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# On $(m, s)$-Support Regular Graphs

N.R.Santhi Maheswari

PG and Research Department of Mathematics
G Venkataswamy Naidu College, Kovilpatti - 628 502, Tamil Nadu, India

L.Subhalakshmi

Research Scholar (Reg. No. 21212052092001)

PG and Research Department of Mathematics
G Venkataswamy Naidu College, Kovilpatti, Abishekapatti-627 012, Tirunelveli, Tamil Nadu, India

E-mail: nrsmaths@yahoo.com, lakshmansubha2@gmail.com

**Abstract**: We define $m-$support of vertex in a graph and $(m, s)$-support regular graph. Then it provides some results on $(2, s)-$support regular graphs and method to construct such graphs as in [6]. Graphs which are induced subgraphs of $(2, s)-$ support regular graphs are constructed and illustrated. The smallest order of $(2, s)-$ support regular graph containing $G$ and $G^c$ as induced subgraphs are constructed.

**Key Words**: Induced graph, distance degree regular graph, support regular, $(2, s)-$support regular graphs.

**AMS(2010)**: 05C12.

## §1. Introduction

Only simple connected graphs are considered. Distance d-regular graphs was introduced by [3]. The concept of 2-regular graphs was introduced by [2]. [15] proved that for any graph with the highest degree as r, $G$ is contained as an induced subgraph (IS) of a r-regular graph. [4] found the least required new vertices which when given to G, we get the r-regular graph. [5], [6] dealt in detail about $(r, m, k)-$regular graphs. [7] constructed minimal $(r, 2, k)-$regular graphs containing $G$ and $G^c$. With these ideas, we define m-support of a vertex, and detail about $(2, s)-$support regular graphs and its construction similar to [5]-[7].

## §2. Preliminaries

**Definition** 2.1 *$G$ is $(k, l)$ regular, when all node of $G$ is at distance k from exactly l nodes.*

**Definition** 2.2 *A graph is $(2, l)$ regular when $d_2(v_i) = 1$, for all nodes $v_i$.*

**Definition** 2.3 *A graph is $(r, k, l)$ regular whenever $d(v) = r$, and $d_k(v) = l$, for all $v \in V$.*

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**Definition** 2.4 *The support of a node is defined as the* $\sum$ *of degrees of its adjacent nodes. That is* $s(z) = \sum\limits_{zx \in E(G)} d(x)$.

**Definition** 2.5 *G is support regular, if the support of all the vertices are alike.*

## §3. $m$-Support of a Vertex

**Definition** 3.1 *The m-support of a node x in a graph G is given as* $s_m(x) = \sum\limits_{xy \in E(G)} d_m(y)$, *where* $d_m(y)$ *represent the counting of nodes at a distance m from y.*

**Definition** 3.2 *In G, 1-support of vertex* $s_1(v) = \sum\limits_{uv \in E(G)} d_1(u)$, *where* $d_1(u)$ *is count of vertices at distance 1 from u.*

**Definition** 3.3 *The* $2-$*support of a node is* $s_2(x) = \sum\limits_{xy \in E(G)} d_2(y)$ $d_2(y) =$ *count of nodes at distance 2 from y.*

**Definition** 3.4 *The* $3-$*support of node is* $s_3(v) = \sum\limits_{uv \in E(G)} d_3(u)$, $d_3(u) =$ *count of vertices at distance 3 from u.*

**Example** 3.5 Consider the following graph.

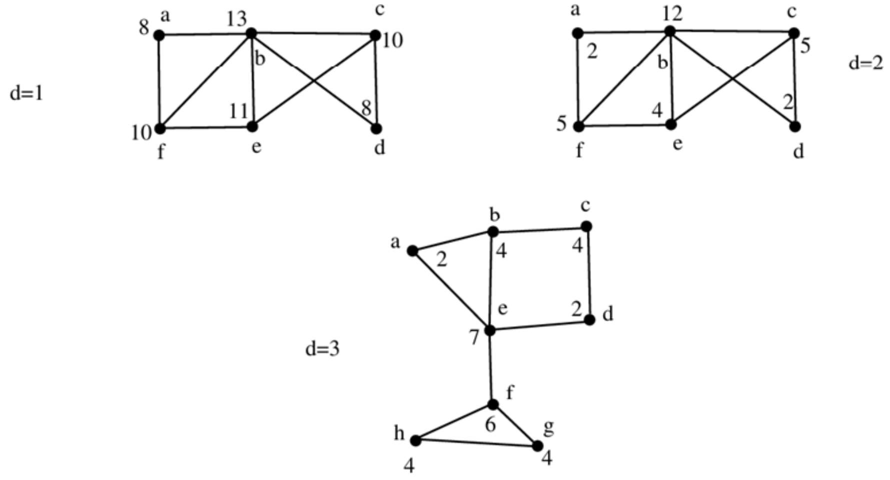


**Figure 1.** The support of vertices are denoted near them at respective distances

## §4. $(m, s)$-Support Regular Graphs

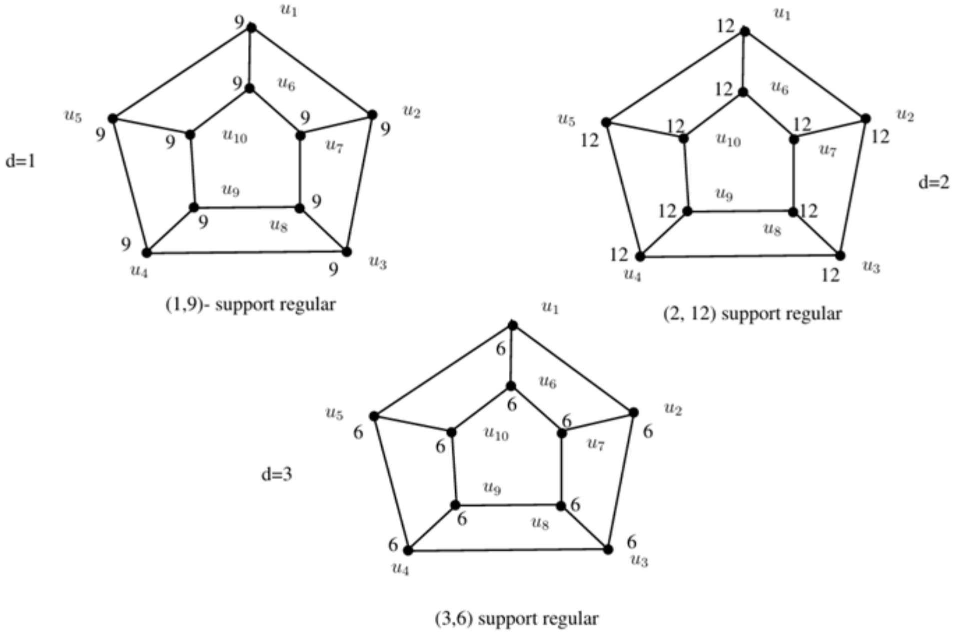
**Definition** 4.1 *G is said to be (m,s)-support regular, if* $s_m(v_i)$ *is same for all* $v_i \in V(G)$.

**Definition** 4.2 *G is (1,s)-support regular, if* $s_1(v_i)$ *are alike for all* $v_i \in V(G)$.

**Definition** 4.3 *$G$ is $(2,s)$ support regular, if $s_2(v_i)$ is same $\forall$ nodes in $G$.*

**Definition** 4.4 *A graph is (3,s)-support regular, if $s_3(v_i)$ is same for all $v_i \in V(G)$.*

**Example** 4.5 Some examples to represent the definitions.



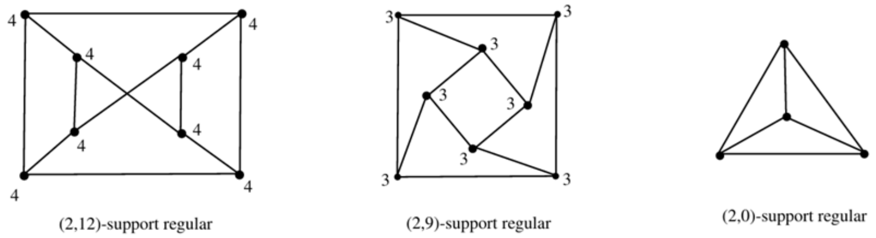
(1,9)- support regular

(2, 12) support regular

(3,6) support regular

**Figure 2.**

**Example** 4.6 All regular graphs which are $(2,s)$-support regular.



(2,12)-support regular

(2,9)-support regular

(2,0)-support regular

**Figure 3.**

**Remark** 4.7 There are no non-regular $(2,k)$ regular graphs which are $(2,s)$-support regular.

**Observation** 4.9 A connected graph is (2,0) support regular, iff it is complete graph $K_n, n \geq 1$.

**Proposition** 4.9 *Every $(r,m,k)$-regular graphs are $(m,rk)$-support regular.*

*Proof* Since $K$ is (r,m,k) regular, i.e., $d(x) = r, d_m(x) = k, \forall$ x in $G$.

Now, for $v \in V$, $s_m(v) = \sum\limits_{uv \in E} d_m(u) = k + k + \cdots + k$ ($r$ times) $= rk$. This is true for all vertices of $G$, i.e., $s_m(v_i) = rk$, $\forall v \in V(G)$.

**Theorem** 4.10 *A $(m, rk)$-support regular graph consist of atleast $r + k + 1$ vertices.*

*Proof* From Result 4.9, $(r, m, k)$ regular graphs are $(m, rk)-$support regular. And the number of vertices in $(r, m, k)$ regular graph is at least r+k+1. ⇒ the number of vertices in $(m, rk)$-support regular graphs is at least $r + k + 1$. □

**Observation** 4.11 The following are some (2,s)-support regular graphs.

(1) $K_{l,l}$ is (2, l(l-1))-support regular;

(2) Cycle $C_{2m}$ is (2,2)-support regular;

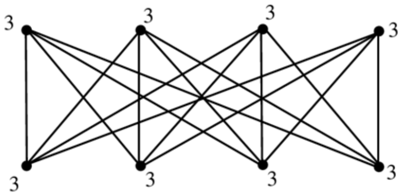
(3) Cycle $C_{2m+1}$ is (2,4)-support regular;

(4) Peterson graph is (2,18)-support regular;

(5) Any r-regular graph with diameter $< a$, is $(a, 0)-$support regular.

**Theorem** 4.12 *For $n \geq 1$, $\exists$ a $(2,(n-1))-$support regular graph with $|G| = 2n$.*

*Proof* For $r \geq 1$, a complete bipartite graph $K_{r,r}$ exist, which is $(2, r(r-1))$-support regular of order $2r$. □



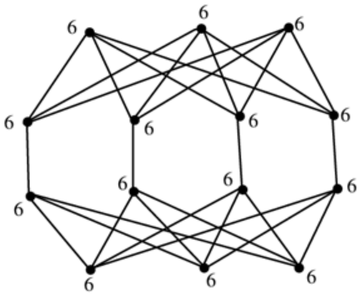
(2,12)-support regular

**Figure 4.**

**Theorem** 4.13 *For $n \geq 1$, $\exists$ a $(2, 2n(n-1))-$support regular graph, $|G| = 4n - 2$.*

*Proof* For any $n \geq 1$, let $F_n$ be formed by 2 disjoint replicas of $K_{n-1,n}$ by adding edge between the nodes of the two replica sets of size n. The new graph is $(2, 2n(n-1))-$support regular and $G = |4r - 2|$. □

**Example** 4.14 Putting $n = 4$ in above theorem we obtain a $(2, 24)$ support regular graph of order 14 shown in Figure 5.



(2,24) support regular

**Figure 5.**

**Observation** 4.15 For $n \geq 1$, the minimal cardinality of $(2, n(n-1))-$ support regular graph, say $H$, which contains $K_{n,n}$ of cardinality 2n is $H$.

**Theorem** 4.16 *Graphs of order* $n \geq 2$ *are induced subgraphs of* $(2, 2n(n+1))-$*support regular graphs of order* $5n$.

*Proof* Take G and $|G| = n \geq 2$, and $V(G) = \{y_1, y_2, \cdots, y_n\}$. Let $G_l$ denote replica of G with $V(G_l) = \left\{y_1^l, y_2^l, \cdots, y_n^l : 1 \leq l \leq 5\right\}$. $H_1$ as the graph whose vertex set is such as

$$V(H_1) = \bigcup_{l=1}^{5} V(G_l) = \left\{y_a^l : 1 \leq a \leq n, 1 \leq l \leq 5\right\}.$$

The edge set is given as

$$\begin{aligned} E(H_1) \quad = \quad & \bigcup_{l=1}^{5} E(G_l) \bigcup_{l=1}^{4} \left\{y_b^l y_a^{l+1}, y_b^5 y_a^1 : y_b^1 y_a^1 \notin E(G_1)\right\} \text{(where } 1 \leq b \leq n, b+1 \leq a \leq n \leq b) \\ & \bigcup_{f=1}^{n} \left\{y_f^a y_f^{a+1}, y_f^5 y_f^1 : 1 \leq a \leq 4\right\}. \end{aligned}$$

So, $H_1$ thus obtained contains G as IS. Now, in $H_1$, $d(y_a^l) = n+1, 1 \leq a \leq n$ and $d_2(y_a^l) = 2n, 1 \leq l \leq 5$. This implies $s_2(y_a) = \sum\limits_{y_a^l y_b^l \in E(H_1)} d_2(y_b) = 2n + 2n + \cdots$ $((n+1)$ times $= 2n(n+1))$. Thus, $H_1$ is $(2, 2n(n+1))-$support regular graph and $|H| = 5n$ containing G as an IS. $\square$

**Example** 4.17 A graph constructed based on above theorem for $n = 4$ is shown in Figure 6..

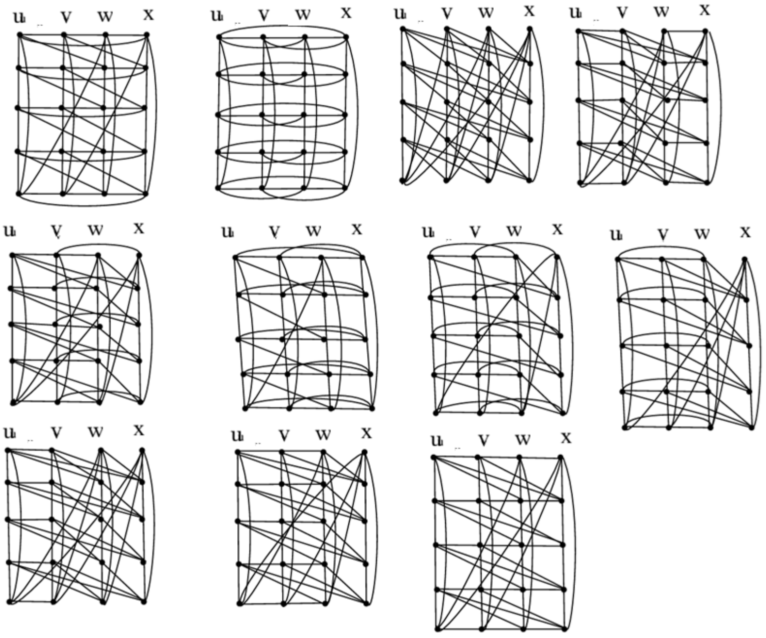


**Figure 6.**

**Corollary** 4.18 *Every $G$ of cardinality $n \geq 2$ are IS of $(2,(2n-2)\,n+1)$-support regular graphs with cardinality $3n$.*

*Proof* We set the value of l as $1 \leq l \leq 3$ in theorem 4.16, we obtain $H$ containing G as an IS. □

**Example** 4.19 An illustration of above corollary, here we have $n = 4$.

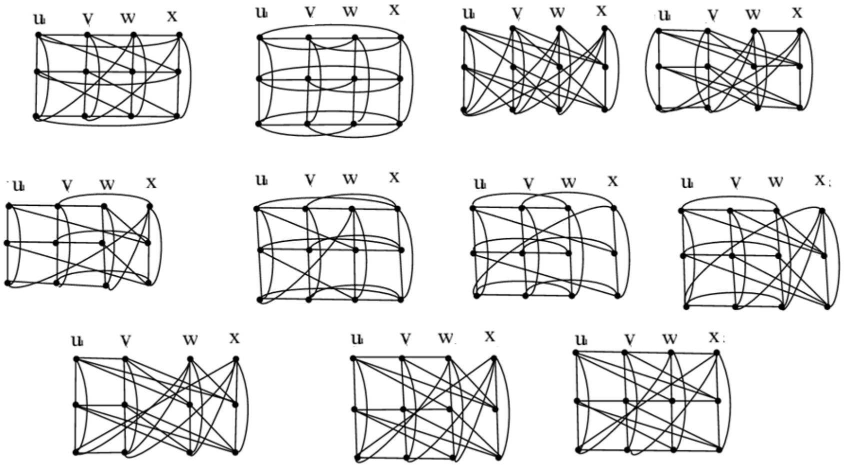


**Figure 7.**

**Corollary** 4.20 *Every $G$ with cardinality $n \geq 2$ are IS of $(2,(n+1)(2n-1))-$ support regular graphs with cardinality $4n$.*

*Proof* Taking 4 replica of $G$ in Theorem 4.16, we get the desired result. □

**Example** 4.21 An illustration of Corollary 4.20, for $n = 3$, we obtain the following $(2, 20)-$support regular graph, say $H$ and $|H| = 12$ which contains $G$ as an IS.

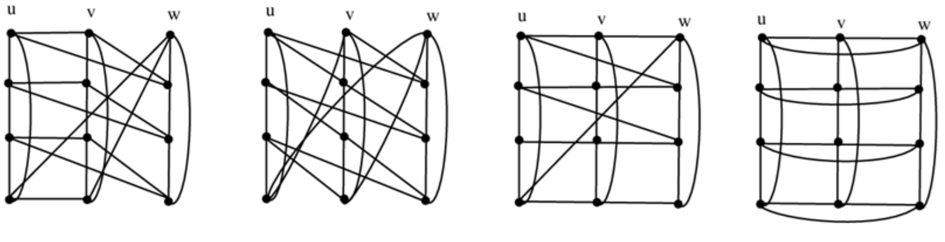


**Figure 8.**

**Theorem** 4.22 *For $r \geq 1$, every $G$ with $|G| = n \geq 2$ is an IS of $(2,(n+r-1)(nr-1))-$support regular graph of order $2nr$.*

*Proof* Let there be $G$ and $|G| = n \geq 2$, whose nodes are $V(G) = \{x_1, x_2, \cdots, x_n\}$. And we denotes the copies of $G$ as $G_l$, whose vertex set is such that $V(G_l) = \left\{x_1^l, x_2^l, \cdots, x_n^l : 1 \leq l \leq r.\right\}$.

And $G_{k+r}$ be another copy of given graph, where the vertex set is as

$$V(G_{k+r}) = \left\{y_1^k, y_2^k, \cdots, y_n^k : 1 \leq k \leq r\right\}.$$

Let $H$ denote the graph with vertex set as $V(H) = \left\{x_a^l, y_a^l : 1 \leq a \leq n, 1 \leq l \leq r\right\}$. Then, the lines of $H$ are given as

$$\begin{aligned} E(H) &= \bigcup_{l=1}^{2r} E(G_l) \bigcup_{l=1}^{r} \left\{x_b^l y_a^l, y_b^l x_a^l : x_b^1 x_a^1 \notin E(G_1), 1 \leq b \leq n, b+1 \leq a \leq n\right\} \\ & \bigcup_{f=1}^{n} \left\{x_f^a y_f^{a+b} : 1 \leq a \leq r, 0 \leq b \leq r-1\right\} \text{(superscripts are taken over modulo r).} \end{aligned}$$

The graph $H$ contains $G$ as an IS. Also, we have $d_H(x_i^l) = d_H(y_i^l) = n+r-1, (1 \leq l \leq r)$ and $d_2(x_i^l) = d_2(y_i^l) = nr-1, (1 \leq a \leq n)$ in $H$. Also, $s_2(x_a^l) = s_2(y_a^l) = (n+r-1)(nr-1)$ in $H$. Therefore $H$ is a $(2, (n+r-1)(nr-1))-$support regular graph. Hence the proof. $\square$

**Example** 6.23 For representation of above theorem, here we have $r = 3, n = 3$ in Figure 9.

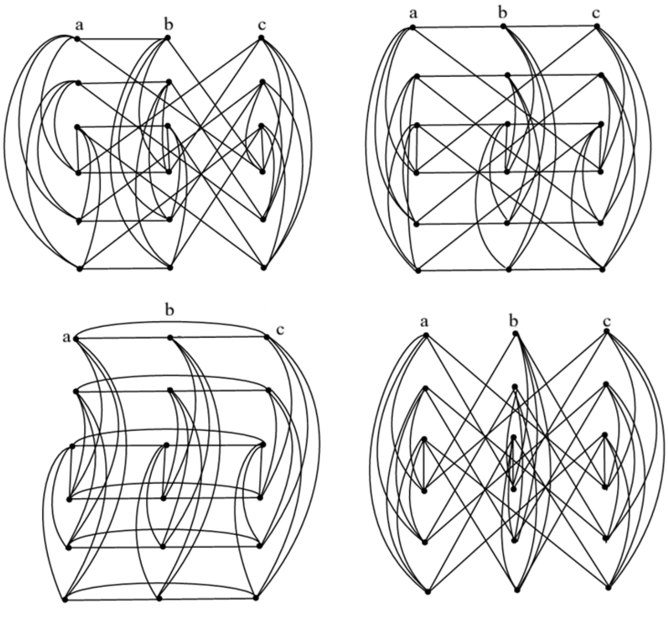


**Figure 9.**

**Corollary** 4.24 *Every graph with* $|G| = n \geq 2$, *is an IS of* $(2, n-1)-$*support regular graph with cardinality* $2n$.

*Proof* The case of theorem 4.22, where r=1. Take $G$ such that $|G| = n \geq 2$ with $V(G) = \{z_1, z_2, \cdots, z_n\}$. $G_1$ be copy of given graph with and $V(G_1) = \left\{z_1^1, z_2^1, \cdots, z_n^1\right\}$ and $G_2$ be another copy such that $V(G_2) = \left\{x_1^1, x_2^1, ...x_n^1\right\}$. Let $H$ be graph whose vertex set is $V(H) =$

$V(G_1)\bigcup V(G_2)$ and the edge set is

$$\begin{aligned} E(H) &= E(G_1)\bigcup E(G_2) \\ & \bigcup \left\{z_b^1x_a^1, x_b^1z_a^1 : z_b^1z_a^1 \notin E(G_1), 1 \leq b \leq n, b+1 \leq a \leq n\right\} \bigcup_{f=1}^{n} \left\{z_f^1x_f^1\right\}. \end{aligned}$$

Then, the graph $H$ contains given G as an IS. Also in $H$, $d_H(z_a^1) = d_H(x_a^1) = n$ and $d_2(z_a^1) = d_2(x_a^1) = n-1$, (i=1 to n) and $s_2(z_a^1) = s_2(x_a^1) = n(n-1)$. $H$ is therefore $(2, n(n-1))-$support regular graph with $|H| = 2n$ containing $G$ as an IS. $\square$

**Example** 4.25 The following explains the above corollary for $n = 4$.

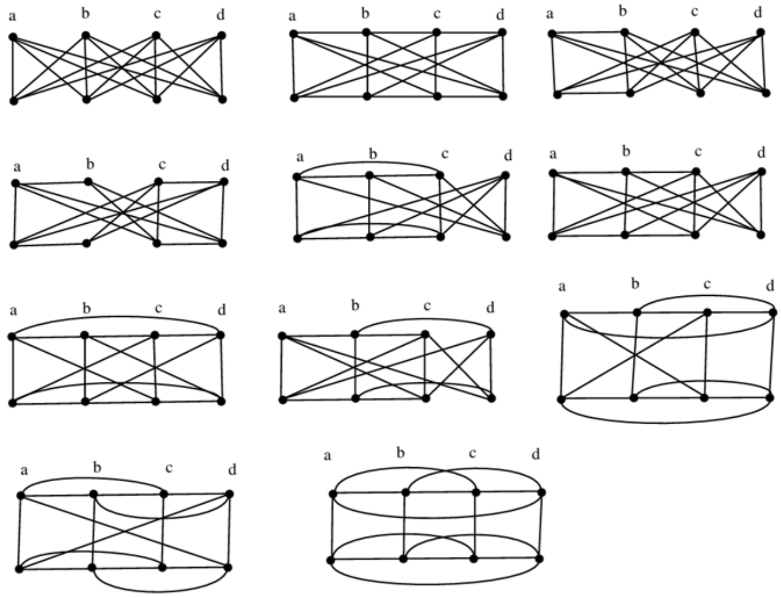


**Figure 10.**

**Corollary** 4.26 *All graphs with* $|G| = n \geq 2$ *are IS of a* $(2, (n+1)(2n-1))-$ *support regular graphs.*

**Corollary** 4.27 *All graphs of order* $n \geq 2$ *are IS of a* $(2, (n+2)(3n-1))-$ *support regular graphs.*

**Remark** 4.28 We can conclude that if $r = 1, 2, 3, \cdots n$, then the given graph of cardinality $n \geq 2$, is contained as $IS$ in $(2, n(n-1))$, $(2, (n+1)(2n-1)), \cdots, (2, 2n(n^2-1))-$ support regular graphs with respective cardinality as $2n, 4n, \cdots, 2n^2$.

## §5. Minimal $(2, s)$-Support Regular Graph containing $G$ as an IS

We construct the minimal $(2, s)-$support regular graph containing a G as an IS, taking inspirations from [5].

**Theorem** 5.1 *For $r > 1$, every $G$ with $|G| = n \geq 2$ is an IS of $(2,(n+r-2)(r-1)(n-1))-$ support regular graph of cardinality $nr$.*

*Proof* Consider $G$ with $|G| = n \geq 2$ and $V(G) = \{y_1, y_2, y_3, \cdots, y_n\}$. $G_l$ be replicas of given graph with $V(G_l) = \left\{y_1^l, y_2^l, \cdots, y_n^l\right\}$ such that l= 1 to r. Let $H$ be graph such that $V(H) = \bigcup\limits_{l=1}^{r} V(G_l)$. The lines of $H$ are

$$\begin{aligned} E(H) &= \bigcup_{l=1}^{r} E(G_l) \bigcup_{l=1}^{r-1} \left\{y_f^l y_e^{l+1}, y_f^r y_e^1 : y_f^1 y_e^1 \notin E(G_1)\right\} \ (f = 1 \text{ to } n, e = f + 1 \text{to } n) \\ & \bigcup_{g=1}^{n} \left\{y_g^e y_g^{e+f}\right\} \end{aligned}$$

where $e = 1$ to $r-1$, $f = 1$ to $r-e$. The resulting is a graph which contains the given graph as an IS. In $H$, $d(y_e^l) = n + r - 2$, (e = 1 to n, l= 1 to r) and of order nr.
The following cases are examined to find the $d_2$ degree and 2-support of the vertices.

**Case** 1. $l = 1$.

Suppose $y \in V(G_1)$, then $y = y_f^1$, for some f. Let $y_f^1 \notin$ [1-neighbourhood of $y_e^1$] but in $V(H)$. Then by construction join $y_f^1$ to $y_e^2$ and $y_e^2$ with $y_e^1$. Then we have $y_f^1$ in [2-neighbourhood of $y_e^1$]. We can infer that $V(H)-$ [1-neighbourhood of $y_e^1$] $\subseteq$ [2- neighbourhood of $y_e^1$].

Consider $y_f^1 \in$ [$2-$neighbourhood of $y_e^1$], then $y_f^1$ not connected to $y_e^1$ in [1-neighbourhood], thus $y_f^1 \in V(H)-$ [1-neighbourhood of $y_e^1$], thus [2-neighbourhood of $y_e^1$] $\subseteq V(H)$- [1-neighbourhood of $y_e^1$], thus [2-neighbourhood of $y_e^1$] $= V(H)$-[1-neighbourhood of $y_e^1$]. Now, $d_2(y_e^1) = (r-1)(n-1)$, where a= 1 to n. The support of vertices are $s_2(y_a^1) = (n+r-2)(n-1)(r-1)$.

**Case** 2. $2 \leq l \leq r-1$.

If $y \in V(G_l)$, then $y = y_f^l$, for some f. In this also we obtain $d_2(y_e^l) = (r-1)(n-1)$, where $e = 1$ to n. The support of vertices are $s_2(y_e^l) = (n+r-2)(n-1)(r-1)$.

**Case** 3. $l = r$.

If $y \in V(G_l)$, then $y = y_f^r$, for some f. Similar to above cases, we get $d_2(v_e^r) = (m-1)(n-1), 1 \leq e \leq n$. The support of the vertices is $s_2(v_e^r) = (n+r-2)(r-1)(n-1)$.

Similarly for $2 \leq l \leq r$, $d_2(y_e^l) = (r-1)(n-1)$, for $1 \leq e \leq n$ and $s_2(y_e^l) = (n+r-2)(r-1)(n-1)$. Therefore $H$ is $(2,(n+r-2)(r-1)(n-1))-$support regular graph containing $G$ as an IS. This completes the proof. □

**Corollary** 5.2 *For $r > 1$, the smallest order of a $(2,(n+r-2)(r-1)(n-1))-$support regular graph containing $G$ with $|G| = n \geq 2$ as an IS, is $nr$.*

*Proof* Consider $H$ constructed in the above theorem which is $(2,(n+r-2)(r-1)(n-1))-$support regular and $|H| = nr$. Suppose it is of order nr-1, i.e., for each $y_e \in H$, $d_2(y_e) = (r-1)(n-1)$ and $s_2(y_e) = (n+r-2)(r-1)(n-1)$. Then $H$ has at least $n+r-2+(r-1)(n-1)+1 = nr$ vertices, which contradicts. Hence the proof. □

**Corollary** 5.3 *Every graph where* $n \geq 2$ *is an IS of a* $(2, n(n-1))-$*support regular graph of minimal cardinality* $2n$.

**Corollary** 5.4 *All graphs with order* $n \geq 2$ *is IS of a* $(2, (n+1)2(n-1))-$*support regular graph of minimal cardinality* $3n$.

**Example** 5.5 The above corollary is explained as follows, for $n = 2, m = 3$.

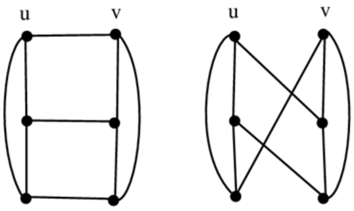


**Figure 11.**

**Corollary** 5.6 *Every* $G$ *such that* $|G| = n \geq 2$ *is an IS of a* $(2, (n+2)3(n-1))-$*support regular graph of minimal order* $4n$.

**Remark** 5.7 When $m$ takes values as $2, 3, 4, 5, \cdots, n$ there exist $(2, n(n-1))$, $(2, (n+1)2(n-1)), \cdots, (2, (2n-2)(n-1)^2)-$support regular graphs of minimalistic order $2n, 3n, 4n, \cdots, nn^2, \cdots$, respectively containing G of order $n \geq 2$ as IS.

## §6. **Minimal $(2, s)$-Support Regular Graph Containing $G$ and $G^c$ as IS**

In this section, as in [7] we construct the minimally existing $(2, s)$ support regular graph containing $G$ and $G^c$ as IS.

**Theorem** 6.1 *For any* $G$*, where* $|G| = n \geq 2$*,* $\exists$ $(2, (r+2n-2)(r-1)(2n-1))-$ *support regular graph, say* $H$ *with* $|H| = 2nr$ *such that it contains* $G$ *and* $G^c$ *as IS.*

*Proof* Take $G$, such that $|G| = n \geq 2$, then node set of $G$ and $G^c$ are alike and let them be $\{x_a^1\}$ where a=1 to n. Let $\breve{G}$, isomorphic to $G^c$ and $V(\breve{G}) = \{y_a^1\}$ and a= 1 to n, also $y_a^1$ correspond to $x_a^1$ ($a = 1$ to $n$). Consider $G_1 = G \bigcup \grave{G}$. Now, $V(G_1) = \{x_a^1, y_a^1\}$ and $a = 1$ to $n$. $G_l$, $t = 2$ to $l$ denote $(l-1)$ copies of $G_1$ and $V(G_l) = \{x_a^l, y_a^l\}$ and $a = 1$ to $n$, $l = 2$ to $r$ and $x_a^l, y_a^l$, ($l = 2$ to $r$) correspond to $x_a^1 y_a^1$ ($a = 1$ to $n$), respectively. The new graph $H$ has the vertex set $V(H) = \bigcup\limits_{l=1}^{r} V(G_l)$, and

$$E(H) = \bigcup_{l=1}^{r} E(G_l) \bigcup_{l=1}^{r-1} \{x_b^l x_a^{r+1}, x_l^r x_a^1 : x_b^1 x_a^1 \notin E(G_1); y_b^l y_a^{l+1}, y_b^r y_a^1 : y_b^1 y_a^1 \notin E(G_1)\}$$
$$(b = 1 \text{ to } n, a = b+1 \text{ to } n)$$

$$\bigcup_{f=1}^{n} \left\{ y_f^a y_f^{a+b}; y_f^a y_f^{a+b} \right\} \ (a = 1 \text{ to } r-1, b = 1 \text{ to } r-a)$$

$$\bigcup_{l=1}^{r-1} \left\{ x_b^l y_a^{l+1}, x_b^l y_a^1 \right\} \ (a, b = 1 \text{ to } n).$$

The resulting graph $H$ contains $G_1$ as IS and in $H$, $d(x_a^l) = r + 2(n-1)$, a= 1 to n, l= 1 to r and $H$ is graph of order 2nr and it contains $G$ and $G^c$ as IS.

The calculation of $d_2$ and $s_2$ of nodes is in the below.

**Case** 1. $l = 1$, if $x \in V(G_1)$ then $x \in V(G)$ or $x \in V(\breve{G})$.

**Subcase 1.1** If $v \in V(G)$, then $x = x_b^1$, for any b. Let $x_b^1 \notin$ [1-neighbourhood of $x_a^1$] but in $V(H)$. Then by construction join $x_b^1$ to $x_a^2$ and $x_a^2$ with $x_a^1$. Then we have $x_b^1$ in [2-neighbourhood of $x_a^1$]. We can infer that $V(H)-$ (1-neighbourhood of $x_a^1$) $\subseteq$ [2-neighbourhood of $x_a^1$].

Consider $x_b^1 \in$ [2-neighbourhood of $x_a^1$], then $x_b^1$ not connected to $x_a^1$ in [1-neighbourhood], thus $x_b^1 \in V(H)$- [1-neighbourhood of $x_a^1$], thus [2-neighbourhood of $x_a^1$] $\in V(H)$-[1-neighbourhood of $x_a^1$], thus [2-neighbourhood of $x_a^1$] $= V(H)$-[1-neighbourhood of $x_a^1$]. Now, $d_2(x_a^1) = (r-1)(n-1)$, where $a = 1$ to $n$. The support of vertices are

$$s_2(x_a^1) = (n + r - 2)(n-1)(r-1).$$

**Subcase 1.2** If $v \in V(\breve{G})$, then $x = x_b^1$, for any b. As in above case, we get $d_2(x_a^1) = (r-1)(2n-1), 1 \leq a \leq n$. The support of the vertices is

$$s_2(x_a^1) = (r + 2n - 2)(r-1)(2n-1).$$

The similar case follows for vertices when $2 \leq l \leq r-1$ and $x \in V(G_l)$ and also when $l = r$ and $x \in V(G_l)$. In all these cases we have $H$ is a $(2, (r+2n-2)(r-1)(2n-1))-$support regular graph with cardinality 2nr containing $G$ and $G^c$ as IS. □

**Corollary** 6.2 *For all $m \geq 1$, the minimal order of $(2, (m+2n-2)(m-1)(2n-1))-$support regular graph which contains $G$ and $G^c$ is $2nr$.*

*Proof* Let us take the graph $H$ as constructed in above theorem, such that $|H| =$ 2nr. Suppose $|H| = 2mn - 1$. For each node, $s_2(v_i) = (m+2n-2)(m-1)(2n-1) \Rightarrow d_2(v_i) = (m-1)(2n-1)$ and $d(v_i) = m + 2n - 2$. Therefore, $H$ has at least

$$(m + 2n - 2) + (m-1)(2n-1) + 1 = 2mn$$

vertices, which contradicts. □

**Corollary** 6.3 *Any $G$ with $|G| = n \geq 2$, then $G$ and $G^c$ are IS of $(2, 2n(2n-1))-$ support regular graph of cardinality $4n$.*

**Example** 6.4 An illustration of above corollary, for $n = 2$ is shown in Figure 12.

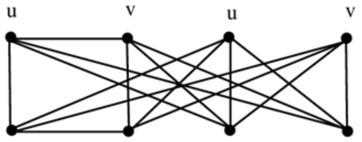


**Figure 12.**

**Corollary** 6.5 *Any $G$ with $|G| = n \geq 2$, then $G$ and $G^c$ are IS of* $(2, 2n + 1(2n - 1))-$ *support regular graphs of minimalistic order* $6n$.

**Example** 6.6 An representation of above corollary is shown in Figure 13.

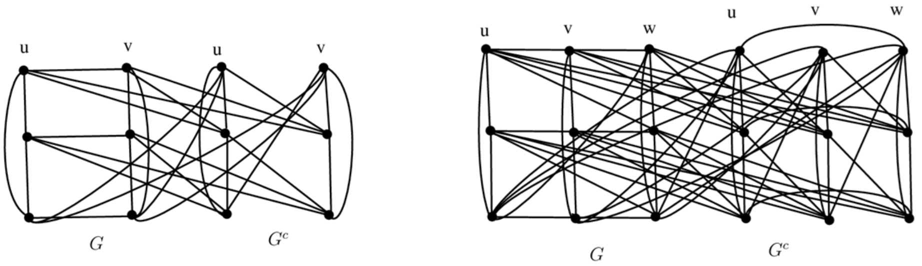


**Figure 13.**

**Remark** 6.7 Setting values for m as, $m = 4, \cdots$ then, there exist $(2, (2n+3)4(2n-1)), (2, (2n+4)5(2n-1)) \cdots -$support regular graphs of minimalistic order $8n, 10n, \cdots$ respectively containing $G$, $(|G| = n \geq 2)$ and $G^c$ as IS.

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