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Some Results on Super Mean Graphs

R. Vasuki[1] and A. Nagarajan[2]

[1] Department of Mathematics, Dr.Sivanthi Aditanar College of Engineering,
Tiruchendur - 628 215, Tamil Nadu, India.

[2] Department of Mathematics, V.O. Chidambaram College,
Thoothukudi - 628 002, Tamil Nadu, India.

E-mail: vasukisehar@yahoo.co.in, nagarajan.voc@gmail.com

Abstract: Let G be a graph and $f : V(G) \to \{1, 2, 3, \ldots, p + q\}$ be an injection. For each edge $e = uv$ and an integer $m \geq 2$, the induced *Smarandachely edge m-labeling* f_S^* is defined by

$$f_S^*(e) = \left\lceil \frac{f(u) + f(v)}{m} \right\rceil.$$

Then f is called a *Smarandachely super m-mean labeling* if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, p + q\}$. Particularly, in the case of $m = 2$, we know that

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even;} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u) + f(v) \text{ is odd.} \end{cases}$$

Such a labeling is usually called a *super mean labeling*. A graph that admits a Smarandachely super mean m-labeling is called *Smarandachely super m-mean graph*. In this paper, we prove that the H-graph, corona of a H-graph, $G \odot S_2$ where G is a H-graph, the cycle C_{2n} for $n \geq 3$, corona of the cycle C_n for $n \geq 3$, mC_n-snake for $m \geq 1, n \geq 3$ and $n \neq 4$, the dragon $P_n(C_m)$ for $m \geq 3$ and $m \neq 4$ and $C_m \times P_n$ for $m = 3, 5$ are super mean graphs, i.e., Smarandachely super 2-mean graphs.

Keywords: Labeling, Smarandachely super mean labeling, Smarandachely super m-mean graph, super mean labeling, super mean graphs

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§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected, simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology we follow [1].

Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the Cartesian

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product $G_1 \times G_2$ has p_1p_2 vertices which are $\{(u,v)/u \in G_1, v \in G_2\}$. The edges are obtained as follows: (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$.

The corona of a graph G on p vertices $v_1, v_2, \ldots, v_p$ is the graph obtained from G by adding p new vertices $u_1, u_2, \ldots, u_p$ and the new edges u_iv_i for $1 \leq i \leq p$, denoted by $G \odot K_1$. For a graph G, the 2-corona of G is the graph obtained from G by identifying the center vertex of the star S_2 at each vertex of G, denoted by $G \odot S_2$. The baloon of a graph G, $P_n(G)$ is the graph obtained from G by identifying an end vertex of P_n at a vertex of G. $P_n(C_m)$ is called a dragon. The join of two graphs G and H is the graph obtained from $G \cup H$ by joining each vertex of G with each vertex of H by means of an edge and it is denoted by $G + H$.

A path of n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. $K_{1,m}$ is called a star, denoted by S_m. The bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the center vertices of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively, denoted by $B(m)$. A triangular snake T_n is obtained from a path $v_1v_2 \ldots v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $1 \leq i \leq n-1$, that is, every edge of a path is replaced by a triangle C_3.

We define the H-graph of a path P_n to be the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even and a cyclic snake mC_n the graph obtained from m copies of C_n by identifying the vertex $v_{(k+2)_j}$ in the j^{th} copy at a vertex $v_{1_{j+1}}$ in the $(j+1)^{th}$ copy if $n = 2k+1$ and identifying the vertex $v_{(k+1)_j}$ in the j^{th} copy at a vertex $v_{1_{j+1}}$ in the $(j+1)^{th}$ copy if $n = 2k$.

A vertex labeling of G is an assignment $f : V(G) \to \{1, 2, 3, \ldots, p+q\}$ be an injection. For a vertex labeling f, the induced *Smarandachely edge m-labeling* f_S^* for an edge $e = uv$, an integer $m \geq 2$ is defined by

$$f_S^*(e) = \left\lceil \frac{f(u) + f(v)}{m} \right\rceil.$$

Then f is called a *Smarandachely super m-mean labeling* if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, p+q\}$. Particularly, in the case of $m = 2$, we know that

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even;} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

Such a labeling is usually called a *super mean labeling*. A graph that admits a Smarandachely super mean m-labeling is called *Smarandachely super m-mean graph*, particularly, *super mean graph* if $m = 2$. A super mean labeling of the graph P_6^2 is shown in Fig.1.1.

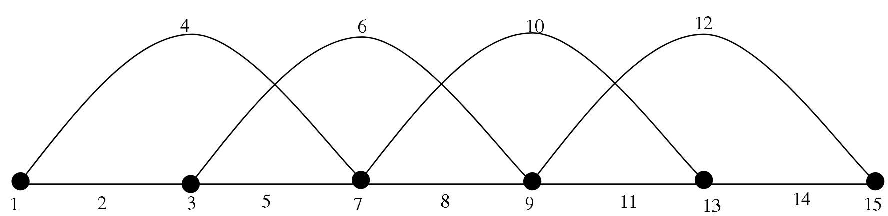


Fig.1.1

The concept of mean labeling was first introduced by S. Somasundaram and R. Ponraj [7]. They have studied in [4,5,7,8] the mean labeling of some standard graphs.

The concept of super mean labeling was first introduced by R. Ponraj and D. Ramya [2]. They have studied in [2,3] the super mean labeling of some standard graphs like $P_n, C_{2n+1}, n \geq 1$, $K_n (n \leq 3)$, $K_{1,n} (n \leq 3)$, T_n, $C_m \cup P_n (m \geq 3, n \geq 1)$, $B_{m,n} (m = n, n+1)$ etc. They have proved that the union of two super mean graph is super mean graph and C_4 is not a super mean graph. Also they determined all super mean graph of order ≤ 5.

In this paper, we establish the super meanness of the graph C_{2n} for $n \geq 3$, the H-graph, Corona of a H- graph, 2-corona of a H-graph, corona of cycle C_n for $n \geq 3$, mC_n-snake for $m \geq 1$, $n \geq 3$ and $n \neq 4$, the dragon $P_n(C_m)$ for $m \geq 3$ and $m \neq 4$ and $C_m \times P_n$ for $m = 3, 5$.

§2. Results

Theorem 2.1 *The H-graph G is a super mean graph.*

Proof Let $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ be the vertices of the graph G. We define a labeling $f : V(G) \to \{1, 2, \ldots, p+q\}$ as follows:

$f(v_i) = 2i - 1, \quad 1 \leq i \leq n$

$f(u_i) = 2n + 2i - 1, \quad 1 \leq i \leq n$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$f^*(v_i v_{i+1}) = 2i, \quad 1 \leq i \leq n-1$

$f^*(u_i u_{i+1}) = 2n + 2i, \quad 1 \leq i \leq n-1$

$f^*(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}) = 2n$ if n is odd

$f^*(v_{\frac{n}{2}+1} u_{\frac{n}{2}}) = 2n$ if n is even

Then clearly it can be verified that the H-graph G is a super mean graph. For example the super mean labelings of H-graphs G_1 and G_2 are shown in Fig.2.1. □

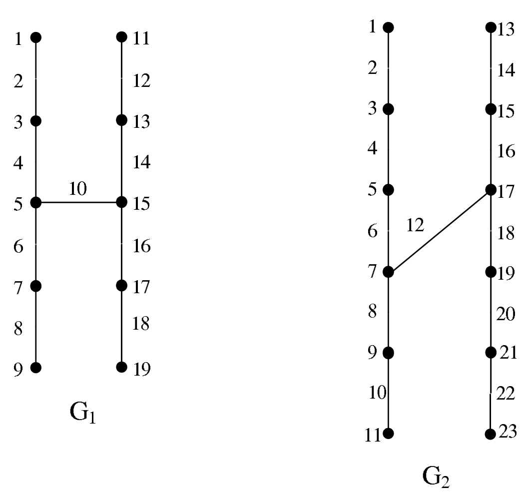


Fig.2.1

Theorem 2.2 *If a H-graph G is a super mean graph, then $G \odot K_1$ is a super mean graph.*

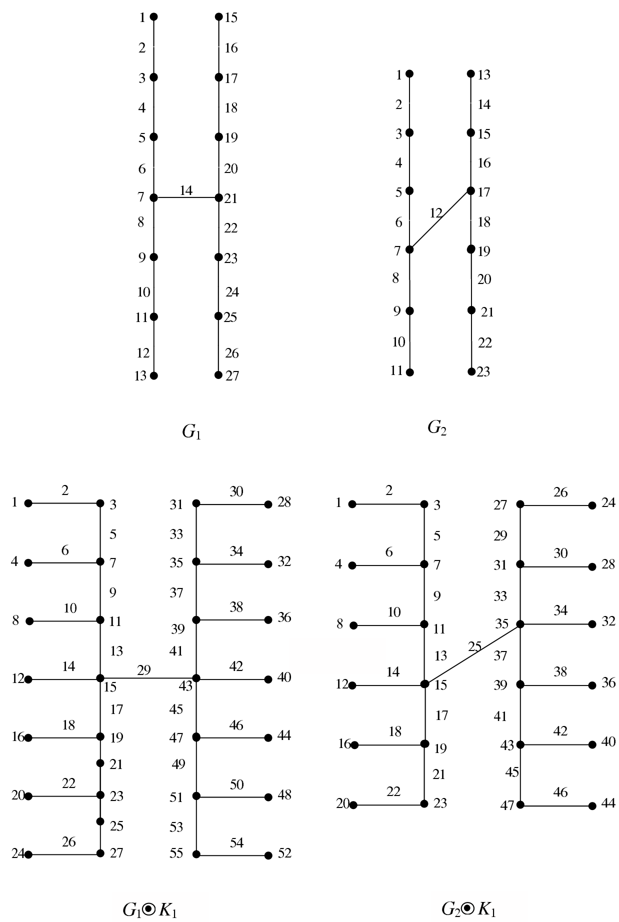


Fig.2.2

Proof Let f be a super mean labeling of G with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$. Let $v'_1, v'_2, \ldots, v'_n$ and $u'_1, u'_2, \ldots, u'_n$ be the corresponding new vertices in $G \odot K_1$.

We define a labeling $g : V(G \odot K_1) \to \{1, 2, \ldots, p+q\}$ as follows:

$$
\begin{aligned}
g(v_i) &= f(v_i) + 2i, && 1 \le i \le n\\
g(u_i) &= f(u_i) + 2n + 2i, && 1 \le i \le n\\
g(v'_1) &= f(v_1)\\
g(v'_i) &= f(v_i) + 2i - 3, && 2 \le i \le n\\
g(u'_i) &= f(u_i) + 2n + 2i - 3, && 1 \le i \le n
\end{aligned}
$$

For the vertex labeling g, the induced edge labeling g^* is defined as follows:

$$
\begin{array}{lll}
g^*(v_iv_{i+1}) & = f^*(v_iv_{i+1}) + 2i + 1, & 1 \le i \le n-1 \\
g^*(u_iu_{i+1}) & = f^*(u_iu_{i+1}) + 2n + 2i + 1, & 1 \le i \le n-1 \\
g^*(v_iv_i') & = f(v_i) + 2i - 1, & 1 \le i \le n \\
g^*(u_iu_i') & = f(u_i) + 2n + 2i - 1, & 1 \le i \le n \\
g^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) & = 2f^*((v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) + 1 & \text{if } n \text{ is odd} \\
g^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) & = 2f^*((v_{\frac{n}{2}+1}u_{\frac{n}{2}}) + 1 & \text{if } n \text{ is even}
\end{array}
$$

It can be easily verified that g is a super mean labeling and hence $G \odot K_1$ is a super mean graph. For example the super mean labeling of H-graphs G_1, G_2, $G_1 \odot K_1$ and $G_2 \odot K_1$ are shown in Fig.2.2. □

Theorem 2.3 *If a H-graph G is a super mean graph, then $G \odot S_2$ is a super mean graph.*

Proof Let f be a super mean labeling of G with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$. Let $v_1', v_2', \ldots, v_n'$, $v_1'', v_2'', \ldots, v_n''$, $u_1', u_2', \ldots, u_n'$ and $u_1'', u_2'', \ldots, u_n''$ be the corresponding new vertices in $G \odot S_2$.

We define $g : V(G \odot S_2) \to \{1, 2, \ldots, p + q\}$ as follows:

$$
\begin{array}{lll}
g(v_i) & = f(v_i) + 4i - 2, & 1 \le i \le n \\
g(v_i') & = f(v_i) + 4i - 4, & 1 \le i \le n \\
g(v_i'') & = f(v_i) + 4i, & 1 \le i \le n \\
g(u_i) & = f(u_i) + 4n + 4i - 2, & 1 \le i \le n \\
g(u_i') & = f(u_i) + 4n + 4i - 4, & 1 \le i \le n \\
g(u_i'') & = f(u_i) + 4n + 4i, & 1 \le i \le n
\end{array}
$$

For the vertex labeling g, the induced edge labeling g^* is defined as follows:

$$
\begin{array}{lll}
g^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) & = 3f^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}}) & \text{if } n \text{ is odd} \\
g^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) & = 3f^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) & \text{if } n \text{ is even}
\end{array}
$$

$$
\begin{array}{lll}
g^*(v_iv_{i+1}) & = f^*(v_iv_{i+1}) + 4i, & 1 \le i \le n-1 \\
g^*(v_iv_i') & = f(v_i) + 4i - 3, & 1 \le i \le n \\
g^*(v_iv_i'') & = f(v_i) + 4i - 1, & 1 \le i \le n \\
g^*(u_iu_{i+1}) & = f^*(u_iu_{i+1}) + 4n + 4i & 1 \le i \le n-1 \\
g^*(u_iu_i') & = f(u_i) + 4n + 4i - 3, & 1 \le i \le n \\
g^*(u_iu_i'') & = f(u_i) + 4n + 4i - 1, & 1 \le i \le n
\end{array}
$$

It can be easily verified that g is a super mean labeling and hence $G \odot S_2$ is a super mean graph. For example the super mean labelings of $G_1 \odot S_2$ and $G_2 \odot S_2$ are shown in Fig.2.3. □

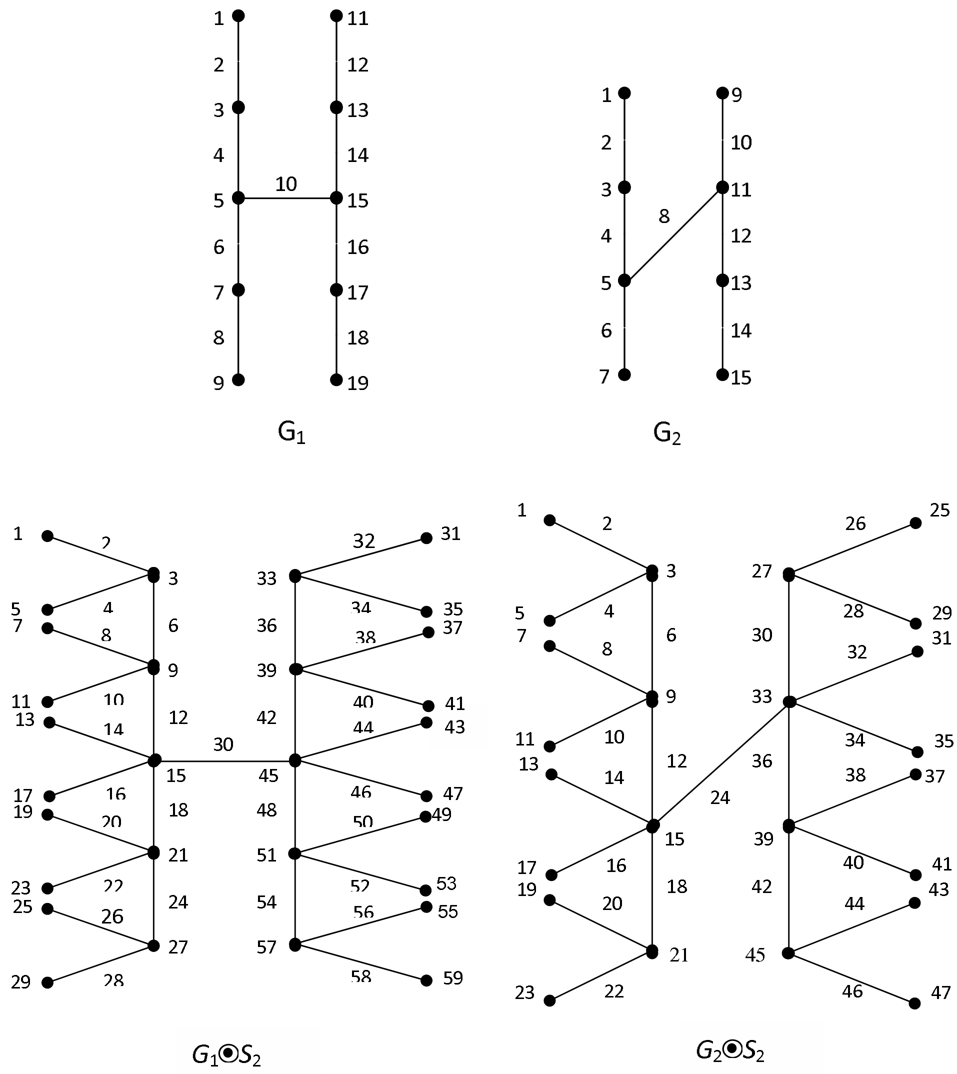


Fig.2.3

Theorem 2.4 *Cycle C_{2n} is a super mean graph for $n \geq 3$.*

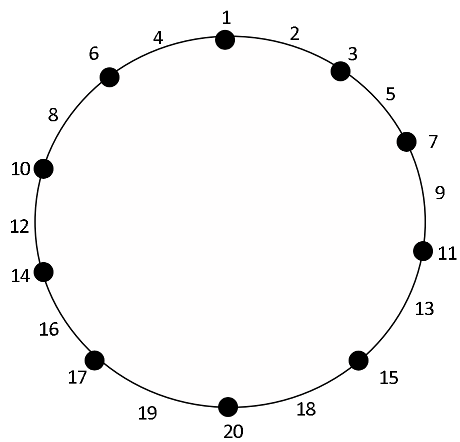
Proof Let C_{2n} be a cycle with vertices $u_1, u_2, \ldots, u_{2n}$ and edges $e_1, e_2, \ldots, e_{2n}$. Define $f : V(C_{2n}) \to \{1, 2, \ldots, p+q\}$ as follows:

$$\begin{aligned} f(u_1) &= 1 \\ f(u_i) &= 4i - 5, & 2 \leq i \leq n \\ f(u_{n+j}) &= 4n - 3j + 3, & 1 \leq j \leq 2 \\ f(u_{n+j+2}) &= 4n - 4j - 2, & 1 \leq j \leq n-2 \end{aligned}$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$$\begin{aligned} f^*(e_1) &= 2 \\ f^*(e_i) &= 4i-3, \qquad 2 \le i \le n-1 \\ f^*(e_n) &= 4n-2, \\ f^*(e_{n+1}) &= 4n-1, \\ f^*(e_{n+j+1}) &= 4n-4j, \qquad 1 \le j \le n-1 \end{aligned}$$

It can be easily verified that f is a super mean labeling and hence C_{2n} is a super mean graph. For example the super mean labeling of C_{10} is shown in Fig.2.4. □



C_{10}

Fig.2.4

Remark 2.5 In [2], it was proved that $C_{2n+1}, n \ge 1$ is a super mean graph and C_4 is not a super mean graph and hence the cycle C_n is a super mean graph for $n \ge 3$ and $n \ne 4$.

Theorem 2.6 *Corona of a cycle C_n is a super mean graph for $n \ge 3$.*

Proof Let C_n be a cycle with vertices $u_1, u_2, \ldots, u_n$ and edges $e_1, e_2, \ldots, e_n$. Let $v_1, v_2, \ldots, v_n$ be the corresponding new vertices in $C_n \odot K_1$ and E_i be the edges joining $u_iv_i, i = 1$ to n.

Define $f : V(C_n \odot K_1) \to \{1, 2, \ldots, p+q\}$ as follows:

Case i When n is odd, $n = 2m+1, m = 1, 2, 3, \ldots$

$$f(u_1) = 3$$

$$f(u_i) = \begin{cases} 5 + 8(i-2) & 2 \le i \le m+1 \\ 12 + 8(2m+1-i) & m+2 \le i \le 2m+1 \end{cases}$$

$$f(v_1) = 1$$

$$f(v_i) = \begin{cases} 7 + 8(i-2) & 2 \le i \le m+1 \\ 10 + 8(2m+1-i) & m+2 \le i \le 2m+1 \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$$f^*(e_1) = 4$$

$$f^*(e_i) = \begin{cases} 9 + 8(i-2) & 2 \le i \le m+1 \\ 8 + 8(2m+1-i) & m+2 \le i \le 2m+1 \end{cases}$$

$$f^*(E_1) = 2$$

$$f^*(E_i) = \begin{cases} 6 + 8(i-2) & 2 \le i \le m+1 \\ 11 + 8(2m+1-i) & m+2 \le i \le 2m+1 \end{cases}$$

Case ii When n is even, $n = 2m, m = 2, 3, \ldots$

$$\begin{aligned} f(u_1) &= 3 \\ f(u_i) &= 5 + 8(i-2), && 2 \le i \le m \\ f(u_{m+1}) &= 8m - 2, \\ f(u_i) &= 12 + 8(2m-i), && m+2 \le i \le 2m \\ f(v_1) &= 1 \\ f(v_i) &= 7 + 8(i-2), && 2 \le i \le m \\ f(v_{m+1}) &= 8m, \\ f(v_{m+2}) &= 8m - 7, \\ f(v_i) &= 10 + 8(2m-i), && m+3 \le i \le 2m \end{aligned}$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$$\begin{aligned} f^*(e_1) &= 4 \\ f^*(e_i) &= 9 + 8(i-2), && 2 \le i \le m-1 \\ f^*(e_m) &= 8m - 6, \\ f^*(e_{m+1}) &= 8m - 3, \\ f^*(e_i) &= 8 + 8(2m-i), && m+2 \le i \le 2m \\ f^*(E_1) &= 2 \\ f^*(E_i) &= 6 + 8(i-2), && 2 \le i \le m \\ f^*(E_{m+1}) &= 8m - 1 \\ f^*(E_i) &= 11 + 8(2m-i), && m+2 \le i \le 2m \end{aligned}$$

It can be easily verified that f is a super mean labeling and hence $C_n \odot K_1$ is a super mean graph. For example the super mean labelings of $C_7 \odot K_1$ and $C_8 \odot K_1$ are shown in Fig.2.5.□

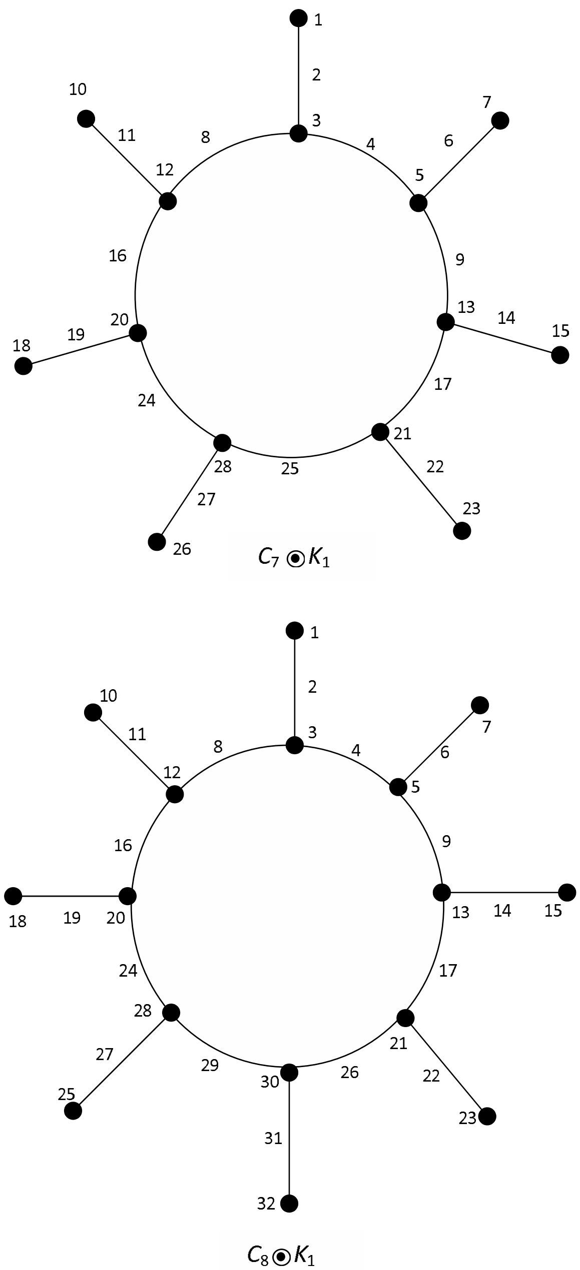


Fig.2.5

Remark 2.7 C_4 is not a super mean graph, but $C_4 \odot K_1$ is a super mean graph.

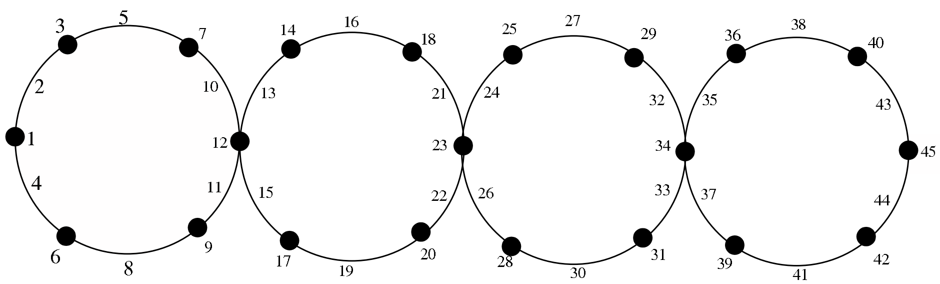
Theorem 2.8 *The graph mC_n- snake, $m \geq 1, n \geq 3$ and $n \neq 4$ has a super mean labeling.*

Proof We prove this result by induction on m.

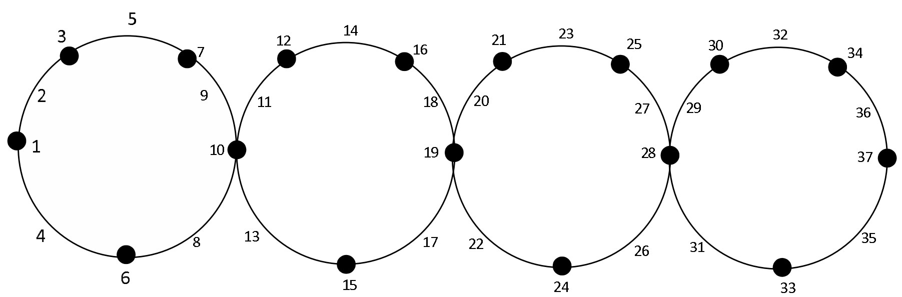
Let $v_{1_j}, v_{2_j}, \ldots, v_{n_j}$ be the vertices and $e_{1_j}, e_{2_j}, \ldots, e_{n_j}$ be the edges of mC_n for $1 \leq j \leq m$. Let f be a super mean labeling of the cycle C_n.

When $m = 1$, by Remark 1.5, C_n is a super mean graph, $n \geq 3, n \neq 4$. Hence the result is true when $m = 1$.

Let $m = 2$. The cyclic snake $2C_n$ is the graph obtained from 2 copies of C_n by identifying the vertex $v_{(k+2)_1}$ in the first copy of C_n at a vertex v_{1_2} in the second copy of C_n when $n = 2k+1$ and identifying the vertex $v_{(k+1)_1}$ in the first copy of C_n at a vertex v_{1_2} in the second copy of C_n when $n = 2k$.



$4C_6$-snake



$4C_5$-snake

Fig.2.6

Define a super mean labeling g of $2C_n$ as follows:

For $1 \leq i \leq n$,

$$g(v_{i_1}) = f(v_{i_1})$$
$$g(v_{i_2}) = f(v_{i_1}) + 2n - 1$$
$$g^*(e_{i_1}) = f^*(e_{i_1})$$
$$g^*(e_{i_2}) = f^*(e_{i_1}) + 2n - 1.$$

Thus, $2C_n$-snake is a super mean graph.

Assume that mC_n-snake is a super mean graph for any $m \geq 1$. We will prove that $(m+1)C_n$-snake is a super mean graph. Super mean labeling g of $(m+1)C_n$ is defined as follows:

$$g(v_{i_j}) = f(v_{i_1}) + (j-1)(2n-1), \qquad 1 \leq i \leq n, 2 \leq j \leq m$$
$$g(v_{i_{m+1}}) = f(v_{i_1}) + m(2n-1), \qquad 1 \leq i \leq n$$

For the vertex labeling g, the induced edge labeling g^* is defined as follows:

$$g^*(e_{i_j}) = f^*(e_{i_1}) + (j-1)(2n-1), \qquad 1 \le i \le n, 2 \le j \le m$$
$$g^*(e_{i_{m+1}}) = f^*(e_{i_1}) + m(2n-1), \qquad 1 \le i \le n$$

Then it is easy to check the resultant labeling g is a super mean labeling of $(m+1)C_n$-snake. For example the super mean labelings of $4C_6$-snake and $4C_5$- snake are shown in Fig.2.6. □

Theorem 2.9 *If G is a super mean graph then $P_n(G)$ is also a super mean graph.*

Proof Let f be a super mean labeling of G. Let $v_1, v_2, \ldots, v_p$ be the vertices and $e_1, e_2, \ldots, e_q$ be the edges of G and let $u_1, u_2, \ldots, u_n$ and $E_1, E_2, \ldots, E_{n-1}$ be the vertices and edge of P_n respectively.

We define g on $P_n(G)$ as follows:

$$g(v_i) = f(v_i), \qquad 1 \le i \le p.$$
$$g(u_j) = p + q + 2j - 2, \qquad 1 \le j \le n.$$

For the vertex labeling g, the induced edge labeling g^* is defined as follows:

$$g^*(e_i) = f(e_i) \qquad 1 \le i \le p.$$
$$g^*(E_j) = p + q + 2j - 1, \qquad 1 \le j \le n-1.$$

Then g is a super mean labeling of $P_n(G)$. □

Corollary 1.10 *Dragon $P_n(C_m)$ is a super mean graph for $m \ge 3$ and $m \ne 4$.*

Proof Since C_m is a super mean graph for $m \ge 3$ and $m \ne 4$, by using the above theorem, $P_n(C_m)$ for $m \ge 3$ and $m \ne 4$ is also a super mean graph. For example, the super mean labeling of $P_5(C_6)$ is shown in Fig.2.7. □

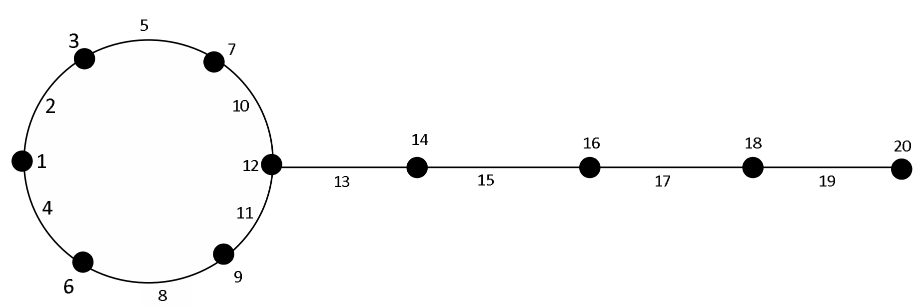


Fig.2.7

Remark 2.11 The converse of the above theorem need not be true. For example consider the graph C_4. $P_n(C_4)$ for $n \ge 3$ is a super mean graph but C_4 is not a super mean graph. The super mean labeling of the graph $P_4(C_4)$ is shown in Fig.2.8

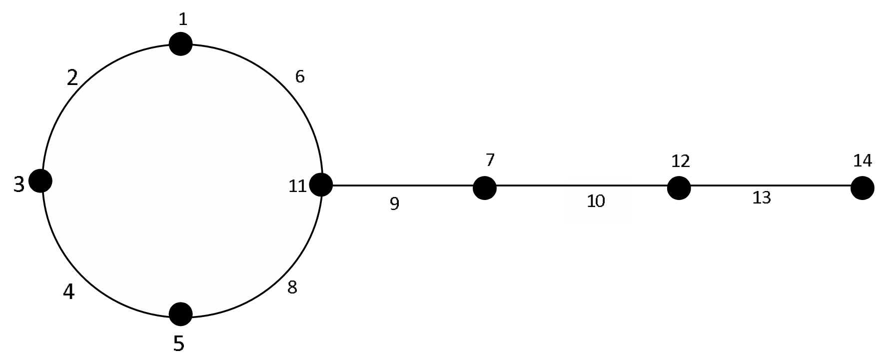


Fig.2.8

Theorem 2.12 *$C_m \times P_n$ for $n \geq 1, m = 3, 5$ are super mean graphs.*

Proof Let $V(C_m \times P_n) = \{v_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(C_m \times P_n) = \{e_{i_j} : e_{i_j} = v_{i_j} v_{(i+1)_j}, 1 \leq j \leq n, 1 \leq i \leq m\} \cup \{E_{i_j} : E_{i_j} = v_{i_j} v_{i_{j+1}}, 1 \leq j \leq n-1, 1 \leq i \leq m\}$ where $i+1$ is taken modulo m.

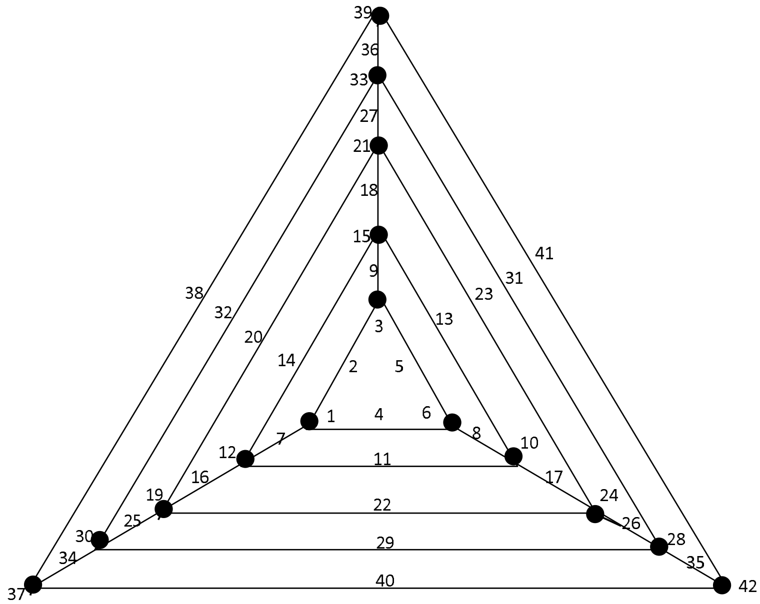
Case i $m = 3$

First we label the vertices of C_3^1 and C_3^2 as follows:

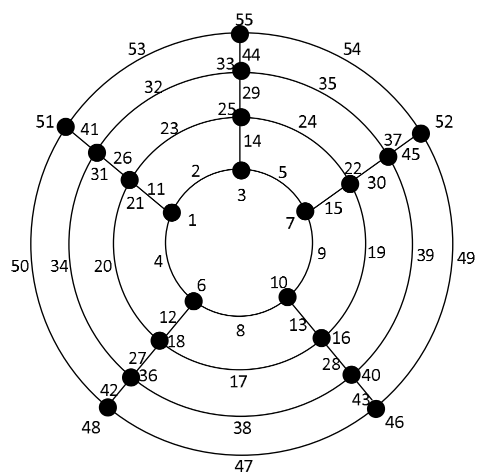
$$\begin{aligned}
f(v_{1_1}) &= 1 \\
f(v_{i_1}) &= 3i - 3, && 2 \leq i \leq 3 \\
f(v_{i_2}) &= 12 + 3(i-1), && 1 \leq i \leq 2 \\
f(v_{3_2}) &= 10
\end{aligned}$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$$\begin{aligned}
f^*(e_{i_1}) &= 2 + 3(i-1), && 1 \leq i \leq 2 \\
f^*(e_{3_1}) &= 4 \\
f^*(e_{1_2}) &= 14 \\
f^*(e_{i_2}) &= 13 - 2(i-2), && 2 \leq i \leq 3 \\
f^*(E_{i_1}) &= 7 + 2(i-1), && 1 \leq i \leq 2 \\
f^*(E_{3_1}) &= 8
\end{aligned}$$



$C_3 \times P_5$



$C_5 \times P_4$

Fig. 2.9

If the vertices and edges of C_3^{2j-1} and C_3^{2j} are labeled then the vertices and edges of C_3^{2j+1} and C_3^{2j+2} are labeled as follows:

$$f(v_{i_{2j+1}}) = f(v_{i_{2j-1}}) + 18, \quad 1 \le i \le 3, 1 \le j \le \tfrac{n-1}{2} \text{ if } n \text{ is odd and } 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even}$$

$$f(v_{i_{2j+2}}) = f(v_{i_{2j}}) + 18, \quad 1 \le i \le 3, 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and } 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even.}$$

$$f^*(e_{i_{2j+1}}) = f^*(e_{i_{2j-1}}) + 18, \quad 1 \le i \le 3, 1 \le j \le \tfrac{n-1}{2} \text{ if } n \text{ is odd and } 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even}$$

$$f^*(e_{j_{2j+2}}) = f^*(e_{i_{2j}}) + 18, \quad 1 \le i \le 3, 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and } 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even}$$

$$f^*(E_{i_{2j+1}}) = f^*(E_{i_{2j-1}}) + 18, \quad 1 \le i \le 3, 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and } 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even}$$

$$f^*(E_{i_{2j+2}}) = f^*(E_{i_{2j}}) + 18, \quad 1 \le i \le 3, 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and } 1 \le j \le \tfrac{n-4}{2} \text{ if } n \text{ is even}$$

Case ii $m = 5$.

First we Label the vertices of C_5^1 and C_5^2 as follows:

$$f(v_{1_1}) = 1$$

$$f(v_{i_1}) = \begin{cases} 4i - 5, & 2 \le i \le 3 \\ 10 - 4(i-4), & 4 \le i \le 5 \end{cases}$$

$$f(v_{1_2}) = 21$$

$$f(v_{i_2}) = \begin{cases} 25 - 3(i-2), & 2 \le i \le 3 \\ 16 + 2(i-4) & 4 \le i \le 5 \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is defined as follows:

$$f^*(e_{i_1}) = 2 + 3(i-1), \quad 1 \le i \le 2$$

$$f^*(e_{3_1}) = 9$$

$$f^*(e_{i_1}) = 8 - 4(i-4), \quad 4 \le i \le 5$$

$$f^*(e_{i_2}) = \begin{cases} 23 + (i-1), & 1 \le i \le 2 \\ 19 - 2(i-3), & 3 \le i \le 4 \end{cases}$$

$$f^*(e_{5_2}) = 20,$$

$$f^*(E_{1_1}) = 11$$

$$f^*(E_{i_1}) = \begin{cases} 14 + (i-2), & 2 \le i \le 3 \\ 13 - (i-4), & 4 \le i \le 5 \end{cases}$$

If the vertices and edges of C_5^{2j-1} and C_5^{2j} are labeled then the vertices and edges of C_5^{2j+1} and C_5^{2j+2} are labeled as follows:

$$
\begin{aligned}
f(v_{i_{2j+1}}) &= l(v_{i_{2j-1}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even and} \\
& && 1 \le j \le \tfrac{n-1}{2} \text{ if } n \text{ is odd} \\
f(v_{i_{2j+2}}) &= l(v_{i_{2j}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even and} \\
& && 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd.} \\
f^*(E_{i_{2j+1}}) &= f^*(E_{i_{2j-1}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and} \\
& && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even} \\
f^*(E_{i_{2j+2}}) &= f^*(E_{i_{2j}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd and} \\
& && 1 \le j \le \tfrac{n-4}{2} \text{ if } n \text{ is even} \\
f^*(e_{i_{2j+1}}) &= f^*(e_{i_{2j-1}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even and} \\
& && 1 \le j \le \tfrac{n-1}{2} \text{ if } n \text{ is odd} \\
f^*(e_{i_{2j+2}}) &= f^*(e_{i_{2j}}) + 30, 1 \le i \le 5, && 1 \le j \le \tfrac{n-2}{2} \text{ if } n \text{ is even and} \\
& && 1 \le j \le \tfrac{n-3}{2} \text{ if } n \text{ is odd.}
\end{aligned}
$$

Then it is easy to check that the labeling f is a super mean labeling of $C_3 \times P_n$ and $C_5 \times P_n$. For example the super mean labeling of $C_3 \times P_5$ and $C_5 \times P_4$ are shown in Fig.2.9. □

§3. Open Problems

We present the following open problem for further research.

Open Problem. *For what values of m (except 3,5) the graph $C_m \times P_n$ is super mean graph.*

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