

## On Translational Hull Of Completely $\mathcal{J}^{*,\sim}$ -Simple Semigroups

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**Abstract:** In this paper, we give a construction theorem about the translational hull of completely  $\mathcal{J}^{*,\sim}$ -simple semigroups which extends the translational hulls of completely  $\mathcal{J}^*$ -simple semigroups and completely simple semigroups.

**Key Words:** Translational hull; completely  $\mathcal{J}^{*,\sim}$ -simple semigroup; construction.

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### §1. Introduction

Let  $S$  be a semigroup. A mapping  $\lambda$  from a semigroup  $S$  to itself is a left translation of  $S$  if  $\lambda(ab) = (\lambda a)b$  for all elements  $a, b$  of  $S$ ; a mapping  $\rho$  from  $S$  to itself is a right translation of  $S$  if  $(ab)\rho = a(b\rho)$  for all elements  $a, b$  of  $S$ . A left translation  $\lambda$  and right translation  $\rho$  are linked if  $a(\lambda b) = (a\rho)b$  for all  $a, b$  of  $S$ , in this case, the pair  $(\lambda, \rho)$  is a bitranslation of  $S$ . The set  $\Lambda(S)$  of all left translations of  $S$  and the set  $P(S)$  of all right translations of  $S$  are semigroups under the composition of mappings. The translational hull of  $S$  is the subsemigroup  $\Omega(S)$  of  $\Lambda(S) \times P(S)$  of all bitranslations of  $S$ . A left translation  $\lambda$  is inner if  $\lambda = \lambda_a$  for some  $a \in S$ , where  $\lambda_a x = ax$  for all  $x \in S$ ; an inner right translation  $\rho_a$  is defined dually; the pair  $\pi_a = (\lambda_a, \rho_a)$  is an inner bitranslation and the set  $\Pi(S)$  of all inner bitranslations is the inner part of  $\Omega(S)$  (actually an ideal of  $\Omega(S)$ ).

The translation hull of semigroups plays an important role in the algebraic theory of semigroups. It is an important tool in the study of ideal extensions. For more related details of translational hulls, the reader is referred to [1], [4], [5], [7],[14],[15].

In order to generalize regular semigroups, new Green's relations, namely, the Green's  $*$ -relations on a semigroup have been introduced as follows ([11], [12]):

$$\mathcal{L}^* = \{(a, b) \in S \times S : (\forall x, y \in S^1) ax = ay \Leftrightarrow bx = by\},$$

$$\mathcal{R}^* = \{(a, b) \in S \times S : (\forall x, y \in S^1) xa = ya \Leftrightarrow xb = yb\},$$

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$$\begin{aligned}\mathcal{H}^* &= \mathcal{L}^* \cap \mathcal{R}^*, \\ \mathcal{D}^* &= \mathcal{L}^* \vee \mathcal{R}^*, \\ (a, b) \in \mathcal{J}^* &\Leftrightarrow J^*(a) = J^*(b),\end{aligned}$$

where  $J^*(a)$  and  $J^*(b)$  are the principal  $*$ -ideals generated by  $a$  and  $b$  respectively.

In [3], Fountain investigated a class of semigroups called abundant semigroups in which each  $\mathcal{L}^*$ -class and each  $\mathcal{R}^*$ -class of  $S$  contain at least an idempotent. And from which, we know that, the class of regular semigroups are properly contained in the one of abundant semigroups.

According to [3], a semigroup in which every idempotent is primitive is said to be a primitive semigroup, and an abundant semigroup  $S$  is called a completely  $\mathcal{J}^*$ -simple semigroup if  $S$  itself is primitive and the idempotents of  $S$  generate a regular subsemigroup of  $S$ . Clearly, completely  $\mathcal{J}^*$ -simple semigroups extend completely simple semigroups studied by Clifford and Petrich in [2].

Later on, Ren and Shum [16] investigated the structure of superabundant semigroups, and generalized the corresponding results of completely regular semigroups in [15].

On the other hand, in order to further generalize completely regular semigroups [superabundant semigroups] in the class of rpp semigroups, Guo, Shum and Gong [10] introduced the so-called  $(*, \sim)$ -Green's relations on a semigroup  $S$ . The relations  $\mathcal{L}^{*, \sim}$  and  $\mathcal{R}^{*, \sim}$  are respectively defined as  $\mathcal{L}^*$  and  $\mathcal{R}$ . The intersection and the join of  $\mathcal{L}^{*, \sim}$  and  $\mathcal{R}^{*, \sim}$  are respectively denoted by  $\mathcal{H}^{*, \sim}$  and  $\mathcal{D}^{*, \sim}$ . The relation  $\mathcal{J}^{*, \sim}$  is defined by the rule that  $a \mathcal{J}^{*, \sim} b$  if and only if  $J^{*, \sim}(a) = J^{*, \sim}(b)$ , Where, for any  $a, b \in S$ ,  $a \mathcal{R} b$  if and only if for all  $e \in E(S)$ ,  $ea = a$  if and only if  $eb = b$ , and  $J^{*, \sim}(a)$  is the smallest ideal containing  $a$  and saturated by  $\mathcal{L}^{*, \sim}$  and  $\mathcal{R}^{*, \sim}$ .

According to [10], a semigroup  $S$  is called an  $r$ -ample semigroup if  $S$  is  $\mathcal{L}^{*, \sim}$ -abundant and  $\mathcal{R}^{*, \sim}$ -abundant, here we call that  $S$  is  $\sigma$ -abundant, if every equivalence  $\sigma$ -class of  $S$  contains idempotents of  $S$ . An  $r$ -ample semigroup is called a super- $r$ -ample semigroup, if  $S$  is  $\mathcal{H}^{*, \sim}$ -abundant. The class of super- $r$ -ample semigroups forms a proper extension class of the class of superabundant semigroups. It was shown in [10] that  $\mathcal{R}^{*, \sim}$  usually is not a left congruence on  $S$  even if  $S$  is an  $\mathcal{R}^{*, \sim}$ -abundant semigroup, but in a super- $r$ -ample semigroup  $S$ , the relation  $\mathcal{R}^{*, \sim}$  is a left congruence on  $S$ .

In [9], the authors defined a class of completely  $\mathcal{J}^{*, \sim}$ -simple semigroups, and give the structure of such semigroups which extended the celebrated Rees theorem for completely simple semigroups. According to [9], a super- $r$ -ample semigroup  $S$  is called a completely  $\mathcal{J}^{*, \sim}$ -simple semigroup if  $S$  is  $\mathcal{J}^{*, \sim}$ -simple. Clearly, a completely  $\mathcal{J}^*$ -simple semigroup must be completely  $\mathcal{J}^{*, \sim}$ -simple.

Note from [15] and [1] that, the translational hulls of completely simple semigroups and completely  $\mathcal{J}^*$ -simple semigroups have been solved, so naturally, we will quote such a question: what is the translational hull of completely  $\mathcal{J}^{*, \sim}$ -simple semigroups, do we have some similar results with the ones of completely  $\mathcal{J}^*$ -simple semigroups or completely simple semigroups?

In this paper, we will set out to discuss the above question, and finally establish a construction theorem about the translational hull of completely  $\mathcal{J}^{*, \sim}$ -simple semigroups which extend the translational hulls of completely  $\mathcal{J}^*$ -simple semigroups and completely simple semigroups.

For notations and terminologies not mentioned in this paper, the readers are referred to [5],[9] or [10].

## §2. Main Results

**Definition 2.1** ([6], Definition 1) *Let  $\mathcal{M}[T; I, \Lambda; P]$  be a Rees matrix semigroup and  $P$  the  $\Lambda \times I$  matrix over a left cancellative monoid  $T$ . Then  $P$  is said to be normalized at 1 if there is an element  $1 \in I \cap \Lambda$  such that  $p_{1i} = p_{\lambda 1} = e$  for all  $i \in I, \lambda \in \Lambda$ , where  $e$  is the identity of the left cancellative monoid  $T$ . Furthermore, the Rees matrix semigroup  $\mathcal{M}[T; I, \Lambda; P]$  is called normalized if  $P$  is normalized.*

**Lemma 2.2** ([6], Theorem 1) *Let  $T$  be a left cancellative monoid with an identity element  $e$  and  $I, \Lambda$  be nonempty sets. Let  $P = (p_{\lambda i})$  be a  $\Lambda \times I$  matrix where each entry in  $P$  is a unit of  $T$ . Suppose that  $P$  is normalized at  $1 \in I \cap \Lambda$ . Then the normalized Rees matrix semigroup  $\mathcal{M} = \mathcal{M}[T; I, \Lambda; P]$  is completely  $\mathcal{J}^{*,\sim}$ -simple semigroup.*

*Conversely, every completely  $\mathcal{J}^{*,\sim}$ -simple semigroup is isomorphic to a normalized Rees matrix semigroup  $\mathcal{M} = \mathcal{M}[T; I, \Lambda; P]$  over a left cancellative monoid  $T$ .*

By Lemma 2.2, we know that if  $S$  is a completely  $\mathcal{J}^{*,\sim}$ -simple semigroup, then it can be isomorphic to a normalized Rees matrix semigroup  $\mathcal{M} = \mathcal{M}[T; I, \Lambda; P]$  over a left cancellative monoid  $T$ . Hence, to discuss the translational hulls of completely  $\mathcal{J}^{*,\sim}$ -simple semigroups, we can also consider the cases of normalized Rees matrix semigroups  $\mathcal{M} = \mathcal{M}[T; I, \Lambda; P]$  over a left cancellative monoid  $T$  for convenience.

In the following, we will establish the translational hull of a normalized Rees matrix semigroup  $\mathcal{M} = \mathcal{M}[T; I, \Lambda; P]$  over a left cancellative monoid  $T$ . Before we give our main result, it will be useful to make use of the following notation.

**Notation** We set  $\mathcal{M}[T; I, \Lambda; P]$  with  $P$  normalized at  $1 \in I \times \Lambda$  and denoted by  $e$  the identity of  $T$ . Let  $T(S) = \{(F, t, \Phi) \in \mathcal{T}'(I) \times T \times \mathcal{T}(\Lambda) \mid \text{for all } i \in I, \mu \in \Lambda, p_{\mu, Fi} t p_{1\Phi, i} = p_{\mu, F1} t p_{\mu\Phi, i}\}$  with multiplication  $(F, t, \Phi)(F', t', \Phi') = (FF', t p_{1\Phi, F'1} t', \Phi\Phi')$  for all  $i \in I$  and  $\lambda \in \Lambda$ , where  $\mathcal{T}'(I)$  ( $\mathcal{T}(\Lambda)$ ) means the semigroup of all full transformations on  $I$  ( $\Lambda$ ) and all of the transformations are written on the left (right).

**Theorem 2.3** *Let  $S = \mathcal{M}[T; I, \Lambda; P]$  with  $P$  normalized, each entry in  $P$  is a unit of  $T$ , and let  $e$  be the identity of  $T$ . Define a mapping  $\sigma$  from  $\Omega(S)$  to  $T(S)$  by*

$$\sigma : (\lambda, \rho) \rightarrow (F, t, \Phi) \quad ((\lambda, \rho) \in \Omega(S))$$

where  $F, t$  and  $\Phi$  are defined by the requirements

$$\lambda(i, e, 1) = (Fi, \dots, \dots) \in S, \quad (1)$$

$$(1, e, 1)\rho = (\dots, t, \dots) \in S, \quad (2)$$

$$(1, e, \mu)\rho = (\cdots, \cdots, \mu\Phi) \in S. \quad (3)$$

Further define a mapping  $\tau$  from  $T(S)$  to  $\Omega(S)$  by

$$\tau : (F, t, \Phi) \rightarrow (\lambda, \rho) \quad ((F, t, \Phi) \in T(S)),$$

where  $\lambda$  and  $\rho$  are defined by the formulas

$$\lambda(i, h, \mu) = (Fi, tp_{1\Phi, i}h, \mu) \quad ((i, h, \mu) \in S), \quad (4)$$

$$(i, h, \mu)\rho = (i, hp_{\mu, F1}t, \mu\Phi) \quad ((i, h, \mu) \in S). \quad (5)$$

Then  $\sigma$  and  $\tau$  are mutually inverse isomorphisms between  $\Omega(S)$  and  $T(S)$ .

*Proof* We will show the theorem by the following steps.

(i)  $\sigma$  is a mapping.

Let  $(\lambda, \rho) \in \Omega(S)$ . For any  $(i, h, \mu) \in S$ , we have

$$\lambda(i, h, \mu) = \lambda[(i, h, 1)(1, e, \mu)] = [\lambda(i, h, 1)](1, e, \mu),$$

so that  $\lambda(i, h, \mu) = (j, h', \mu)$  for some  $j \in I$  and  $h' \in T$ . Similarly, we have  $(i, h, \mu)\rho = (i, h'', \nu)$  for some  $h'' \in T$  and  $\nu \in \Lambda$ . In the following, we will use the above statements repeatedly. In particular, we may define  $s_i$  and  $r_\mu$  by

$$\lambda(i, e, 1) = (Fi, s_i, 1) \quad (i \in I),$$

$$(1, e, \mu)\rho = (1, r_\mu, \mu\Phi) \quad (\mu \in \Lambda).$$

By the definition of  $t$  in this theorem, we have  $t = r_1$ . Also, notice that

$$[(1, e, 1)\rho](i, e, 1) = (1, t, 1\Phi)(i, e, 1) = (1, tp_{1\Phi, i}, 1),$$

$$(1, e, 1)[\lambda(i, e, 1)] = (1, e, 1)(Fi, s_i, 1) = (1, s_i, 1),$$

we have  $s_i = tp_{1\Phi, i}$ . Thus,

$$\begin{aligned} \lambda(i, h, \mu) &= \lambda[(i, e, 1)(1, h, \mu)] = [\lambda(i, e, 1)](1, h, \mu) \\ &= (Fi, s_i, 1)(1, h, \mu) = (Fi, tp_{1\Phi, i}, 1)(1, h, \mu) \\ &= (Fi, tp_{1\Phi, i}h, \mu). \end{aligned}$$

This proves (4). With a similar argument, we can establish (5). Hence,

$$(1, e, \mu)[\lambda(i, e, 1)] = (1, e, \mu)(Fi, tp_{1\Phi, i}, 1) = (1, p_{\mu, Fi}tp_{1\Phi, i}, 1),$$

$$[(1, e, \mu)\rho](i, e, 1) = (1, p_{\mu, F1}, \mu\Phi)(i, e, 1) = (1, p_{\mu, F1}tp_{\mu\Phi, i}, 1).$$

Since  $(\lambda, \rho) \in \Omega(S)$ , we have  $p_{\mu,Fi}tp_{1\Phi,i} = p_{\mu,F1}tp_{\mu\Phi,i}$ , and then  $(F, t, \Phi) \in T(S)$ , and (i) holds.

(ii)  $\tau$  is a mapping.

Let  $(F, t, \Phi) \in T(S)$ , and let  $\lambda$  and  $\rho$  be defined as (4) and (5) respectively. Then for  $(i, h, \mu), (j, k, \nu) \in S$ , we have

$$\begin{aligned} \lambda(i, h, \mu)(j, k, \nu) &= (Fi, tp_{1\Phi,i}h, \mu)(j, k, \nu) = (Fi, tp_{1\Phi,i}hp_{\mu j}k, \nu) \\ &= \lambda(i, hp_{\mu j}k, \nu) = \lambda[(i, h, \mu)(j, k, \nu)]. \end{aligned}$$

Hence,  $\lambda$  is a left translation. Similarly, we can show that  $\rho$  is a right translation. Further, on the one hand,

$$(i, h, \mu)[\lambda(j, k, \nu)] = (i, h, \mu)(Fj, tp_{1\Phi,j}k, \nu) = (i, hp_{\mu, Fj}tp_{1\Phi,j}k, \nu), \quad (6)$$

on the other hand,

$$[(i, h, \mu)\rho](j, k, \nu) = (i, hp_{\mu, F1}t, \mu\Phi)(j, k, \nu) = (i, hp_{\mu, F1}tp_{\mu\Phi,j}k, \nu), \quad (7)$$

and notice that  $(F, t, \Phi) \in T(S)$ , we can immediately obtain that (6) and (7) are equal. And then  $(\lambda, \rho) \in \Omega(S)$ . Thus,  $\tau$  is a mapping from  $T(S)$  to  $\Omega(S)$ , and (ii) holds.

(iii)  $\sigma\tau$  is an identity mapping on  $\Omega(S)$ .

Let  $(\lambda, \rho) \in \Omega(S)$ , and let  $(\lambda, \rho)\sigma\tau = (F, t, \Phi)\tau = (\lambda', \rho')$  so that  $\lambda'(i, h, \mu) = (Fi, tp_{1\Phi,i}h, \mu)$ . By the proof of (i), we know (4) holds for  $\lambda$ , thus, we have  $\lambda = \lambda'$ . Similarly, we have  $\rho = \rho'$ . Hence, (iii) holds.

(iv)  $\tau\sigma$  is an identity mapping on  $T(S)$ .

Let  $(F, t, \Phi) \in T(S)$ , and let  $(F, t, \Phi)\tau\sigma = (\lambda, \rho)\sigma = (F', t', \Phi')$ . Then (4) and (5) are satisfied, and thus  $\lambda(i, e, 1) = (Fi, tp_{1\Phi,1}1)$ ,  $(1, e, \mu)\rho = (1, p_{\mu, F1}t, \mu\Phi)$ . By (1), (2) and (3), we immediately obtain that  $F = F', t = t'$ , and  $\Phi = \Phi'$ . Consequently,  $\tau\sigma$  is the identity mapping on  $T(S)$ .

(v)  $\tau$  is a homomorphism.

Let  $(F, t, \Phi)\tau = (\lambda, \rho)$ ,  $(F', t', \Phi')\tau = (\lambda', \rho')$ , and  $(FF', tp_{1\Phi, F'1}t', \Phi\Phi')\tau = (\xi, \eta)$ . On the one hand, we have

$$\lambda\lambda'(i, h, \mu) = \lambda(F'i, t'p_{1\Phi',i}h, \mu) = (FF'i, tp_{1\Phi, F'1}t'p_{1\Phi',i}h, \mu). \quad (8)$$

On the other hand,

$$\xi(i, h, \mu) = (FF'i, tp_{1\Phi, F'1}t'p_{1\Phi\Phi',i}h, \mu). \quad (9)$$

Since  $(F', t', \Phi') \in T(S)$ , we have  $p_{1\Phi, F'i}t'p_{1\Phi',i} = p_{1\Phi, F'1}t'p_{1\Phi\Phi',i}$ , and then (8) and (9) are equal. That is,  $\lambda\lambda' = \xi$ . Similarly, we can prove that  $\rho\rho' = \eta$ . Therefore,  $\tau$  is a homomorphism.

(vi) Analogous with the proof of (v), we can prove that  $\sigma$  is a homomorphism.

Summing up the six steps above, we have shown that both  $\sigma$  and  $\tau$  are isomorphisms.  $\square$

**Remark 2.4** From Theorem 2.3, we know that, under the isomorphism, the translational hull of a normalized Rees matrix semigroup  $\mathcal{M} = \mathcal{M}[T; I, \Lambda; P]$  over a left cancellative monoid  $T$  can regard as the semigroup  $T(S)$ , whose elements and multiplications are defined in the Notation. And then by Lemma 2.2, the translational hull of a completely  $\mathcal{J}^{*,\sim}$ -simple semigroup can be also regarded as this form up to isomorphism.

Further, from Remark 1 in [9], we know that if  $S$  is an abundant semigroup, then  $\mathcal{R}^{*,\sim} = \mathcal{R}^*$ . Hence  $S$  is a completely  $\mathcal{J}^{*,\sim}$ -simple semigroup if and only if  $S$  is a completely  $\mathcal{J}^*$ -simple semigroup;  $S$  is a left cancellative monoid if and only if  $S$  is a cancellative monoid. If  $S$  is a regular semigroup, then  $\mathcal{R}^{*,\sim} = \mathcal{R}^*$ ,  $\mathcal{L}^{*,\sim} = \mathcal{L}^*$ . Hence  $S$  is a completely  $\mathcal{J}^{*,\sim}$ -simple semigroup if and only if  $S$  is a completely simple semigroup;  $S$  is a left cancellative monoid if and only if  $S$  is a group.

Now, if we let left cancellative monoid  $T$  be a cancellative monoid in Theorem 2.3, then we can immediately get the translational hull of a completely  $\mathcal{J}^*$ -simple semigroup which is the main theorem in [1].

**Corollary 2.5** *Let  $S = \mathcal{M}[T; I, \Lambda; P]$  with  $P$  normalized, each entry in  $P$  is a unit of cancellative monoid  $T$ , and let  $e$  be the identity of  $T$ . Define a mapping  $\sigma$  by*

$$\sigma : (\lambda, \rho) \rightarrow (F, t, \Phi) \quad ((\lambda, \rho) \in \Omega(S))$$

where  $F, t$  and  $\Phi$  are defined by the requirements

$$\lambda(i, e, 1) = (Fi, \dots, \dots) \in S, \quad (1)$$

$$(1, e, 1)\rho = (\dots, t, \dots) \in S, \quad (2)$$

$$(1, e, \mu)\rho = (\dots, \dots, \mu\Phi) \in S. \quad (3)$$

Further define a mapping  $\tau$  by

$$\tau : (F, t, \Phi) \rightarrow (\lambda, \rho) \quad ((F, t, \Phi) \in T(S)),$$

where  $\lambda$  and  $\rho$  are defined by the formulas

$$\lambda(i, h, \mu) = (Fi, tp_{1\Phi, i}h, \mu) \quad ((i, h, \mu) \in S), \quad (4)$$

$$(i, h, \mu)\rho = (i, hp_{\mu, F1}t, \mu\Phi) \quad ((i, h, \mu) \in S). \quad (5)$$

Then  $\sigma$  and  $\tau$  are mutually inverse isomorphisms between  $\Omega(S)$  and  $T(S)$ .

Also, if we let  $T$  be a group  $G$  in Theorem 2.3, then we can immediately get the translational hull of a completely simple semigroup which is the Theorem III.7.2 in [15].

**Corollary 2.6** Let  $S = \mathcal{M}[G; I, \Lambda; P]$  with  $P$  normalized, and let  $e$  be the identity of group  $G$ . Define a mapping  $\sigma$  by

$$\sigma : (\lambda, \rho) \rightarrow (F, g, \Phi) \quad (\lambda, \rho) \in \Omega(S)$$

where  $F, g$  and  $\Phi$  are defined by the requirements

$$\lambda(i, e, 1) = (Fi, \dots, \dots) \quad (1)$$

$$(1, e, 1)\rho = (\dots, g, \dots) \quad (2)$$

$$(1, e, \mu)\rho = (\dots, \dots, \mu\Phi) \quad (3)$$

Further define a mapping  $\tau$  by

$$\tau : (F, g, \Phi) \rightarrow (\lambda, \rho) \quad ((F, g, \Phi) \in T(S)),$$

where  $\lambda$  and  $\rho$  are defined by the formulas

$$\lambda(i, h, \mu) = (Fi, gp_{1\Phi, i}h, \mu) \quad (i, h, \mu) \in S, \quad (4)$$

$$(i, h, \mu)\rho = (i, hp_{\mu, F}g, \mu\Phi), \quad (i, h, \mu) \in S. \quad (5)$$

Then  $\sigma$  and  $\tau$  are mutually inverse isomorphisms between  $\Omega(S)$  and  $T(S)$ .

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