

Some Results on 4-Total Difference Cordial Graphs

R. Ponraj

(Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India)

S.Yesu Doss Philip and R. Kala

(Department of Mathematics, Manonmaniam Sundarnar University, Tirunelveli-627012, Tamilnadu, India)

E-mail: ponrajmaths@gmail.com, jesuphilip09@gmail.com, karthipyi91@yahoo.co.in

Abstract: Let G be a graph. Let $f : V(G) \rightarrow \{0, 1, 2, \dots, k-1\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge uv , assign the label $|f(u) - f(v)|$. f is called k -total difference cordial labeling of G if $|t_{df}(i) - t_{df}(j)| \leq 1$, $i, j \in \{0, 1, 2, \dots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labeled with x . A graph with admits a k -total difference cordial labeling is called k -total difference cordial graphs.

Key Words: Difference cordial labeling, Smarandachely difference cordial labeling, star, path, cycle, bistar, crown, comb.

AMS(2010): 05C78.

§1. Introduction

We consider here finite, simple and undirected graphs only. Ponraj etl., has been introduced the concept of k -total difference cordial graph in [4]. In [4,5], 3-total difference cordial labeling path, complete graph, comb, armed crown, crown, wheel, star etc have been investigate and also we prove that every graph is a subgraph of a connected k -total difference cordial graphs in. In this paper we investigate 4-total difference of cordial labeling of some graphs like star, path, cycle, bistar, crown, comb, etc.

§2. K -Total Difference Cordial Labeling

Definition 2.1 Let G be a graph. Let $f : V(G) \rightarrow \{0, 1, 2, \dots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv , assign the label $|f(u) - f(v)|$. f is called k -total difference cordial labeling of G if $|t_{df}(i) - t_{df}(j)| \leq 1$, $i, j \in \{0, 1, 2, \dots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labelled with x . A graph with a k -total difference cordial labeling is called k -total difference cordial graph. Otherwise, if there is a pair $\{i, j\} \subset \{0, 1, 2, \dots, k-1\}$ such that $|t_{df}(i) - t_{df}(j)| > 1$, such a labeling is called a Smarandachely k -total difference cordial labeling of G .

Remark 2.2([6]) 2-total difference cordial graph is 2-total product cordial graph.

¹Received July 13, 2019, Accepted March 16, 2020.

§3. Preliminaries

Definition 3.1 The corona of G_1 with $G_2, G_1 \odot G_2$ is the graph obtained by taking one copy of G_2 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2 .

Definition 3.2 Armed crown AC_n is the graph obtained from the cycle $C_n : u_1u_2 \cdots u_nu_1$ with $V(AC_n) = V(C_n) \cup \{v_i, w_i : 1 \leq i \leq n\}$ and $E(AC_n) = E(C_n) \cup \{u_iv_i, v_iw_i : 1 \leq i \leq n\}$.

Definition 3.3 An edge $x = uv$ of G is said to be subdivided if it is replaced by the edges uw and wv where w is a vertex not in $V(G)$. If every edge of G is subdivided, the resulting graph is called the subdivision graph $S(G)$.

Definition 3.4 Jelly fish graphs $J(m, n)$ obtained from a cycle $C_4 : uxvyu$ by joining x and y with an edge and appending m pendent edges to u and n pendent edges to v .

Definition 3.5 Triangular snake T_n is obtained from the path $P_n : u_1u_2 \cdots u_n$ with $V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n-1\}$ and $E(T_n) = E(P_n) \cup \{u_iv_i, u_{i+1}v_i : (1 \leq i \leq n-1)\}$.

Definition 3.6 Double Triangular snake $D(T_n)$ is obtained from the Path $P_n : u_1u_2 \cdots u_n$ with $V(D(T_n)) = V(P_n) \cup \{v_i, w_i : 1 \leq i \leq n-1\}$ and $E(D(T_n)) = E(P_n) \cup \{u_iv_i, u_iw_i : 1 \leq i \leq n-1\} \cup \{v_iu_{i+1}, w_iu_{i+1} : 1 \leq i \leq n-1\}$.

§4. Main Results

Theorem 4.1 Any star $K_{1,n}$ is 4-total difference cordial.

Proof Let $V(K_{1,n}) = \{u, u_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{uu_i : 1 \leq i \leq n\}$.

Case 1. $n \equiv 0 \pmod{4}$.

Let $n = 4r, r \in \mathbb{N}$. Assign the label 1 to the central vertex. Next assign the label 0 to the vertices u_1, u_2, \dots, u_{2r} and assign the label 3 to the remaining vertices.

Case 2. $n \equiv 1 \pmod{4}$.

Let $n = 4r + 1, r \in \mathbb{N}$. Assign the label 1 to the central vertex u . We now move to the pendent vertices. Assign the label 0 to the vertices u_1, u_2, \dots, u_{2r} and assign the label 3 to the next remaining vertices $u_{2r+1}, u_{2r+2}, \dots, u_{4r}$ and u_{4r+1} .

Case 3. $n \equiv 2 \pmod{4}$.

Let $n = 4r + 2, r \in \mathbb{N}$. In this case assign the label 0 to the vertices u_1, u_2, \dots, u_{2r} and u_{2r+1} . Next assign the label 3 to the vertices $u_{2r+2}, u_{2r+3}, \dots, u_{4r+2}$. Finally assign 1 to the central vertex u .

Case 4. $n \equiv 3 \pmod{4}$.

As in case (3) assign the label to $u, u_1, u_2, \dots, u_{n-1}$. Next assign the label 3 to the vertex

u_n .

Table 1 given below establish that this vertex labeling pattern is a 4-total difference cordial labeling.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod{4}$	$2r$	$2r + 1$	$2r$	$2r$
$n \equiv 1 \pmod{4}$	$2r$	$2r + 1$	$2r + 1$	$2r + 1$
$n \equiv 2 \pmod{4}$	$2r + 1$	$2r + 2$	$2r + 1$	$2r + 1$
$n \equiv 3 \pmod{4}$	$2r + 1$	$2r + 2$	$2r + 2$	$2r + 2$

Table 1

A 4-total difference cordial labeling of $K_{1,n}(n = 1, 2, 3)$ is given in Table 2.

Values of n	u	u_1	u_2	u_3
1	1	3		
2	1	0	3	
3	1	0	3	3

Table 2

This completes the proof. □

Theorem 4.2 *The path P_n is 4-total difference cordial for all values of n .*

Proof Let P_n be the path u_1, u_2, \dots, u_n .

Case 1. $n \equiv 0 \pmod{4}$ $n > 3$.

Let $n = 4r, r \in \mathbb{N}$, Assign the labels 3, 1, 1 and 3 respectively to the vertices u_1, u_2, u_3, u_4 . Next assign the labels 3, 1, 1 and 3 to the next 4 vertices u_5, u_6, u_7, u_8 respectively. Proceeding like this until we reach the vertex u_n . That is in this process the last 4 vertices $u_{n-3}, u_{n-2}, u_{n-1}$ and u_n receive the labels 3, 1, 1 and 3.

Case 2. $n \equiv 1 \pmod{4}$ $n > 3$.

Let $n = 4r + 1, r \in \mathbb{N}$. As in Case 1, assign the label to the vertices u_1, u_2, \dots, u_{n-1} . Next assign the label 3 to the vertex u_n .

Case 3. $n \equiv 2 \pmod{4}$, $n > 3$.

Let $n = 4r + 2, r \in \mathbb{N}$. Assign the label to the vertices u_1, u_2, \dots, u_{n-1} as in Case 2. Next assign the label 1 to the vertices u_n .

Case 4. $n \equiv 3 \pmod{4}$, $n > 3$.

Let $n = 4r + 3, r \in \mathbb{N}$. Assign the label to the vertices u_1, u_2, \dots, u_{n-1} as in Case 3. Next assign the label 1 to the vertex u_n . This vertex labels is a 4-total difference cordial labels follows from Table 3 for $n > 3$.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod{4}$	$2r - 1$	$2r$	$2r$	$2r$
$n \equiv 1 \pmod{4}$	$2r$	$2r$	$2r$	$2r + 1$
$n \equiv 2 \pmod{4}$	$2r$	$2r + 1$	$2r + 1$	$2r + 1$
$n \equiv 3 \pmod{4}$	$2r + 1$	$2r + 2$	$2r + 1$	$2r + 1$

Table 3

A 4-total difference cordial labeling of $P_n(n = 1, 2, 3)$ is given in Table 4.

Values of n	u_1	u_2	u_3
1	0		
2	0	2	
3	0	2	3

Table 4

This completes the proof. □

Theorem 4.3 *The cycle C_n is 4-total difference cordial if $n \equiv 0, 1, 3 \pmod{4}$*

Proof Let C_n be the cycle $u_1u_2 \cdots u_nu_1$. Assign the label to the vertices u_1, u_2, \dots, u_n as in Theorem 4.2. Table 5 given below shows that this labeling of C_n is a 4-total difference cordial.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod{4}$	$2r$	$2r$	$2r$	$2r$
$n \equiv 1 \pmod{4}$	$2r$	$2r$	$2r$	$2r + 1$
$n \equiv 3 \pmod{4}$	$2r + 1$	$2r + 2$	$2r + 1$	$2r + 1$

Table 5

This completes the proof. □

Theorem 4.4 *The bistar $B_{n,n}$ is 4-total different cordial for all n .*

Proof Let $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(B_{n,n}) = \{uu_i, vv_i, uv : (1 \leq i \leq n)\}$. Clearly $B_{n,n}$ has $2n + 2$ vertices and $2n + 1$ edges. Assign the label 1 to the central vertices u and v . Assign the label 3 to the vertices u_1, u_2, \dots, u_n and v_1 . We now assign the label 1 to the vertices v_2, v_3, \dots, v_n . Clearly $t_{df}(0) = n, t_{df}(1) = t_{df}(2) = t_{df}(3) = n + 1$. Therefore f is a 4-total difference cordial labeling. □

Theorem 4.5 *The crown $C_n \odot K_1$ is 4-total difference cordial labeling for all values of n .*

Proof Let C_n be the cycle $u_1u_2 \cdots u_nu_1$. Let $V(C_n \odot K_1) = V(C_n) \cup \{v_i : 1 \leq i \leq n\}$ and $E(C_n \odot K_1) = E(C_n) \cup \{u_i v_i : 1 \leq i \leq n\}$. Assign the label 1 to the cycle vertices u_1, u_2, \dots, u_n . Next we move to the pendent vertices v_i . Assign the label 3 to all pendent vertices v_1, v_2, \dots, v_n .

Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = t_{df}(3) = n$. Hence f is a 4-total difference cordial labeling. \square

Corollary 4.1 *All combs are 4-total difference cordial labeling.*

Proof Clearly the vertex labeling in theorem 4.5 is also a 4-total difference cordial labeling of $P_n \odot K_1$. \square

Theorem 4.6 *The armed crown AC_n is 4-total difference cordial for all n .*

Proof Clearly AC_n has $3n$ vertices and $3n$ edges. Let the vertex set and edge set as in Definition 3.2. Assign the label 1 to the all the cycle vertices u_1, u_2, \dots, u_n . Next we assign the label 3 to the vertices v_1, v_2, \dots, v_n .

Case 1. n is even.

In this case assign the label 3 to the pendent vertices $w_1 w_2 \dots w_{\frac{n}{2}}$ and 1 to the remaining pendent vertices $w_{\frac{n}{2}+1}, w_{\frac{n}{2}+2}, \dots, w_n$.

Case 2. n is odd.

Assign the label 3 to the vertices $w_1, w_2, \dots, w_{\frac{n+1}{2}}$ and 1 to the vertices $w_{\frac{n+3}{2}}, w_{\frac{n+5}{2}}, \dots, w_n$. The table 6 given below establish that this vertex labeling pattern is a 4-total difference cordial labeling.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
n is even	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n}{2}$
n is odd	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

Table 6

This completes the proof. \square

Theorem 4.7 *The double triangular snake DT_n is 4-total difference cordial for all n .*

Proof Let the vertex set and edge set as in Definition 3.6.

Case 1. $n \equiv 0 \pmod{3}$.

Assign the labels 3, 2, 3 to the path vertices u_1, u_2, u_3 . Next assign the labels 3, 2, 3 to the next 3 vertices u_4, u_5, u_6 respectively. Proceeding like this until we reach the vertices u_n . That is in the process the last three vertices u_{n-2}, u_{n-1}, u_n receive the label 3, 2, 3. Next assign the label 0 to the vertices v_1, v_2, \dots, v_n and assign the label 2 to the vertices w_1, w_2, \dots, w_n .

Case 2. $n \equiv 1 \pmod{3}$.

In this case assign the labels to the vertices $u_i, (1 \leq i \leq n-1), v_i, w_i, (1 \leq i \leq n-1)$ as in Case 1. Next assign the labels 3, 0, 2 respectively to the vetices u_n, v_{n-1}, w_n .

Case 3. $n \equiv 2 \pmod{3}$.

As in Case 2 assign the labels to the vertices $u_1, u_2, \dots, u_{n-1}, v_1, v_2, \dots, v_{n-2}$ and w_1, w_2, \dots, w_{n-2} .

Finally assign the label 2, 0 and 2 to the vertices u_n, v_{n-1} and w_{n-1} . Table 7 given below establish that this labeling scheme is a 4-total difference cordial labeling of DT_n .

Nature of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod{3}$	$2n - 2$	$2n - 2$	$2n - 1$	$2n - 2$
$n \equiv 1 \pmod{3}$	$2n - 2$	$2n - 2$	$2n - 2$	$2n - 1$
$n \equiv 2 \pmod{3}$	$2n - 2$	$2n - 2$	$2n - 1$	$2n - 2$

Table 7

This completes the proof. □

Example 4.1 A 4-total difference cordial labeling of $D(T_6)$ is shown in Figure 1.

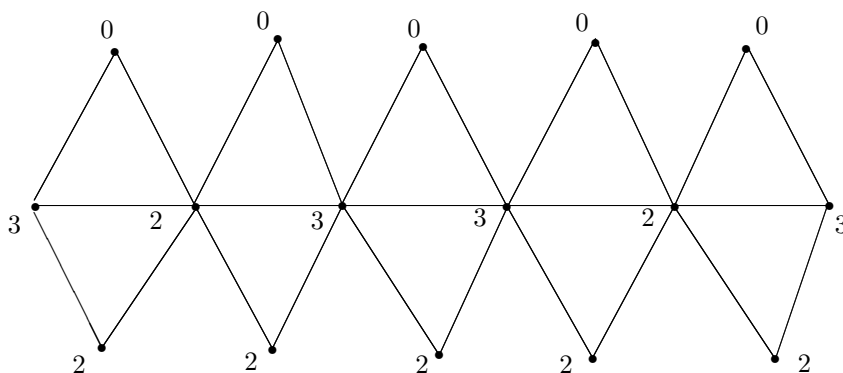


Figure 1

Theorem 4.8 The jelly fish $J(n, n)$ is 4-total difference cordial for all n .

Proof Let C_4 be a cycle $uxvyu$. Let $V(J(n, n)) = V(C_4) \cup \{u_i, v_i : 1 \leq i \leq n\}$ and $E(J(n, n)) = E(C_4) \cup \{xy, xu_i, yv_i : 1 \leq i \leq n\}$. Assign the label 1 to the all cycle vertices u, x, y, v . Next we move to the pendent vertices. Assign the label 3 to the u_1, u_2, \dots, u_n and v_1, v_2 . Assign the label 1 to the v_3, v_4, \dots, v_n . Since $t_{df}(0) = n + 3, t_{df}(1) = t_{df}(2) = t_{df}(3) = n + 2$, f is a 4-total difference cordial labeling. □

Theorem 4.9 The subdivision of the bistar $B_{n,n}$, $S(B_{n,n})$ is 4-total different cordial for all n .

Proof Let $V(S(B_{n,n})) = \{u, w, v, u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and $E(S(B_{n,n})) = \{uu_i, u_i x_i, uw, ww, vv_i, v_i y_i : 1 \leq i \leq n\}$. Assign the label 1 to the vertices u, w and v . Next assign the label 3 to the vertices $u_1, u_2, \dots, u_i, x_1, x_2, \dots, x_i$ and v_1 . We now assign the label 2 to the vertices y_1, y_2, \dots, y_n and v_2 . Finally assign the label 1 to the vertices v_3, v_4, \dots, v_n . Since $t_{df}(0) = t_{df}(1) = t_{df}(3) = 2n + 1, t_{df}(2) = 2n + 2$. The labeling f is a 4-total difference cordial labeling. □

Theorem 4.10 $P_n \odot 2K_1$ is 4-total difference cordial for all n .

Proof Let P_n be the path u_1, u_2, \dots, u_n . Let v_i, w_i be the pendent vertices adjacent to u_i ($1 \leq i \leq n$). Assign the label 1 to the path vertices u_1, u_2, \dots, u_n .

Case 1. n is even.

Assign the label 3 to all the vertices v_1, v_2, \dots, v_n and $w_1, w_2, \dots, w_{\frac{n}{2}}$. We now assign the label 1 to the vertices $w_{\frac{n}{2}+1}, w_{\frac{n}{2}+2}, \dots, w_n$.

Case 2. n is odd.

As in Case 1 assign the label to the vertices u_i, v_i, w_i ($1 \leq i \leq n$). Next assign the label 3 to the vertices u_n and assign the label 1 to the vertex w_n .

Table 8 given below establish that this vertex labeling pattern is a 4-total difference cordial labeling.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
n is even	$\frac{3n}{2} - 1$	$\frac{3n}{2}$	$\frac{3n}{2}$	$\frac{3n}{2}$
n is odd	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$

Table 8

This completes the proof. □

Theorem 4.11 $S(P_n \odot K_1)$ is 4-total difference cordial for all n .

Proof Let P_n be the path u_1, u_2, \dots, u_n . Let $V(P_n \odot K_1) = V(P_n) \cup \{v_i : 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = \{u_i : 1 \leq i \leq n\}$. Let x_i be the vertex which subdivide the edge $u_i u_{i+1}, \{1 \leq i \leq n-1\}$ and y_i be the vertex which subdivide $u_i v_i : \{1 \leq i \leq n\}$. Assign the label 3 to the all path vertices u_1, u_2, \dots, u_n and x_1, x_2, \dots, x_n and v_2 . Next we assign the label 1 to the vertices y_1, y_2, \dots, y_n and v_1 . Finally we assign the label 2 to the remaining vertices v_3, v_4, \dots, v_n . Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = 2n - 1, t_{df}(3) = 2n$. Therefore, f is a 4-total difference cordial labeling of $S(P_n \odot K_1)$. □

Theorem 4.12 $S(C_n \odot K_1)$ is 4-total difference cordial for all values of n .

Proof Let $C_n : u_1 u_2 \dots u_n u_1$ be the cycle. Let $V(C_n \odot K_1) = V(C_n) \cup \{v_i : (1 \leq i \leq n)\}$ and $E(C_n \odot K_1) = E(C_n) \cup \{u_i v_i : 1 \leq i \leq n\}$. Let x_i, y_i be the vertices which subdivide the edges $u_i u_{i+1} (1 \leq i \leq n-1)$, $u_i v_i (1 \leq i \leq n)$ respectively. First we assign the label 3 to the cycle vertices u_1, u_2, \dots, u_n and x_1, x_2, \dots, x_n . Next we assign the label 1 to the y_1, y_2, \dots, y_n . Finally assign the label 2 to the all pendent vertices v_1, v_2, \dots, v_n . Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = t_{df}(3) = 2n$. Therefore f is a 4-total difference cordial labeling of $S(C_n \odot K_1)$. □

Theorem 4.13 $S(AC_n)$ is 4-total difference cordial for all n .

Proof Let the vertex set and edge set of AC_n as in definition 3.2. let $x_i : (1 \leq i \leq n-1), y_i : (1 \leq i \leq n-1)$ and $z_i : (1 \leq i \leq n-1)$ be the vertex which subdivide the edges $u_i u_{i+1} : (1 \leq i \leq n-1), u_i v_i : (1 \leq i \leq n-1)$ and $v_i w_i : (1 \leq i \leq n-1)$ respectively. Assign the label 3 to the vertices u_1, u_2, \dots, u_n and x_1, x_2, \dots, x_n and w_1, w_2, \dots, w_n . Next assign

the label 1 to the vertices y_1, y_2, \dots, y_n . Then assign the label 2 to the vertices v_1, v_2, \dots, v_n and z_1, z_2, \dots, z_n . obviously $t_{df}(0) = t_{df}(1) = t_{df}(2) = t_{df}(3) = 3n$. Therefore f is a 4-total difference cordial labeling of $S(AC_n)$. \square

Example 4.2 A 4-total difference cordial labeling of $S(AC_n)$ is shown in Figure 2.

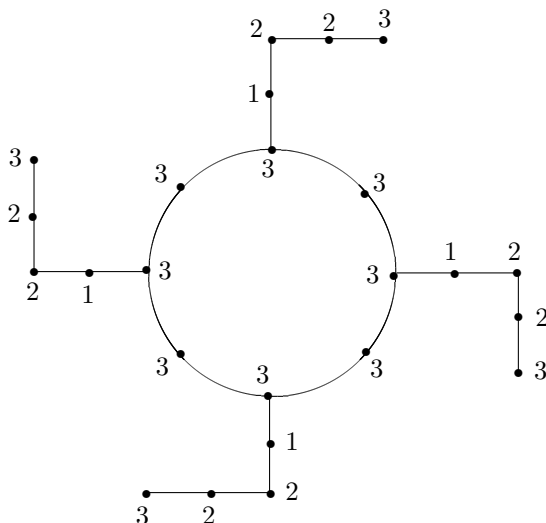


Figure 2

Theorem 4.14 $S(T_n)$ is 4-total difference cordial.

Proof Let the vertex set and edge set of T_n as in Definition 3.7. Let x_i, y_i and z_i be the vertices which subdivide the edges $u_i u_{i+1}, u_i, v_i$ and $u_{i+1} v_i, (1 \leq i \leq n)$.

Case 1. $n \equiv 0 \pmod{4}$.

Assign the label 3 to the vertices u_1, u_2, \dots, u_n and x_1, x_2, \dots, x_{n-1} . Assign the label 1 to the vertices $y_1, y_2 \dots y_{n-1}$. Next assign the label 2, 3, 1 and 3 to the vertices z_1, z_2, z_3, z_4 then assign the label 2, 3, 1 and 3 to the next 4 vertices z_5, z_6, z_7, z_8 respectively. Proceeding like this until we reach the vertices z_{n-1} . That is in the process the last four vertices are $z_{n-4}, z_{n-3}, z_{n-2}, z_{n-1}$ receive the label 2, 3, 1, 3. Next assign the label 0, 2, 3, 2 to the vertices v_1, v_2, v_3, v_4 then assign the label 0, 2, 3, 2 to the next 4 vertices v_5, v_6, v_7, v_8 respectively. Proceeding like this until we reach the vertices v_{n-1} . That is in the process the last 4 vertices $v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}$ receive the label 0, 2, 3, 2.

Case 2. $n \equiv 1 \pmod{4}$.

As in Case 1 assign the labels to the vertices $u_i, (1 \leq i \leq n-1), v_i, x_i, y_i, z_i, (1 \leq i \leq n-2)$. Next assign the labels 3, 0, 3, 1 and 2 respect to the vertices $u_n, v_{n-1}, x_{n-1}, y_{n-1}$ and z_{n-1} .

Case 3. $n \equiv 2 \pmod{4}$.

In this case, assign the labels to the vertices $u_i, (1 \leq i \leq n-1), v_i, x_i, y_i, z_i, (1 \leq i \leq n-2)$ as in Case 2. Finally assign the labels 3, 2, 3, 1 and 3 to the vertices $u_n, v_{n-1}, x_{n-1}, y_{n-1}$ and

z_{n-1} respectively.

Case 4. $n \equiv 3 \pmod{4}$.

As in Case 3, assign the label to $u_i, (1 \leq i \leq n-1), v_i, x_i, y_i, z_i, (1 \leq i \leq n-2)$. Next assign the labels 3, 3, 3, 1 and 1 to the vertices $u_n, v_{n-1}, x_{n-1}, y_{n-1}$ and z_{n-1} respectively. Table 9 given below establish that this vertex labeling pattern is a 4-total difference cordial labeling.

Nature of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod{4}$	$\frac{11n-8}{4}$	$\frac{11n-8}{4}$	$\frac{11n-12}{4}$	$\frac{11n-12}{4}$
$n \equiv 1 \pmod{4}$	$\frac{11n-9}{4}$	$\frac{11n-9}{4}$	$\frac{11n-5}{4}$	$\frac{11n-9}{4}$
$n \equiv 2 \pmod{4}$	$\frac{11n-10}{4}$	$\frac{11n-10}{4}$	$\frac{11n-10}{4}$	$\frac{11n-10}{4}$
$n \equiv 3 \pmod{4}$	$\frac{11n-9}{4}$	$\frac{11n-9}{4}$	$\frac{11n-9}{4}$	$\frac{11n-13}{4}$

Table 9

This completes the proof. □

References

- [1] I.Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, *Ars Combinatoria*, 23(1987), 201-207.
- [2] J.A.Gallian, A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 19 (2017) #Ds6.
- [3] F.Harary, *Graph theory*, Addison wesley, New Delhi (1969).
- [4] R.Ponraj, S.Yesu Doss Philip and R.Kala, k -total difference cordial graphs, *Journal of Algorithms and Computation*, 51(2019), 121-128.
- [5] R.Ponraj, S.Yesu Doss Philip and R.Kala, 3-total difference cordial graphs, *Global Engineering Science and research*, 6(2019), 46-51.
- [6] M.Sundaram, R.Ponraj and S.Somasundaram, Total product cordial labeling of graphs, *Bulletin of Pure and Applied Sciences (Section E - Maths and Statistics)*, 25E(2005), 199-203.