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On the Uniqueness and Value Distribution of Entire Functions With Their Derivatives

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Abstract: In this article, we study the uniqueness problem of entire functions sharing a value with their derivatives. The result of this paper extends the result due to Zhang.

Key Words: Value distribution, entire functions, weighted sharing, Nevanlinna theory, multiplicity.

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§1. Introduction

The uniqueness theory of meromorphic functions is an interesting problem in the value distribution theory and also the uniqueness theory of algebroid functions is an interesting problem in the value distribution theory. Ming-Liang Fang [11] and Q Zhang [9] and several other authors proved some interesting results on uniqueness and value sharing of entire functions and also meromorphic function that shares one small function with its derivative (see [3-5, 7-8, 10,12-31).

Let f be a transcendental meromorphic function in the plane and $m(r, f)$, $N(r, f)$ and $T(r, f)$ be the usual notations used in the Nevanlinna theory. Let $S(r, f)$ denote any quantity satisfying $S(r, f)$=$o[T(r, f)]$ as $r \longrightarrow \infty$ except possibly for a set of r of finite linear measure. Throughout this paper we denote by $a, a_0, a_1, \cdots, a_n$ meromorphic functions (or constants) for smaller growth than f, that is $T(r, f) = S(r, f)$.

Let f and g be two non-constant meromorphic functions. Let a be a finite complex number. We denote by $E(a, f)$ the set of zeros of $f - a$ (counting multiplicity), by $\overline{E}(a, f)$ the set of zeros of $f - a$ (ignoring multiplicity). We say f and g share a CM (IM), if $E(a, f) = E(a, g)$ ($\overline{E}(a, f) = \overline{E}(a, g)$). Similarly, we define that f and g share a small function $a(z)$ CM (IM), if $E(a(z), f) = E(a(z), g)$ ($\overline{E}(a(z), f) = \overline{E}(a(z), g)$). Moreover, $GCD(n_1, n_2, \cdots, n_k)$ denotes the greatest common divisor of positive integers $n_1, n_2, \cdots, n_k$.

In 2005, Zhang [9] obtained the following result.

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Theorem A. *Let f be a non-constant meromorphic function and $k(\geq 1)$, $l(\geq 0)$ be integers. Also, let $a \equiv a(z)(\not\equiv 0, \infty)$ be a meromorphic function such that $T(r,a) = S(r,f)$. Suppose that $f - a$ and $f^{(k)} - a$ share $(0,l)$. If $l \geq 2$ and*

$$(3+k)\Theta(\infty, f) + 2\delta_{2+k}(0,f) > k+4, \tag{1.1}$$

or if $l = 1$ and

$$(4+k)\Theta(\infty, f) + 3\delta_{2+k}(0,f) > k+6, \tag{1.2}$$

or if $l = 0$ and

$$(6+2k)\Theta(\infty, f) + 5\delta_{2+k}(0,f) > 2k+10, \tag{1.3}$$

then $f \equiv f^{(k)}$.

Let

$$\mathcal{P}(w) = a_{n+m}w^{n+m} + \cdots + a_n w^n + \cdots + a_0 = a_{n+m}\prod_{i=1}^{s}(w - w_{p_i})^{p_i},$$

where $a_j(j = 0,1,2,\cdots,n+m-1)$, $a_{n+m} \neq 0$ and $w_{p_i}(i = 1,2,\cdots,s)$ are distinct finite complex numbers and $2 \leq s \leq n+m$ and $p_1, p_2, \cdots, p_s$, $s \geq 2$, n, m and k are all positive integers with $\sum_{i=1}^{s} p_i = n+m$. Let $p > \max_{p \neq p_i, i=1,2,\cdots,r}\{p_i\}$, $r = s-1$, where s and r are two positive integers.

Let

$$P(w_{1)} = a_{n+m}\prod_{i=1}^{s-1}(w_1 + w_p - w_{p_i})^{p_i} = b_q w_1^q + b_{q-1}w_1^{q-1} + \cdots + b_0,$$

where $a_{n+m} = b_q$, $w_1 = w - w_p$, $q = n+m-p$. Therefore, $\mathcal{P}(w) = w_1^p P(w_1)$. We assume $P(w_1) = b_q \prod_{i=1}^{r}(w_1 - \alpha_i)^{p_i}$, where $\alpha_i = w_{p_i} - w_p$, $(i = 1,2,\cdots,r)$, be distinct zeros of $P(w_1)$.

Definition 1.1([2]) *For two positive integers n, p we define $\mu_p = \min\{n,p\}$ and $\mu_p^* = p+1-\mu_p$. Then it is clear that*

$$N_p\left(r, \frac{1}{f^n}\right) \leq \mu_p N_{\mu_p^*}\left(r, \frac{1}{f}\right). \tag{1.4}$$

In the present paper, we extend Theorem A by investigating the uniqueness of meromorphic functions of the form $f_1^p P(f_1) - a$ and $(f_1^p P(f_1))^{(k)} - a$ and obtain the following result.

Theorem 1.1 *Let $k(\geq 1)$, $l(\geq 0)$, $n(\geq 1)$, $p(\geq 1)$ and $m(\geq 0)$ be integers, f and $f_1 = f - w_p$ be two non-constant entire functions. Let $\mathcal{P}(z) = a_{m+n}z^{m+n} + \cdots + a_n z^n + \cdots + a_0$, $a_{m+n} \neq 0$, be a polynomial in z of degree $m+n$ such that $\mathcal{P}(f) = f_1^p P(f_1)$. Suppose $\mathcal{P}(f)$ and $(\mathcal{P}(f))^{(k)}$ share $(1,l)$.*

If $l \geq 2$ and

$$\mu_2\delta_{\mu_2^*}(w_p, f) + \mu_{k+2}\delta_{\mu_{k+2}^*}(w_p, f) > m+n-2p+\mu_2+\mu_{k+2} \tag{1.5}$$

or $l = 1$ *and*

$$\frac{1}{2}\Theta(w_p, f) + \mu_2\delta_{\mu_2^*}(w_p, f) + \mu_{k+2}\delta_{\mu_{k+2}^*}(w_p, f) > \frac{3(m+n)-5p}{2} + \mu_2 + \mu_{k+2} + \frac{1}{2} \tag{1.6}$$

or $l = 0$ *and*

$$2\Theta(w_p, f)+\mu_2\delta_{\mu_2^*}(w_p, f)+\mu_{k+1}\delta_{\mu_{k+1}^*}(w_p, f)+\mu_{k+2}\delta_{\mu_{k+2}^*}(w_p, f) > 4(m+n)-5p+2+\mu_2+\mu_{k+1}+\mu_{k+2} \tag{1.7}$$

then $\mathcal{P}(f) \equiv (\mathcal{P}(f))^{(k)}$.

§2. **Preliminary Lemmas**

Let F and G be two non-constant meromorphic functions. We denote by H the following function

$$H = \left(\frac{F''}{F'} - \frac{2F'}{F-1}\right) - \left(\frac{G''}{G'} - \frac{2G'}{G-1}\right). \tag{2.1}$$

Lemma 2.1 ([9]) *Let f be a non constant meromorphic function, k, p, be two positive integers, then*

$$N_p\left(r, \frac{1}{f^{(k)}}\right) \le N_{p+k}\left(r, \frac{1}{f}\right) + k\overline{N}(r, f) + S(r, f).$$

Clearly,

$$\overline{N}\left(r, \frac{1}{f^{(k)}}\right) = N_1\left(r, \frac{1}{f^{(k)}}\right).$$

Lemma 2.2 ([6] *Let H be defined as in* (2.1). *If F and G share 1 IM and $H \not\equiv 0$, then*

$$N_{11}\left(r, \frac{1}{F-1}\right) \le N(r, H) + S(r, F) + S(r, G).$$

Lemma 2.3 ([1]) *Let F and G share $(1, l)$ and $\overline{N}(r, F) = \overline{N}(r, G)$ and $H \not\equiv 0$, then*

$$\begin{aligned} N(r, H) \quad \le \quad & \overline{N}(r, F) + \overline{N}_{(2}\left(r, \frac{1}{F}\right) + \overline{N}_{(2}\left(r, \frac{1}{G}\right) + \overline{N}_0\left(r, \frac{1}{F'}\right) \\ & + \overline{N}_0\left(r, \frac{1}{G'}\right) + \overline{N}_L\left(r, \frac{1}{F-1}\right) + \overline{N}_L\left(r, \frac{1}{G-1}\right) + S(r, f). \end{aligned}$$

§3. **Proof of Theorem** 1.1

Proof of Theorem 1.1 Let $F = \mathcal{P}(f) = f_1^p P(f_1)$ and $G = (\mathcal{P}(f))^{(k)} = (f_1^p P(f_1))^{(k)}$. Since $\mathcal{P}(f)$ and $[\mathcal{P}(f)]^{(k)}$ share $(1, l)$, F, G share $(1, l)$ except the zeros and poles of $a(z)$. Also, let's note that

$$\overline{N}(r, F) = \overline{N}(r, f) + S(r, f) \quad \text{and} \quad \overline{N}(r, G) = \overline{N}(r, f) + S(r, f).$$

Let H be defined as in (2.1). We consider the following cases.

Case 1. Suppose $H \not\equiv 0$.

By the second fundamental theorem of Nevanlinna, we have

$$\begin{aligned}
T(r,F)+T(r,G) \quad \leq \quad & \overline{N}(r,F)+\overline{N}(r,G)+\overline{N}\left(r,\frac{1}{F}\right)+\overline{N}\left(r,\frac{1}{G}\right) \\
& +\overline{N}\left(r,\frac{1}{F-1}\right)+\overline{N}\left(r,\frac{1}{G-1}\right) \\
& -\overline{N}_0\left(r,\frac{1}{F'}\right)-\overline{N}_0\left(r,\frac{1}{G'}\right)+S(r,F)+S(r,G), \qquad (3.1)
\end{aligned}$$

where $\overline{N}_0\left(r,\frac{1}{F'}\right)$ denotes the reduced counting function of the zeros of F' which are not the zeros of $F(F-1)$.

Since F and G share 1 IM, it is easy to verify that

$$\begin{aligned}
\overline{N}\left(r,\frac{1}{F-1}\right) \quad = \quad & N_{11}\left(r,\frac{1}{F-1}\right)+\overline{N}_L\left(r,\frac{1}{F-1}\right) \\
& +\overline{N}_L\left(r,\frac{1}{G-1}\right)+N_E^{(2}\left(r,\frac{1}{G-1}\right)=\overline{N}\left(r,\frac{1}{G-1}\right). \qquad (3.2)
\end{aligned}$$

Using Lemmas 2.2 and 2.3, (3.1) and (3.2), we get

$$\begin{aligned}
T(r,F)+T(r,G) \quad \leq \quad & 3\overline{N}(r,F)+N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right) \\
& +N_{11}\left(r,\frac{1}{F-1}\right)+2N_E^{(2}\left(r,\frac{1}{G-1}\right) \\
& +3\overline{N}_L\left(r,\frac{1}{F-1}\right) \\
& +3\overline{N}_L\left(r,\frac{1}{G-1}\right)+S(r,F)+S(r,G). \qquad (3.3)
\end{aligned}$$

Subcase 1.1 $l \geq 2.$.

Obviously,

$$\begin{aligned}
N_{11}\left(r,\frac{1}{F-1}\right) \quad + \quad & 2N_E^{(2}\left(r,\frac{1}{G-1}\right)+3\overline{N}_L\left(r,\frac{1}{F-1}\right)+3\overline{N}_L\left(r,\frac{1}{G-1}\right) \\
& \leq N\left(r,\frac{1}{G-1}\right)+S(r,F) \\
& \leq T(r,G)+S(r,F)+S(r,G). \qquad (3.4)
\end{aligned}$$

Using (3.3) and (3.4), we get

$$T(r,F) \leq N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)+3\overline{N}(r,F)+S(r,F). \qquad (3.5)$$

Using Lemma 2.1, (1.4) and (3.5), we get

$$\begin{aligned}(n+m)T(r,f) &\leq N_2\left(r,\frac{1}{f_1^pP(f_1)}\right)+N_2\left(r,\frac{1}{(f_1^pP(f_1))^{(k)}}\right)+3\overline{N}(r,f)+S(r,f)\\ &\leq 3\overline{N}(r,f)+\mu_2N_{\mu_2^*}\left(r,\frac{1}{f-w_p}\right)+(n+m-p)T(r,f)\\ &\quad +N_{k+2}\left(r,\frac{1}{f_1^pP(f_1)}\right)+k\overline{N}(r,f)+S(r,f)\\ &\leq (k+3)\overline{N}(r,f)+\mu_2N_{\mu_2^*}\left(r,\frac{1}{f-w_p}\right)+2(n+m-p)T(r,f)\\ &\quad +\mu_{k+2}N_{\mu_{k+2}^*}\left(r,\frac{1}{f-w_p}\right)+S(r,f).\end{aligned}$$

So, $\mu_2\delta_{\mu_2^*}(w_p,f)+\mu_{k+2}\delta_{\mu_{k+2}^*}(w_p,f)\leq m+n-2p+\mu_2+\mu_{k+2}$, which contradicts with (1.5).

Subcase 1.2 $l=1$.

It is easy to verify that

$$\begin{aligned}N_{11}\left(r,\frac{1}{F-1}\right) &+2N_E^{(2}\left(r,\frac{1}{G-1}\right)+2\overline{N}_L\left(r,\frac{1}{F-1}\right)+3\overline{N}_L\left(r,\frac{1}{G-1}\right)\\ &\leq N\left(r,\frac{1}{G-1}\right)+S(r,F)\\ &\leq T(r,G)+S(r,F)+S(r,G). \qquad (3.6)\end{aligned}$$

$$\begin{aligned}\overline{N}_L\left(r,\frac{1}{F-1}\right) &\leq \frac{1}{2}N\left(r,\frac{F}{F'}\right)\\ &\leq \frac{1}{2}N\left(r,\frac{F'}{F}\right)+S(r,F)\\ &\leq \frac{1}{2}\left(\overline{N}\left(r,\frac{1}{F}\right)+\overline{N}(r,F)\right)+S(r,F). \qquad (3.7)\end{aligned}$$

Using (3.3), (3.6) and (3.7), we get

$$T(r,F)\leq N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)+\frac{7}{2}\overline{N}(r,F)+\frac{1}{2}\overline{N}\left(r,\frac{1}{F}\right)+S(r,F). \qquad (3.8)$$

Using Lemma (2.1), (1.4) and (3.8), we get

$$\begin{aligned}(n+m)T(r,f) \leq &\left(k+\frac{7}{2}\right)\overline{N}(r,f)+\mu_2N_{\mu_2^*}\left(r,\frac{1}{f-w_p}\right)+\mu_{k+2}N_{\mu_{k+2}^*}\left(r,\frac{1}{f-w_p}\right)\\ &+\frac{1}{2}\overline{N}\left(r,\frac{1}{f-w_p}\right)+\frac{5}{2}(n+m-p)T(r,f)+S(r,f).\end{aligned}$$

So,

$$\frac{1}{2}\Theta(w_p,f)+\mu_2\delta_{\mu_2^*}(w_p,f)+\mu_{k+2}\delta_{\mu_{k+2}^*}(w_p,f)\leq\frac{3(m+n)-5p}{2}+\mu_2+\mu_{k+2}+\frac{1}{2}$$

which contradicts with (1.6).

Subcase 1.3 $l=0$.

It is easy to verify that

$$\begin{aligned}N_{11}\left(r,\frac{1}{F-1}\right)&+2N_E^{(2}\left(r,\frac{1}{G-1}\right)+\overline{N}_L\left(r,\frac{1}{F-1}\right)+2\overline{N}_L\left(r,\frac{1}{G-1}\right)\\&\leq N\left(r,\frac{1}{G-1}\right)+S(r,F)\leq T(r,G)+S(r,F)+S(r,G).\end{aligned}\tag{3.9}$$

$$\begin{aligned}\overline{N}_L\left(r,\frac{1}{F-1}\right)&\leq N\left(r,\frac{1}{F-1}\right)-\overline{N}\left(r,\frac{1}{F-1}\right)\\&\leq N\left(r,\frac{F}{F'}\right)\leq N\left(r,\frac{F'}{F}\right)+S(r,F)\\&\leq\overline{N}\left(r,\frac{1}{F}\right)+\overline{N}(r,F)+S(r,F).\end{aligned}\tag{3.10}$$

Using (3.3), (3.9) and (3.10), we get

$$\begin{aligned}T(r,F)\quad\leq\quad&N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)\\&+6\overline{N}(r,F)+2\overline{N}\left(r,\frac{1}{F}\right)+N_1\left(r,\frac{1}{G}\right)+S(r,F).\end{aligned}\tag{3.11}$$

Using Lemma 2.1 and (3.11), we get

$$\begin{aligned}(n+m)T(r,f)\quad\leq\quad&N_2\left(r,\frac{1}{f_1^pP(f_1)}\right)+N_2\left(r,\frac{1}{(f^pP(f_1))^{(k)}}\right)+6\overline{N}(r,f)\\&+2\overline{N}\left(r,\frac{1}{f_1^pP(f_1)}\right)+N_1\left(r,\frac{1}{(f_1^pP(f_1))^{(k)}}\right)+S(r,f).\end{aligned}$$

So,

$$\begin{aligned}2\Theta(w_p,f)+\mu_2\delta_{\mu_2^*}(w_p,f)+\mu_{k+1}\delta_{\mu_{k+1}^*}(w_p,f)+\mu_{k+2}\delta_{\mu_{k+2}^*}(w_p,f)\\\leq4(m+n)-5p+2+\mu_2+\mu_{k+1}+\mu_{k+2}.\end{aligned}$$

which contradicts with (1.7).

Case 2. Suppose $H\equiv0$.

Using (2.1), we get

$$\frac{F''}{F'} - \frac{2F'}{F-1} = \frac{G''}{G'} - \frac{2G'}{G-1}.$$

Hence,

$$\frac{1}{F-1} \equiv C\frac{1}{G-1} + D, \tag{3.12}$$

where C, D are constants and $C \neq 0$.

We discuss the following three cases:

Subcase 2.1 $D \neq 0, -1$.

Rewrite (3.12) as,

$$\frac{G-1}{C} = \frac{F-1}{D+1-DF},$$

we have,

$$\overline{N}(r, G) = \overline{N}\left(r, \frac{1}{F - \frac{(D+1)}{D}}\right).$$

By using second fundamental theorem of Nevanlinna, we get

$$\begin{aligned}
(n+m)T(r,f) &= T(r,F) + S(r,f) \\
&\leq \overline{N}(r,F) + \overline{N}\left(r, \frac{1}{F}\right) + \overline{N}\left(r, \frac{1}{F - \frac{(D+1)}{D}}\right) + S(r,f) \\
&\leq \overline{N}(r,F) + \overline{N}\left(r, \frac{1}{F}\right) + \overline{N}(r,G) + S(r,f) \\
&\leq 2\overline{N}(r,f) + \overline{N}\left(r, \frac{1}{f_1^p P(f_1)}\right) + S(r,f) \\
&\leq 2\overline{N}(r,f) + \overline{N}\left(r, \frac{1}{f - w_p}\right) + (n+m-p)T(r,f) + S(r,f).
\end{aligned}$$

So, $\Theta(w_p, f) \leq 1 - p$,

which contradicts with (1.5), (1.6) and (1.7).

Subcase 2.2 $D = 0$.

Then from (3.12), we get

$$G = CF - (C-1). \tag{3.13}$$

If $C \neq 1$, then

$$\overline{N}\left(r, \frac{1}{G}\right) = \overline{N}\left(r, \frac{1}{F - \frac{(C-1)}{C}}\right).$$

Proceeding as in Subcase 2.1, we get

$$\begin{aligned}(n+m)T(r,f) \leq\ & (k+1)\overline{N}(r,f)+\overline{N}\left(r,\frac{1}{f-w_p}\right)\\ & +2(n+m-p)T(r,f)+N_{k+1}\left(r,\frac{1}{f-w_p}\right)+S(r,f).\end{aligned}$$

So,

$$\Theta(w_p,f)+\mu_{k+1}\delta_{\mu^*_{k+1}}(w_p,f)\leq 1+\mu_{k+1}+n+m-2p,$$

which contradicts with (1.5), (1.6) and (1.7).

Therefore, $C=1$. By using (3.13), we get $F\equiv G$ and so, $f_1^pP(f_1)=(f_1^pP(f_1))^{(k)}$.

Subcase 2.3 $D=-1$.

Then from (3.12) we get

$$\begin{aligned}\frac{1}{F-1}&=\frac{C}{G-1}-1\\ &\Rightarrow\frac{F}{F-1}=\frac{C}{G-1}.\end{aligned}$$

Hence we have $\overline{N}\left(r,\frac{1}{F}\right)=\overline{N}(r,G)=S(r,f)$ and hence $\overline{N}\left(r,\frac{1}{f}\right)=S(r,f)$.

If $C\neq -1$, then

$$\overline{N}\left(r,\frac{1}{G}\right)=\overline{N}\left(r,\frac{1}{F-\frac{C}{C+1}}\right).$$

Proceeding as in Subcase 2.1, we get

$$\begin{aligned}(n+m)T(r,f)\leq (k+1)\overline{N}(r,f)+\overline{N}\left(r,\frac{1}{f-w_p}\right)\\ +2(n+m-p)T(r,f)+N_{k+1}\left(r,\frac{1}{f-w_p}\right)+S(r,f).\end{aligned}$$

So,

$$\Theta(w_p,f)+\mu_{k+1}\delta_{\mu^*_{k+1}}(w_p,f)\leq 1+\mu_{k+1}+n+m-2p$$

which contradicts with (1.5), (1.6) and (1.7).

Therefore, $C=-1$. By using (3.13), we get $FG\equiv 1$. Hence, $\mathcal{P}(f)(\mathcal{P}(f))^{(k)}=1$. Thus in this case,

$$\overline{N}(r,f)+\overline{N}\left(r,\frac{1}{f}\right)=S(r,f).$$

We have,

$$\frac{(\mathcal{P}(f))^{(k)}}{\mathcal{P}(f)} = \frac{1}{(\mathcal{P}(f))^2}. \tag{3.14}$$

From first fundamental theorem and (3.14), we get

$$\begin{aligned} 2T(r,\mathcal{P}(f)) &\leq T\left(r, \frac{(\mathcal{P}(f))^{(k)}}{\mathcal{P}(f)}\right) \\ &\leq N\left(r, \frac{(\mathcal{P}(f))^{(k)}}{\mathcal{P}(f)}\right) + S(r,f) \\ &\leq k\left(\overline{N}(r,\mathcal{P}(f)) + \overline{N}\left(r, \frac{1}{\mathcal{P}(f)}\right)\right) + S(r,f) \\ &\leq S(r,f), \end{aligned}$$

which is impossible. This completes the Proof. □

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