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On the SuperHyperStructure and Uncertain SuperHyperStructure

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Abstract: In this paper, we introduce the SuperHyperStructure and Neutrosophic SuperHyperStructure on n-th powerset of a set H and discuss how to generalize the classical structures, also the HyperStructures to the more general case, i.e., the SuperHyperStructures and Neutrosophic SuperHyperStructure.

Key Words: HyperStructure, SuperHyperStructure, Neutrosophic SuperHyperStructure, PowerSet.

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§1. Introduction

Notice that the *n-th powersets* that are the bases on the *SuperHyperStructure* and *Uncertain SuperHyperStructure* [2, 3, 15], together with their particular cases such as the *SuperHyperAlgebra* and *Uncertain SuperHyperAlgebra* (endowed with *SuperHyperOperations* and *SuperHyper-Axioms*) [2016, 2022], *SuperHyperGraph* (including *SuperHyperTree*) and *Uncertain SuperHyperGraph* (including *Uncertain SuperHyperTree*) [2019 - 2022], *SuperHyperSoft Set*, *SuperHyperFunction* and *Uncertain SuperHyperFunction* [2022], *SuperHyperTopology* (all built on the powersets of $P(H)$, or $P^n(H)$, for $n \geq 1$) and *Uncertain SuperHyperTopology* [2022] were all founded by Smarandache [2] and developed between 2016 - 2024.

By *Uncertain* we mean all types of Fuzzy and fuzzy-extensions *Intuitionistic Fuzzy*, *Inconsistent Intuitionistic Fuzzy* (*Picture Fuzzy, Ternary Fuzzy*), *Pythagorean Fuzzy (Atanassov's Intuitionistic Fuzzy* of second type), *q-Rung Orthopair Fuzzy*, *Spherical Fuzzy*, *n-HyperSpherical Fuzzy,Neutrosophic,Refined Neutrosophic,Plithogenic*, etc.) Sets/Logics/Measures/Probability/ Statistics.

Definition 1.1 *A SuperHyperStructure is a structure built on the n-th PowerSet of a Set H, for $n \geq 1$, as in our real world (because a set (or system) H (that may be a set of items, an organization, country, etc.) is composed by sub-sets that are parts of $P(H)$, which in their turn are organized in sub-sub-sets that are parts of $P(P(H)) = P^2(H)$, then in sub-sub-sub-sets that are parts of $P^3(H)$, and so on, $P^{n+1}(H) = P(P^n(H))$.*

The powerset $P(H)$ means all non-empty and empty subsets of H, including the empty set

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$\emptyset$, which represents the indeterminacy that occurs into the set H (as in our real world where we deal with unclear/indeterminate/unknwon information in any space/set); and similarly for $P^n(H)$.

While $P_*(H)$ means all non-empty subsets of H, or $P_*(H) = P(H) - \emptyset$. And similarly for $P_*^n(H)$.

A structure built on $P_*^n(H)$ is called a SuperHyperStructure (has no indeterminacy), while built on $P^n(H)$ it is called Neutrosophic SuperHyperStructure (does have indeterminacy).

In a SuperHyperStructure we deal with SuperHyperAxioms, SuperHyperLaws, SuperHyperTheorems, SuperHyperProperties, SuperHyperOperators, etc. Similarly for a Neutrosophic SuperHyperStructure.

§2. “Hyper” and “Super” Prefixes

The prefix “*Hyper*” (Marty [1], 1934) stand for the codomain of the functions and operations to be $P(H)$, or the powerset of the set H.

While the prefix “*Super*” (Smarandache [2], 2016) stands for using the $P^n(H), n \geq 2$, or the n-th PowerSet of the Set H because the set (or system) H (that may be a set of items, a company, institution, country, region, etc.) is organized in sub-systems that are part of $P(H)$, which in their turn are organized in sub-sub-systems, that are part of $P(P(H)) = P^2(H)$ and so on in the domain and/or codomain of the functions and operations and axioms.

§3. SuperHyperStructure

A *SuperHyperStructure* [2, 2016] is a structure built on the n-th powerset $P_*^n(H)$ of a non-empty set H, for integer $n \geq 1$, whose *SuperHyperOperators* are defined as follows:

$$\sharp_{SHS} : (P_*^r(H))^m \to P_*^n(H),$$

where $P_*^r(H)$ is the r-powerset of H, for integer $r \geq 1$, while similarly $P_*^n(H)$ is the n-th powerset of H, and the SuperHyperAxioms act on it. Indeterminacy is not allowed on this structure.

§4. Neutrosophic SuperHyperStructure

A *Neutrosophic SuperHyperStructure* is a structure built on the n-th powerset $P^n(H)$ of a non-empty set H, for integer $n \geq 1$, whose *Neutrosophic SuperHyperOperators* are defined as follows:

$$\sharp_{SHS} : (P^r(H))^m \to P^n(H),$$

where $P^r(H)$ is the r-powerset of H, for integer $r \geq 1$, while similarly $P^n(H)$ is the n-th powerset of H, and the SuperHyperAxioms act on it.

Indeterminacy is allowed on this structure.

§5. History: From the classic and HyperStructures to SuperHyperStructures

(i) Classical Structure

A classical *Structure* is built on a non-empty set H, whose classical operations $\sharp_S$ are defined as:

$$\sharp_S : H^m \to H, \quad \text{for integer } m \geq 1,$$

and with classical Axioms acting on it.

(ii) Classical HyperStructure

A classical *HyperStructure* (Marty [1], 1934) is built on a non-empty set H, whose *HyperOperations* are defined as:

$$\sharp_{HS} : H^m \to P*(H),$$

where $P*(H)$ is the powerset of H, without the empty set, and the HyperAxioms acting on it.

(iii) Neutrosophic HyperStructure

The *Neutrosophic SuperHyperStructure* is an extension of the Classical HyperStructure, because it allows indeterminacy (uncertainty, unknown), denoted by the empty set $\emptyset$, into the the powerset of H, that it is denoted by $P(H)$, without $*$.

(iv) SuperHyperStructure

A *SuperHyperStructure* is built on n-th powerset $P^n(H)$ of a non-empty set H, for integer $n \geq 1$, whose *SuperHyperOperators* are defined as follows:

$$\sharp_{SHS} : (P^r(H))^m \to P^n(H),$$

where $P^r(H)$ is the r-powerset of H, for integer $r \geq 1$, while similarly $P^n(H)$ is the n-th powerset of H, and the SuperHyperAxioms act on it.

Indeterminacy is not allowed in this structure.

(v) Neutrosophic SuperHyperStructure

A *Neutrosophic SuperHyperStructure* is built on n-th powerset $P^n(H)$ of a non-empty set H, for integer $n \geq 1$, whose *SuperHyperOperators* are defined as follows:

$$\sharp_{SHS} : (P^r(H))^m \to P^n(H),$$

where $P^r(H)$ is the r-powerset of H, for integer $r \geq 1$, while similarly $P^n(H)$ is the n-th powerset of H, and the SuperHyperAxioms act on it.

Indeterminacy (denoted by the empty set $\emptyset$) is allowed in this structure.

(vi) The n-th powerset of H with indeterminacy (denoted by the empty set $\emptyset$)

The n-th powerset $P^n(H)$ of the set H, that the Neutrosophic SuperHyperStructure is built on, best describe our real world where always the indeterminacy occurs, and a set (or system) H (that may be a set of items, company, institution, country, etc.) is organized in sub-systems, which in their turn are organized each of them in sub-sub-systems, and so on.

The n-th PowerSet $P^n(H)$ is defined recursively:

$$\begin{aligned}
P^0(H) &\stackrel{\text{def}}{=} H, \\
P^1(H) &= P(H), \\
P^2(H) &= P(P(H)), \\
P^3(H) &= P(P^2(H)) = P(P(P(H))), \\
\cdots \quad \cdots & \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots, \\
P^n(H) &= P(P^{n-1}(H)) = \underbrace{P(P(\cdots P(H)\cdots))}_{n},
\end{aligned}$$

where P is repeated n times into the last formula, and the empty set $\emptyset$ (that represents indeterminacy, uncertainty) is allowed in all sequence terms: $H, P^1(H), P^2(H), P^3(H), \cdots, P^n(H)$.

(vii) The n-th powerset of H without Indeterminacy

The n-th powerset $P_*^n(H)$ of the set H, that the SuperHyperStructure is built on, describe a world that does not contain indeterminacy , where similarly the set (or system) H (that may be a set of items, a company, institution, country, region, etc.) is organized in sub-systems, which in their turn are organized each of them in sub-sub-systems, and so on.

The n-th powerSet $P_*^n(H)$ is also defined recursively:

$$\begin{aligned}
P_*^0(H) &\stackrel{\text{def}}{=} H, \\
P_*^1(H) &= P_*(H), \\
P_*^2(H) &= P_*(P_*(H)), \\
P_*^3(H) &= P_*(P_*^2(H)) = P_*(P_*(P_*(H))), \\
\cdots \quad \cdots & \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots, \\
P_*^n(H) &= P_*(P_*^{n-1}(H)) = \underbrace{P_*(P_*(\cdots P_*(H)\cdots))}_{n},
\end{aligned}$$

where P_* is repeated n times into the last formula, and the empty set $\emptyset$ (that represents indeterminacy, uncertainty) is not allowed in none of the sequence terms:

$$H, P_*^1(H), P_*^2(H), P_*^3(H), \cdots, P_*^n(H).$$

The n-th powerSet $P^n(H)$ and $P_*^n(H)$ of a non-empty set H, were introduced by Smarandache [2] in 2016.

§6. Particular Cases of SuperHyperStructure, Neutrosophic SuperHyperStructure

Depending on the type of Structure, in any field of knowledge, on has multiple types of *SuperHyperStructures* and respectively their corresponding *Neutrosophic SuperHyperStructures*, such as:

– *SuperHyperAlgebra* (when the structure is an Algebra), *SuperHyperGraph* (when the structure is a Graph), *SuperHyperGeometry* (when the structure is a type of Geometry), *SuperHyperCalculus* etc.

Further on, a *SuperHyperAlgebra* may be as particular cases: *SuperHyper Semigroup*, *SuperHyper Group*, *SuperHyper Field*, etc.

A *SuperHyperGeometry* may be as particular cases: *SuperHyper EuclideanGeometry*, *Superhyper NonEuclideanGeometry*, *SuperHyper ProjectiveGeometry*, etc.

See *https://fs.unm.edu/SHS/* for more details on the SuperHyperStructure and Neutrosophic SuperHyperStructure.

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