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On Mean Graphs

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Abstract: Let $G(V,E)$ be a graph with p vertices and q edges. For every assignment $f : V(G) \rightarrow \{0,1,2,3,\ldots,q\}$, an induced edge labeling $f^* : E(G) \rightarrow \{1,2,3,\ldots,q\}$ is defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u) \text{ and } f(v) \text{ are of the same parity} \\ \dfrac{f(u)+f(v)+1}{2} & \text{otherwise} \end{cases}$$

for every edge $uv \in E(G)$. If $f^*(E) = \{1,2,\ldots,q\}$, then we say that f is a mean labeling of G. If a graph G admits a mean labeling, then G is called a mean graph. In this paper, we prove that the graphs double sided step ladder graph $2S(T_m)$, Jelly fish graph $J(m,n)$ for $|m-n| \leq 2$, $P_n(+)N_m$, $(P_2 \cup kK_1) + N_2$ for $k \geq 1$, the triangular belt graph $TB(\alpha)$, $TBL(n,\alpha,k,\beta)$, the edge mC_n- snake, $m \geq 1, n \geq 3$ and $S_t(B(m)_{(n)})$ are mean graphs. Also we prove that the graph obtained by identifying an edge of two cycles C_m and C_n is a mean graph for $m,n \geq 3$.

Key Words: Smarandachely edge 2-labeling, mean graph, mean labeling, Jelly fish graph, triangular belt graph.

AMS(2010): 05C78

§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected, simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology we follow [1].

Path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. $K_{1,m}$ is called a star and it is denoted by S_m. The bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the center vertices of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively. $B_{m,m}$ is often denoted by $B(m)$. The join of two graphs G and H is the graph obtained from $G \cup H$ by joining each vertex of G with each vertex of H by means of an edge and it is denoted by $G + H$. The edge mC_n- snake is a graph obtained from m copies of C_n by identifying the edge $v_{k+1}v_{k+2}$ in each copy of C_n, n is either $2k+1$ or $2k$ with the edge v_1v_2 in the successive

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copy of C_n. The graph $P_n \times P_2$ is called a ladder. Let P_{2n} be a path of length $2n-1$ with $2n$ vertices $(1,1),(1,2),\ldots,(1,2n)$ with $2n-1$ edges $e_1, e_2, \ldots, e_{2n-1}$ where e_i is the edge joining the vertices $(1,i)$ and $(1,i+1)$. On each edge e_i, for $i = 1,2,\ldots,n$, we erect a ladder with $i+1$ steps including the edge e_i and on each edge e_i, for $i = n+1, n+2, \ldots, 2n-1$, we erect a ladder with $2n+1-i$ steps including the edge e_i. The resultant graph is called double sided step ladder graph and is denoted by $2S(T_m)$, where $m = 2n$ denotes the number of vertices in the base.

A vertex labeling of G is an assignment $f : V(G) \to \{0,1,2,\ldots,q\}$. For a vertex labeling f, the induced edge labeling f^* is defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u) \text{ and } f(v) \text{ are of the same parity} \\ \dfrac{f(u)+f(v)+1}{2} & \text{otherwise} \end{cases}$$

A vertex labeling f is called a mean labeling of G if its induced edge labeling $f^* : E(G) \to \{1,2,\ldots,q\}$ is a bijection, that is, $f^*(E) = \{1,2,\ldots,q\}$. If a graph G has a mean labeling, then we say that G is a mean graph. It is clear that a mean labeling is a Smarandachely edge 2-labeling of G.

A mean labeling of the Petersen graph is shown in Figure 1.

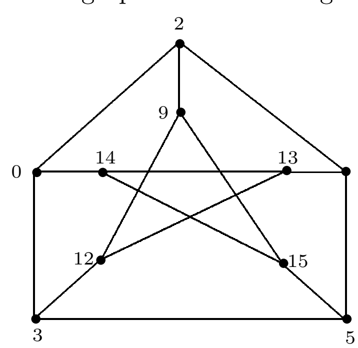

Figure 1

The concept of mean labeling was introduced and studied by S.Somasundaram and R.Ponraj [4]. Some new families of mean graphs are studied by S.K.Vaidya et al. [6], [7]. Further some more results on mean graphs are discussed in [2], [3], [5].

In this paper, we establish the meanness of the graphs double sided step ladder graph $2S(T_m)$, Jelly fish graph $J(m,n)$ for $|m-n| \leq 2$, $P_n(+)N_m$, $(P_2 \cup kK_1) + N_2$ for $k \geq 1$, the triangular belt graph $TB(\alpha)$, $TBL(n,\alpha,k,\beta)$, the edge mC_n-snake $m \geq 1, n \geq 3$ and $S_t(B(m)_{(n)})$. Also we prove that the graph obtained by identifying an edge of two cycles C_m and C_n is a mean graph for $m,n \geq 3$.

§2. Mean Graphs

Theorem 2.1 *The double sided step ladder graph $2S(T_m)$ is a mean graph where $m = 2n$ denotes the number of vertices in the base.*

Proof Let P_{2n} be a path of length $2n-1$ with $2n$ vertices $(1,1),(1,2),\cdots,(1,2n)$ with $2n-1$ edges, $e_1,e_2,\cdots,e_{2n-1}$ where e_i is the edge joining the vertices $(1,i)$ and $(1,i+1)$. On each edge e_i, for $i=1,2,\cdots,n$, we erect a ladder with $i+1$ steps including the edge e_i and on each edge e_i, for $i=n+1,n+2,\cdots,2n-1$, we erect a ladder with $2n+1-i$ steps including the edge e_i.

The double sided step ladder graph $2S(T_m)$ has vertices denoted by $(1,1),(1,2),\ldots,(1,2n)$, $(2,1),(2,2),\cdots,(2,2n),(3,2),(3,3),\cdots,(3,2n-1),(4,3),(4,4),\cdots,(4,2n-2),\cdots,(n+1,n),(n+1,n+1)$. In the ordered pair (i,j), i denotes the row (counted from bottom to top) and j denotes the column (from left to right) in which the vertex occurs. Define $f: V(2S(T_m)) \to \{0,1,2,\ldots,q\}$ as follows:

$$f(i,j)=(n+1-i)(2n-2i+3)+j-1, \quad 1\le j\le 2n, i=1,2$$
$$f(i,j)=(n+1-i)(2n-2i+3)+j+1-i, \quad i-1\le j\le 2n+2-i, 3\le i\le n+1.$$

Then, f is a mean labeling for the double sided step ladder graph $2S(T_m)$. Thus $2S(T_m)$ is a mean graph. □

For example, a mean labeling of $2S(T_{10})$ is shown in Figure 2.

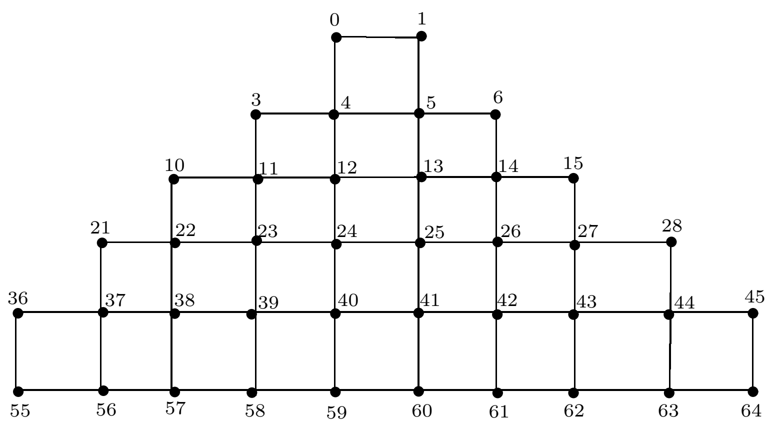

Figure 2

For integers $m,n\ge 0$ we consider the graph $J(m,n)$ with vertex set $V(J(m,n)) = \{u,v,x,y\}\cup\{x_1,x_2,c\ldots,x_m\}\cup\{y_1,y_2,\cdots,y_n\}$ and edge set $E(J(m,n))=\{(u,x),(u,v),(u,y),(v,x),(v,y)\}\cup\{(x_i,x): i=1,2,\cdots,m\}\cup\{(y_i,y): i=1,2,\cdots,n\}$. We will refer to $J(m,n)$ as a Jelly fish graph.

Theorem 2.2 *A Jelly fish graph $J(m,n)$ is a mean graph for $m,n\ge 0$ and $|m-n|\le 2$.*

Proof The proof is divided into cases following.

Case 1 $m=n$.

Define a labeling $f : V(J(m,n)) \to \{0,1,2,\ldots,q=m+n+5\}$ as follows:

$$f(u)=2,\ f(y)=0,$$
$$f(v)=m+n+4,\ f(x)=m+n+5,$$
$$f(x_i)=4+2(i-1),\quad 1\le i\le m$$
$$f(y_{n+1-i})=3+2(i-1),\quad 1\le i\le n$$

Then f provides a mean labeling.

Case 2 $\quad m=n+1$ or $n+2$

Define $f : V(J(m,n)) \to \{0,1,2,\ldots,q=m+n+5\}$ as follows:

$$f(u)=2,\ f(v)=2n+4,\ f(y)=0,$$
$$f(x)=\begin{cases} m+n+5 & \text{if } m=n+1\\ m+n+4 & \text{if } m=n+2\end{cases}$$
$$f(x_i)=\begin{cases} 4+2(i-1), & 1\le i\le n\\ 2n+5+2(i-(n+1)), & n+1\le i\le m\end{cases}$$
$$f(y_{n+1-i})=3+2(i-1),\quad 1\le i\le n.$$

Then f gives a mean labeling. Thus $J(m,n)$ is a mean graph for $m,n\ge 0$ and $|m-n|\le 2$. □

For example, a mean labeling of $J(6,6)$ and $J(9,7)$ are shown in Figure 3.

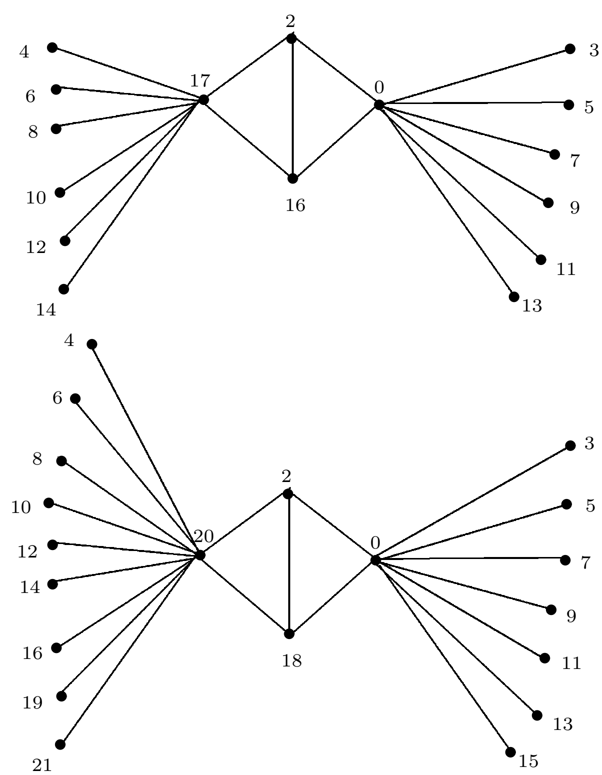

Figure 3

Let $P_n(+)N_m$ be the graph with $p = n + m$ and $q = 2m + n - 1$. $V(P_n(+)N_m) = \{v_1, v_2, \cdots, v_n, y_1, y_2, \cdots, y_m\}$, where $V(P_n) = \{v_1, v_2, \cdots, v_n\}$, $V(N_m) = \{y_1, y_2, \cdots, y_m\}$ and

$$E(P_n(+)N_m) = E(P_n) \bigcup \left\{ \begin{array}{l} (v_1, y_1), (v_1, y_2), \cdots, (v_1, y_m), \\ (v_n, y_1), (v_n, y_2), \cdots, (v_n, y_m). \end{array} \right\}$$

Theorem 2.3 *$P_n(+)N_m$ is a mean graph for all $n, m \geq 1$.*

Proof Let us define $f : V(P_n(+)N_m) \to \{1, 2, 3, \cdots, 2m + n - 1\}$ as follows:

$$\begin{aligned} f(y_i) &= 2i - 1, \ 1 \leq i \leq m, \\ f(v_1) &= 0, \\ f(v_i) &= 2m + 1 + 2(i - 2), \quad 2 \leq i \leq \left\lceil \frac{n+1}{2} \right\rceil \\ f(v_{n+1-i}) &= 2m + 2 + 2(i - 1), \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor. \end{aligned}$$

Then, f gives a mean labeling. Thus $P_n(+)N_m$ is a mean graph for $n, m \geq 1$. □

For example, a mean labeling of $P_8(+)N_5$ and $P_7(+)N_6$ are shown in Figure 4.

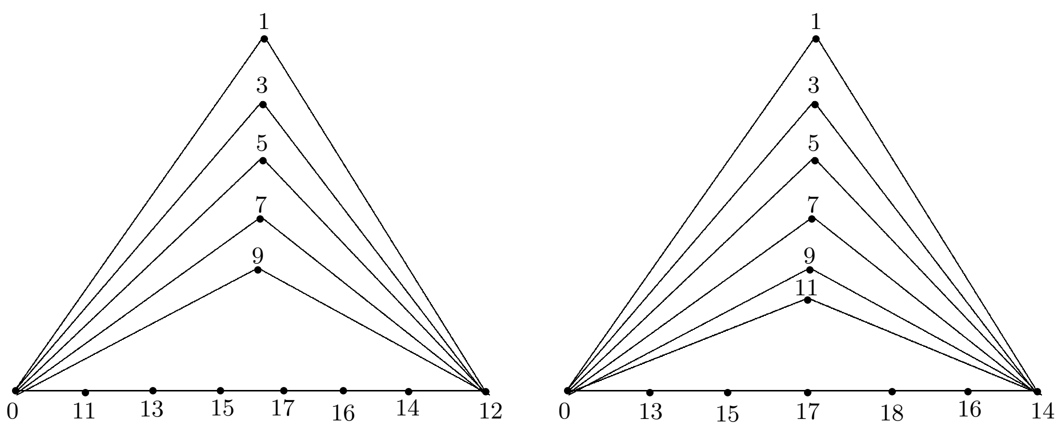

Figure 4

Theorem 2.4 *For $k \geq 1$, the planar graph $(P_2 \cup kK_1) + N_2$ is a mean graph.*

Proof Let the vertex set of $P_2 \cup kK_1$ be $\{z_1, z_2, x_1, x_2, \cdots, x_k\}$ and $V(N_2) = \{y_1, y_2\}$. We have $q = 2k + 5$. Define a labeling $f : V((P_2 \cup kK_1) + N_2) \to \{1, 2, \cdots, 2k + 5\}$ by

$$\begin{aligned} &f(y_1) = 0, \ f(y_2) = 2k + 5, \ f(z_1) = 2 \\ &f(z_2) = 2k + 4 \\ &f(x_i) = 4 + 2(i - 1), \quad 1 \leq i \leq k \end{aligned}$$

Then, f is a mean labeling and hence $(P_2 \cup kK_1) + N_2$ is a mean graph for $k \geq 1$. □

For example, a mean labeling of $(P_2 \cup 5K_1) + N_2$ is shown in Figure 5.

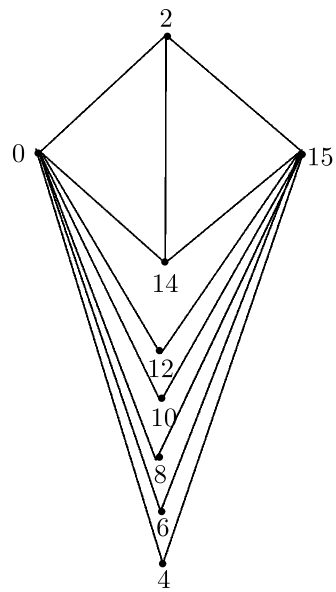

Figure 5

Let $S = \{\uparrow, \downarrow\}$ be the symbol representing, the position of the block as given in Figure 6.

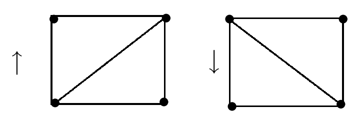

Figure 6

Let α be a sequence of n symbols of S, $\alpha \in S^n$. We will construct a graph by tiling n blocks side by side with their positions indicated by α. We will denote the resulting graph by $TB(\alpha)$ and refer to it as a triangular belt.

For example, the triangular belts corresponding to sequences $\alpha_1 = \{\downarrow\uparrow\uparrow\}$, $\alpha_2 = \{\downarrow\downarrow\uparrow\downarrow\}$ respectively are shown in Figure 7.

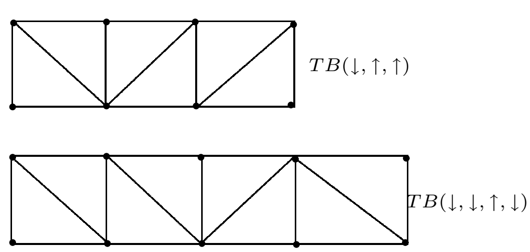

Figure 7

Theorem 2.5 *A triangular belt $TB(\alpha)$ is a mean graph for any α in S^n with the first and last block are being $\downarrow$ for all $n \geq 1$.*

Proof Let $u_1, u_2, \ldots, u_n, u_{n+1}$ be the top vertices of the belt and $v_1, v_2, \ldots, v_n, v_{n+1}$ be the bottom vertices of the belt. The graph $TB(\alpha)$ has $2n+2$ vertices and $4n+1$ edges. Define $f : V(TB(\alpha)) \rightarrow \{0, 1, 2, \ldots, q = 4n+1\}$ as follows :

$$f(u_i) = 4i, \quad 1 \leq i \leq n$$
$$f(u_{n+1}) = 4n+1$$
$$f(v_1) = 0$$
$$f(v_i) = 2 + 4(i-2), \quad 2 \leq i \leq n$$

Then f gives a mean labeling. Thus $TB(\alpha)$ is a mean graph for all $n \geq 1$. □

For example, a mean labeling of $TB(\alpha), TB(\beta)$ and $TB(\gamma)$ are shown in Figure 8.

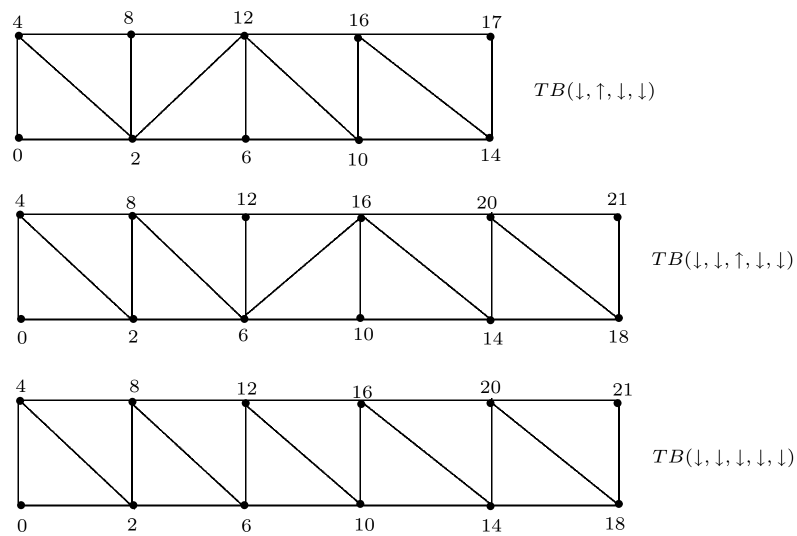

Figure 8

Corollary 2.6 *The graph P_n^2 is a mean graph.*

Proof The graph P_n^2 is isomorphic to $TB(\downarrow, \downarrow, \downarrow, \ldots, \downarrow)$ or $TB(\uparrow, \uparrow, \uparrow, \ldots, \uparrow)$. Hence the result follows from Theorem 2.5. □

We now consider a class of planar graphs that are formed by amalgamation of triangular belts. For each $n \geq 1$ and α in S^n n blocks with the first and last block are $\downarrow$ we take the triangular belt $TB(\alpha)$ and the triangular belt $TB(\beta)$, β in S^k where $k > 0$.

We rotate $TB(\beta)$ by 90 degrees counter clockwise and amalgamate the last block with the first block of $TB(\alpha)$ by sharing an edge. The resulting graph is denoted by $TBL(n, \alpha, k, \beta)$, which has $2(nk+1)$ vertices, $3(n+k)+1$ edges with

$$V(TBL(n, \alpha, k, \beta)) = \{u_{1,1}, u_{1,2}, \cdots, u_{1,n+1}, u_{2,1}, u_{2,2}, \cdots, u_{2,n+1}, v_{3,1}, v_{3,2}, \cdots, v_{3,k-1}, v_{4,1}, v_{4,2}, \cdots, v_{4,k-1}\}.$$

Theorem 2.7 *The graph $TBL(n,\alpha,k,\beta)$ is a mean graph for all α in S^n with the first and last block are $\downarrow$ and β in S^k for all $k>0$.*

Proof Define $f : V(TBL(n,\alpha,k,\beta)) \to \{0,1,2,\ldots,3(n+k)+1\}$ as follows:

$$f(u_{1,i}) = 4k+4i, \quad 1 \le i \le n$$
$$f(u_{1,n+1}) = 4(n+k)+1$$
$$f(u_{2,1}) = 4k$$
$$f(u_{2,i}) = 4k+2+4(i-2), \quad 2 \le i \le n+1$$
$$f(v_{3,i}) = 4i-4, \quad 1 \le i \le k$$
$$f(v_{4,i}) = 4i-2, \quad 1 \le i \le k$$

Then f provides a mean labeling and hence $TBL(n,\alpha,k,\beta)$ is a mean graph. □

For example, a mean labeling of $TBL(4,\downarrow,\uparrow,\uparrow,\downarrow,2,\uparrow,\uparrow)$ and $TBL(5,\downarrow,\uparrow,\downarrow,\uparrow,\downarrow,3,\uparrow,\downarrow,\uparrow)$ is shown in Figure 9.

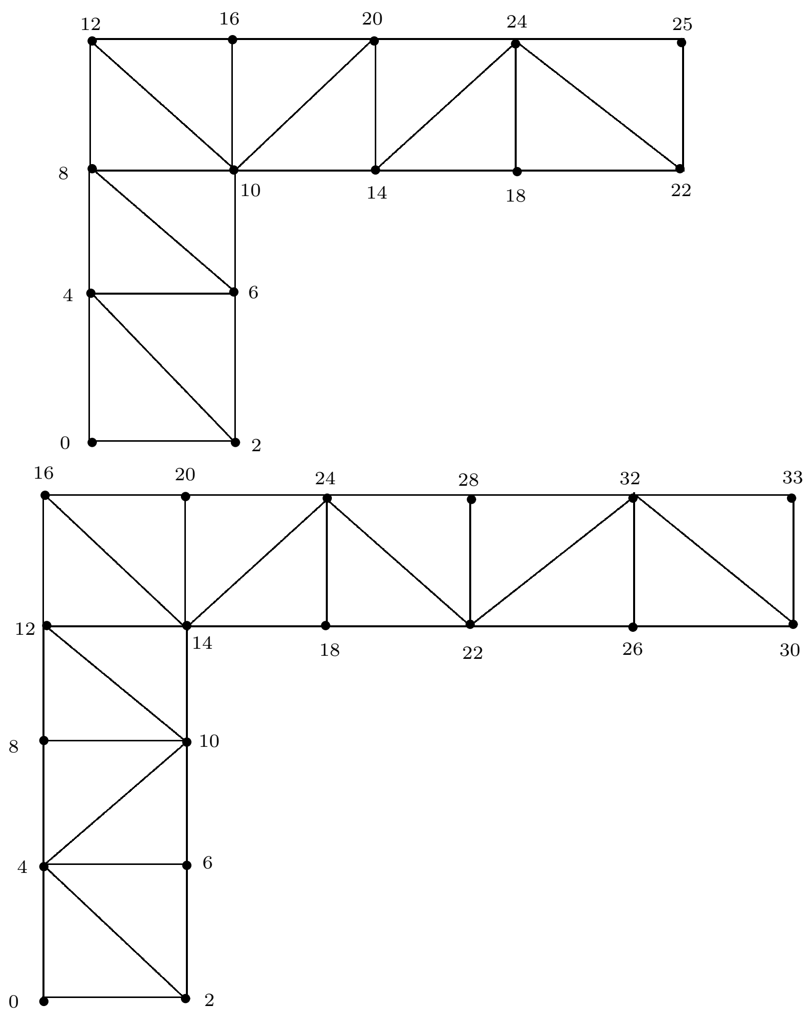

Figure 9

Theorem 2.8 *The graph edge mC_n-snake, $m \geq 1, n \geq 3$ has a mean labeling.*

Proof Let $v_{1_j}, v_{2_j}, \ldots, v_{n_j}$ be the vertices and $e_{1_j}, e_{2_j}, \ldots, e_{n_j}$ be the edges of edge mC_n-snake for $1 \leq j \leq m$.

Case 1 n is odd

Let $n = 2k+1$ for some $k \in Z^+$. Define a vertex labeling f of edge mC_n-snake as follows:

$$\begin{aligned}
f(v_{1_1}) &= 0,\ f(v_{2_1}) = 1\\
f(v_{i_1}) &= 2i-2, \quad 3 \leq i \leq k+1\\
f(v_{(k+1+i)_1}) &= n-2(i-1), \quad 1 \leq i \leq k\\
f(v_{1_2}) &= f(v_{(k+2)_1}),\ f(v_{2_2}) = f(v_{(k+1)_1}),\\
f(v_{i_2}) &= n+4+2(i-3), \quad 3 \leq i \leq k+1\\
f(v_{(k+1+i)_2}) &= 2n-2-2(i-1), \quad 1 \leq i \leq k-1\\
f(v_{n_2}) &= n+2\\
f(v_{i_j}) &= f(v_{i_{j-2}}) + 2n-2, \quad 3 \leq j \leq m, \quad 1 \leq i \leq n.
\end{aligned}$$

Then f gives a mean labeling.

Case 2 n is even

Let $n = 2k$ for some $k \in Z^+$. Define a labeling f of edge mC_n-snake as follows:

$$\begin{aligned}
f(v_{1_1}) &= 0,\ f(v_{2_1}) = 1,\\
f(v_{i_1}) &= 2i-2, \quad 3 \leq i \leq k+1\\
f(v_{(k+1+i)_1}) &= n-1-2(i-1), \quad 1 \leq i \leq k-1\\
f(v_{i_j}) &= f(v_{i_{j-1}}) + n-1, \quad 2 \leq j \leq m, \quad 1 \leq i \leq n
\end{aligned}$$

Then f is a mean labeling. Thus the graph edge mC_n-snake is a mean graph for $m \geq 1$ and $n \geq 3$. □

For example, a mean labeling of edge $4C_7$-snake and $5C_6$-snake are shown in Figure 10.

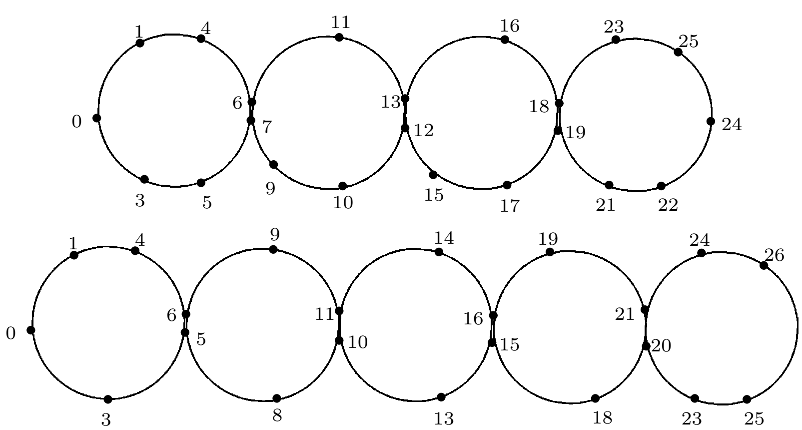

Figure 10

Theorem 2.9 *Let G' be a graph obtained by identifying an edge of two cycles C_m and C_n. Then G' is a mean graph for $m, n \geq 3$.*

Proof Let us assume that $m \leq n$.

Case 1 m is odd and n is odd

Let $m = 2k+1$, $k \geq 1$ and $n = 2l+1$, $l \geq 1$. The G' has $m+n-2$ vertices and $m+n-1$ edges. We denote the vertices of G' as follows:

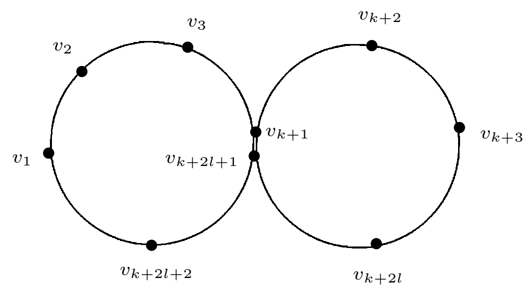

Figure 11

Define $f : V(G') \to \{0, 1, 2, 3, \ldots, q = m+n-1\}$ as follows:

$$f(v_1) = 0,\ f(v_i) = 2i-1, \quad 2 \leq i \leq k+1$$
$$f(v_i) = m+3+2(i-k-2), \quad k+2 \leq i \leq k+l$$
$$f(v_i) = m+n-1-2(i-k-l-1), \quad k+l+1 \leq i \leq k+2l$$
$$f(v_i) = m-1-2(i-k-2l-1), \quad k+2l+1 \leq i \leq 2k+2l$$

Then f is a mean labeling.

Case 2 m is odd and n is even

Let $m = 2k+1$, $k \geq 1$ and $n = 2l$, $l \geq 2$. Define $f : V(G') \to \{0, 1, 2, 3, \ldots, q = m+n-1\}$ as follows:

$$f(v_1) = 0,\ f(v_i) = 2i-1, \quad 2 \leq i \leq k+1$$
$$f(v_i) = m+3+2(i-k-2), \quad k+2 \leq i \leq k+l$$
$$f(v_i) = m+n-2-2(i-k-l-1), \quad k+l+1 \leq i \leq k+2l-1$$
$$f(v_i) = m-1-2(i-k-2l), \quad k+2l \leq i \leq 2k+2l-1$$

Then, f gives a mean labeling.

Case 3 m and n are even

Let $m = 2k$, $k \geq 2$ and $n = 2l$, $l \geq 2$. Define f on the vertex set of G' as follows:

$$f(v_1) = 0,\ f(v_i) = 2i - 2, \quad 2 \leq i \leq k+1$$
$$f(v_i) = m + 3 + 2(i - k - 2), \quad k+2 \leq i \leq k+l$$
$$f(v_i) = m + n - 2 - 2(i - k - l - 1), \quad k+l+1 \leq i \leq k+2l-1$$
$$f(v_i) = m - 1 - 2(i - k - 2l), \quad k+2l \leq i \leq 2k+2l-2$$

Then, f is a mean labeling. Thus G' is a mean graph. □

For example, a mean labeling of the graph G' obtained by identifying an edge of C_7 and C_{10} are shown in Figure 12.

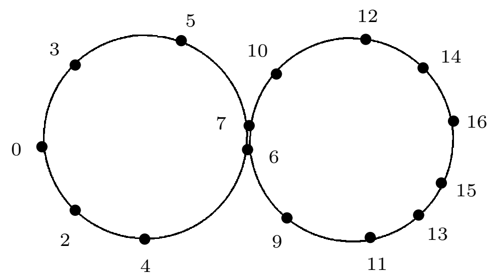

Figure 12

Theorem 2.10 *Let $\{u_iv_iw_iu_i : 1 \leq i \leq n\}$ be a collection of n disjoint triangles. Let G be the graph obtained by joining w_i to $u_{i+1}, 1 \leq i \leq n-1$ and joining u_i to u_{i+1} and $v_{i+1}, 1 \leq i \leq n-1$. Then G is a mean graph.*

Proof The graph G has $3n$ vertices and $6n-3$ edges respectively. We denote the vertices of G as in Figure 13.

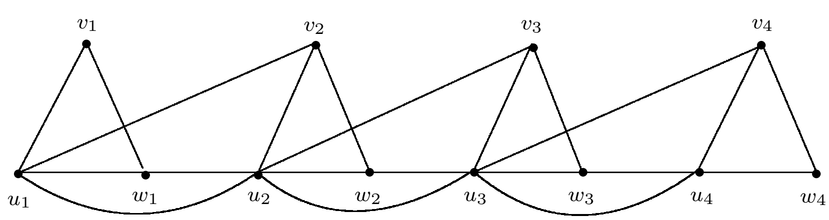

Figure 13

Define $f : V(G) \to \{0, 1, 2, \ldots, 6n-3\}$ as follows:

$$f(u_i) = 6i - 4, 1 \leq i \leq n$$
$$f(v_i) = 6i - 6, 1 \leq i \leq n$$
$$f(w_i) = 6i - 3, 1 \leq i \leq n.$$

Then f gives a mean labeling and hence G is a mean graph. □

For example, a mean labeling of G when $n = 6$ is shown Figure 14.

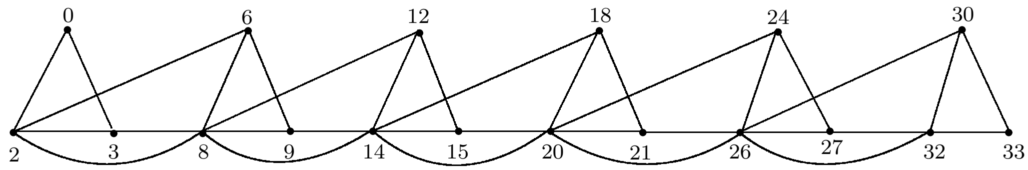

Figure 14

The graph obtained by attaching m pendant vertices to each vertex of a path of length $2n-1$ is denoted by $B(m)_{(n)}$. Dividing each edge of $B(m)_{(n)}$ by t number of vertices, the resultant graph is denoted by $S_t(B(m)_{(n)})$.

Theorem 2.11 *The $S_t(B(m)_{(n)})$ is a mean graph for all $m, n, t \geq 1$.*

Proof Let $v_1, v_2, \ldots, v_{2n}$ be the vertices of the path of length $2n-1$ and $u_{i,1}, u_{i,2}, \ldots, u_{i,m}$ be the pendant vertices attached at $v_i, 1 \leq i \leq 2n$ in the graph $B(m)_{(n)}$. Each edge $v_iv_{i+1}, 1 \leq i \leq 2n-1$, is subdivided by t vertices $x_{i,1}, x_{i,2}, \ldots, x_{i,t}$ and each pendant edge $v_iu_{i,j}$, $1 \leq i \leq 2n, 1 \leq j \leq m$ is subdivided by t vertices $y_{i,j,1}, y_{i,j,2}, \ldots, y_{i,j,t}$.

The vertices and their labels of $S_t(B(m)_{(1)})$ are shown in Figure 15.

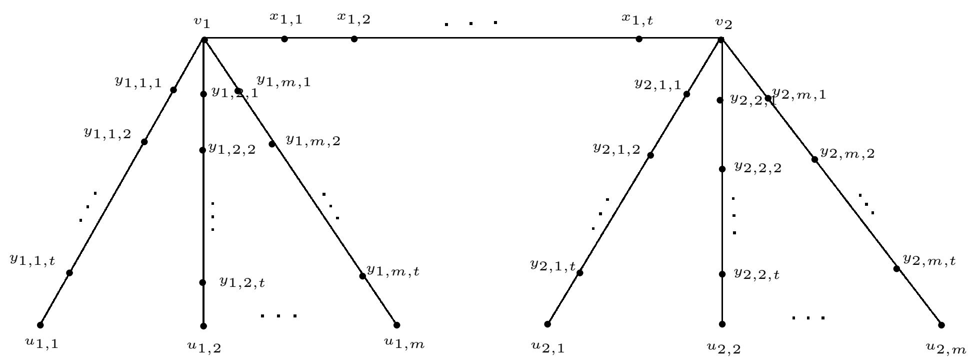

Figure 15

Define $f : V(S_t(B(m)_{(n)})) \to \{0, 1, 2, \ldots, (t+1)(2mn+2n-1)\}$ as follows:

$$f(v_i) = \begin{cases} (t+1)(m+1)(i-1) & \text{if } i \text{ is odd and } 1 \leq i \leq 2n-1 \\ (t+1)[(m+1)i-1] & \text{if } i \text{ is even and } 1 \leq i \leq 2n-1 \end{cases}$$

$$f(x_{i,k}) = \begin{cases} (t+1)[(m+1)i+m-1]+k & \text{if } i \text{ is odd, } 1 \leq i \leq 2n-1 \text{ and } 1 \leq k \leq t \\ (t+1)[(m+1)i-1]+k & \text{if } i \text{ is even, } 1 \leq i \leq 2n-1 \text{ and } 1 \leq k \leq t \end{cases}$$

$$f(y_{i,j,k}) = \begin{cases} (t+1)(m+1)(i-1) & \text{if } i \text{ is odd,} \\ +(2t+2)(j-1)+k, & 1 \leq i \leq 2n, 1 \leq j \leq m \text{ and } 1 \leq k \leq t \\ (t+1)[(m+1)(i-2)+1] & \text{if } i \text{ is even,} \\ +(2t+2)(j-1)+k, & 1 \leq i \leq 2n, 1 \leq j \leq m \text{ and } 1 \leq k \leq t \end{cases}$$

$$
\text{and } f(u_{i,j}) = \begin{cases} (t+1)[(m+1)(i-1)+1] & \text{if } i \text{ is odd,} \\ +(2t+2)(j-1), & 1 \leq i \leq 2n \text{ and } 1 \leq j \leq m \\ (t+1)[(m+1)(i-2)+2] & \text{if } i \text{ is even,} \\ +(2t+2)(j-1), & 1 \leq i \leq 2n \text{ and } 1 \leq j \leq m. \end{cases}
$$

Then, f is a mean labeling. Thus $S_t(B(m)_{(n)})$ is a mean graph. □

For example, a mean labeling of $S_3(B(4)_{(2)})$ is shown in Figure 16.

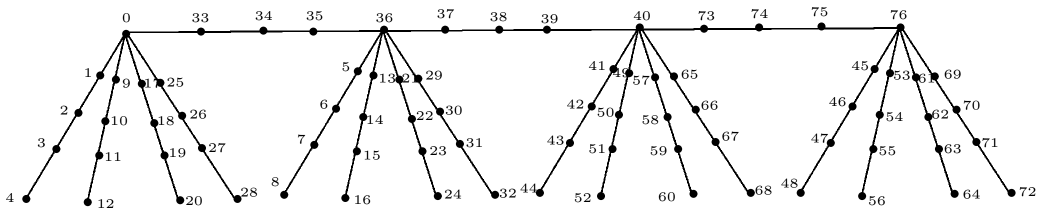

Figure 16

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