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On the Edge Geodetic and k-Edge Geodetic Number of a Graph

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Abstract: For vertices u and v in a connected graph $G = (V, E)$, the distance $d(u, v)$ is the length of a shortest $u - v$ path in G. A $u - v$ path of length $d(u, v)$ is called a $u - v$ *geodesic*. For an integer $k \geq 1$, a geodesic of length k in G is called a k-geodesic. A set $S \subseteq V$ is a k-edge geodetic set of G if each edge $e \in E - E(< S >)$ lies on a k-geodesic of some pair of vertices in S and a set $T \subseteq V$ is an *edge geodetic set* of G if each edge of G lies on a geodesic of some pair of vertices in T, and Smarandache edge-geodetic set of G if each edge of G lies on at least two geodesics of T. The minimum cardinality of a k-edge geodetic set of G is the *k-edge geodetic number* $eg_k(G)$ and the minimum cardinality of an edge geodetic set is the *edge geodetic number* $eg(G)$. In this paper we investigate how the edge geodetic number and the k-edge geodetic number of a graph G are affected by adding a pendant edge to G. It is proved that if G' is a graph obtained from G by adding a pendant edge, then $eg(G) \leq eg(G') \leq eg(G) + 1$ and $eg_2(G) \leq eg_2(G') \leq eg_2(G) + 1$. For any integer $k \geq 2$, it is also proved that $eg_k(G') \leq eg_k(G) + 2$. It is shown that for any integer $k \geq 4$ and for every pair a, b of integers with $4 \leq a \leq b + 2$, there is a connected graph G such that $eg_k(G) = b$ and $eg_k(G') = a$, where G' is a graph obtained from G by adding a pendant edge.

Key Words: Smarandache edge-geodetic set, geodetic number, edge geodetic number, k-geodetic number, k-edge geodetic number, k-extreme edge.

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§1. Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by n and m respectively. For basic graph theoretic terminology, we refer to Harary [3]. For vertices u and v in a connected graph G, the *distance* $d(u, v)$ is the length of a shortest u-v path in G. It is known that the distance is a metric on the vertex set of G. A u-v path of length $d(u, v)$ is called a u-v *geodesic*. A vertex x is said to lie on a u-v geodesic P if x is a vertex of P including the vertices u and v. For any path P in a graph and two vertices x, y on P, we use $P[x, y]$ to denote the portion of P between x and y, inclusive of x and y. The *neighborhood* of a vertex v is the set $N(v)$ consisting of all vertices u which are adjacent with v. A vertex v is an *extreme vertex* of G if the subgraph induced by its neighbors is complete. The *closed interval* $I[u, v]$ consists of all vertices lying on

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some u-v geodesic of G, while for $S \subseteq V$, $I[S] = \bigcup_{u,v \in S} I[u,v]$. A set S of vertices is a *geodetic set* if $I[S] = V$, and the minimum cardinality of a geodetic set is the *geodetic number* $g(G)$. A geodetic set of cardinality $g(G)$ is called a *g-set* of G. The geodetic number of a graph was introduced in [1], [4] and further studied in [2], [5]. It was shown in [4] that determining the geodetic number of a graph is an NP-hard problem. A set S of vertices is an *edge geodetic set* of a graph G if each edge of G lies on a geodesic of vertices in S, and Smarandache edge-geodetic set of G if each edge of G lies on at least two geodesics of S. The minimum cardinality of an edge geodetic set is the *edge geodetic number* $eg(G)$. An edge geodetic set of cardinality $eg(G)$ is called *eg-set* of G. Edge geodetic sets and the edge geodetic number of a graph with several interesting applications are investigated in [7].

For an integer $k \geq 1$, a geodesic in G of length k is called *k-geodesic*. A vertex v is called *k-extreme vertex* if v is not the internal vertex of a k-geodesic joining any pair of distinct vertices of G. Obviously, each extreme vertex of a connected graph G is k-extreme vertex of G. In particular, each end vertex of G is a k-extreme vertex of G. A set $S \subseteq V$ is called a *k-geodetic set* of G if each vertex v in $V - S$ lies on a k-geodesic of vertices in S. The minimum cardinality of a k-geodetic set of G is its *k-geodetic number* $g_k(G)$. A k-geodetic set of cardinality $g_k(G)$ is called g_k*-set*. The k-geodetic number of a graph was refereed to as *k-geodeomination number* and studied in [6].

For any $S \subseteq V$, let $E(< S >)$ denote the edge set of the subgraph induced by S. A set $S \subseteq V$ is called a *k-edge geodetic set* of G if each edge in $E - E(< S >)$ lies on a k-geodesic of vertices in S. The minimum cardinality of a k-edge geodetic set of G is its *k-edge geodetic number* $eg_k(G)$. A k-edge geodetic set of cardinality $eg_k(G)$ is called eg_k*-set* of G. For $k \geq 2$, an edge of G is called *k-extreme edge* if it does not lie on any k-geodesic of vertices of G.

For the graph G given in Fig.1.1, it is easy to see that the set $S = \{v_1, v_2, v_5, v_6\}$ of end vertices is a g_2-set and so $g_2(G) = 4$. Since the edge v_3v_4 does not lie on any 2-geodesic of vertices of S, S is not a 2-edge geodetic set of G. It is easily seen that $S_1 = \{v_1, v_2, v_3, v_5, v_6\}$ is a minimum 2-edge geodetic set of G so that $eg_2(G) = 5$. Also, $S_2 = \{v_1, v_2, v_4, v_5, v_6\}$ is another eg_2-set of G.

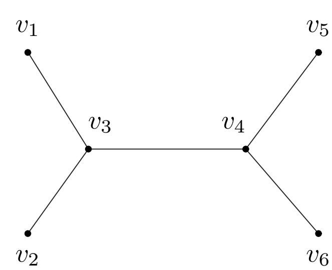

Fig.1.1

The k-edge geodetic number of a graph was introduced and studied in [8]. It is proved in [8] that each triple a, b, k of integers with $2 \leq a \leq b$ and $k \geq 2$ is realizable as the k-geodetic number and k-edge geodetic number of a graph respectively. Also it is shown in [8] that for given integers a, b, c and $k \geq 2$ with $3 \leq a \leq b \leq c$, there is a connected graph G with

$g(G) = a$, $eg(G) = b$ and $eg_k(G) = c$. These concepts have many applications in location theory and convexity theory. There are interesting applications of these concepts to the problem of designing the route for a shuttle and communication network design.

A fundamental question in graph theory concerns how the value of a parameter is affected by making a small change in the graph. The geodetic number and the k-geodetic number, affected by adding a pendant edge, was discussed in [5] and [6] respectively. In this paper we study how the edge geodetic number and the k-edge geodetic number of a graph are affected by adding a pendant edge to the graph.

Throughout the following G denotes a connected graph with at least two vertices. The following theorems will be used in the sequel.

Theorem 1.1([7]) *Each extreme vertex of a connected graph G belongs to every edge geodetic set of G. In particular, if the set of all extreme vertices W is an edge geodetic set of G, then W is the unique eg-set of G.*

Theorem 1.2([8]) *Every k-edge geodetic set contains both the ends of each k-extreme edge. If the set W of the ends of all the k-extreme edges together with the set of k-extreme vertices is a k-edge geodetic set, then W is the unique eg_k-set of G and so $eg_k(G) = |W|$.*

Theorem 1.3([7]) *For any tree T with k end vertices, $eg(T) = k$.*

Theorem 1.4([7]) *For a connected graph G, $eg(G) = 2$ if and only if there exist two antipodal vertices u and v such that every edge lies on a $u - v$ geodesic of G.*

Theorem 1.5([7]) *For a connected graph G, no cut vertex belongs to any eg-set of G.*

Theorem 1.6([7]) *For the complete bipartite graph $K_{m,n} (m, n \geq 2)$, $eg(K_{m,n}) = min\{m, n\}$.*

Theorem 1.7([7]) *If a connected graph G of order n has exactly one vertex v of degree $n - 1$ then $eg(G) = n - 1$.*

§2. How the edge geodetic number of a connected graph is affected by adding a pendant edge

In this section we discuss how the edge geodetic number of a connected graph G is affected by adding a pendant edge to G. Let G' be a graph obtained from a connected graph G by adding a pendant edge uv, where u is not vertex of G and v is a vertex of G.

Theorem 2.1 *If G' is a graph obtained from a connected graph G by adding a pendant edge uv at a vertex v of G, then $eg(G) \leq eg(G') \leq eg(G) + 1$.*

Proof Let S be any eg-set of G and let $S' = S \cup \{u\}$. We claim that S' is an edge geodetic set of G'. Let e be an edge of G'. If $e \in E(G)$, then e lies on a geodesic of vertices in S. If $e = uv$, then, since every edge geodetic set of G is a geodetic set of G, it follows that the vertex v lies on a $x - y$ geodesic P with $x, y \in S$. Then, it is clear that the portion $P[x, v]$ of the $x - v$ path on P together with the edge uv is a $x - u$ geodesic of G', which contains the edge e with

$x, u \in S'$. Hence S' is an edge geodetic set of G' and so $eg(G') \leq eg(G)+1$. Let S' be an eg-set of G'. By Theorems 1.1 and 1.5, $u \in S'$ and $v \notin S'$. Also, it is clear that $S = (S' - \{u\}) \cup \{v\}$ is an edge geodetic set of G so that $eg(G) \leq |S'| - 1 + 1 = |S'| = eg(G')$. Hence the result. □

Remark 2.2 The bounds for $eg(G')$ in Theorem 2.1 are sharp. If the graph G is the path $P_n(n \geq 3)$ on n vertices, then, by Theorem 1.3, $eg(P_n) = 2$. Let G' be the path obtained from P_n by adding a pendant edge at one of its end vertices. Then, by Theorem 1.3, $eg(G') = 2 = eg(G)$. If G' is the tree obtained from P_n by adding a pendant edge at a cut vertex of P_n, then by Theorem 1.3, $eg(G') = 3 = eg(G) + 1$.

Theorem 2.3 *Let G' be a graph obtained from a connected graph G by adding a pendant edge uv at a vertex v of G. Then $eg(G) = eg(G')$ if and only if v is a vertex of some eg-set of G.*

Proof First, assume that there is an eg-set S of G such that $v \in S$. Let $S' = (S - \{v\}) \cup \{u\}$. We show that S' is an eg-set of G'. If $e = uv$, then it is clear that e lies on every $w - u$ geodesic of G, where $w \in S'(w \neq u)$. Let e be any edge of G. Since S is an eg-set of G, e lies on a $x - y$ geodesic in G with $x, y \in S$. If both $x, y \in S - \{v\}$, then e also lies on a $x - y$ geodesic in G' with $x, y \in S'$. If e lies on a $x - v$ geodesic in G with $x \in S - \{v\}$, then e also lies on $x - u$ geodesic in G'. Thus S' is an edge geodetic set of G' so that $eg(G') \leq |S'| = |S| = eg(G)$. Now, the result follows from Theorem 2.1.

Conversely, suppose that $eg(G) = eg(G')$. Suppose that v does not belong to any eg-set of G. Let S' be an eg-set of G'. Since u is an end vertex of G' and v is a cut vertex of G', by Theorems 1.1 and 1.5, $u \in S'$ and $v \notin S'$. Let $S = (S' - \{u\}) \cup \{v\}$. Then $S \subseteq V(G)$ and $|S| = |S'| = eg(G') = eg(G)$. Let e be any edge of G. Then e is also an edge of G' and so e lies on a geodesic P in G' joining a pair of vertices $x, y \in S'$. If $x \neq u$ and $y \neq u$, then $x \in S$ and $y \in S$ so that e lies on a geodesic joining a pair of vertices in S. Otherwise, let $x \neq u$ and $y = u$. Then it follows that e lies on a geodesic in G joining x and v in S. Thus, S is an edge geodetic set of G and since $|S| = eg(G)$, it follows that S is an eg-set of G. Since $v \in S$, this is contradiction to our assumption. This completes the proof. □

Remark 2.4 If a vertex v is added to a connected graph G such that more than one edge is incident with v, then the edge geodetic number of the resulting graph can stay the same, increase significantly or decrease significantly. For example, for the complete bipartite graph $K_{m,n}$ we have, by Theorem 1.6, $eg(K_{m,n}) = m$ for all $2 \leq m \leq n$. However, if we add a new vertex to $K_{m,n}$ and join this vertex to all the vertices of the minimum partite set containing m vertices, the resulting graph is $K_{m,n+1}$ and again by Theorem 1.6, the edge geodetic number is m. Hence a new vertex may be added to a graph along with a large number of edges such that it does not affect the edge geodetic number. On the other hand, it is clear that $eg(C_n) = 2$ for all even $n \geq 4$. If we add a vertex v to this C_n and join v to all the vertices of C_n, the resulting graph is the wheel $K_1 + C_n$. Now, it follows from Theorem 1.7 that $eg(K_1 + C_n) = n$ and so the edge geodetic number of the resulting graph increases significantly. Also, it is clear that $eg(K_{1,n}) = n$ for all $n \geq 2$. If we add a vertex v and join it to all the end vertices of $K_{1,n}$ then we obtain the graph $K_{2,n}$. By Theorem 1.6, $eg(K_{2,n}) = 2$, and so the edge geodetic number of the resulting graph decreases significantly for large n.

§3. How the k-edge geodetic number of a connected graph is affected by adding a pendant edge

We now consider how the k-edge geodetic number of a connected graph G is affected by the addition of a pendant edge.

Proposition 3.1 *Let G' be a graph obtained from a connected graph G by adding a pendant edge uv at a vertex v of G. Then $eg_k(G') \leq eg_k(G) + 2$.*

Proof Let S be an eg_k-set of G. Then $S \cup \{u, v\}$ is a k-edge geodetic set of G' and so $eg_k(G') \leq |S \cup \{u, v\}| \leq eg_k(G) + 2$. □

Proposition 3.2 *There is no connected graph G with $diam(G) \geq k$ such that $eg_k(G') = 2$, where G' is a graph obtained from G by adding a pendant edge at a vertex of G.*

Proof Suppose that there exists a connected graph G with $diam(G) \geq k$ such that $eg_k(G') = 2$. Let G' be a graph obtained from G by adding a pendant edge uv at a vertex v of G. By Theorem 1.1, u belongs to every edge geodetic set of G'. Let $S' = \{u, y\}$ be an eg_k-set of G'. Then $y \neq v$ and it is clear that $S = \{v, y\}$ is a eg_{k-1}- set of G. Hence S is an eg-set of G and $d(v, y) = k - 1$. Now, by Theorem 1.4, v and y are antipodal vertices and so $diam(G) = k - 1$, which is a contradiction. Hence the result follows. □

Observation 3.3 In a connected graph G, each edge in G has at least one end in every 2-edge geodetic set of G.

Theorem 3.4 *If G' is a graph obtained from a connected graph G by adding a pendant edge at a vertex of G, then $eg_2(G) \leq eg_2(G') \leq eg_2(G) + 1$.*

Proof Let G' be the graph obtained from G by adding a pendant edge uv at a vertex v of G. Let S be an eg_2-set of G. Let $S' = S \cup \{u\}$. We claim that S' is a 2-edge geodetic set of G'. Let e be an edge of G' be such that $e \notin E(< S' >)$. If $e \in E(G)$, then e lies on a 2-geodesic of vertices in S. If $e \notin E(G)$, then $e = uv$ and $v \notin S$. Let vw be an edge of G. Then, by Observation 3.3, we have $w \in S$. Now, it is clear that the edge uv lies on the 2-geodesic $P : w, v, u$ of G' with $w, u \in S'$. Hence S' is a 2-edge geodetic set of G' and so $eg_2(G') \leq |S'| = eg_2(G) + 1$.

Now, let T' be any eg_2-set of G'. Then by Theorem 1.2, $u \in T'$. Let $T = (T' - \{u\}) \cup \{v\}$. Then $|T| \leq |T'|$. We show that T is a 2-edge geodetic set of G. Let $e = xy$ be any edge of G such that $e \notin E(< T >)$. Then it is clear that $e \notin E(< T' >)$. Now, since $e \in E(G')$ and T' is a eg_2-set of G', we see that $e = xy$ lies on a 2-geodesic P of vertices in T'. By Observation 3.3, we may assume that $x \in T'$. Assume that the geodesic P is $P : x, y, z$ with $x, z \in T'$. Since $xy \in E(G)$, we have $x \neq u$ and so $x \in T$. Now, if $z = u$, then $y = v$ and so $xy \in E(< T >)$, which is a contradiction. Hence $z \neq u$ and so $z \in T$. Thus T is a 2-edge geodetic set of G so that $eg_2(G) \leq |T| \leq |T'| = eg_2(G')$. □

Proposition 3.5 *Let G' be a graph obtained from a connected graph G by adding a pendant* *edge uv at a vertex v of G. If v belongs to some eg_2-set of G', then $eg_2(G') = eg_2(G) + 1$.*

Proof Let T' be an eg_2-set of G' such that $v \in T'$. By Theorem 1.2, $u \in T'$. Now, let $T = T' - \{u\}$. Then $|T| = |T'| - 1 = eg_2(G') - 1$ and as in the proof of Theorem 3.4, T is a 2-edge geodetic set of G so that $eg_2(G) \le |T| = eg_2(G') - 1$ Now, the result follows from Theorem 3.4. □

Remark 3.6 The converse of Theorem **??** is not true. For the graph $G = K_{1,n} (n \ge 2)$, we have that $eg_2(G) = n$. However, if we add a pendant edge to the cut vertex of $K_{1,n}$, then the resulting graph G' is $K_{1,n+1}$ and so $eg_2(G') = n + 1 = eg_2(G) + 1$. However, the cut vertex of $K_{1,n+1}$ does not belong to any eg_2-set of $K_{1,n+1}$.

Problem 3.7 *Characterize graphs G for which $eg_2(G') = eg_2(G)$, where G' is a graph obtained from G by adding a pendant edge.*

In view of Proposition 3.1, we have the following realization theorem.

Theorem 3.8 *Let $k \ge 4$ be an integer. For each pair a, b of integers with $4 \le a \le b + 2$, there is a connected graph G with $eg_k(G) = b$ and $eg_k(G') = a$, where G' is a graph obtained from G by adding a pendant edge.*

Proof We prove the theorem by considering five cases.

Case 1. Let $a = b$. Let G be the graph obtained from the path $P : v_0, v_1, \ldots, v_k$ by adding $b - 3$ new vertices $u_1, u_2, \ldots, u_{b-3}$ and joining them to v_2. The graph G is shown in Fig.3.1. It is clear that the edges $u_i v_2 (1 \le i \le b - 3)$ are the only k-extreme edges of G. Hence $S = \{v_0, v_k, u_1, u_2, \ldots, u_{b-3}, v_2\}$ is the set of all k-extreme vertices and the ends of all k-extreme edges of G. Since S is a k-edge geodetic set of G, it follows from Theorem 1.2 that $eg_k(G) = |S| = b$.

Now, let G' be the graph obtained from G by adding a pendant edge $v_k x$. It is clear that G' has no k-extreme edges. Let $S' = \{v_0, u_1, u_2, \ldots, u_{b-3}, x\}$ be the set of all k-extreme vertices of G'. Since the edges $v_0 v_1$ and $v_1 v_2$ do not lie on any k-geodesic joining a pair of vertices in S', we have S' is not a k-edge geodetic set of G'. Since $S' \cup \{v_k\}$ is a k-edge geodetic set of G', it follows from Theorem 1.2 that $eg_k(G') = |S'| + 1 = b = a$.

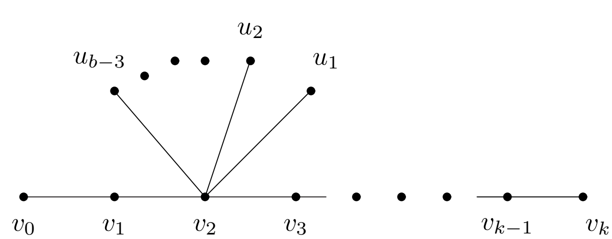

Fig.3.1

Case 2. $a = b + 1$. Let G be the graph obtained from the path $P : v_0, v_1, \ldots, v_k$ by adding $b - 2$ new vertices $u_1, u_2, \ldots, u_{b-2}$ and joining each u_i to v_1. Let $S = \{u_1, u_2, \ldots, u_{b-2}, v_0, v_k\}$.

Then S is the set of all k-extreme vertices of G. It is clear that G has no k-extreme edges and S is a k-edge geodetic set of G and so by Theorem 1.2, $eg_k(G) = |S| = b$. Now, let G' be the graph obtained from G by adding a new vertex x and joining it to v_1. Then, just as above, the set $S' = \{u_1, u_2, \ldots, u_{b-2}, x, v_0, v_k\}$ of all k-extreme vertices of G' is the eg_k-set of G'. Hence $eg_k(G') = |S'| = b + 1 = a$.

Case 3. $a = b+2$. Let G be the graph constructed in Case 2. Then, as in Case 2, $eg_k(G) = b$. Now, let G' be the graph obtained from G by adding a new vertex x and joining it to v_2. Then the edge xv_2 is the only k-extreme edge in G'. Since the set $S' = \{u_1, u_2, \ldots, u_{b-2}, v_0, v_k, x, v_2\}$ of all k-extreme vertices together with the ends of the k-extreme edge xv_2 is a k-edge geodetic set, it follows from Theorem 1.2 that $eg_k(G') = |S'| = b + 2 = a$.

Case 4. $a = b - 1$. Let G_1 be the graph obtained from the path $P : v_0, v_1, \ldots, v_k$ by adding a new vertex w and joining it to the vertices v_1, v_2 and v_3. Let $Q : x_0, x_1, \ldots, x_{k-2}$ be a path such that it is vertex disjoint with G_1. Let G_2 be the graph obtained from G_1 and Q by identifying the vertices v_2 and x_0. Let G be the graph obtained from G_2 by adding $b - 5$ new vertices $z_1, z_2, \ldots, z_{b-5}$ and joining each z_i to v_1. The graph is G shown in Fig.3.2. It is clear that the edge v_2w is the only k-extreme edge of G. Since the set $S = \{v_0, z_1, z_2, \ldots, z_{b-5}, v_k, x_{k-2}, v_2, w\}$ of all k-extreme vertices and the ends of the k-extreme edge v_2w of G is a k-edge geodetic set, it follows from Theorem 1.2 that $eg_k(G) = |S| = b$.

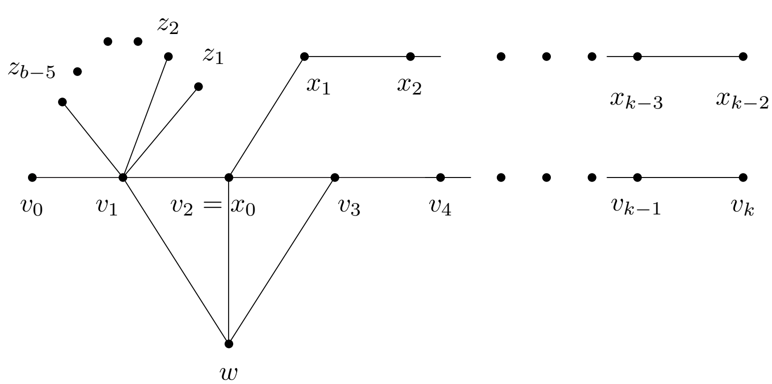

Fig.3.2

Now, let G' be the graph obtained from G by adding a pendant edge wx. It is clear that G' has no k-extreme edges. Since the set $S' = \{v_0, z_1, z_2, \ldots, z_{b-5}, x, v_k,$
$x_{k-2}\}$ of all k-extreme vertices of G' is a k-edge geodetic set of G', it follows from Theorem 1.2 that $eg_k(G') = |S'| = b - 1 = a$.

Case 5. $4 \leq a \leq b - 2$. Let G_1 be the graph obtained from the path $P : v_0, v_1, \ldots, v_k$ by adding a new vertex w and joining it to both v_2 and v_4. Let G_2 be the graph obtained from G_1 by adding $b - a - 1$ new vertices $u_1, u_2, \ldots, u_{b-a-1}$ and joining each u_i to the vertices v_2, v_3, v_4 and w. Let G_3 be the graph obtained from G_2 by adding $a - 4$ new vertices $z_1, z_2, \ldots, z_{a-4}$ and joining each z_i to v_1. Let $Q : x_0, x_1, \ldots, x_{k-3}$ be a path such that it is vertex disjoint with G_3. Let G be the graph obtained from G_3 and Q by identifying the vertices v_3 and x_0. The graph G is shown in Fig.3.3. It is clear that the edges u_iv_3 and u_iw $(1 \leq i \leq b-a-1)$ are the only k-extreme edges of G and so by Theorem 1.2, the vertices $u_1, u_2, \ldots, u_{b-a-1}, v_3, w$ belong to every k-edge geodetic set of G.

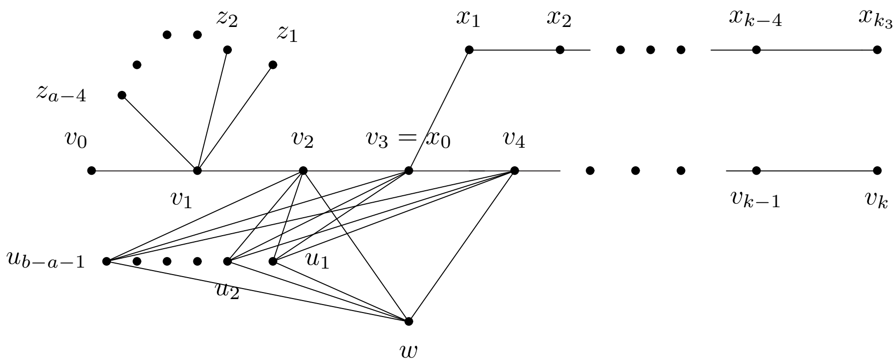

Fig.3.3

First, suppose that $k = 4$. Let $S = \{v_0, u_1, u_2, \ldots, u_{b-a-1}, v_3, w, z_1, z_2, \ldots, z_{a-4}, x_1\}$. Then S is the set of all k-extreme vertices and the ends of all k-extreme edges of G. It is clear that S is not a k-edge geodetic set of G and $S \cup \{v_4\}$ is a k-edge geodetic set of G so that by Theorem 1.2, $eg_k(G) = |S| + 1 = b - 1 + 1 = b$. Now, let G' be the graph obtained from G by adding a new vertex x and joining it to w. Then the graph G' has no k-extreme edges. Let $S' = \{v_0, z_1, z_2, \ldots, z_{a-4}, x, x_1\}$. Then S' is the set of all k-extreme vertices of G'. It is clear that S' is not a k-edge geodetic set of G' and $S' \cup \{v_4\}$ is a k-edge geodetic set of G' so that by Theorem 1.2, $eg_k(G') = |S'| + 1 = a - 1 + 1 = a$. Next, suppose that $k \geq 5$. Let $T = \{v_0, u_1, u_2, \ldots, u_{b-a-1}, v_3, w, z_1, z_2, \ldots, z_{a-4}, x_{k-3}, v_k\}$. Then T is the set of all k-extreme vertices and the ends of all k-extreme edges of G. It is clear that T is a k-edge geodetic set of G and so by Theorem 1.2, $eg_k(G) = |T| = b$.

Let G' be the graph obtained from G by adding a new vertex x and joining it to w. Then G' has no k-extreme edges and $T' = \{v_0, z_1, z_2, \ldots, z_{a-4}, x, x_{k-3}, v_k\}$ is the set of all k-extreme vertices of G'. Since T' is a k-edge geodetic set of G', it follows from Theorem 1.2 that $eg_k(G') = |T'| = a$. Thus the proof is complete. □

References

[1] F. Buckley and F. Harary, *Distance in Graphs*, Addison-Wesley, Redwood City, CA, 1990.

[2] G. Chartrand, F. Harary and P. Zhang, On the Geodetic Number of a Graph, *Networks*, **39**(1)(2002), 1-6.

[3] F.Harary, *Graph Theory,* Addison-Wesley, 1969.

[4] F. Harary, E. Loukakis, C. T. Souros, The geodetic number of a graph, *Mathl. Comput.Modeling*,**17**(11) (1993), 89-95.

[5] R. Muntean, P. Zhang, On Geodomination in Graphs, *Congr. Numer.*, 143 (2000),161-174.

[6] R. Muntean, P. Zhang, k-Geodomination in Graphs, *ARS Combinatoria*, 63 (2002), 33-47.

[7] A. P. Santhakumaran and J. John, Edge Geodetic Number of a Graph, *Journal of Discrete Mathematical Sciences & Cryptography*, **10**(3) (2007),415-432.

[8] A. P. Santhakumaran and S. V. Ullas Chandran, The k-edge geodetic number of a graph, (communicated).