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Pair Difference Cordial Labeling of Franklin Graph, Heawood Graph, Tietze Graph and Durer Graph

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Abstract: Let $G = (V, E)$ be a (p, q) graph. Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ called the set of labels. Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1. A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph. In this paper we investigate the pair difference cordial labeling behavior of Pair Difference Cordial Labeling of Franklin graph, Heawood graph, Tietze graph and Durer graph.

Key Words: Path, cycle, wheel, gear graph, ladder.

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§1. Introduction

In this paper we consider only finite, undirected and simple graphs. Cordial labeling was introduced in [2] by Cahit. Also cordial related labeling was studied in [15,16]. In [7] the notion of pair difference cordial labeling of a graph was introduced and also the pair difference cordial labeling behaviour of path, cycle, star, ladder have been studied. In [8,9,10,11,12,13,14], the

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pair difference cordial labeling behaviour of snake related graph and butterfly graph have been investigated. In this paper we have study about the pair difference cordiality of some named graphs like Franklin graph, Heawood graph, Tietze graph and Durer graph.

§2. Preliminaries

Definition 2.1([3]) *The corona graph $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and n copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy G_2, where G_1 is graph of order n.*

Definition 2.2([4]) *Let C_n be the cycle $a_1a_2a_3\cdots a_na_1$ where $n \equiv 0(mod12)$. Then FG_n is the graph with the vertex set $V(FG_n) = V(C_n)$ and $E(FG_n) = E(C_n) \cup\{a_{4i+1}a_{4i+4} : 0 \leq i \leq \frac{n-4}{4}\}$ $\cup\{a_{4i+3}a_{4i+10} : 0 \leq i \leq \frac{n-8}{4}\}\cup\{a_2a_{n-5}, a_{n-1}a_6\}$. FG_n has n vertices and $\frac{3n}{2}$ edges. Note that FG_{12} is called the Franklin graph.*

Definition 2.3([3]) *Let C_n be the cycle $a_1a_2a_3\cdots a_na_1$ where $n \equiv 0(mod14)$. Then HG_n is the graph with the vertex set $V(HG_n) = V(C_n)$ and $E(HG_n) = E(C_n) \cup\{a_{2i+1}a_{2i+6} : 0 \leq i \leq \frac{n-4}{2}\}$ $\cup\{a_2a_{n-3}, a_{n-1}a_4\}$. HG_n has n vertices and $\frac{3n}{2}$ edges. The graph HG_{14} is called the Heawood graph which is given in Figure 1.*

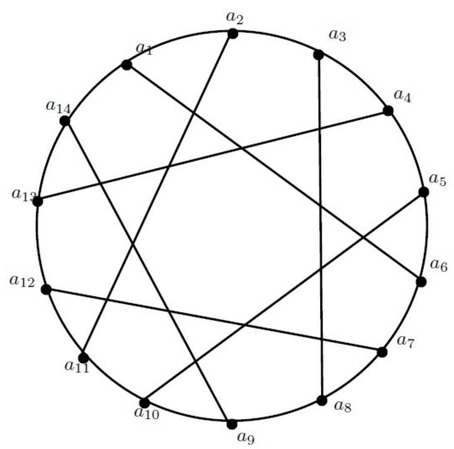

Figure 1

Definition 2.4([3]) *Let C_n be the cycle $v_1v_2v_3\cdots v_nv_1$ where $n \equiv 0(mod9)$. Then TG_n is the graph with the vertex set $V(TG_n) = V(C_n) \cup\{u_{3i-2} : 1 \leq i \leq \frac{n}{3}\}$ and $E(TG_n) = E(C_n)$ $\cup\{u_{3i-2}v_{3i-2} : 1 \leq i \leq \frac{n}{3}\}\cup\{u_{3i-1}u_{3i+3} : 1 \leq i \leq \frac{n-3}{3}\} \cup\{u_{n-1}u_3\}$. TG_n has $\frac{4n}{3}$ vertices and 2n edges. Note that TG_9 is called the Tietze graph.*

Definition 2.5([3]) *Let C_n be the cycle $v_1v_2v_3\cdots v_nv_1$ where $n \equiv 0(mod6)$. Then DG_n is the graph with the vertex set $V(DG_n) = V(C_n) \cup\{g_i : 1 \leq i \leq n\}$ and $E(DG_n) = E(C_n)$ $\cup\{g_iv_i : 1 \leq i \leq n\}\cup\{g_ig_{i+2} : 1 \leq i \leq n-2\} \cup\{g_ng_2, g_{n-1}g_1\}$. DG_n has 2n vertices and 3n edges. The graph DG_6 is called the Durer graph.*

§3. Main Results

Theorem 3.1 *FG_n is pair difference cordial for all values of $n \equiv 0(mod12)$.*

Proof Take the vertex set and edge set from definition 2.2.

Assign the labels $1, 2, 3, \cdots, \frac{n}{2}$ to the vertices $a_1, a_2, a_3, \cdots, a_{\frac{n}{2}}$ respectively and assign the labels $-1, -2, -3, , \cdots, -\frac{n}{2}$ respectively to the vertices $a_{\frac{n+2}{2}}, a_{\frac{n+4}{2}}, a_{\frac{n+6}{2}}$. Next assign the labels $-4, -6, -5, -7$ respectively to the vertices $a_{\frac{n+8}{2}}, a_{\frac{n+10}{2}}, a_{\frac{n+12}{2}}, a_{\frac{n+14}{2}}$ and assign the labels $-8, -10, -9, -11$ respectively to the vertices $a_{\frac{n+16}{2}}, a_{\frac{n+18}{2}}, a_{\frac{n+20}{2}}, a_{\frac{n+22}{2}}$. Proceeding like this until we reach the vertex a_n. Here $\Delta_{f_1^c} = \Delta_{f_1} = \frac{3n}{4}$. □

Theorem 3.2 *HG_n is pair difference cordial for all values of $n \equiv 0(mod14)$.*

Proof Take the vertex set and edge set from Definition 2.3.

Assign the labels $1, 2, 3, \cdots, \frac{n}{2}$ to the vertices $a_1, a_2, a_3, \cdots, a_{\frac{n}{2}}$ respectively and assign the labels $-1, -2, -3, , \cdots, -\frac{n}{2}$ respectively to the vertices $a_{\frac{n+2}{2}}, a_{\frac{n+4}{2}}, a_{\frac{n+6}{2}}$. Next assign the labels $-4, -6, -5, -7$ respectively to the vertices $a_{\frac{n+8}{2}}, a_{\frac{n+10}{2}}, a_{\frac{n+12}{2}}, a_{\frac{n+14}{2}}$ and assign the labels $-8, -10, -9, -11$ respectively to the vertices $a_{\frac{n+16}{2}}, a_{\frac{n+18}{2}}, a_{\frac{n+20}{2}}, a_{\frac{n+22}{2}}$. Proceeding like this until we reach the vertex a_n.

The Table 1 given below establish that this vertex labeling f is a pair difference cordial of HG_n for $n \equiv 0(mod14)$.

Nature of $n = 14k$	$\Delta_{f_1^c}$	Δ_{f_1}
$n = 14k$, k is odd	$\frac{3n-2}{4}$	$\frac{3n+2}{4}$
$n = 14k$, k is even	$\frac{3n}{4}$	$\frac{3n}{4}$

Table 1

This completes the proof. □

Theorem 3.3 *TG_n is pair difference cordial for all values of $n \equiv 0(mod9)$.*

Proof Take the vertex set and edge set from Definition 2.4.

Assign the labels $1, 2, 3, \cdots, \frac{2n}{3}$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{2n}{3}}$ and assign the labels $-1, -2, -3, \cdots, -\frac{n}{3}$ to the vertices $v_{\frac{2n+3}{3}}$, $v_{\frac{2n+6}{3}}$, $v_{\frac{2n+9}{3}}$, $\cdots, v_n$ respectively. Next assign the labels to the vertices $u_i, 1 \leq i \leq n$. There are two cases arises.

Case 1. n is even.

Assign the labels $-\frac{n+3}{3}, -\frac{n+9}{3}, -\frac{n+15}{3}, \cdots, -(\frac{2n}{3} - 3)$ respectively to the vertices $u_1, u_2, u_3, \cdots, u_{\frac{n-3}{3}}$ and assign the labels $-\frac{n+6}{3}, -\frac{n+12}{3}, -\frac{n+18}{3}, \cdots, -(\frac{2n}{3} - 2)$ to the vertices $u_{\frac{n}{3}}$, $u_{\frac{n+3}{3}}$, $u_{\frac{n+6}{3}}$, $\cdots, u_{n-2}$ respectively. Finally assign the labels $-(\frac{2n}{3} - 1), -(\frac{2n}{3})$ respectively to the vertices u_{n-1}, u_n.

Case 2. n is odd.

Assign the labels $-\frac{n+6}{3}, -\frac{n+12}{3}, -\frac{n+18}{3}, \cdots, -(\frac{2n}{3} - 3)$ respectively to the vertices $u_1, u_2, u_3, \cdots, u_{\frac{n-3}{3}}$ and assign the labels $-\frac{n+3}{3}, -\frac{n+9}{3}, -\frac{n+15}{3}, \cdots, -(\frac{2n}{3} - 2)$ to the vertices $u_{\frac{n+3}{3}}$, $u_{\frac{n+3}{3}}$,

$u_{\frac{n+6}{3}}$, $\cdots$, u_{n-2} respectively. Lastly assign the labels $-(\frac{2n}{3}-1)$,$-(\frac{2n}{3})$ respectively to the vertices u_{n-1}, u_n. In both cases, we get $\Delta_{f_1^c} = \Delta_{f_1} = n$. □

Theorem 3.4 *DG_n is pair difference cordial for all values of $n \equiv 0(mod 6)$.*

Proof Take the vertex set and edge set from Definition 2.5.

Assign the labels $1, 2, 3, \cdots, n$ to the vertices $v_1, v_2, v_3, \cdots, v_n$ respectively and assign the labels $-1, -2, -3, , \cdots, -\frac{n}{2}$ respectively to the vertices g_1,g_3, g_5, $\cdots$,g_{n-1}. Now assign the labels $-\frac{n+4}{2}$,$-\frac{n+8}{2}$,$-\frac{n+12}{2}$, $\cdots$,$-(n-3)$ respectively to the vertices g_2,g_4, g_6, $\cdots$,$g_{\frac{n-6}{2}}$ and assign the labels $-\frac{n+2}{2}$,$-\frac{n+6}{2}$,$-\frac{n+10}{2}$, $\cdots$,$-(n-2)$ respectively to the vertices $g_{\frac{n-2}{2}}$,$g_{\frac{n+2}{2}}$,$g_{\frac{n+6}{2}}$,$\cdots$, g_{n-2}. Finally assign the labels $-(n-1), -n$ respectively to the vertices g_{n-1}, g_n.

Clearly $\Delta_{f_1^c} = \Delta_{f_1} = \frac{3n}{2}$. A pair difference cordial labeling of DG_{18} is given in Figure 2.

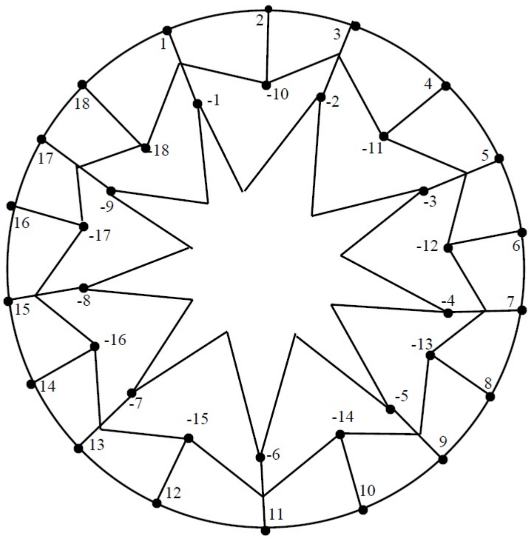

Figure 2

Theorem 3.5 *$FG_n \odot K_1$ is pair difference cordial for all values of $n \equiv 0(mod 12)$.*

Proof Let $V(FG_n \odot K_1) = V(FG_n) \cup \{x_i : 1 \leq i \leq n\}$ and $E(FG_n \odot K_1) = E(FG_n) \cup \{a_i x_i : 1 \leq i \leq n\}$. Note that $FG_n \odot K_1$ has $2n$ vertices and $\frac{5n}{2}$ edges.

Assign the labels $1, 5, 9, \cdots, n-3$ respectively to the vertices $x_1, x_3, x_5, \cdots, x_{n-1}$ and assign the labels $4, 8, 12, \cdots, n$ to the vertices $x_2, x_4, x_6, \cdots, x_n$ respectively. Next assign the labels $2, 6, 10, \cdots, n-2$ respectively to the vertices $a_1, a_3, a_5, \cdots, a_{n-1}$ and assign the labels $3, 7, 11, \cdots, n-1$ to the vertices $a_2, a_4, a_6, \cdots, a_n$ respectively.

Now assign the labels $-1, -3, -5, \cdots, -(n-1)$ respectively to the vertices $x_1, x_2, x_3, \cdots, x_n$ and assign the labels $-2, -4, -6, \cdots, -(n)$ to the vertices $a_1, a_2, a_3, \cdots, a_n$ respectively.

Clearly $\Delta_{f_1^c} = \Delta_{f_1} = \frac{5n}{4}$. □

Theorem 3.6 *$HG_n \odot K_1$ is pair difference cordial for all values of $n \equiv 0(mod 14)$.*

Proof Let $V(HG_n \odot K_1) = V(HG_n) \cup \{x_i : 1 \leq i \leq n\}$ and $E(HG_n \odot K_1) = E(HG_n) \cup$

$\{a_ix_i : 1 \leq i \leq n\}$. Note that $HG_n \odot K_1$ has $2n$ vertices and $\frac{5n}{2}$ edges.

There are two cases arises.

Case 1. $n = 14k$ where k is odd.

Assign the labels $1, 5, 9, \cdots, n-5$ respectively to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-4}{2}}$ and assign the labels $4, 8, 12, \cdots, n-2$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n-2}{2}}$ respectively. Next assign the labels $-1, -3, -5, \cdots, -(n-3)$ respectively to the vertices $x_{\frac{n}{2}}$, $x_{\frac{n+2}{2}}$,$x_{\frac{n+4}{2}}$,$\cdots, x_{n-2}$ and assign the labels $-2, -4, -6, \cdots, -(n-2)$ to the vertices $a_{\frac{n}{2}}$, $a_{\frac{n+2}{2}}$,$a_{\frac{n+4}{2}}$,$\cdots, a_{n-2}$ respectively.

Now assign the labels $2, 6, 10, \cdots, n-4$ respectively to the vertices $a_1, a_3, a_5, \cdots, a_{\frac{n-4}{2}}$ and assign the labels $3, 7, 11, \cdots, n-3$ to the vertices $a_2, a_4, a_6, \cdots, a_{\frac{n-2}{2}}$ respectively. Lastly assign the labels $n-1, n, -(n-1), -n$ respectively to the vertices $a_{n-1}, a_n, x_{n-1}, x_n$.

Case 2. $n = 14k$ where k is even.

Assign the labels $1, 5, 9, \cdots, n-3$ respectively to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-2}{2}}$ and assign the labels $4, 8, 12, \cdots, n$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n}{2}}$ respectively. Next assign the labels $-1, -3, -5, \cdots, -(n-1)$ respectively to the vertices $x_{\frac{n+2}{2}}$, $x_{\frac{n+4}{2}}$,$x_{\frac{n+6}{2}}$,$\cdots, x_n$ and assign the labels $-2, -4, -6, \cdots, -(n)$ to the vertices $a_{\frac{n+2}{2}}$, $a_{\frac{n+4}{2}}$,$a_{\frac{n+6}{2}}$,$\cdots, a_n$ respectively.

Now assign the labels $2, 6, 10, \cdots, n-2$ respectively to the vertices $a_1, a_3, a_5, \cdots, a_{\frac{n-2}{2}}$ and assign the labels $3, 7, 11, \cdots, n-1$ to the vertices $a_2, a_4, a_6, \cdots, a_{\frac{n}{2}}$ respectively. Lastly assign the labels $n-1, n, -(n-1), -n$ respectively to the vertices $a_{n-1}, a_n, x_{n-1}, x_n$.

The Table 2 given below establish that this vertex labeling f is a pair difference cordial of $HG_n \odot K_1$, $n \equiv 0(mod 14)$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
$n = 14k$, k is odd	$\frac{5n+2}{4}$	$\frac{5n-2}{4}$
$n = 14k$, k is even	$\frac{5n}{4}$	$\frac{5n}{4}$

Table 2

This completes the proof. □

Theorem 3.7 *$DG_n \odot K_1$ is pair difference cordial for all values of $n \equiv 0(mod 6)$.*

Proof Let $V(DG_n \odot K_1) = V(DG_n) \cup \{x_i, y_i : 1 \leq i \leq n\}$ and $E(DG_n \odot K_1) = E(DG_n) \cup \{g_ix_i, v_iy_i : 1 \leq i \leq n\}$. Note that $HG_n \odot K_1$ has $4n$ vertices and $5n$ edges.

Assign the labels $1, 5, 9, \cdots, 2n-3$ respectively to the vertices $y_1, y_3, y_5, \cdots, y_{n-1}$ and assign the labels $4, 8, 12, \cdots, 2n$ to the vertices $y_2, y_4, y_6, \cdots, y_n$ respectively. Next assign the labels $2, 6, 10, \cdots, 2n-2$ respectively to the vertices $v_1, v_3, v_5, \cdots, v_{n-1}$ and assign the labels $3, 7, 11, \cdots, 2n-1$ to the vertices $v_2, v_4, v_6, \cdots, v_n$ respectively.

Now assign the labels $-1, -3, -5, \cdots, -(2n-1)$ respectively to the vertices $x_1, x_2, x_3, \cdots, x_n$ and assign the labels $-2, -4, -6, \cdots, -(2n)$ to the vertices $g_1, g_2, g_3,$ $\cdots, g_n$ respectively.

Obviously $\Delta_{f_1^c} = \Delta_{f_1} = \frac{5n}{2}$. □

Theorem 3.8 *$TG_n \odot K_1$ is pair difference cordial for all values of $n \equiv 0(mod 9)$.*

Proof Let $V(TG_n \odot K_1) = V(TG_n) \cup \{x_i, y_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n}{3}\}$ and $E(TG_n \odot K_1) = E(TG_n) \cup \{v_i x_i, u_j y_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n}{3}\}$. Note that $TG_n \odot K_1$ has $\frac{8n}{3}$ vertices and $\frac{10n}{3}$ edges. Our proof is divided into three cases.

Case 1. $n \equiv 0 (mod 36)$.

Assign the labels $1, 5, 9, \cdots, n-3$ respectively to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-2}{2}}$ and assign the labels $4, 8, 12, \cdots, n$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n}{2}}$ respectively. Next assign the labels $2, 6, 10, \cdots, n-2$ respectively to the vertices $v_1, v_3, v_5, \cdots, v_{\frac{n-2}{2}}$ and assign the labels $3, 7, 11, \cdots, n-1$ to the vertices $v_2, v_4, v_6, \cdots, v_{\frac{n}{2}}$ respectively.

Assign the labels $-1, -5, -9, \cdots, -(n-3)$ respectively to the vertices $x_{\frac{n+2}{2}}, x_{\frac{n+6}{2}}, x_{\frac{n+10}{2}}, \cdots, x_{n-1}$ and assign the labels $-4, -8, -12, \cdots, -n$ to the vertices $x_{\frac{n+4}{2}}, x_{\frac{n+8}{2}}, x_{\frac{n+12}{2}}, \cdots, x_n$ respectively. Next assign the labels $-2, -6, -10, \cdots, -(n-2)$ respectively to the vertices $v_{\frac{n+2}{2}}, v_{\frac{n+6}{2}}, v_{\frac{n+10}{2}}, \cdots, v_{n-1}$ and assign the labels $-3, -7, -11, \cdots, -(n-1)$ to the vertices $v_{\frac{n+4}{2}}, v_{\frac{n+8}{2}}, v_{\frac{n+12}{2}}, \cdots, v_n$ respectively.

Now we assign the labels $(n+1), (n+3), (n+5), \cdots, \frac{4n-3}{3}$ respectively to the vertices $u_1, u_2, u_3, \cdots, u_{\frac{n}{6}}$ and assign the labels $(n+2), (n+4), (n+6), \cdots, \frac{4n}{3}$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n}{6}}$ respectively. Next assign the labels $-(n+1), -(n+5), -(n+9), \cdots, -(\frac{4n-9}{3})$ respectively to the vertices $u_{\frac{n+6}{6}}, u_{\frac{n+12}{6}}, u_{\frac{n+18}{6}}, \cdots, u_{\frac{3n}{12}}$ and assign the labels $-(n+3), -(n+7), -(n+11), \cdots, -(\frac{4n-3}{3})$ respectively to the vertices $y_{\frac{n+6}{6}}, y_{\frac{n+12}{6}}, y_{\frac{n+18}{6}}, \cdots, y_{\frac{3n}{12}}$. Now assign the labels $-(n+2), -(n+6), -(n+10), \cdots, -(\frac{4n-6}{3})$ respectively to the vertices $u_{\frac{3n+12}{12}}, u_{\frac{3n+24}{12}}, u_{\frac{3n+36}{12}}, \cdots, u_n$ and assign the labels $-(n+4), -(n+8), -(n+12), \cdots, -(\frac{4n}{3})$ respectively to the vertices $y_{\frac{3n+12}{12}}, y_{\frac{3n+24}{12}}, y_{\frac{3n+36}{12}}, \cdots, y_n$.

Case 2. $n \equiv 9 (mod 36)$.

Assign the labels $1, 5, 9, \cdots, n-4$ respectively to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-3}{2}}$ and assign the labels $4, 8, 12, \cdots, n-1$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n-1}{2}}$ respectively. Next assign the labels $2, 6, 10, \cdots, n-3$ respectively to the vertices $v_1, v_3, v_5, \cdots, v_{\frac{n-3}{2}}$ and assign the labels $3, 7, 11, \cdots, n-2$ to the vertices $v_2, v_4, v_6, \cdots, v_{\frac{n-1}{2}}$ respectively.

Assign the labels $-1, -5, -9, \cdots, -(n-4)$ respectively to the vertices $x_{\frac{n+1}{2}}, x_{\frac{n+5}{2}}, x_{\frac{n+9}{2}}, \cdots, x_{n-2}$ and assign the labels $-4, -8, -12, \cdots, -(n-1)$ to the vertices $x_{\frac{n+3}{2}}, x_{\frac{n+7}{2}}, x_{\frac{n+11}{2}}, \cdots, x_{n-1}$ respectively. Next assign the labels $-2, -6, -10, \cdots, -(n-3)$ respectively to the vertices $v_{\frac{n+1}{2}}, v_{\frac{n+5}{2}}, v_{\frac{n+9}{2}}, \cdots, v_{n-2}$ and assign the labels $-3, -7, -11, \cdots, -(n-2)$ to the vertices $v_{\frac{n+3}{2}}, v_{\frac{n+7}{2}}, v_{\frac{n+11}{2}}, \cdots, v_{n-1}$ respectively. Assign the labels $n, -n$ respectively to the vertices v_n, x_n.

Now we assign the labels $(n+1), (n+3), (n+5), \cdots, \frac{4n-6}{3}$ respectively to the vertices $u_1, u_2, u_3, \cdots, u_{\frac{n-3}{6}}$ and assign the labels $(n+2), (n+4), (n+6), \cdots, \frac{4n-3}{3}$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-3}{6}}$ respectively. Next assign the labels $-(n+1), -(n+5), -(n+9), \cdots, -(\frac{4n-18}{3})$ respectively to the vertices $u_{\frac{n+3}{6}}, u_{\frac{n+9}{6}}, u_{\frac{n+15}{6}}, \cdots, u_{\frac{n-5}{4}}$ and assign the labels $-(n+3), -(n+7), -(n+11), \cdots, -(\frac{4n-12}{3})$ respectively to the vertices $y_{\frac{n+3}{6}}, y_{\frac{n+9}{6}}, y_{\frac{n+15}{6}}, \cdots, y_{\frac{n-5}{4}}$. Now assign the labels $-(n+2), -(n+6), -(n+10), \cdots, -(\frac{4n-15}{3})$ respectively to the vertices $u_{\frac{n-1}{4}}, u_{\frac{n+3}{4}}, u_{\frac{n+7}{4}}, \cdots, u_{\frac{n-6}{3}}$ and assign the labels $-(n+4), -(n+8), -(n+12), \cdots, -(\frac{4n-9}{3})$ respectively to the vertices $y_{\frac{n-1}{4}}, y_{\frac{n+3}{4}}, y_{\frac{n+7}{4}}, \cdots, y_{\frac{n-6}{3}}$. Now assign the labels $-(\frac{4n-3}{3}), -(\frac{4n-6}{3}), -(\frac{4n}{3}), (\frac{4n}{3})$

respectively to the vertices $u_{\frac{n-3}{3}}, y_{\frac{n-3}{3}}, u_{\frac{n}{3}}, y_{\frac{n}{3}}$.

Case 3.$n \equiv 18(mod 36)$.

Assign the labels $1, 5, 9, \cdots, n-5$ respectively to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n42}{2}}$ and assign the labels $4, 8, 12, \cdots, n-3$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n-2}{2}}$ respectively. Next assign the labels $2, 6, 10, \cdots, n-4$ respectively to the vertices $v_1, v_3, v_5, \cdots, v_{\frac{n-4}{2}}$ and assign the labels $3, 7, 11, \cdots, n-2$ to the vertices $v_2, v_4, v_6, \cdots,$
$v_{\frac{n-2}{2}}$ respectively.

Assign the labels $-1, -5, -9, \cdots, -(n-5)$ respectively to the vertices $x_{\frac{n}{2}}, x_{\frac{n+4}{2}}, x_{\frac{n+8}{2}},$ $\cdots, x_{n-3}$ and assign the labels $-4, -8, -12, \cdots, -(n-3)$ to the vertices $x_{\frac{n+2}{2}}, x_{\frac{n+6}{2}},$ $x_{\frac{n+10}{2}}, \cdots, x_{n-2}$ respectively. Next assign the labels $-2, -6, -10, \cdots, -(n-4)$ respectively to the vertices $v_{\frac{n}{2}}, v_{\frac{n+4}{2}}, v_{\frac{n+8}{2}}, \cdots, v_{n-3}$ and assign the labels $-3, -7, -11,$ $\cdots, -(n-2)$ to the vertices $v_{\frac{n+2}{2}}, v_{\frac{n+6}{2}}, v_{\frac{n+10}{2}}, \cdots, v_{n-2}$ respectively. Next we assign the labels $n-1, n, -(n-1), -n$ respectively to the vertices $x_{n-1}, v_{n-1}, x_n, v_n$.

Now we assign the labels $(n+1), (n+3), (n+5), \cdots, \frac{4n-3}{3}$ respectively to the vertices $u_1, u_2, u_3, \cdots, u_{\frac{n}{6}}$ and assign the labels $(n+2), (n+4), (n+6), \cdots, \frac{4n}{3}$ to the vertices $y_1, y_2, y_3,$ $\cdots, y_{\frac{n}{6}}$ respectively. Next assign the labels $-(n+1), -(n+5), -(n+9), \cdots, -(\frac{4n-12}{3})$ respectively to the vertices $u_{\frac{n+6}{6}}, u_{\frac{n+12}{6}}, u_{\frac{n+18}{6}}, \cdots, u_{\frac{n-2}{4}}$ and assign the labels $-(n+3), -(n+7), -(n+11), \cdots, -(\frac{4n-6}{3})$ respectively to the vertices $y_{\frac{n+6}{6}}, y_{\frac{n+12}{6}}, y_{\frac{n+18}{6}}, \cdots, y_{\frac{n-2}{4}}$. Now assign the labels $-(n+2), -(n+6), -(n+10), \cdots, -(\frac{4n-15}{3})$ respectively to the vertices $u_{\frac{n+2}{4}}, u_{\frac{n+6}{4}}, u_{\frac{n+10}{4}},$ $\cdots, u_{\frac{n-3}{3}}$ and assign the labels $-(n+4), -(n+8), -(n+12), \cdots, -(\frac{4n-9}{3})$ respectively to the vertices $y_{\frac{n+2}{4}}, y_{\frac{n+6}{4}}, y_{\frac{n+10}{4}}, \cdots, y_{\frac{n-3}{3}}$. Finally assign the labels $-(\frac{4n-3}{3}), -(\frac{4n}{3})$ to the vertices $u_{\frac{n}{3}}, v_{\frac{n}{3}}$.

In all the three cases, $\Delta_{f_1^c} = \Delta_{f_1} = \frac{10n}{6}$. □

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