i\in\mathbb{N}}$ a family of additive mappings of R onto R, such that $2t_n(xyz) = \sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}(x) + t_{i-1}(x)t_{i-1}(y)t_i(x)$, holds for all $x, y \in R$ and $n \in \mathbb{N}$, then T is Jordan higher centralizer of R.

Key Words: Semiprime rings, higher centralizer, higher triple centralizer, Jordan higher triple centralizer.

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§1. Introduction

Let R be a ring. An additive mapping $T : R \to R$ is called a left (right) centralizer of R if $T(xy) = T(x)y\,(T(xy) = xT(y))$,is called a left (right) Jordan centralizer of R in case $T\left(x^2\right) = T(x)x(T\left(x^2\right) = xT(x))$, holds for all $x, y \in R$. We follow Zalar [7] and call T a centralizer in case T is both a left and a right centralizer. Vukman [5] proved that if $T : R \to R$ is an additive mapping such that $2T\left(x^2\right) = T(x)x + xT(x)$ holds for all $x \in R$ then T is a centralizer. In [1] and [2] A.M. Ibraheem and S.M. Salih presented and studied the concepts of higher triple left (resp. right) centralizer and Jordan higher triple left (resp. right) centralizer of rings and Γ-ring, and in [3] they provide that: if M is a 2-torsion free semiprime Γ-ring satisfying the assumption $p\alpha r\beta q = p\beta r\alpha q$, for all $p, r, q \in M$ and $\alpha, \beta \in \Gamma$, $F \in (f_i)_{i\in\mathbb{N}}$ is a family of additive mapping associated with a Jordan higher triple centralizer $T = (t_i)_{i\in\mathbb{N}}$ of M such that

$$2f_n(p\alpha r\beta q) = \sum_{i=1}^{n} f_i(p)\alpha t_{i-1}(r)\beta t_{i-1}(p) + t_{i-1}(p)\alpha t_{i-1}(pr)f_i(p)$$

hold for all $p, r \in M$ and $\alpha, \beta \in \Gamma$, and $n \in \mathbb{N}$ of M, then F is Jordan generalized higher triple centralizer of M. In this paper we prove that: if R is a 2-torsion free semiprime ring and $T \in (t_i)_{i\in\mathbb{N}}$ a family of additive mappings of R onto R, such that $2t_n(xyz) = \sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}(x) + t_{i-1}(x)t_{i-1}(y)t_i(x)$, holds for all $x, y \in R$ and $n \in \mathbb{N}$, then T is Jordan higher centralizer of R.

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§2. Main Results

For proving our main results, we need the following Lemmas:

Lemma 2.1([6]) *Let R be a 2-torsion free semiprime ring. Suppose that the identity $axb+bxc$ holds for all $x \in R$ and for some $a, b, c \in R$, then $(a+c)xb = 0$ for all $x \in R$.*

Lemma 2.2 *Let R be a semiprime ring, and $T = (t_i)_{i\in\mathbb{N}}$ be a family of an additive mappings of R onto R, such that*

$$2t_n(xyx) = \sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}(x) + t_{i-1}(x)t_{i-1}(y)t_i(x)$$

for all $x, y \in R$ and $n \in \mathbb{N}$, then

(i) $2t_n(xyz+zyx) = \sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}(z) + t_i(z)t_{i-1}(y)t_{i-1}(x) + t_{i-1}(x)t_{i-1}(y)t_i(z) + t_{i-1}(z)t_{i-1}(y)t_i(x)$*;*

(ii) $\left(t_n\left(x^2-\right)\sum_{i=1}^{n} t_i(x)t_{i-1}(x)\right)t_{i-1}(y)t_{i-1}(x)+t_{i-1}(x)t_{i-1}(y)\left(t_n\left(x^2\right)\sum_{i=1}^{n} t_{i-1}(x)t_i(x)\right) = 0$*;*

(iii) If R is a 2-torsion free ring, then

$$2t_n\left(x^2\right) = \sum_{i=1}^{n} t_i(x)t_{i-1}(x) + t_{i-1}(x)t_i(x);$$

(iv) In particular if R is a 2-torsion free commutative ring, then

$$2t_n(xyz) = \sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}(z) + t_{i-1}(x)t_{i-1}(y)t_i(z).$$

Proof (*i*) Linearizing the hypothesis on x, we get

$$\begin{aligned}2t_n\left((x+z)y(x+z)\right) &= \sum_{i=1}^{n} t_i(x+z)t_{i-1}(y)t_{i-1}(x+z) + t_{i-1}(x+z)t_{i-1}(y)t_i(x+z),\\ &= \sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}(x) + t_i(x)t_{i-1}(y)t_{i-1}(z) + t_i(z)t_{i-1}(y)t_{i-1}(x)\\ &\quad + t_i(z)t_{i-1}(y)t_{i-1}(z) + t_{i-1}(x)t_{i-1}(y)t_i(x) + t_{i-1}(x)t_{i-1}(y)t_i(z)\\ &\quad + t_{i-1}(z)t_{i-1}(y)t_i(x) + t_{i-1}(z)t_{i-1}(y)t_i(z)\cdots \qquad (1)\end{aligned}$$

On the other hand

$$\begin{aligned}2t_n\left((x+z)y(x+z)\right) &= 2t_n\left(xyx+xyz+zyx+zyz\right)\\ &= 2t_n(xyx)+2t_n(zyz)+2t_n(xyz+zyx). \qquad (2)\end{aligned}$$

Comparing (1), (2) and with the hypothesis, we get

$$\begin{aligned} 2t_n\left((x+z)y(x+z)\right) &= \sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}(z) \\ &\quad + t_i(z)t_{i-1}(y)t_{i-1}(x)+t_{i-1}(x)t_{i-1}(y)t_i(z)+t_{i-1}(z)t_{i-1}(y)t_i(x). \end{aligned}$$

(ii) Putting x^2 for z in part (i), we get

$$\begin{aligned} 2t_n\left(x^2yx+xyx^2\right) &= \sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}\left(x^2\right)+t_i\left(x^2\right)t_{i-1}(y)t_{i-1}\left(x\right) \\ &\quad + t_{i-1}(x)t_{i-1}(y)t_{i-1}\left(x^2\right)+t_{i-1}\left(x^2\right)t_{i-1}(y)t_{i-1}\left(x\right)\ldots \qquad (3) \end{aligned}$$

Replacing $t_{i-1}\left(x^2\right)$ by $t_{i-1}(x)t_{i-1}(x)$ in (3), we get

$$\begin{aligned} 2t_n\left(x^2yx+xyx^2\right) &= \sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}t_{i-1}(x)t_{i-1}(x)+t_{i-1}(x)t_{i-1}(x)t_{i-1}(y)t_{i-1}\left(x\right) \\ &\quad + t_{i-1}(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x)+t_{i-1}(x)t_{i-1}(x)t_{i-1}(y)t_{i-1}\left(x\right)\cdots \qquad (4) \end{aligned}$$

Replacing y by $yx+xy$ in the hypothesis, we get

$$\begin{aligned} 2t_n\left(x(yx+xy)x\right) &= \sum_{i=1}^{n} t_i(x)t_{i-1}(yx)t_{i-1}(x)+t_i(x)t_{i-1}(xy)t_{i-1}(x) \\ &\quad + t_{i-1}(x)t_{i-1}\left(yx\right)t_i(x)+t_{i-1}(x)t_{i-1}(yx)t_i(x)\ldots \qquad (5) \end{aligned}$$

Replacing $t_{i-1}(yx)$ in (5) by $t_{i-1}(x)t_{i-1}(y)$, and $t_{i-1}(xy)$ by $t_{i-1}(y)t_{i-1}(x)$, we get

$$\begin{aligned} 2t_n\left(x^2yx+xyx^2\right) &= \sum_{i=1}^{n} t_i(x)t_{i-1}(x)t_{i-1}(y)t_{i-1}(x)+t_i(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x) \\ &\quad + t_{i-1}(x)t_{i-1}(y)t_{i-1}(x)t_i(x)+t_{i-1}(x)t_{i-1}(x)t_{i-1}(y)t_i(x)\ldots \qquad (6) \end{aligned}$$

Comparing (4) and (6), we get

$$\left(t_n\left(x^2\right)-\sum_{i=1}^{n} t_i(x)t_{i-1}(x)\right)t_{i-1}(y)t_{i-1}(x)+t_{i-1}(x)t_{i-1}(y)\left(t_n\left(x^2\right)\sum_{i=1}^{n} t_{i-1}(x)t_i(x)\right)=0.$$

(iii) Taking $a=t_n\left(x^2\right)-\sum_{i=1}^{n} t_i(x)t_{i-1}(x)$, $b=t_n\left(x^2\right)-\sum_{i=1}^{n} t_{i-1}(x)t_i(x)$, $c=t_{i-1}(y)$ and $d=t_{i-1}(x)$ in (ii) we find that $acd+dcb=0$ for all $c\in R$. Then, by Lemma 2.1, we have

$$\left(t_n\left(x^2\right)-\sum_{i=1}^{n} t_i(x)t_{i-1}(x)+t_n\left(x^2\right)-\sum_{i=1}^{n} t_{i-1}(x)t_i(x)\right)t_{i-1}(y)t_{i-1}(x)=0,$$

$$\left(2t_n\left(x^2\right)-\sum_{i=1}^{n} t_i(x)t_{i-1}(x)-\sum_{i=1}^{n} t_{i-1}(x)t_i(x)\right)t_{i-1}(y)t_{i-1}(x)=0 \qquad (7)$$

for all $x, y \in R$ and $n \in \mathbb{N}$. Let

$$A_n(x) = \left(t_n\left(x^2\right) - \sum_{i=1}^{n} t_i(x)t_{i-1}(x) + t_n\left(x^2\right) - \sum_{i=1}^{n} t_{i-1}(x)t_i(x) \right),$$

then (7) becomes

$$A_n(x)t_{n-1}(y)t_{n-1}(x) = 0. \tag{8}$$

Replacing $t_{n-1}(y)$ by $t_{n-1}(x)t_{n-1}(y)A_n(x)$ in (8), we get

$$A_n(x)t_{n-1}(x)t_{n-1}(y)A_n(x)t_{n-1}(x) = 0$$

Since $t_{n-1}(y)$ is onto and R is semiprime, we have

$$A_n(x)t_{n-1}(x) = 0, \text{ for all } x \in R. \tag{9}$$

Left multiplying (8) by $t_{n-1}(x)$ and right multiplying by $A_n(x)$, we get

$$t_{n-1}(x)A_n(x)t_{n-1}(y)t_{n-1}(x)A_n(x) = 0.$$

Since $t_{n-1}(y)$ is onto and R is semiprime, we have

$$t_{n-1}(x)A_n(x) = 0, \text{ for all } x \in R. \tag{10}$$

Linearizing (9), we get

$$A_n(x+y)t_{n-1}(x+y) = 0, \text{ for all } x \in R$$

that leads to

$$\left(A_n(x) + A_n(y) + B_n(x,y)\right)\left(t_{n-1}(x) + t_{n-1}(y)\right),$$

where

$$\begin{aligned} B_n(x,y) &= 2t_n(xy+yx) - \sum_{i=1}^{n} t_i(x)t_{i-1}(y) \\ &\quad - \sum_{i=1}^{n} t_i(y)t_{i-1}(x) - \sum_{i=1}^{n} t_{i-1}(x)t_i(y) - \sum_{i=1}^{n} t_{i-1}(y)t_i(x) = 0. \end{aligned}$$

This yields that

$$\begin{aligned} &A_n(x)t_{n-1}(x) + A_n(y)t_{n-1}(x) + A_n(x)t_{n-1}(x) + B_n(x,y)t_{n-1}(x) \\ &\qquad + A_n(x)t_{n-1}(y) + A_n(y)t_{n-1}(y) + B_n(x,y)t_{n-1}(y) = 0. \end{aligned} \tag{11}$$

Using (9) in (11), we get

$$A_n(y)t_{n-1}(x) + A_n(x)t_{n-1}(y) + B_n(x,y)t_{n-1}(x) + B_n(x,y)t_{n-1}(y) = 0. \tag{12}$$

Replacing x by $-x$ in (12), we get

$$A_n(x)t_{n-1}(y) - A_n(y)t_{n-1}(x) + B_n(x,y)t_{n-1}(x) - B_n(x,y)t_{n-1}(y) = 0. \tag{13}$$

Comparing (13) with (12), and since R is 2-torsion free, we arrive at

$$A_n(x)t_{n-1}(y) + B_n(x,y)t_{n-1}(x) = 0. \tag{14}$$

Right multiplying (14) by $A_n(x)$ and using (10), we get

$$A_n(x)t_{n-1}(y)A_n(x) = 0.$$

Since $t_{n-1}(y)$ is onto and R is semiprime, we have $A_n(x) = 0$ for all $x \in R$. That is,

$$2t_n\left(x^2\right) = \sum_{i=1}^{n} t_i(x)t_{i-1}(x) + t_{i-1}(x)t_i(x).$$

(iv) By using (i), and since R is commutative, we have

$$2t_n(xyz + xyz) = 2(2t_n(xyz)) = 2\left(\sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}(z) + t_{i-1}(x)t_{i-1}(y)t_i(z)\right)$$

for all $x, y \in R$ and $n \in \mathbb{N}$. Since R is a 2-torsion free we get the require result. □

Theorem 2.3 *Let R be a 2-torsion free semiprime ring, and $T = (t_i)_{i\in\mathbb{N}}$ be a family of additive mappings of R onto R such that*

$$2t_n(xyz) = \sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}(x) + t_{i-1}(x)t_{i-1}(y)t_i(x)$$

holds for all $x, y \in R$ and $n \in \mathbb{N}$, then T is Jordan higher centralizer of R.

Proof Taking *(iii)* of Lemma 2.2 and linearizing it, we have

$$\begin{aligned} 2t_n((x+y)(x+y)) &= 2t_n\left(x^2\right) + 2t_n(xy+yx) + 2t_n\left(y^2\right) \\ &= \sum_{i=1}^{n} t_i(x)t_{i-1}(x) + t_i(y)t_{i-1}(y) + t_i(x)t_{i-1}(y) + t_i(y)t_{i-1}(x) \\ &\quad + t_{i-1}(x)t_i(x) + t_{i-1}(y)t_i(y) + t_{i-1}(x)t_i(y) + t_{i-1}(y)t_i(x) \end{aligned}$$

By applying *(iii)*, we get

$$2t_n(xy+yx) = \sum_{i=1}^{n} t_i(x)t_{i-1}(y) + t_i(y)t_{i-1}(x) + t_{i-1}(x)t_i(y) + t_{i-1}(y)t_i(x) \tag{15}$$

for all $x, y \in R$. Replacing y by $2xyx$ in (15), we get

$$\begin{aligned} 2(2t_n\left(x(xyx)+xyx)x\right)) &= \sum_{i=1}^{n} t_i(x)t_{i-1}(xyx) \\ &\quad + t_i(xyx)t_{i-1}(x) + t_{i-1}(x)t_i(xyx) + t_{i-1}(xyx)t_i(x). \end{aligned} \tag{16}$$

Now, replacing $t_{i-1}(xyx)$ by $t_{i-1}(x)t_{i-1}(y)t_{i-1}(x)$ in (16) and applying the condition of the theorem, we ge

$$\begin{aligned} 4t_n\left(x^2yx+xyx^2\right) = 2\sum_{i=1}^{n} & t_i(x)t_{i-1}(x)t_{i-1}(y)t_{i-1}(x) \\ & + t_i(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x) + t_{i-1}(x)t_{i-1}(y)t_{i-1}(x)t_i(x) \\ & + t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x) + t_{i-1}(x)t_{i-1}(x)t_{i-1}(y)t_{i-1}(x) \\ & + 2t_{i-1}(x)t_{i-1}(y)t_{i-1}(x)t_i(x). \end{aligned} \tag{17}$$

Comparing (17) with equation (6), we have

$$\begin{aligned} \sum_{i=1}^{n} & t_i(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x) + t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x) \\ & - t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x) - t_{i-1}(x)t_i(x)t_{i-1}(y)t_i(x) = 0. \end{aligned} \tag{18}$$

Replacing $t_{i-1}(y)$ by $t_{i-1}(y)t_{i-1}(x)$ in (18), we get

$$\begin{aligned} \sum_{i=1}^{n} & t_i(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x)t_{i-1}(x) + t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x) \\ & - t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x) - t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x)t_i(x) = 0. \end{aligned} \tag{19}$$

Right multiplying (18) by $t_{i-1}(x)$, we get

$$\begin{aligned} \sum_{i=1}^{n} & t_i(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x)t_{i-1}(x) + t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x) \\ & - t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x) - t_{i-1}(x)t_i(x)t_{i-1}(y)t_i(x)t_{i-1}(x) = 0. \end{aligned} \tag{20}$$

Subtracting (19) from (20), we get

$$\sum_{i=1}^{n} t_{i-1}(x)t_{i-1}(x)t_{i-1}(y)\left[t_i(x), t_{i-1}(x)\right] - t_{i-1}(x)t_{i-1}(y)\left[t_i(x), t_{i-1}(x)\right]t_{i-1}(x) = 0 \tag{21}$$

for all $x, y \in R$. Replacing $t_{i-1}(y)$ by $t_i(y)t_{i-1}(y)$ in (21), we get

$$\sum_{i=1}^{n} t_{i-1}(x)t_{i-1}(x)t_i(y)t_{i-1}(y)\left[t_i(x), t_{i-1}(x)\right] - t_{i-1}(x)t_i(y)t_{i-1}(y)\left[t_i(x), t_{i-1}(x)\right]t_{i-1}(x) = 0. \tag{22}$$

Left multiplying (21) by t_i, we get

$$\sum_{i=1}^{n} t_i(x)t_{i-1}(x)t_{i-1}(x)t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]-t_i(x)t_{i-1}(x)t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(x)=0. \tag{23}$$

Subtracting (22) from (23), we get

$$\begin{aligned}\sum_{i=1}^{n} &\left[t_i(x),t_{i-1}(x)t_{i-1}(x)\right]t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]\\ &-\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(x)=0.\end{aligned} \tag{24}$$

Now, in (24) if we take

$$\begin{aligned} a &= \left[t_i(x),t_{i-1}(x)t_{i-1}(x)\right],\\ b &= \left[t_i(x),t_{i-1}(x)\right],\\ c &= \left[t_i(x),t_{i-1}(x)\right]t_{i-1}(x)\end{aligned}$$

and $d=t_{i-1}(x)$. Hence, we have $adb+bdc=0$ implies that $(a+c)db=0$, by Lemma 2.1 and so we get

$$\sum_{i=1}^{n}\left(\left[t_i(x),t_{i-1}(x)t_{i-1}(x)\right]-\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(x)\right)t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]=0$$

This implies that,

$$\sum_{i=1}^{n} t_{i-1}(x)\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]=0 \tag{25}$$

for all $x,y\in R$. Replacing $t_{i-1}(y)$ by $t_{i-1}(y)t_{i-1}(x)$ in (25), we get

$$\sum_{i=1}^{n} t_{i-1}(x)\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)t_{i-1}(x)\left[t_i(x),t_{i-1}(x)\right]=0$$

for all $x,y\in R$. Since $t_{i-1}(y)$ is onto, and R is semiprime ring, we have

$$\sum_{i=1}^{n} t_{i-1}(x)\left[t_i(x),t_{i-1}(x)\right]=0 \tag{26}$$

for all $x,y\in R$. Replacing $t_{i-1}(y)$ by $t_{i-1}(x)t_{i-1}(y)$ in (18), we get

$$\begin{aligned}\sum_{i=1}^{n} &t_i(x)t_{i-1}(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x)+t_{i-1}(x)t_i(x)t_{i-1}(x)t_{i-1}(y)t_{i-1}(x)\\ &-t_{i-1}(x)t_i(x)t_{i-1}(x)t_{i-1}(y)t_{i-1}(x)-t_{i-1}(x)t_i(x)t_{i-1}(x)t_{i-1}(y)t_i(x)=0.\end{aligned} \tag{27}$$

Left multiplying (18) by $t_{i-1}(x)$, we get

$$\begin{aligned}\sum_{i=1}^{n} & t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x)t_{i-1}(x)+t_{i-1}(x)t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x)\\ & -t_{i-1}(x)t_{i-1}(x)t_i(x)t_{i-1}(y)t_{i-1}(x)-t_{i-1}(x)t_{i-1}(x)t_i(x)t_{i-1}(y)t_i(x)=0. \qquad (28)\end{aligned}$$

Subtracting (27) from (28), we get

$$\sum_{i=1}^{n}\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)t_{i-1}(x)t_{i-1}(x)-t_{i-1}(x)\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)t_{i-1}(x)=0, \qquad (29)$$

for all $x,y\in R$. Using (26) in (29), we get

$$\sum_{i=1}^{n}\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)t_{i-1}(x)t_{i-1}(x)=0. \qquad (30)$$

Replacing $t_{i-1}(y)$ by $t_{i-1}(y)t_i(x)$ in (30), we get

$$\sum_{i=1}^{n}\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)t_i(x)t_{i-1}(x)t_{i-1}(x)=0. \qquad (31)$$

Right multiplying (30) by $t_i(x)$, we get

$$\sum_{i=1}^{n}\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)t_{i-1}(x)t_{i-1}(x)t_i(x)=0. \qquad (32)$$

Subtracting (31) from (32), we get

$$\sum_{i=1}^{n}\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)\left[t_i(x),t_{i-1}(x)t_{i-1}(x)\right]=0. \qquad (33)$$

Now, we can rewritten (33) and using the relation (26), we get

$$\sum_{i=1}^{n}\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(x)=0. \qquad (34)$$

Replacing $t_{i-1}(y)$ by $t_{i-1}(x)t_{i-1}(y)$ in (34), we get

$$\sum_{i=1}^{n}\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(x)t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(x)=0.$$

Since $t_{i-1}(y)$ is onto, and R is semiprime ring, we have

$$\sum_{i=1}^{n}\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(x)=0,\ \text{for all}\ \ x,y\in R. \qquad (35)$$

Linearizing (26) and using (26) again, we have

$$\sum_{i=1}^{n} t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]+t_{i-1}(x)\left[t_i(y),t_{i-1}(x)\right]+t_{i-1}(y)\left[t_i(y),t_{i-1}(x)\right]$$
$$+\,t_{i-1}(x)\left[t_i(x),t_{i-1}(y)\right]+t_{i-1}(y)\left[t_i(x),t_{i-1}(y)\right]+t_{i-1}(x)\left[t_i(y),t_{i-1}(y)\right]=0. \tag{36}$$

Replacing x by $-x$ in (36), we get

$$\sum_{i=1}^{n} t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]+t_{i-1}(x)\left[t_i(y),t_{i-1}(x)\right]+t_{i-1}(x)\left[t_i(x),t_{i-1}(y)\right]$$
$$=t_{i-1}(y)\left[t_i(y),t_{i-1}(x)\right]+t_{i-1}(y)\left[t_i(x),t_{i-1}(y)\right]+t_{i-1}(x)\left[t_i(y),t_{i-1}(y)\right]=0. \tag{37}$$

Putting (37) in (36) and since R is 2-torsion free, we have

$$\sum_{i=1}^{n} t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]+t_{i-1}(x)\left[t_i(y),t_{i-1}(x)\right]+t_{i-1}(x)\left[t_i(x),t_{i-1}(y)\right]=0. \tag{38}$$

Left multiplying (38) by $[t_i(x),t_{i-1}(x)]$, and using (35), we get

$$\sum_{i=1}^{n}\left[t_i(x),t_{i-1}(x)\right]t_{i-1}(y)\left[t_i(x),t_{i-1}(x)\right]=0, \text{ for all } x,y\in R.$$

Since $t_{i-1}(y)$ is onto, and R is semiprime ring, we have

$$\sum_{i=1}^{n}\left[t_i(x),t_{i-1}(x)\right]=0. \tag{39}$$

Comparing (39) with part (iii) of Lemma 2.2, we get

$$t_n(x^2)=\sum_{i=1}^{n} t_i(x)t_{i-1}(x) \text{ and } t_n(x^2)=\sum_{i=1}^{n} t_{i-1}(x)t_i(x)$$

That is, T is a Jordan higher left and right centralizer of R, hold for all $x\in R$, and $n\in\mathbb{N}$. Therefore, T is Jordan higher centralizer of R. □

Proposition 2.4 *Let R be a 2-torsion free semiprime ring, and $T=(t_i)_i\in\mathbb{N}$ be a family of additive mappings of R onto R, such that*

$$2t_n(xyx)=\sum_{i=1}^{n} t_i(x)t_{i-1}(y)t_{i-1}(x)+t_{i-1}(x)t_{i-1}(y)t_i(x)$$

for all $x,y\in R$ and $n\in\mathbb{N}$,then T is Jordan higher triple centralizer of R.

Proof From [2] we have T is Jordan higher left and right centralizer of R, and by [4] we get, T is a Jordan higher triple left centralizer of R.

Similarly, we can prove that T is a Jordan higher triple right centralizer of R, Therefore, we have T is Jordan higher triple centralizer of R.

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