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Reciprocal Status-Distance Index of Mycielskian and its Complement

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Abstract: The reciprocal status-distance (RSD) index of a connected graph G is defined as

$$RSD(G) = \sum_{\{u,v\} \subseteq V(G)} \frac{\sigma(u) + \sigma(v)}{d_G(u,v)},$$

where, $\sigma(u) = \sum_{v \in V(G)} d_G(u,v)$ is the status of a vertex u in $V(G)$. In this paper, we find RSD index of Mycielskian graphs and its complement in terms of Zagreb indices.

Key Words: Distance, status of a vertex, reciprocal status-distance index, Mycielskian graph.

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§1. Introduction

Consider the graph G, that has n vertices and m edges. Let's call its vertex and edge sets $V(G)$ and $E(G)$, respectively. The number of edges joining a vertex u in a graph G is indicated by $deg_G(u)$, which represents the degree of that vertex. The distance between the vertex u and v is given by $d_G(u,v)$, which is the length of the shortest path connecting u and v. The diameter of G is the largest distance between any two vertices in G and is denoted by $diam(G)$. For a graph theoretic terminology, we refer the books [4, 18].

A chemical graph is a graph in which the vertices represent atoms and the edges represent bonds between those atoms in a chemical structure. A topological index for a (*chemical*) graph G is a numerical quantity invariant under automorphisms of G and it does not depend on the labeling or pictorial representation of the graph. Topological indices and graph invariants based on the distances between vertices of a graph or vertex degrees are widely used for characterizing molecular graphs, establishing relationships between structure and properties of molecules, predicting biological activity of chemical compounds and making their chemical applications [3].

The status [16] of a vertex $u \in V(G)$ is defined as the sum of its distance from every other

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vertex in $V(G)$ and is denoted by $\sigma(u)$. That is,

$$\sigma(u) = \sum_{v\in V(G)} d_G(u,v).$$

More results and applications on status related indices can be found in [19, 20, 25, 29, 16].

The Wiener index $W(G)$ of a connected graph G is defined as the sum of the distances between all pairs of vertices of G [35]. That is,

$$W(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u,v) = \frac{1}{2}\sum_{u\in V(G)} \sigma(u).$$

The Wiener index is also called as gross status [16] and total status [4]. For more about the Wiener index one can refer [5, 8, 15, 28, 30, 31, 35].

The first and second Zagreb indices of a graph G are defined as [38]

$$M_1(G) = \sum_{uv\in E(G)} [d_G(u) + d_G(v)]$$

and

$$M_2(G) = \sum_{uv\in E(G)} [d_G(u)d_G(v)].$$

The Zagreb indices were used in the structure property model [13, 33]. Recent results on the Zagreb indices can be found in [6, 11, 12, 23, 27, 36].

One of the well known index called as degree distance was introduced by Dobrynin and Kochetova [2] and is defined as,

$$DD(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u,v)(d_G(u) + d_G(v)).$$

More on degree distance can be found in [2, 37].

In a search for triangle-free graphs with arbitrarily large chromatic numbers, Mycielski [10], developed an interesting graph transformation [32] as follows. For a graph $G = (V,E)$, the Mycielskian of G is the graph $\psi(G)$ (or simply, ψ) with the disjoint union $V\cup Y\cup \{y\}$ as its vertex set and $E \cup \{v_p y_q : v_q v_p \in E\} \cup \{yy_p : 1 \le p \le n\}$ as its edge set, where $V = \{v_1, v_2, v_3, ...v_n\}$ and $Y = \{y_1, y_2, y_3, \cdots, y_n\}$ [22]. The Mycielskian and generalized Mycielskians have fascinated graph theorists a great deal. This has resulted in studying several graph parameters of these graphs. Fisher et al. [9] determine the domination number of the Mycielskian in 1998, Taeri et al. [26] determine the Wiener index of the Mycielskian in 2012, and Ashrafi et al. [17] determine Zagreb coindices of the Mycielskian in 2012 [3].

Recently, Kishori P. N et al. introduced reciprocal status-distance index of a graph in [24] and is defined as

$$RSD(G) = \sum_{\{u,v\}\subseteq V(G)} \frac{\sigma(u) + \sigma(v)}{d_G(u,v)},$$

where $\sigma(u)$ and $\sigma(v)$ are the status of the vertex u and v, respectively.

In this paper, determined the reciprocal status-distance index of the Mycielskian of each graph with diameter two. Also, computed the reciprocal status-distance index of the complement of Mycielskian of arbitrary graphs in terms of Zagreb indices.

§2. Reciprocal Status-Distance Index of the Mycielskian Graph

To determine the reciprocal status-distance index of Mycielskian graphs,we need following observations. Here on wards we will treat that G is a connected graph, for any vertex u of G there are $deg_G(u)$ which are at distance at 1 from u and the remaining $(n-1-deg_G(u))$ vertices are at distance at least 2. Therefore, $\sigma_u \leq deg_G(u)+(n-1-deg_G(u))$.

Let $V(G)=\{v_1,v_2,v_3,\cdots,v_n\}$, $Y=\{y_1,y_2,y_3,\cdots,y_n\}$, $V(G)\cap Y=\phi$, $y\notin V(G)\cup Y$ and ψ is the Mycielskain of G, where $V(\psi)=\{v_1,v_2,v_3,\cdots,v_n,y_1,y_2,y_3,\cdots,y_n,\{y\}\}$ and $E(\psi)=E(G)\cup\{v_py_q : v_pv_q\in E(G)\}\cup\{yy_p : 1\leq p\leq n\}$.

We begin with the following straight forward, previously known auxiliary result.

Observation 2.1([3]) Let ψ be the Mycielskian of G. Then for each $v\in V(\psi)$ we have

$$deg_\psi(v)=\begin{cases} n, & v=y\\ 1+deg_G(v_p), & v=y_p\\ 2deg_G(v_p), & v=v_p\end{cases}$$

Observation 2.2([3]) In the Mycielskian ψ of G, the distance between two vertices $u,v\in V(\psi)$ are given as follows

$$d_\psi(u,v)=\begin{cases} 1, & u=y,v=y_p\\ 2, & u=y,v=y_p\\ 2, & u=y_p,v=y_q\\ d_G(v_p,v_q), & u=y_p,v=y_q,d_G(v_p,v_q)\leq 3\\ 4, & u=v_p,v=v_q,d_G(v_p,v_q)\geq 4\\ 2, & u=v_p,v=y_q,p=q\\ d_G(v_p,v_q), & u=y_p,v=y_q,p\neq q,d_G(v_p,v_q)\leq 2\\ 3, & u=v_p,v=y_q,p\neq q,d_G(v_p,v_q)\geq 3\end{cases}$$

Specially, the diameter of the Mycielskian graph is at most four [3]. There are m unordered pairs of vertices in V whose distance is one and

$$\sum_{(u,v)\in V\times V,d_G(u,v)=1}(deg_G(u)+deg_G(v))=2\sum_{uv\in E(G)}(deg_G(u)+deg_G(v))=2M_1(G).$$

lemma 2.1([3]) *Let G be a graph of size m whose vertex set is $V = \{v_1, v_2, v_3, \cdots, v_n\}$. Then,*

$$\sum_{\{v_p,v_q\}\subseteq V} (deg_G(u) + deg_G(v)) = 2m(n-1)$$

lemma 2.2([3]) *Let G be a graph of size m. Then,*

$$\sum_{\{v_p,v_q\}\notin E(G)} (deg_G(u_p) + deg_G(v_q)) = 2m(n-1) - M_1(G).$$

Theorem 2.1 *Let G be a graph of order n and size m whose $diam(G) = 2$. If ψ is the Mycielskain of G, Then,*

$$\begin{aligned} RSD(\psi(G)) &= \frac{13}{2}n^2 - \frac{17}{2}n - 15m + 6mn \\ &+ 4(n-1)\left[H(G) + (\binom{n}{2} - m)\right] \\ &+ \binom{n}{2}(2n-3) - \left[M_1(G) + \frac{\overline{M_1}(G)}{2}\right] \end{aligned}$$

Proof By the definition of reciprocal status-distance index, we have

$$RSD(\psi(G)) = \sum_{\{u,v\}\subseteq V(\psi)} \frac{\sigma_\psi(u) + \sigma_\psi(v)}{d_\psi(u,v)}.$$

In respect of various viable cases which u and v can be taken from the set $V(\psi)$, considered the following cases. In what follows, the symbols are as before and two observations 1 and 2 are applied to determine the reciprocal status-distance of the $\psi(G)$.

Case 1. $u = y$ and $v \in Y$:

$$\begin{aligned} \sum_{p=1}^{n} \frac{\sigma_\psi(y) + \sigma_\psi(y_p)}{d_\psi(y, y_p)} &= \sum_{p=1}^{n} \sigma_\psi(y) + \sigma_\psi(y_p) \\ &= \sum_{p=1}^{n} 4(n-1) - (deg_\psi(y) + deg_\psi(y_p)) \\ &= \sum_{p=1}^{n} 4(n-1) - \sum_{p=1}^{n}(deg_\psi(y) + deg_\psi(y_p)) \\ &= 4n(n-1) - \sum_{p=1}^{n}(1 + n + deg_G(v_p)) \\ &= 4n(n-1) - (n(n+1) + 2m) \\ &= 3n^2 - 5n - 2m \end{aligned}$$

Case 2. $u = y$ and $v \in V(G)$:

$$\begin{aligned}\sum_{p=1}^{n} \frac{\sigma_\psi(y) + \sigma_\psi(v_p)}{d_\psi(y, v_p)} &= \sum_{p=1}^{n} \frac{4(n-1) - (deg_\psi(y) + deg_\psi(v_p))}{d_\psi(y, v_p)} \\ &= \sum_{p=1}^{n} \frac{4(n-1) - (n + 2deg_G(v_p))}{2}\end{aligned}$$

$$\begin{aligned}\sum_{p=1}^{n} \frac{\sigma_\psi(y) + \sigma_\psi(v_p)}{d_\psi(y, v_p)} &= \sum_{p=1}^{n} 2(n-1) - \frac{1}{2}\sum_{p=1}^{n}(n + 2deg_G(v_p)) \\ &= 2n(n-1) - \frac{1}{2}(\sum_{p=1}^{n} n + 2\sum_{p=1}^{n} deg_G(v_p)) \\ &= 2n(n-1) - \frac{1}{2}(n^2 + 2.2m) \\ &= 2n(n-1) - \frac{1}{2}n^2 - 2m.\end{aligned}$$

Case 3. $\{u, v\} \subseteq Y$:

Using lemma 2.1, we see that

$$\begin{aligned}&\sum_{\{y_p, y_q\} \subseteq Y} \frac{4(n-1) - (deg_\psi(y_p) + deg_\psi(y_q))}{d_\psi(y_p, y_q)} \\ &= \sum_{\{y_p, y_q\} \subseteq Y} \frac{4(n-1) - (1 + deg_G(v_p) + 1 + deg_G(v_q))}{2} \\ &= \sum_{\{y_p, y_q\} \subseteq Y} \frac{4(n-1) - (2 + deg_G(v_p) + deg_G(v_q))}{2} \\ &= 2\binom{n}{2}(2n-3) + \sum_{\{y_p, y_q\} \subseteq [n]} (d_G(v_p) + d_G(v_q)) \\ &= \binom{n}{2}(2n-3) + \sum_{p=1}^{n}(n-1)deg_G(v_p) \\ &= \binom{n}{2}(2n-3) - 2m(n-1)\end{aligned}$$

Case 4. $\{u, v\} \subseteq V(G)$. Since the diameter of G is two, Observation 2.2 implies that $d_\psi(v_p, v_q) = d_G(v_p, v_q)$. Hence,

$$\begin{aligned}&\sum_{\{v_p, v_q\} \subseteq V(G)} \frac{4(n-1) - (deg_\psi(v_p) + deg_\psi(v_q))}{d_\psi(v_p, v_q)} \\ &= \sum_{\{v_p, v_q\} \subseteq V(G)} \frac{4(n-1) - (2deg_G(v_p) + 2deg_G(v_q))}{d_G(v_p, v_q)} \\ &= 4(n-1)H(G) - 2RSD(G).\end{aligned}$$

Case 5. $u = v_p$ and $v = y_p$, $1 \leq p \leq n$

$$\begin{aligned}\sum_{p=1}^{n} \frac{4(n-1) - (d_\psi(v_p) + deg_\psi(y_p)}{d_\psi(v_p, y_p)} &= 2n(n-1) - \frac{1}{2}\sum_{p=1}^{n}(deg_\psi(v_p) + deg_\psi(y_p)) \\ &= 2n(n-1) - \frac{1}{2}\sum_{p=1}^{n}(2deg_G(v_p) + deg_G(v_p) + 1) \\ &= 2n(n-1) - \frac{1}{2}\sum_{p=1}^{n}(3deg_G(v_p) + 1) \\ &= 2n(n-1) - \frac{1}{2}n - 3m = 2n^2 - \frac{3}{2}n - 3m\end{aligned}$$

Case 6. $u = v_p$ and $v = y_q$, $p \neq q$

$$\begin{aligned}&\sum_{\{v_p, y_q\}_{p\neq q} \subseteq V(\psi)} \frac{4(n-1) - (deg_\psi(v_p)) + deg_\psi(y_q)}{d_\psi(v_p, y_q)} \\ &= \sum_{\{v_p, y_q, p\neq q\} \subseteq V(\psi)} \frac{4(n-1) - (2deg_G(v_p) + deg_G(v_p) + 1)}{d_\psi(v_p, y_q)} \\ &= \sum_{\{v_p, y_q\} \subseteq V(\psi)} \frac{4(n-1) - (deg_G(v_p) + 1)}{d_\psi(v_p, y_q)} \\ &\quad - \sum_{\{v_p, y_q\} \subseteq V(\psi)} \frac{deg_G(v_p) + deg_G(y_q)}{d_\psi(v_p, y_q)}\end{aligned}$$

Since $d_\psi(v_p, y_q) = d_\psi(v_q, y_p), d_\psi(v_p, v_p) = 0$, using observation 2.2, we have

$$\sum_{\{v_p, y_q, p\neq q\} \subseteq V(\psi)} \frac{deg_G(v_p) + deg_G(v_q)}{d_\psi(v_p, y_q)} = 2 \sum_{\{v_p, y_q,\} \subseteq V(G)} \frac{deg_G(v_p) + deg_G(v_q)}{d_G(v_p, v_q)} = 2RSD(G)$$

Each edge $v_pv_q = v_qv_p \in E(G)$ corresponds to two pairs $\{v_p, y_q\}$ and $\{v_q, y_p\}$ of distance 1 in the Mycielskian graph ψ. Since the diameter of G is 2 and using Lemma 2.2, we obtain [3]

$$\begin{aligned}&\sum_{\{v_p, y_q, p\neq q\} \subseteq V(\psi)} \frac{4(n-1) - (deg_G(v_p) + 1)}{d_\psi(v_p, y_q)} \\ &= \sum_{\{v_p, y_q\} \subseteq V(\psi), v_pvq \in E(G)} (4(n-1) - (1 + deg_G(v_p)) \\ &\quad + \sum_{\{v_p, y_q\} \subseteq V(\psi), v_pv_q \notin E(G)} \frac{(4(n-1) - (deg_G(v_p) + 1))}{2} \\ &= \sum_{\{v_p, y_q\} \subseteq V(\psi), v_pvq \in E(G)} ((4n-5) - deg_G(v_p)) \\ &\quad + \sum_{\{v_p, y_q\} \subseteq V(\psi), v_pv_q \notin E(G)} \frac{(4(n-1) - (deg_G(v_p) + 1))}{2}\end{aligned}$$

$$\begin{aligned}
&= (4n-5)2m - \sum_{v_pv_q\in E(G)} (deg_G(v_p)+deg_G(v_q)) \\
&+ 4(n-1)(\binom{n}{2} - m) - \sum_{v_pvq\notin E(G)} \frac{deg_G(v_p)+deg_G(v_q)}{2} \\
&= 2m(4n-5) + 4(n-1)(\binom{n}{2} - m) - M_1(G) - \frac{\overline{M_1}(G)}{2}
\end{aligned}$$

$$\begin{aligned}
\sum_{\{v_p,y_q\}_{p\neq q}\subseteq V(\psi)} & \frac{4(n-1)-(deg_\psi(v_p))+deg_\psi(v_q)}{d_\psi(v_p,y_q)} \\
&= 2m(4n-5) + 4(n-1)(\binom{n}{2} - m) - M_1(G) - \frac{\overline{M_1}(G)}{2} \\
&+ 2RSD(G)
\end{aligned}$$

$$\begin{aligned}
RSD(\psi(G)) &= 3n^2 - 5n - 2m + 2n(n-1) - \frac{1}{2}n^2 - 2m + \binom{n}{2}(2n-3) - 2m(n-1) \\
&+ 4(n-1)H(G) - 2RSD(G) + 2n^2 - \frac{3}{2}n - 3m + 2m(4n-5) \\
&+ 4(n-1)(\binom{n}{2} - m) - M_1(G) - \frac{\overline{M_1}(G)}{2} + 2RSD(G) \\
RSD(\psi(G)) &= \frac{13}{2}n^2 - \frac{17}{2}n - 15m + 6mn + 4(n-1)[H(G) + (\binom{n}{2} - m)] + \binom{n}{2}(2n-3) \\
&- [M_1(G) + \frac{\overline{M_1}(G)}{2}].
\end{aligned}$$

This completes the proof. □

§3. Reciprocal Status-Distance Index of the Complement of Mycielskian

In order to determine the reciprocal status-distance index of the complement of Mycielskian graphs, we required the following two observations [3].

Observations 3.1 Let $\overline{\psi}$ be the complement of Mycielskian ψ of G, Then for each $v \in V(\overline{\psi})$ we have [3]

$$deg_{\overline{\psi}}(v) = \begin{cases} n, & v = y \\ 2n - (1 + deg_G(v_p), & v = y_p \\ 2n - 2deg_G(v_p), & v = v_p \end{cases}$$

Observations 3.2 In the complement of Mycielskian ψ of G, the distance between two vertices

$u, v \in V(\overline{\psi})$ are given as follows [3]

$$d_{\overline{\psi}}(u,v) = \begin{cases} 2, & u = y, v = y_p \\ 1, & u = y, v = v_p \\ 1, & u = y_p, v = y_q \\ 1, & u = y_p, v = y_q, d_G(v_p, v_q) > 1 \\ 2, & u = v_p, v = v_q, d_G(v_p, v_q) = 1 \\ 1, & u = v_p, v = y_q, p = q \\ 1, & u = v_p, v = y_q, p \neq q, d_G(v_p, v_q) > 1 \\ 2, & u = v_p, v = y_q, p \neq q, d_G(v_p, v_q) = 1. \end{cases}$$

Specially, the diameter of $\overline{\psi}$ is exactly 2 [3].

Theorem 3.1 *Let G be a graph of order m and size n and let $\overline{\psi}$ be the complement of the Mycielskian ψ of G. Then the reciprocal status -distance index of $\overline{\psi}$ is given by*

$$RSD(\overline{\psi}(G)) = \frac{n^2}{2} - \frac{15}{2}n + 7m + 2mn + 4(n-1)(2\binom{n}{2} - m) - 3\binom{n}{2} + 2M_1(G).$$

Proof By the definition of reciprocal status-distance index, we have

$$RSD(\overline{\psi}) = \sum_{\{u,v\} \subseteq V(\overline{\psi})} \frac{4(n-1) - (deg_{\overline{\psi}}(u) + deg_{\overline{\psi}}(v))}{d_{\overline{\psi}}(u,v)}.$$

Case 1. $u = y$ and $v \in Y$. In this case,

$$\begin{aligned} \sum_{p=1}^{n} \frac{4(n-1) - (deg_{\overline{\psi}}(y) + deg_{\overline{\psi}}(y_p))}{d_{\overline{\psi}}(y, y_p)} &= 2n(n-1) - \frac{1}{2}\sum_{p=1}^{n}(3n - deg_G(v_p) - 1) \\ &= 2n^2 - 2n - \frac{3}{2}n^2 + m - \frac{n}{2} \\ &= \frac{7}{2}n^2 - \frac{5}{2}n + m. \end{aligned}$$

Case 2. $u = y$ and $v \in V(G)$. In this case,

$$\begin{aligned} &\sum_{p=1}^{n} \frac{4(n-1) - (deg_{\overline{\psi}}(y) + deg_{\overline{\psi}}(v_p))}{d_{\overline{\psi}}(y, y_p)} \\ &= 4n(n-1) - \sum_{p=1}^{n}(3n - 2deg_G(v_p)) \\ &= 4n^2 - 4n - 3n^2 + 4m = n^2 - 4n + 4m. \end{aligned}$$

Case 3. $\{u, v\} \subseteq Y$. using Lemma 2.1 we see that

$$\begin{aligned}
&\sum_{\{y_p,y_q\}\subseteq V(Y)} \frac{4(n-1)-(deg_{\overline{\psi}}(y_p)+deg_{\overline{\psi}}(y_q))}{d_{\overline{\psi}}(y_p,y_q)}\\
&= \sum_{\{y_p,y_q\}\subseteq V(Y)} 4(n-1)-(deg_{\overline{\psi}}(y_p)+deg_{\overline{\psi}}(y_q))\\
&= 4\binom{n}{2}(n-1) - \sum_{\{p,q\}\subseteq[n]} (2n-(1+deg_G(v_p))\\
&\quad - \sum_{\{p,q\}\subseteq[n]} (2n-(1+deg_G(v_p)))\\
&= 4\binom{n}{2}(n-1) - \sum_{\{p,q\}\subseteq[n]} (4n-2)\\
&\quad + \sum_{\{p,q\}\subseteq[n]} (deg_G(v_p)+deg_G(v_q))\\
&= 4\binom{n}{2}(n-1) - 4n^2 + 2n + 2m(n-1).
\end{aligned}$$

Case 4. $\{u, v\} \subseteq V(G)$. Using Lemma 2.2 we have

$$\begin{aligned}
&\sum_{\{v_p,v_q\}\subseteq V(G)} \frac{4(n-1)-(deg_{\overline{\psi}}(v_p)+deg_{\o\psi}(v_q))}{d_{\overline{\psi}}(v_p,v_q)}\\
&= \sum_{v_pv_q\notin E(G)} 4(n-1)-2(deg_G(v_p)+deg_G(v_q))\\
&\quad + \sum_{v_p,v_q\in E(G)} \frac{4(n-1)-(4n-2(deg_G(v_p)+deg_G(v_q)))}{2}\\
&= 4(n-1)(\binom{n}{2}-m) - 2\overline{M_1}(G)\\
&\quad + 2(n-1)m - 2mn + M_1(G).
\end{aligned}$$

Case 5. $u = v_p$ and $v = y_p$, $1 \leq p \leq n$.

$$\begin{aligned}
&\sum_{p=1}^{n} \frac{4(n-1)-(deg_{\overline{\psi}}(v_p)+deg_{\overline{\psi}}(y_p))}{d_{\overline{\psi}}(v_p,y_p)}\\
&= \sum_{p=1}^{n} 4(n-1)-(4n-3deg_G(v_p)-1)\\
&= 4n(n-1) - 4n^2 + 6m + n\\
&= 6m - 3n
\end{aligned}$$

Case 6. $u = v_p$ and $v = y_q$, $p \neq q$. by Observation 3.2 [3], $d_{\overline{\psi}}(v_p, y_q) = d_{\overline{\psi}}(v_q, y_p)$ is 1 when

$v_pv_q \notin E(G)$,

$$\begin{aligned}
&\sum_{\{v_p,y_q\}\subseteq V(\psi)} \frac{4(n-1)-(deg_{\overline{\psi}}(v_p)+deg_{\overline{\psi}}(y_q))}{d_{\overline{\psi}}(v_p,y_q)} \\
&\quad= \sum_{(v_p,v_q),v_pv_q\notin E(G)} 4(n-1)-(4n-1-2deg_G(v_p)-deg_G(v_q)) \\
&\quad\quad+ \sum_{(v_p,v_q),v_p,v_q\in E(G)} \frac{4(n-1)-(4n-1-2deg_G(v_p)-deg_G(v_q))}{2}
\end{aligned}$$

and each vertex v_p can be paired with $n-1-deg_G(v_p)$ vertices as (v_p,v_q) with the condition $v_pv_q \notin E(G)$. Also note that $\sum_{(v_p,v_q)}(deg_G(v_p)+deg_G(v_q))$ is equal to $2\sum_{\{v_pv_q\}}(deg_G(v_p)+deg_G(v_q))$. Hence, using Lemma 2.2 we obtain

$$\sum_{\{v_p,y_q\}\subseteq V(\psi)} \frac{4(n-1)-(deg_\psi(v_p)+deg_\psi(y_q))}{d_\psi(v_p,y_q)} = a+\frac{1}{2}b,$$

where,

$$\begin{aligned}
a &= \sum_{(v_p,v_q),v_pv_q\notin E(G)} 4(n-1)-(4n-1-2deg_G(v_p)-deg_G(v_q)) \\
&= 4(n-1)\left(\binom{n}{2}-m\right)-(4n-1)\left(\binom{n}{2}-m\right) \\
&\quad+\{\sum_{v_pv_q\notin E(G)}(deg_G(v_p)+deg_G(v_q))+\sum_{(v_p,v_q)\in E(G)} deg_G(v_p)\} \\
&= \left(\binom{n}{2}-m\right)[4n-4-4n+1]+\text{ø}M_1(G)+\sum_{p=1}^{n}(deg_G(v_p))^2 \\
&= 3\left(m-\binom{n}{2}\right)+2\overline{M_1}(G)
\end{aligned}$$

$$\begin{aligned}
b &= \sum_{(v_p,v_q),v_pv_q\in E(G)} 4(n-1)-(4n-1-2deg_G(v_p)-deg_G(v_q)) \\
&= 4(2m(n-1))-2m(4n-1)+\sum_{v_pv_q\in E(G)}(deg_G(v_p)+deg_G(v_q))+\sum_{p=1}^{n}(deg_G(v_p))^2 \\
&= 2m(4n-4-4n+1)+2M_1(G)=2m(-3)+2M_1(G) \\
&= 2M_1(G)-6m
\end{aligned}$$

$$\sum_{\{v_p,y_q\}\subseteq V(\overline{\psi})} \frac{4(n-1)-(deg_{\overline{\psi}}(v_p)+deg_{\overline{\psi}}(y_q))}{d_{\overline{\psi}}(v_p,y_q)}$$

$$\begin{aligned}
&= 3(m - \binom{n}{2}) + 2\overline{M_1}(G) + M_1(G) - 3m \\
&= 3m - 3\binom{n}{2} + 2\overline{M_1}(G) + M_1(G) - 3m \\
&= 2\overline{M_1}(G) + M_1(G) - 3\binom{n}{2}
\end{aligned}$$

$$\begin{aligned}
RSD(\overline{\psi}(G)) = {} & \frac{7}{2}n^2 - \frac{5}{2}n + m + n^2 - 4n + 4m + 4\binom{n}{2}(n-1) - 4n^2 + 2n + 2m(n-1) \\
& + 4(n-1)(\binom{n}{2} - m) - 2\overline{M_1}(G) + 2(n-1)m - 2mn + M_1(G) + 6m - 3n \\
& + 2\overline{M_1}(G) + M_1(G) - 3\binom{n}{2} \\
RSD(\overline{\psi}(G)) = {} & \frac{n^2}{2} - \frac{15}{2}n + 7m + 2mn + 4(n-1)(2\binom{n}{2} - m) - 3\binom{n}{2} + 2M_1(G),
\end{aligned}$$

This completes the proof. ☐

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