International J.Math. Combin. Vol.1(2015), 65-73

Some Minimal $(r, 2, k)$-Regular Graphs Containing a Given Graph and its Complement

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Abstract: A graph G is called $(r, 2, k)$-regular graph if each vertex of G is at a distance 1 away from r vertices of G and each vertex of G is at a distance 2 away from k vertices of G [9]. This paper suggest a method to construct a $((m+2(n-1)), 2, (m-1)(2n-1)))$-regular graph H_4 of smallest order $2mn$ containing a given graph G of order $n \geq 2$, and its complement G^c as induced subgraphs, for any $m > 1$. Also, in this paper we calculate the topological indices Wiener index W, hyper Wiener index WW, degree distance DD, variance of degrees, first, second and third Zagreb indexes of the graphs H_4 which we constructed in this paper.

Key Words: Induced subgraph; clique number; independent number; (d, k)-regular graphs; $(2, k)$-regular graphs;$(r, 2, k)$-regular graphs; semiregular.

AMS(2010): 05C12

§1. Introduction

In this paper, we consider only finite, simple, connected graphs. For basic definitions and terminologies we refer Harary [7] and J.A.Bondy and U.S.R.Murty [4]. We denote the vertex set and edge set of a graph G by $V(G)$ and $E(G)$ respectively. The degree of a vertex v is the number of edges incident at v. A graph G is regular if all its vertices have the same degree.

For a connected graph G, the distance $d(u, v)$ between two vertices u and v is the length of a shortest (u, v) path. Therefore, the degree of a vertex v is the number of vertices at a distance 1 from v, and it is denoted by $d(v)$. This observation suggests a generalization of degree. That is, $d_d(v)$ is defined as the number of vertices at a distance d from v. Hence $d_1(v) = d(v)$ and $N_d(v)$ denote the set of all vertices that are at a distance d away from v in a graph G. That is, $N_1(v) = N(v)$ and $N_2(v)$ denotes the set of all vertices that are at a distance 2 away from v in a graph G and closed neighbourhood $N[v] = N(v) \cup \{v\}$.

The concept of distance d-regular graph was introduced and studied by G.S. Bloom, J.K. Kennedy and L.V.Quintas [3]. A graph G is said to be distance d-regular if every vertex of G has the same number of vertices at a distance d from it. A graph G is said to be (d, k)-regular

[1]Received August 12, 2014, Accepted February 22, 2015.

graph if $d_d(v) = k$, for all v in $V(G)$. A graph G is $(2, k)$ regular if $d_2(v) = k$, for all v in $V(G)$. The concept of the semiregular graph was introduced and studied by Alison Northup [2]. We observe that $(2, k)$ - regular graph and k - semiregular graph are the same. A graph G is said to be $(r, 2, k)$-regular if $d(v) = r$ and $d_2(v) = k$, for all $v \in V(G)$.

An induced subgraph of G is a subgraph H of G such that $E(H)$ consists of all edges of G whose end points belong to $V(H)$.In 1936, Konig [8] proved that if G is any graph, whose largest degree is r, then there is an r-regular graph H containing G as an induced subgraph. In 1963, Paul Erdos and Paul Kelly [6] determined the smallest number of new vertices which must be added to a given graph G to obtain such a graph. We now suggest a method that may be considered an analogue to Konig's theorem for $(r, 2, k)$-regular graph.

With this motivation, already we have constructed a $(m+n-2), 2, (m-1)(n-1))$-regular graph S of order mn containing a given graph G of order $n \geq 2$ as an induced subgraph, for any $m > 1$ [12]. In this paper, our main objective is to construct a $((m+2(n-1)), 2, (m-1)(2n-1))$-regular graph of smallest order $2mn$ containing the given graph G of order $n \geq 2$, and its complement G^c as induced subgraphs, for any $m > 1$.

§2. $(r, 2, k)$-Regular Graph

Definition 2.1 *A graph G is called $(r, 2, k)$-regular if each vertex in graph G is at a distance one from exactly r-vertices and at a distance two from exactly k vertices. That is, $d(v) = r$ and $d_2(v) = k$, for all v in G.*

Example 2.2 A few $(r, 2, k)$-regular graphs are listed following.

(1) The Peterson graph is a (3, 2, 6)-regular graph .

(2) A complete bipartite graph $K_{n,n}$ is a $(n, 2, (n-1))$-regular graph.

Observation 2.3 For any $n \geq 1$, the smallest order of $(n, 2, (n-1))$- regular graph containing the complete bipartite graph $K_{n,n}$ of order $2n$ is $K_{n,n}$ itself.

The following facts can be verified easily.

Observation 2.4([9]) If G is $(r, 2, k)$-regular graph, then $0 \leq k \leq r(r-1)$.

Observation 2.5([10]) For any $r > 1$, a graph G is $(r, 2, r(r-1))$-regular if G is r-regular with girth at least five.

Observation 2.6([11]) For any odd $r \geq 3$, there is no $(r, 2, 1)$-regular graph.

Observation 2.7([11]) Any $(r, 2, k)$-regular graph has at least $k + r + 1$ vertices.

Observation 2.8([11]) If r and k are odd, then $(r, 2, k)$-regular graph has at least $k + r + 2$ vertices.

Observation 2.9([12]) For any $m \geq 1$, every graph G of order $n \geq 2$ is an induced subgraph of $(n + m - 1, 2, (mn - 1))$-regular graph H of order $2mn$.

Observation 2.10([13]) For any $m > 1$, every graph G of order $n \geq 2$ is an induced subgraph of $(n + m - 2, 2, (m-1)(n-1))$-regular graph H of order mn.

§3. Minimal $(r, 2, k)$-Regular Graphs Containing Given Graph and Its Complement as an Induced Subgraph

In this section we construct a smallest $(r, 2, k)$-regular graphs containing given graph and its complement as an induced subgraph.

Theorem 3.1 *For a graph G of order $n \geq 2$, there exists a $(m+2(n-1), 2, (m-1)(2n-1))$-regular graph H_4 of order $2mn$ such that G and G^c are the induced subgraphs of H_4.*

Proof Let G be a graph of order $n \geq 2$, G and G^c has the same vertex set $\{v_i^1 : 1 \leq i \leq n\}$. Take a graph G' which is isomorphic to G^c. The vertex set of G' is denoted as $\{u_i^1 : 1 \leq i \leq n\}$ and u_i^1 corresponds to $v_i^1 (1 \leq i \leq n)$. Let $G_1 = G \cup G'$. Then $V(G_1) = \{v_i^1, u_i^1 : 1 \leq i \leq n\}$. Let $G_t (2 \leq t \leq m)$ be the $(m-1)$ copies of G_1 with the vertex set $V(G_t) = \{v_i^t, u_i^t : 1 \leq i \leq n\}$, for $(2 \leq t \leq m)$ and $v_i^t, u_i^t (2 \leq t \leq m)$ correspond to $v_i^1, u_i^1 (1 \leq i \leq n)$ respectively. The desired graph H_4 has the vertex set $V(H_4) = \bigcup_{t=1}^{m} V(G_t)$, and edge set $E(H_4) = \bigcup_{t=1}^{m} E(G_t) \bigcup E_1 \bigcup E_2 \bigcup E_3 \bigcup E_4 \bigcup E_5$, where,

$$E_1 = \bigcup_{t=1}^{m-1} \{v_j^t v_i^{t+1}, v_j^m v_i^1 : v_j^1 v_i^1 \notin E(G_1)(1 \leq j \leq n), (j+1 \leq i \leq n)\},$$

$$E_2 = \bigcup_{k=1}^{n} \{v_k^i v_k^{i+j} : (1 \leq i \leq m-1), (1 \leq j \leq m-i)\},$$

$$E_3 = \bigcup_{t=1}^{m-1} \{u_j^t u_i^{t+1}, u_j^m u_i^1 : u_j^1 u_i^1 \notin E(G_1)(1 \leq j \leq n), (j+1 \leq i \leq n)\},$$

$$E_4 = \bigcup_{k=1}^{n} \{u_k^i u_k^{i+j} : (1 \leq i \leq m-1), (1 \leq j \leq m-i)\},$$

$$E_5 = \bigcup_{t=1}^{m-1} \{v_j^t u_i^{t+1}, v_j^m u_i^1 : (1 \leq i, j \leq n)\}.$$

The resulting graph H_4 contains G_1 as an induced subgraph. More over in $H_4, (1 \leq t \leq m), d(v_i^t) = m + 2(n-1)$, for $(1 \leq i \leq n)$. Then H_4 is $m + 2(n-1)$, regular graph with $2mn$ vertices. Hence H_4 contains G and G^c as induced subgraphs. In $H_4, d(v_i) = d(v_i^1) = d(u_i) = d(u_i^1) = m + 2n - 2$, $1 \leq i \leq n$. To find the d_2 degree of each vertex in H_4, the following cases are examined.

Case 1. $t = 1$. If $v \in V(G_1)$, then $v \in V(G)$ (or) $v \in V(G')$.

Subcase 1.1 If $v \in V(G)$, then $v = v_j^1$, for some j. Let $v_j^1 \in V(H_4) - N[v_i^1]$. Then v_j^1 and v_i^1 are non-adjacent vertices in H_4. By our construction, v_j^1 is adjacent to v_i^2 and v_i^2 is adjacent to v_i^1. Then $d(v_j^1, v_i^1) = 2$. Hence $v_j^1 \in N_2(v_i^1)$. This implies that $V(H_4) - N[v_i^1] \subseteq N_2(v_i^1)$. If $v_j^1 \in N_2(v_i^1)$, then v_j^1 is non-adjacent with v_i^1. This implies that $v_j^1 \in V(H_4) - N[v_i^1]$. Hence $N_2(v_i^1) = V(H_4) - N[v_i^1]$, $(1 \leq i \leq n)$ and $d_2(v_i^1) = (m-1)(2n-1), (1 \leq i \leq n)$.

Subcase 1.2 If $v \in V(G')$, then $v = u_j^1$, for some j.

Let $u_j^1 \in V(H_4) - N[u_i^1]$. Then, u_j^1 and u_i^1 are non-adjacent vertices in H_4. By our construction, u_j^1 is adjacent to u_i^2 and u_i^2 is adjacent to u_i^1. Then $d(u_j^1, u_i^1) = 2$. Hence $u_j^1 \in N_2(u_i^1)$. This implies that $V(H_4) - N[u_i^1] \subseteq N_2(u_i^1)$. if $u_j^1 \in N_2(u_i^1)$, then u_j^1 is non-adjacent with u_i^1. Hence $u_j^1 \in V(H_4) - N[u_i^1]$. This implies that $N_2(u_i^1) = V(H_4) - N[u_i^1]$, $(1 \leq i \leq n)$ and

$d_2(u_i^1) = (m-1)(2n-1), (1 \leq i \leq n)$.

Case 2. $2 \leq t \leq m-1$. If $v \in V(G_t)$, then $v = v_j^t$ (or) $v = u_j^t$, for some j.

Subcase 2.1 If $v = v_j^t$ and if $v_j^t \in V(H_4) - N[v_i^1]$, then v_j^t and v_i^1 are non-adjacent vertices in H_4. By our construction, v_j^t is adjacent to v_i^t and v_i^t is adjacent to v_i^1. Then $d(v_j^t, v_i^1) = 2$. Hence $v_j^t \in N_2(v_i^1)$. This implies that $V(H_4) - N[v_i^1] \subseteq N_2(v_i^1)$. If $v_j^t \in N_2(v_i^1)$, then, v_j^t is non-adjacent with v_i^1. This implies that $v_j^t \in V(H_4) - N[v_i^1]$. Hence $N_2(v_i^1) = V(H_4) - N[v_i^1]$, $(1 \leq i \leq n)$ and $d_2(v_i^1) = (m-1)(2n-1), (1 \leq i \leq n)$.

Subcase 2.2 If $v = u_j^t$ and if $u_j^t \in V(H_4) - N[u_i^1]$, then u_j^t and u_i^1 are non-adjacent vertices in H_4. By our construction, u_j^t is adjacent to u_i^{t+1} and u_i^{t+1} is adjacent to u_i^1. Then $d(u_j^t, u_i^1) = 2$. Hence $u_j^t \in N_2(u_i^1)$. This implies that $V(H_4) - N[u_i^1] \subseteq N_2(u_i^1)$. If $u_j^t \in N_2(u_i^1)$, then, u_j^t is non-adjacent with u_i^1. Hence $u_j^t \in V(H_4) - N[u_i^1]$. This implies that $N_2(u_i^1) = V(H_4) - N[u_i^1]$, $(1 \leq i \leq n)$ and $d_2(u_i^1) = (m-1)(2n-1), (1 \leq i \leq n)$.

Case 3. $t = m$. If $v \in V(G_m)$, then $v = v_j^m$ (or) $v = u_j^m$ for some j.

Subcase 3.1 If $v = v_j^m$ and if $v_j^m \in V(H_4) - N[v_i^1]$, then v_j^m and v_i^1 are non-adjacent vertices in H_4. By our construction, v_j^m is adjacent to v_i^m and v_i^m is adjacent to v_i^1. Then $d(v_j^m, v_i^1) = 2$. Hence $v_j^m \in N_2(v_i^1)$. This implies that $V(H_4) - N[v_i^1] \subseteq N_2(v_i^1)$. If $v_j^m \in N_2(v_i^1)$, then v_j^m is non-adjacent with v_i^1. Hence $v_i^m \in V(H_4) - N[v_i^1]$. This implies that $N_2(v_i^1) = V(H_4) - N[v_i^1]$, $1 \leq i \leq n$ and $d_2(v_i^1) = (m-1)(2n-1), (1 \leq i \leq n)$.

Subcase 3.2 If $v = u_j^m$ and if $u_j^m \in V(H_4) - N[u_i^1]$, then u_j^m and u_i^1 are non-adjacent vertices in H_4. By our construction, u_j^m is adjacent to u_i^m and u_i^m is adjacent to u_i^1. Then $d(u_j^m, u_i^1) = 2$. Hence $u_j^m \in N_2(v_i)$. This implies that $V(H_4) - N[u_i] \subseteq N_2(u_i)$. If $u_j^m \in N_2(u_i^1)$, then u_j^m is non-adjacent with u_i^1. Hence $u_j^m \in V(H_4) - N[u_i^1]$. This implies that $N_2(u_i^1) = V(H_4) - N[u_i^1]$, $1 \leq i \leq n$ and $d_2(u_i^1) = (m-1)(2n-1), (1 \leq i \leq n)$. Similarly for $(1 \leq t \leq m) d_2(v_i^t) = d_2(u_i^t) = (m-1)(2n-1), (1 \leq i \leq n)$. H_4 is $(m + 2(n-1), 2, (m-1)(2n-1))$-regular graph of order $2mn$ containing a given graph G of order $n \geq 2$ and its complement as induced subgraphs. □

Corollary 3.2 *For any $m \geq 1$, the smallest order of $(m + 2(n-1), 2, (m-1)(2n-1))$-regular graph containing a given graph of order $n \geq 2$ and its complement is $2mn$.*

Proof For the graph H_4 constructed in Theorem 3.1 is $(m + 2(n-1), 2, (m-1)(2n-1))$-regular graph of order $2mn$. Suppose H_4 is $(m + 2(n-1), 2, (m-1)(2n-1))$-regular graph of order $2mn - 1$. Then, for each $v_i \in H_4$, $d_2(v_i) = (m-1)(2n-1)$ and $d(v_i) = m + 2(n-1)$. Hence H_4 has at least $((m-1)(2n-1) + (m + 2(n-1) + 1) = 2mn$ vertices, a contradiction.□

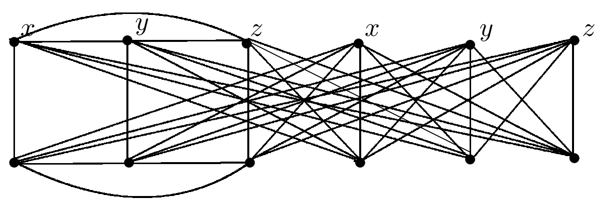

Figure 1

Corollary 3.3 *Every graph G of order $n \geq 2$, and its complement G^c are the induced sub-graphs* *of* $(2n, 2, (2n-1))$*-regular graph of smallest order* $4n$.

In Figure 1, Corollary 3.3 is illustrated for $G = K_3$, in which the graph G is induced by the vertices x, y, z.

Corollary 3.4 *Every graph* G *of order* $n \geq 2$, *and its complement* G^c *are the induced subgraphs of* $(2n+1, 2, 2(2n-1))$*-regular graph of smallest order* $6n$.

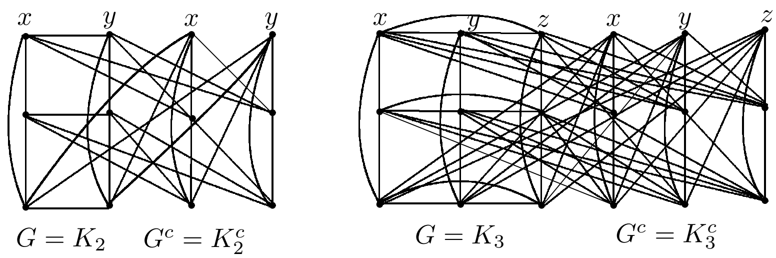

Figure 2

In Figure 2, Corollary 3.4 is illustrated for $G = K_2$ and $G = K_3$, in which the graph G and $G^c$is induced by the vertices x, y for $G = K_2$. In the second graph, the graph G and $G^c$is induced by the vertices x, y, z for $G = K_3$.

Corollary 3.5 *Every graph* G *of order* $n \geq 2$, *and its complement* G^c *are the induced subgraphs of* $(2n+2, 2, 3(2n-1))$*-regular graph of smallest order* $8n$.

Corollary 3.6 *Every graph* G *of order* $n \geq 2$, *and its complement* G^c *are the induced subgraph of* $(2n+3, 2, 4(2n-1))$*-regular graph of smallest order* $10n$.

Remark 3.7 There are at least as many $(m+2(n-1), 2, (m-1)(2n-1))$-regular of order $2mn$ as there are graphs G of order $n \geq 2$. If $m = 2, 3, 4, 5, \ldots$, then there are $(2n, 2, (2n-1)), (2n+1, 2, 2(2n-1)), (2n+2, 2, 3(2n-1)), (2n+3, 2, 4(2n-1)), \ldots$ regular graphs of smallest order $4n, 6n, 8n, 10n, 12n \ldots$ respectively containing any graph G of order $n \geq 2$ and its complement as induced subgraphs.

§4. Topological Indices of the Graph H_4

The topological indices Wiener Index W, Hyper Wiener Index WW, Degree Distance DD, Variance of degrees, The first Zagreb index, The second Zagreb Index and the third Zagreb Index of the graph H_4, which was constructed in Theorem 3.1 are calculated in this section.

Topological index $Top(G)$ of a graph G is a number with this property that for every graph H isomorphic to G, $Top(G) = Top(H)$. For historical background, computational techniques and mathematical properties of Zagreb indices and Wiener, Hyper Wiener one can refer to [21,22,23,24,25].

The graph H_4 is $(m+2n-2, 2, (m-1)(2n-1))$-regular graph having $2mn$ vertices and $mn(m+2n-2))$ edges with diameter 2. Also, for each $v \in H_4$, $d_2(v) = (m-1)(2n-1)$ and

$d(v) = m + 2n - 2$.

Computation of W, WW and DD for H_4 is done by using the following theorem [14]:

Let G be a graph with n vertices, m edges and with diameter 2, then

(1) $W(G) = n(n-1) - m$;

(2) $WW(G) = 3/2(n(n-1)) - 2m$;

(3) $DD(G) = 4(n-1)m - M_1(G)$.

The Wiener index W is the first and important topological index in chemistry which was introduced by H. Wiener in 1947 to study the boiling points of parafins. This index is useful to describe molecular structures and also crystal lattice that depends on its W value.

Definition 4.1 *The Wiener index, $W(G)$ of a finite, connected graph G is defined to be $W(G) = \frac{1}{2}\sum d(u,v)$, where $d(u,v)$ denotes the distance between u and v in G.*

$$\begin{aligned} \text{Wiener Index of a graph } H_4 &= W(H_4) = 2mn(2mn-1) - ((mn)(m+2(n-1)) \\ &= mn(4mn - 2 - m - 2n + 2) = (mn)(4mn - (m+2n)) \end{aligned}$$

The Hyper Wiener index WW was introduced by Randic. The Hyper Wiener Index WW is used as a structure descriptor for predicting physicochemical properties of organic compounds.

Definition 4.2 *The Hyper Wiener index $WW(G)$ of a finite, connected graph G is defined to be $WW(G) = \frac{1}{2}(W_1(G) + W_2(G))$, where $W_1(G) = W(G)$ and $W_\lambda(G) = \sum d_G(k)(k^\lambda)$ is called the Wiener-type invariant of G associated to a real number.*

$$\begin{aligned} \text{Hyper Wiener Index of a graph } H_4 &= WW(H_4) \\ &= (3/2)(2mn(2mn-1) - 2mn(m+2(n-1))) \\ &= (mn)(6mn - 3 - 2m - 4n + 4) \\ &= (mn)(6mn - (2m+4n) + 1) \end{aligned}$$

The Zagreb indices were introduced by Gutman and Trinajestic [7,10,14].

Definition 4.3 *The oldest and most investigated topological graph indices are defined as: First Zagreb index $M_1(G) = \sum_{v\in V(G)}(d_G(v))^2$, second Zagreb index $M_2(G) = \sum_{uv\in E(G)}(d_G(u)d_G(v))$ and third Zagreb index $M_3(G) = \sum |d(u) - d(v)|, uv \in E(G)$.*

The Zagreb Indices of graph H_4 are

$$\begin{aligned}
1.\ M_1(H_4) &= \sum d(u)d(u) = \sum d(u)^2 = 2mn((m+2n-2)^2) \\
2.\ M_2(H_4) &= \sum d(u)d(v), uv \in E(H_4) \\
&= (mn)(m+2(n-1)(m+2(n-1)(m+2(n-1) \\
&= (mn)((m+2(n-1))^3) \\
3.\ M_3(H_4) &= \sum |d(u)-d(v)||, uv \in E(H_4) \\
&= \sum |(m+2(n-1))-(m+2(n-1))| = 0.
\end{aligned}$$

Definition 4.4([4]) *The degree distance (Schultz index) of G was introduced by Dobrynin and Kochetova and Gutman as a weighted version of the Wiener index defined as $DD(G) = \sum(d(u)+d(v))d(u,v)$. It is to be noted that $DD(G)$ and $W(G)$ are closely mutually related for certain classes of molecular graphs.*

The degree distance of graph H_4 is

$$\begin{aligned}
DD(H_4) &= 4(2mn-1)(mn)(m+2(n-1)) - M_1(H_4) \\
&= 2mn(m+2n-2)[2(2mn-1)-(m+2n-2)) \\
&= 2mn(m+2n-2)[4mn-(m+2n)]
\end{aligned}$$

Definition 4.5([13]) *The status, or distance sum of a vertex v in a graph is defined by $s(v) = \sum d(u,v)$, where $d(u,v)$ is the distance between the vertices u and v and $u \neq v$. The status sequence of a graph consists of a list of the stati of all the vertices.*

Since diameter of H_4 is two, the status of a vertex v in H_4 is

$$\begin{aligned}
s(v) &= (m+2(n-1)+2(m-1)(2n-1) \\
&= m+2n-2+4mn-2m-4n+2 = 4mn-2(m+n)
\end{aligned}$$

Definition 4.6 *A graph is said to be self-median, or SM, if the stati of its vertices are all equal.*

Every vertex in H_4 has the same status $4mn-2(m+n)$. Whence, H_4 is a self-median graph.

§5. Open Problems

For further investigation, the following open problem is suggested:

(1) Construct (r, m, k)-regular graphs containing a given graph G and its complement of order $n \geq 2$, as induced subgraph, for $m \geq 3$.

(2) Construct (r, m, k)-regular graphs containing a given graph G and its complement of order $n \geq 2$, as induced subgraph, for all values of k.

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