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Some Generalized Result on Fixed Point Theorem in Complex Valued Intuitionistic Fuzzy Metric Space

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Abstract: In this paper, we will present a number of common fixed point theorems for contraction conditions that satisfy specific requirements in complex valued intuitionistic fuzzy metric spaces.

Key Words: Common fixed point, intuitionistic fuzzy set, complex valued, continuous t-norm.

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§1. Introduction

In 1965, Zadeh [12] proposed the concept of fuzzy sets. Fuzzy set theory is a useful tool for describing situations involving imprecise or ambiguous data. Fuzzy sets deal with situations like these by assigning a degree of belonging to a set to each object. Since then, it has become a burgeoning field of study in engineering, medicine, social science, graph theory, metric space theory, and complex analysis, among other fields. Kramosil and Michalek [6] introduced fuzzy metric spaces in a variety of ways in 1975. With the help of continuous t-norms, George and Veermani [4] improved the concept of fuzzy metric spaces in 1994.

Buckley [3] was the one who originally established the concept of fuzzy complex numbers and fuzzy complex analysis. 1987. Some authors were influenced by Buckley's work. Re-examination of fuzzy complex numbers continues. The year was 2002, and Fuzzy sets were extended to complicated fuzzy sets by Ramot et al. [8]. as though it were a blanket statement Ramot et al. claim that a membership function defines a sophisticated fuzzy set. function with a range that extends beyond [0, 1] the complicated plane's unit circle Singh was born in the year 2016. The concept of complex valued fuzzy was introduced by et al.[10]. Using complex valued continuous to create metric spaces t -norm as well as the concept of convergent convergence. In a complex valued fuzzy sequence, Cauchy sequence in complex valued fuzzy metric spaces. By introducing the concept of non-membership grade to fuzzy set theory, Atanassov [1] created a stir in 1983.

In the complex valued intuitionistic fuzzy metric spaces, this work gives some common fixed point theorems for pairs of occasionally weakly compatible mappings satisfying various requirements.

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§2. Preliminaries

Definition 2.1 *A binary operation* $* : r_s(\cos\theta + i\sin\theta) \times r_s(\cos\theta + i\sin\theta) \to r_s(\cos\theta + i\sin\theta)$, *where* $r_s \in [0,1]$ *and a fix* $\theta \in \left[0, \frac{\pi}{2}\right]$, *is called complex valued continuous t-norm if it satisfies the followings:*

(1) $*$ *is associative and commutative;*

(2) $*$ *is continuous;*

(3) $a * e^{i\theta} = a, \forall a \in r_s(\cos\theta + i\sin\theta)$*;*

(4) $\mathrm{a} * \mathrm{b} \le c * d$ *whenever* $a \le c$ *and* $b \le d, \forall \mathrm{a,b,c,d} \in r_s(\cos\theta + i\sin\theta)$.

Definition 2.2 *A binary operation* $: r_s(\cos\theta + i\sin\theta) \times r_s(\cos\theta + i\sin\theta) \to r_s(\cos\theta + i\sin\theta)$*,where* $r_s \in [0,1]$ *and a fix* $\theta \in \left[0, \frac{\pi}{2}\right]$ *is called complex valued continuous t-co norm if it satisfies the followings:*

(1) *is associative and commutative;*

(2) *is continuous;*

(3) $\mathrm{a} \diamond 0 = a, \forall a \in r_s(\cos\theta + i\sin\theta)$*;*

(4) $\mathrm{a} \diamond \mathrm{b} \le \mathrm{c} \diamond \mathrm{d}$ *whenever* $a \le c$ *and* $b \le d, \forall \mathrm{a,b,c,d} \in r_s(\cos\theta + i\sin\theta\ )$*.*

Definition 2.3 *The following are examples for complex valued continuous t-norm:*

(*i*) $a * b = \min\{a, b\}, \forall a, b \in r_s(\cos\theta + i\sin\theta)$ *and a fix* $\theta \in \left[0, \frac{\pi}{2}\right]$*;*

(*ii*) $a * b = \max(a + b - (\cos\theta + i\sin\theta), 0)$, *for all* $a, b \in r_s(\cos\theta + i\sin\theta)$ *and a fix* $\theta \in \left[0, \frac{\pi}{2}\right]$.

Definition 2.4 *The following are examples for complex valued continuous t-conorm:*

(*i*) $\mathrm{a} \diamond b = \max\{a, b\}, \forall a, b \in r_s(\cos\theta + i\sin\theta)$ *and a fix* $\theta \in \left[0, \frac{\pi}{2}\right]$*;*

(*ii*) $a \diamond b = \min(a + b, 1)$, *for all* $a, b \in r_s(\cos\theta + i\sin\theta)$ *and a fix* $\theta \in \left[0, \frac{\pi}{2}\right]$.

Definition 2.5 *The 5-triplet (* $X, \mathrm{M}, \mathrm{N}, *, \diamond)$ *is said to be complex valued intuitionistic fuzzy metric space if* X *is an arbitrary non empty set,* $*$ *is a complex valued continuous* t*-norm,* $\diamond$ *is a complex valued continuous t-conorm and* $M, N : X \times X \times (0, \infty) \to r_s(\cos\theta + i\sin\theta)$ *are complex valued fuzzy sets, where* $r_s \in [0,1], r_s(\cos\theta + i\sin\theta)$ *are complex valued fuzzy sets, where* $r_s \in$ *and* $\theta \in \left[0, \frac{\pi}{2}\right]$, *satisfying the following conditions:*

for all $\mathrm{x,y,z} \in X, \mathrm{t,s} \in (0, \infty), r_s \in [0,1]$ *and* $\theta \in \left[0, \frac{\pi}{2}\right]$,

$(cf1)$ $M(a, b, p) + M(a, b, p) \le (\cos\theta + i\sin\theta)$*;*

$(cf2)$ $M(a, b, p) > 0$*;*

$(cf3)$ $M(a, b, p) = (\cos\theta + i\sin\theta)$, *for all* $p \in (0, \infty)$ *if and only if* $\mathrm{a} = \mathrm{b}$*;*

$(cf4)$ $M(a, b, p) = M(b, a, p)$*;*

$(cf5)$ $M(a, b, p + s) \ge M(a, c, p) * M(c, b, s)$*;*

$(cf6)$ $M(a, b, p) : (0, \infty) \to r_s(\cos\theta + i\sin\theta)$ *is continuous;*

$(cf7)$ $N(a, b, p) < (\cos\theta + i\sin\theta)$*;*

$(cf8)$ $N(a, b, p) = 0$ *for all* $p \in (0, \infty)$ *if and only if* $\mathrm{a} = \mathrm{b}$*;*

$(cf9)$ *$N(a,b,p) = N(b,a,p)$;*

$(cf10)$ *$N(a,b,p+s) \leq N(a,c,p) \diamond N(c,b,s)$;*

$(cf11)$ *$N(a,b,p) : (0,\infty) \to r_s(\cos\theta + i\sin\theta)$ is continuous.*

The pair (M,N) is called a complex valued intuitionistic fuzzy metric space. The functions $M(a,b,p)$ and $N(a,b,p)$ denotes the degree of nearness and non-nearness between a and b with respect to t. It is noted that if we take $\theta = 0$, then complex valued intuitionistic fuzzy metric simply goes to real valued intuitionistic fuzzy metric.

§3. Main Results

Theorem 3.1 *Let $(X,M,N,*,\diamond)$ be a complex valued intuitionistic fuzzy metric space with*

$$\lim_{p\to\infty} M(a,b,p) = (\cos\theta + i\sin\theta) \quad and \quad \lim_{p\to\infty} N(a,b,p) = 0$$

for all $a,b \in X$ and let P,Q,A and B be self - mappings on X. Let the pairs $\{P,A\}$ and $\{Q,B\}$ be occasionally weakly compatible. If there exists $d \in (0,1)$ such that

$$M(Pa,Qb,dp) \geq \left\{ \begin{array}{c} M(Aa,Bb,p) * M(Pa,Ab,p)* \\ M(Qb,Bb,p) * M(Ps,Bb,p) \end{array} \right\}, \tag{3.1}$$

$$N(Px,By,kt) \leq \left\{ \begin{array}{c} N(Aa,Bb,p) \diamond N(Pa,Ab,p) \\ N(Qb,Bb,p) \diamond N(Ps,Bb,p) \end{array} \right\} \tag{3.2}$$

for all $a,b \in X$ and for all $p > 0$. Then P,Q,A and B have a unique common fixed point in X.

Proof The pairs $\{P,A\}$ and $\{Q,B\}$ be occasionally compatible, so there are points $a,b \in X$ such that Aa and $Qb = Bb$. Now, from (3.1) and (3.2) we have

$$\begin{aligned} M(Pa,Qb,dp) &\geq \left\{ \begin{array}{c} M(Aa,Bb,p) * M(Pa,Ab,p)* \\ M(Qb,Bb,p) * M(Ps,Bb,p) \end{array} \right\} \\ N(Px,By,kt) &\leq \left\{ \begin{array}{c} N(Aa,Bb,p) \diamond N(Pa,Ab,p)\diamond \\ N(Qb,Bb,p) \diamond N(Ps,Bb,p) \end{array} \right\} \\ M(Pa,Qb,dp) &= \left\{ \begin{array}{c} M(Pa,Qb,p) * M(Pa,Pa,p)* \\ M(Qb,Qb,p) * M(Pa,Qb,p) \end{array} \right\} \\ N(Px,By,kt) &= \left\{ \begin{array}{c} N(Pa,Qb,p) \diamond N(Pa,Pa,p)\diamond \\ N(Qb,Qb,p) \diamond N(Pa,Qb,p) \end{array} \right\} \end{aligned}$$

$$\begin{aligned}
M(Pa,Qb,dp) &= \left\{ \begin{array}{c} M(Pa,Qb,p)*(\cos\theta+i\sin\theta)* \\ (\cos\theta+i\sin\theta)*M(Pa,Qb,p) \end{array} \right\} \\
N(Px,By,kt) &= \left\{ \begin{array}{c} N(Pa,Qb,p)\diamond 0\diamond \\ 0\circ N(Pa,Qb,p) \end{array} \right\} \\
M(Pa,Qb,dp) &= M(Pa,Qb,p) \\
N(Px,By,kt) &= N(Pa,Qb,p)
\end{aligned}$$

but $\{a_n\}$ be a sequence in a complex valued intuitionistic fuzzy metric space $(X,M,N,*,\diamond)$ with $\lim_{p\to\infty} M(a,b,p)=\cos\theta+\mathrm{i}\sin\theta$ and $\lim_{p\to\infty} N(a,b,p)=0, \forall a,b\in X$.

If $\lim_{p\to 0} N(a,b,p)=0$, there exists $d\in(0,1)$ such that $M(a_{n+1},a_{n+2},dp)\geqslant M(a_n,a_{n+1},p)$ and $N(a_{n+1},a_{n+2},dp)\leqslant(a_n,a_{n+1}p)$, for all $p>0$, then $\{a_n\}$ is a cauchy sequence in X. Then $Pc=Qb$ and consequently

$$Pa=Aa=Qb=Bb. \tag{3.3}$$

Now suppose that the pair P,A have another coincidence point $Pc=Ac$.

$$\begin{aligned}
M(Pc,Qb,dp) &\geq \left\{ \begin{array}{c} M(Ac,Bb,p)*M(Pc,Ac,p)* \\ M(Qb,Bb,p)*M(Pc,Bb,p) \end{array} \right\} \\
N(Pc,Qb,dp) &\leq \left\{ \begin{array}{c} N(Ac,Bb,p)\diamond N(Pc,Ac,p)\diamond \\ N(Qb,Bb,p)\diamond N(Pc,Bb,p) \end{array} \right\} \\
M(Pc,Qb,dp) &= \left\{ \begin{array}{c} M(Pc,Qb,p)*M(Pc,Pc,p)* \\ M(Qb,Qb,p)*M(Pc,Qb,p) \end{array} \right\} \\
N(Pc,Qb,dp) &= \left\{ \begin{array}{c} N(Pc,Qb,p)\diamond N(Pc,Pc,p)\diamond \\ N(Qb,Qb,p)\diamond N(Pc,Qb,p) \end{array} \right\} \\
M(Pc,Qb,dp) &= \left\{ \begin{array}{c} M(Pc,Qb,p)*(\cos\theta+i\sin\theta)* \\ (\cos\theta+i\sin\theta)*M(Pc,Qb,p) \end{array} \right\} \\
N(Pc,Qb,dp) &= \left\{ \begin{array}{c} N(Pc,Qb,p)\diamond 0\diamond \\ 0\vee N(Pc,Qb,p) \end{array} \right\} \\
M(Pc,Qb,dp) &= M(Pc,Qb,p) \\
N(Pc,Qb,dp) &= N(Pc,Qb,p)
\end{aligned}$$

but $\{a_n\}$ be a sequence in a complex valued intuitionistic fuzzy metric space $(X,\mathrm{M},\mathrm{N},*,\diamond)$ with $\lim_{p\to\infty} M(a,b,p)=\cos\theta+\mathrm{i}\sin\theta$ and $\lim_{p\to\infty} N(a,b,p)=0, \forall a,b\in X$.

If $\lim_{p\to 0} N(a,b,p) = 0$, there exists $d \in (0,1)$ such that $M(a_{n+1}, a_{n+2}, dp) \geqslant M(a_n, a_{n+1}, p)$ and $N(a_{n+1}, a_{n+2}, dp) \leqslant (a_n, a_{n+1}, p)$, for all $p > 0$, then $\{a_n\}$ is a cauchy sequence in X. Then $Pc = Qb$ and consequently

$$Pc = Ac = Qb = Bb. \tag{3.4}$$

From (3.3) and (3.4) we have $\mathrm{Pa} = \mathrm{Pc}$ and therefore the pair $\{\mathrm{P}, \mathrm{A}\}$ have a unique point of coincidence $\mathrm{v} = \mathrm{Pa} = \mathrm{Aa}$, v is the common fixed point of $\{\mathrm{P}, \mathrm{A}\}$.

But, $\{a_n\}$ be a sequence in a complex valued intuitionistic fuzzy metric space $(X, \mathrm{M}, \mathrm{N}, *, \diamond)$ with $\lim_{p\to\infty} M(a,b,p) = \cos\theta + \mathrm{i}\sin\theta$ and $\lim_{p\to\infty} N(a,b,p) = 0, \forall a, b \in X$.

If $\lim_{p\to 0} N(a,b,p) = 0$, there exists $d \in (0,1)$ such that $M(a_{n+1}, a_{n+2}, dp) \geqslant M(a_n, a_{n+1}, p)$ and $N(a_{n+1}, a_{n+2}, dp) \leqslant (a_n, a_{n+1}, p)$, for all $p > 0$, then $\{a_n\}$ is a cauchy sequence in X.Then we have v = w and v is the common fixed point of P, Q, A and B. For uniqueness, let u is an another common fixed point of P, Q, A and B.

Therefore,

$$\begin{aligned}
M(v,w,dp) &= M(Pv,Qu,dp) \\
&\geq \left\{ \begin{array}{c} M(Av,Bu,p) * M(Pv,Av,p) * \\ M(Qu,Bu,p) * M(Pv,Bu,p) \end{array} \right\} \\
&= \left\{ \begin{array}{c} M(v,u,p) * M(v,v,p) * \\ M(v,v,p) * M(v,u,p) \end{array} \right\} \\
&= \left\{ \begin{array}{c} M(v,u,p) * (\cos\theta + i\sin\theta) \\ *(\cos\theta + i\sin\theta) * M(v,u,p) \end{array} \right\} \\
&= M(v,u,p) \\
N(v,w,dp) &= N(Pv,Qu,dp) \\
&\leq \left\{ \begin{array}{c} N(Av,Bu,p) \diamond N(Pv,Av,p) \diamond \\ N(Qu,Bu,p) \diamond N(Pv,Bu,p) \end{array} \right\} \\
&= \left\{ \begin{array}{c} N(v,u,p) \diamond N(v,v,p) \diamond \\ N(v,v,p) \diamond N(v,u,p) \end{array} \right\} \\
&= \left\{ \begin{array}{c} N(v,u,p) \diamond 0 \diamond 0 \diamond \\ N(v,u,p) \end{array} \right\} = N(v,u,p) \\
M(v,w,dp) &= M(Pv,Qw,dp) \\
&\geq \left\{ \begin{array}{c} M(Av,Bw,p) * M(Pv,Av,p) * \\ M(Qw,Bw,p) * M(Pv,Bw,p) \end{array} \right\} \\
&= \{M(Pv,Qw,p) * M(Pv,Pv,p) *\}
\end{aligned}$$

$$
\begin{aligned}
&= \{M(Qw,Qw,p) * M(Pv,Bw,p)\} \\
&= \left\{ \begin{array}{c} M(Pv,Qw,p) * (\cos\theta + i\sin\theta) \\ *(\cos\theta + i\sin\theta) * M(Pv,Qw,p) \end{array} \right\} \\
&= M(Pv,Qw,p) = M(v,w,p) \\
N(v,w,dp) &= N(Pv,Qw,dp) \\
&\leq \left\{ \begin{array}{c} N(Av,Bw,p) \diamond N(Pv,Av,p) \diamond \\ N(Qw,Bw,p) \diamond N(Pv,Bw,p) \end{array} \right\} \\
&= \{N(Pv,Qw,p) \diamond N(Pv,Pv,p) \diamond \\
&= \{N(Pv,Qw,p) \diamond 0 \diamond 0\} \\
&= \left\{ \begin{array}{c} N(Pv,Qw,p) \diamond 0 \diamond 0 \diamond \\ N(Pv,Qw,p) \end{array} \right\} \\
&= N(Pv,Qw,p) = N(v,w,p)
\end{aligned}
$$

Consequently, $v = u$ and P, Q, A and B have a unique common fixed point. □

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