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The General Leap-Zagreb-Type Indices of Some Chemical Graphs*

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Abstract: The leap Zagreb indices have recently found remarkable applications in predicting the physico-chemical properties of alkanes and benzene systems. Subsequently, topological index related to the leap Zagreb indices emerge endlessly. In order to unify the study of these topological indices, we introduce the general leap Zagreb-type indices from the point of mathematics, and give the calculation formulas of the general leap Zagreb-type indices for some chemical trees, chemical chains, benzenoid systems and nanostructures, which extends the known results.

Key Words: Topological index, general leap-Zagreb-type indices, chemical graph.

AMS(2010): 05C09, 05C92.

§1. Introduction

Throughout the article, G is a simple undirected connected graph with vertex set $V(G)$ and edge set $E(G)$. The distance $d(u,v)$ between any two vertices u and v of a graph G is equal to the length of the shortest path connecting them. The 2-distance degree of a vertex u, denoted by τ_u, is the number of vertices $v \in V(G)$ such that $d(u,v) = 2$. Let $K = \{(i, j) \in \mathbb{N} \times \mathbb{N} : 1 \leq i \leq j \leq n-2\}$ and $m(i,j)$ be the number of edges in G joining vertices of 2-distance degree i and j.

In 2017, the leap Zagreb indices of a graph are introduced by Naji, Soner and Gutman [11] based on the 2-distance degree of the vertices. For a graph G, the first, second and third leap Zagreb indices are defined by

$$LM_1(G) = \sum_{u\in V(G)} \tau_u^2, \quad LM_2(G) = \sum_{uv\in E(G)} \tau_u\tau_v, \quad LM_3(G) = \sum_{uv\in E(G)} (\tau_u + \tau_v).$$

The leap Zagreb indices attracted a considerable attention in the researchers' cycles, one may refer to [3, 9, 10, 13, 15, 16] and the references therein. In particular, Basavanagoud, Mondal and Das et al. [2, 4, 8] showed that the leap Zagreb indices have very good correlation with physical properties of chemical compound like entropy, boiling point, accentric factor,

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enthalpy of vaporization and standard enthalpy of vaporization. Inspired by the leap Zagreb indices, many related topological indices have been proposed and studied by scholars, such as

$$\begin{aligned}
HLM_1(G) &= \sum_{uv\in E(G)} (\tau_u+\tau_v)^2 \ ([6]), & HLM_2(G) &= \sum_{uv\in E(G)} (\tau_u\tau_v)^2 \ ([6]),\\
SL(G) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{\tau_u+\tau_v}} \ ([7]), & GAL(G) &= \sum_{uv\in E(G)} \frac{2\sqrt{\tau_u\tau_v}}{\tau_u+\tau_v} \ ([7]),\\
AL_4(G) &= \sum_{uv\in E(G)} \sqrt{\tau_u\tau_v} \ ([14]), & AL_5(G) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{\tau_u\tau_v}} \ ([14]),\\
AL_6(G) &= \sum_{uv\in E(G)} \left(\frac{\tau_u}{\tau_v}+\frac{\tau_v}{\tau_u}\right) \ ([14]), & AL_7(G) &= \sum_{uv\in E(G)} \frac{\tau_u\tau_v}{\tau_u+\tau_v} \ ([14]),\\
LSO(G) &= \sum_{uv\in E(G)} \sqrt{\tau_u^2+\tau_v^2} \ ([5]), & HLF(G) &= \sum_{uv\in E(G)} (\tau_u^2+\tau_v^2)^2 \ ([5]),\\
LF(G) &= \sum_{uv\in E(G)} (\tau_u^2+\tau_v^2) \ ([5]), & LY(G) &= \sum_{uv\in E(G)} (\tau_u^3+\tau_v^3) \ ([15]).
\end{aligned}$$

In order to unify the study of these topological indices, the general leap Zagreb-type indices of a connected graph G are introduced, and defined as

$$LZ_{\alpha,\beta}(G) = \sum_{uv\in E(G)} (\tau_u^{\alpha}\tau_v^{\beta}+\tau_u^{\beta}\tau_v^{\alpha}), \quad LRZ_{\alpha,\beta,\gamma}(G) = \sum_{uv\in E(G)} (\tau_u\tau_v)^{\alpha}(\tau_u^{\beta}+\tau_v^{\beta})^{\gamma},$$

where α, β and γ are arbitrary real numbers. It is clear that, the topological indices discussed previously, can be obtained from the general leap Zagreb-type indices for some particular values of α, β and γ. In this paper, we compute the expressions of the general leap Zagreb-type indices for some molecular trees, chemical chains, benzenoid systems and nanostructures, which extends the results of the references [1, 5, 12].

§2. General Leap Zagreb-Type Indices of Some Chemical Trees

The structures of three types molecular trees with n vertices are shown in Figure 1.

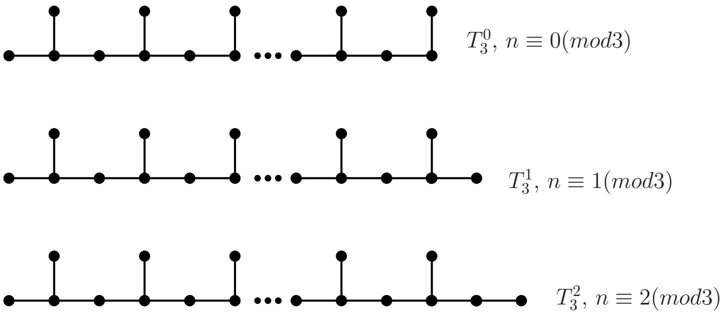


Figure 1 Three types molecular trees with n vertices.

Theorem 2.1 *The general leap Zagreb-type indices of* T_3^0, *shown in Figure 1, are given by*

$$\begin{aligned}
LZ_{\alpha,\beta}(T_3^0) &= \frac{2n-15}{3}\cdot 2^{\alpha+\beta}(2^\alpha+2^\beta)+\frac{n-6}{3}\cdot 2^{1+\alpha+\beta}+2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha\\
&\quad +2^{\alpha+1}+2^{\beta+1}+2^{2\alpha}+2^{2\beta}+3^\alpha+3^\beta+2,\\
LRZ_{\alpha,\beta,\gamma}(T_3^0) &= 2^{\alpha+1}(1+2^\beta)^\gamma+2^{2\alpha}(1+4^\beta)^\gamma+\frac{2n-15}{3}\cdot 2^{3\alpha+\beta\gamma}(1+2^\beta)^\gamma\\
&\quad +\frac{n-6}{3}\cdot 2^{2\alpha+(\beta+1)\gamma}+6^\alpha(2^\beta+3^\beta)^\gamma+3^\alpha(1+3^\beta)^\gamma+2^\gamma.
\end{aligned}$$

Proof By the definition of T_3^0, we obtain the basic information on T_3^0 in the following table.

$m(1,2)$	$m(1,4)$	$m(2,4)$	$m(2,2)$	$m(2,3)$	$m(1,3)$	$m(1,1)$
2	1	$\frac{2n-15}{3}$	$\frac{n-6}{3}$	1	1	1

Thus, we have

$$\begin{aligned}
LZ_{\alpha,\beta}(T_3^0) &= 2(1^\alpha\cdot 2^\beta+1^\beta\cdot 2^\alpha)+(1^\alpha\cdot 4^\beta+1^\beta\cdot 4^\alpha)+\frac{2n-15}{3}\cdot(2^\alpha\cdot 4^\beta+2^\beta\cdot 4^\alpha)\\
&\quad +\frac{n-6}{3}\cdot(2^\alpha\cdot 2^\beta+2^\beta\cdot 2^\alpha)+(2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha)+(1^\alpha\cdot 3^\beta+1^\beta\cdot 3^\alpha)\\
&\quad +(1^\alpha\cdot 1^\beta+1^\beta\cdot 1^\alpha)\\
&= \frac{2n-15}{3}\cdot 2^{\alpha+\beta}(2^\alpha+2^\beta)+\frac{n-6}{3}\cdot 2^{1+\alpha+\beta}+2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha\\
&\quad +2^{\alpha+1}+2^{\beta+1}+2^{2\alpha}+2^{2\beta}+3^\alpha+3^\beta+2,
\end{aligned}$$

$$\begin{aligned}
LRZ_{\alpha,\beta,\gamma}(T_3^0) &= 2(1\cdot 2)^\alpha(1^\beta+2^\beta)^\gamma+(1\cdot 4)^\alpha(1^\beta+4^\beta)^\gamma+\frac{2n-15}{3}\cdot(2\cdot 4)^\alpha(2^\beta+4^\beta)^\gamma\\
&\quad +\frac{n-6}{3}\cdot(2\cdot 2)^\alpha(2^\beta+2^\beta)^\gamma+(2\cdot 3)^\alpha(2^\beta+3^\beta)^\gamma+(1\cdot 3)^\alpha(1^\beta+3^\beta)^\gamma\\
&\quad +(1\cdot 1)^\alpha(1^\beta+1^\beta)^\gamma\\
&= 2^{\alpha+1}(1+2^\beta)^\gamma+2^{2\alpha}(1+4^\beta)^\gamma+\frac{2n-15}{3}\cdot 2^{3\alpha+\beta\gamma}(1+2^\beta)^\gamma\\
&\quad +\frac{n-6}{3}\cdot 2^{2\alpha+(\beta+1)\gamma}+6^\alpha(2^\beta+3^\beta)^\gamma+3^\alpha(1+3^\beta)^\gamma+2^\gamma.
\end{aligned}$$

This completes the proof. □

Theorem 2.2 *The general leap Zagreb-type indices of* T_3^1, *shown in Figure 1, are given by*

$$\begin{aligned}
LZ_{\alpha,\beta}(T_3^1) &= \frac{2n-14}{3}\cdot 2^{\alpha+\beta}(2^\alpha+2^\beta)+\frac{n-7}{3}\cdot 2^{1+\alpha+\beta}+2^{\alpha+2}+2^{\beta+2}+2^{2\alpha+1}+2^{2\beta+1},\\
LRZ_{\alpha,\beta,\gamma}(T_3^1) &= 2^{\alpha+2}(1+2^\beta)^\gamma+2^{2\alpha+1}(1+4^\beta)^\gamma+\frac{2n-14}{3}\cdot 2^{3\alpha+\beta\gamma}(1+2^\beta)^\gamma\\
&\quad +\frac{n-7}{3}\cdot 2^{2\alpha+(\beta+1)\gamma}.
\end{aligned}$$

Proof By the definition of T_3^1, we obtain the basic information on T_3^1 in the following table.

$m(1,2)$	$m(1,4)$	$m(2,4)$	$m(2,2)$
4	2	$\frac{2n-14}{3}$	$\frac{n-7}{3}$

Thus, we have

$$\begin{aligned}LZ_{\alpha,\beta}(T_3^1) &= 4(1^\alpha\cdot 2^\beta+1^\beta\cdot 2^\alpha)+2(1^\alpha\cdot 4^\beta+1^\beta\cdot 4^\alpha)+\frac{2n-14}{3}\cdot(2^\alpha\cdot 4^\beta+2^\beta\cdot 4^\alpha)\\ &+\frac{n-7}{3}\cdot(2^\alpha\cdot 2^\beta+2^\beta\cdot 2^\alpha)\\ &= \frac{2n-14}{3}\cdot 2^{\alpha+\beta}(2^\alpha+2^\beta)+\frac{n-7}{3}\cdot 2^{1+\alpha+\beta}+2^{\alpha+2}+2^{\beta+2}+2^{2\alpha+1}+2^{2\beta+1},\end{aligned}$$

$$\begin{aligned}LRZ_{\alpha,\beta,\gamma}(T_3^1) &= 4(1\cdot 2)^\alpha(1^\beta+2^\beta)^\gamma+2(1\cdot 4)^\alpha(1^\beta+4^\beta)^\gamma\\ &+\frac{2n-14}{3}\cdot(2\cdot 4)^\alpha(2^\beta+4^\beta)^\gamma+\frac{n-7}{3}\cdot(2\cdot 2)^\alpha(2^\beta+2^\beta)^\gamma\\ &= 2^{\alpha+2}(1+2^\beta)^\gamma+2^{2\alpha+1}(1+4^\beta)^\gamma+\frac{2n-14}{3}\cdot 2^{3\alpha+\beta\gamma}(1+2^\beta)^\gamma\\ &+\frac{n-7}{3}\cdot 2^{2\alpha+(\beta+1)\gamma}.\end{aligned}$$

This completes the proof. □

Theorem 2.3 *The general leap Zagreb-type indices of T_3^2, shown in Figure 1, are given by*

$$\begin{aligned}LZ_{\alpha,\beta}(T_3^2) &= \frac{2n-13}{3}\cdot 2^{\alpha+\beta}(2^\alpha+2^\beta)+\frac{n-2}{3}\cdot 2^{1+\alpha+\beta}+3\cdot(2^\alpha+2^\beta)+2^{2\alpha}+2^{2\beta},\\ LRZ_{\alpha,\beta,\gamma}(T_3^2) &= 3\cdot 2^\alpha(1+2^\beta)^\gamma+2^{2\alpha}(1+4^\beta)^\gamma+\frac{2n-13}{3}\cdot 2^{3\alpha+\beta\gamma}(1+2^\beta)^\gamma\\ &+\frac{n-2}{3}\cdot 2^{2\alpha+(\beta+1)\gamma}.\end{aligned}$$

Proof By the definition of T_3^2, we obtain the basic information on T_3^2 in the following table.

$m(1,2)$	$m(1,4)$	$m(2,4)$	$m(2,2)$
3	1	$\frac{2n-13}{3}$	$\frac{n-2}{3}$

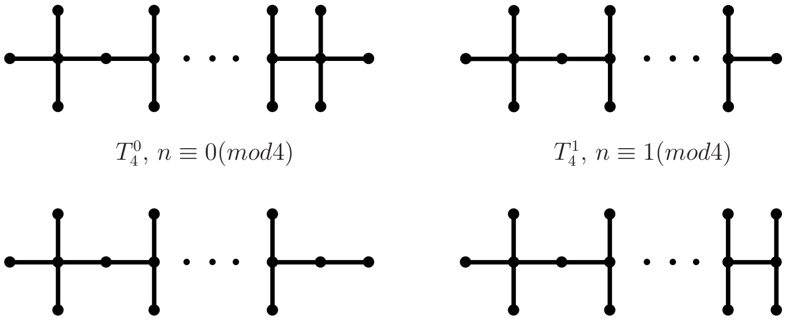
Thus, we have

$$\begin{aligned}LZ_{\alpha,\beta}(T_3^2) &= 3(1^\alpha\cdot 2^\beta+1^\beta\cdot 2^\alpha)+(1^\alpha\cdot 4^\beta+1^\beta\cdot 4^\alpha)+\frac{2n-13}{3}\cdot(2^\alpha\cdot 4^\beta+2^\beta\cdot 4^\alpha)\\ &+\frac{n-2}{3}\cdot(2^\alpha\cdot 2^\beta+2^\beta\cdot 2^\alpha)\\ &= \frac{2n-13}{3}\cdot 2^{\alpha+\beta}(2^\alpha+2^\beta)+\frac{n-2}{3}\cdot 2^{1+\alpha+\beta}+3\cdot(2^\alpha+2^\beta)+2^{2\alpha}+2^{2\beta},\\ LRZ_{\alpha,\beta,\gamma}(T_3^2) &= 3(1\cdot 2)^\alpha(1^\beta+2^\beta)^\gamma+(1\cdot 4)^\alpha(1^\beta+4^\beta)^\gamma\end{aligned}$$

$$
\begin{aligned}
&+\frac{2n-13}{3}\cdot(2\cdot 4)^{\alpha}(2^{\beta}+4^{\beta})^{\gamma}+\frac{n-2}{3}\cdot(2\cdot 2)^{\alpha}(2^{\beta}+2^{\beta})^{\gamma}\\
=\;& 3\cdot 2^{\alpha}(1+2^{\beta})^{\gamma}+2^{2\alpha}(1+4^{\beta})^{\gamma}+\frac{2n-13}{3}\cdot 2^{3\alpha+\beta\gamma}(1+2^{\beta})^{\gamma}+\frac{n-2}{3}\cdot 2^{2\alpha+(\beta+1)\gamma}.
\end{aligned}
$$

This completes the proof. □

There are four types molecular trees with n vertices shown in Figure 2.



T_4^0, $n \equiv 0(mod4)$ T_4^1, $n \equiv 1(mod4)$

T_4^2, $n \equiv 2(mod4)$ T_4^3, $n \equiv 3(mod4)$

Figure 2 Four types molecular trees with n vertices.

Theorem 2.4 *The general leap Zagreb-type indices of* T_4^0*, shown in Figure 2, are given by*

$$
\begin{aligned}
LZ_{\alpha,\beta}(T_4^0) &= \frac{n-12}{2}\cdot 2^{\alpha+\beta}(3^{\alpha}+3^{\beta})+\frac{n-12}{2}\cdot(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+3(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})\\
&\quad +2^{2\alpha}\cdot 6^{\beta}+2^{2\beta}\cdot 6^{\alpha}+3^{\alpha+1}+3^{\beta+1}+6^{\alpha}+6^{\beta}+2\cdot 3^{1+\alpha+\beta},\\
LRZ_{\alpha,\beta,\gamma}(T_4^0) &= 3^{\alpha+1}(1+3^{\beta})^{\gamma}+6^{\alpha}(1+6^{\beta})^{\gamma}+\frac{n-12}{2}\cdot 12^{\alpha}(2^{\beta}+6^{\beta})^{\gamma}+\frac{n-12}{2}\\
&\quad \times 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+24^{\alpha}(4^{\beta}+6^{\beta})^{\gamma}+3\cdot 12^{\alpha}(3^{\beta}+4^{\beta})^{\gamma}+2^{\gamma}\cdot 3^{1+2\alpha+\beta\gamma}.
\end{aligned}
$$

Proof By the definition of T_4^0, we obtain the basic information on T_4^0 in the following table.

$m(1,3)$	$m(1,6)$	$m(2,6)$	$m(2,3)$	$m(4,6)$	$m(3,4)$	$m(3,3)$
3	1	$\frac{n-12}{2}$	$\frac{n-12}{2}$	1	3	3

Thus, we have

$$
\begin{aligned}
LZ_{\alpha,\beta}(T_4^0) &= 3(1^{\alpha}\cdot 3^{\beta}+1^{\beta}\cdot 3^{\alpha})+(1^{\alpha}\cdot 6^{\beta}+1^{\beta}\cdot 6^{\alpha})+\frac{n-12}{2}\cdot(2^{\alpha}\cdot 6^{\beta}+2^{\beta}\cdot 6^{\alpha})\\
&\quad +\frac{n-12}{2}\cdot(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+(4^{\alpha}\cdot 6^{\beta}+4^{\beta}\cdot 6^{\alpha})+3(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})\\
&\quad +3(3^{\alpha}\cdot 3^{\beta}+3^{\beta}\cdot 3^{\alpha})\\
&= \frac{n-12}{2}\cdot 2^{\alpha+\beta}(3^{\alpha}+3^{\beta})+\frac{n-12}{2}\cdot(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+3(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})\\
&\quad +2^{2\alpha}\cdot 6^{\beta}+2^{2\beta}\cdot 6^{\alpha}+3^{\alpha+1}+3^{\beta+1}+6^{\alpha}+6^{\beta}+2\cdot 3^{1+\alpha+\beta},
\end{aligned}
$$

$$\begin{aligned} LRZ_{\alpha,\beta,\gamma}(T_4^0) &= 3(1\cdot 3)^{\alpha}(1^{\beta}+3^{\beta})^{\gamma}+(1\cdot 6)^{\alpha}(1^{\beta}+6^{\beta})^{\gamma}+\frac{n-12}{2}\cdot(2\cdot 6)^{\alpha}(2^{\beta}+6^{\beta})^{\gamma} \\ &\quad +\frac{n-12}{2}\cdot(2\cdot 3)^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+(4\cdot 6)^{\alpha}(4^{\beta}+6^{\beta})^{\gamma}+3(3\cdot 4)^{\alpha}(3^{\beta}+4^{\beta})^{\gamma} \\ &\quad +3(3\cdot 3)^{\alpha}(3^{\beta}+3^{\beta})^{\gamma} \\ &= 3^{\alpha+1}(1+3^{\beta})^{\gamma}+6^{\alpha}(1+6^{\beta})^{\gamma}+\frac{n-12}{2}\cdot 12^{\alpha}(2^{\beta}+6^{\beta})^{\gamma}+\frac{n-12}{2} \\ &\quad \times 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+24^{\alpha}(4^{\beta}+6^{\beta})^{\gamma}+3\cdot 12^{\alpha}(3^{\beta}+4^{\beta})^{\gamma}+2^{\gamma}\cdot 3^{1+2\alpha+\beta\gamma}. \end{aligned}$$

This completes the proof. □

Theorem 2.5 *The general leap Zagreb-type indices of T_4^1, shown in Figure 2, are given by*

$$\begin{aligned} LZ_{\alpha,\beta}(T_4^1) &= \frac{n-9}{2}\cdot(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+\frac{n-9}{2}\cdot 2^{\alpha+\beta}(3^{\alpha}+3^{\beta}) \\ &\quad +6(3^{\alpha}+3^{\beta})+2(6^{\alpha}+6^{\beta}), \\ LRZ_{\alpha,\beta,\gamma}(T_4^1) &= 6\cdot 3^{\alpha}(1+3^{\beta})^{\gamma}+2\cdot 6^{\alpha}(1+6^{\beta})^{\gamma}+\frac{n-9}{2}\cdot 12^{\alpha}(2^{\beta}+6^{\beta})^{\gamma} \\ &\quad +\frac{n-9}{2}\cdot 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}. \end{aligned}$$

Proof By the definition of T_4^1, we obtain the basic information on T_4^1 in the following table.

$m(1,3)$	$m(1,6)$	$m(2,6)$	$m(2,3)$
6	2	$\frac{n-9}{2}$	$\frac{n-9}{2}$

Thus, we have

$$\begin{aligned} LZ_{\alpha,\beta}(T_4^1) &= 6(1^{\alpha}\cdot 3^{\beta}+1^{\beta}\cdot 3^{\alpha})+2(1^{\alpha}\cdot 6^{\beta}+1^{\beta}\cdot 6^{\alpha})+\frac{n-9}{2}\cdot(2^{\alpha}\cdot 6^{\beta}+2^{\beta}\cdot 6^{\alpha}) \\ &\quad +\frac{n-9}{2}\cdot(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha}) \\ &= \frac{n-9}{2}\cdot(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+\frac{n-9}{2}\cdot 2^{\alpha+\beta}(3^{\alpha}+3^{\beta})+6(3^{\alpha}+3^{\beta})+2(6^{\alpha}+6^{\beta}), \end{aligned}$$

$$\begin{aligned} LRZ_{\alpha,\beta,\gamma}(T_4^1) &= 6(1\cdot 3)^{\alpha}(1^{\beta}+3^{\beta})^{\gamma}+2(1\cdot 6)^{\alpha}(1^{\beta}+6^{\beta})^{\gamma} \\ &\quad +\frac{n-9}{2}\cdot(2\cdot 6)^{\alpha}(2^{\beta}+6^{\beta})^{\gamma}+\frac{n-9}{2}\cdot(2\cdot 3)^{\alpha}(2^{\beta}+3^{\beta})^{\gamma} \\ &= 6\cdot 3^{\alpha}(1+3^{\beta})^{\gamma}+2\cdot 6^{\alpha}(1+6^{\beta})^{\gamma} \\ &\quad +\frac{n-9}{2}\cdot 12^{\alpha}(2^{\beta}+6^{\beta})^{\gamma}+\frac{n-9}{2}\cdot 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}. \end{aligned}$$

This completes the proof. □

Theorem 2.6 *The general leap Zagreb-type indices of T_4^2, shown in Figure 2, are given by*

$$\begin{aligned}LZ_{\alpha,\beta}(T_4^2) &= \frac{n-4}{2}\cdot(2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha)+\frac{n-8}{2}\cdot 2^{\alpha+\beta}(3^\alpha+3^\beta)+4(3^\alpha+3^\beta)\\ &\quad +6^\alpha+6^\beta,\\ LRZ_{\alpha,\beta,\gamma}(T_4^2) &= 4\cdot 3^\alpha(1+3^\beta)^\gamma+6^\alpha(1+6^\beta)^\gamma+\frac{n-8}{2}\cdot 12^\alpha(2^\beta+6^\beta)^\gamma\\ &\quad +\frac{n-4}{2}\cdot 6^\alpha(2^\beta+3^\beta)^\gamma.\end{aligned}$$

Proof By the definition of T_4^2, we obtain the basic information on T_4^2 in the following table.

$m(1,3)$	$m(1,6)$	$m(2,6)$	$m(2,3)$
4	1	$\frac{n-8}{2}$	$\frac{n-4}{2}$

Thus, we have

$$\begin{aligned}LZ_{\alpha,\beta}(T_4^2) &= 4(1^\alpha\cdot 3^\beta+1^\beta\cdot 3^\alpha)+(1^\alpha\cdot 6^\beta+1^\beta\cdot 6^\alpha)+\frac{n-8}{2}\cdot(2^\alpha\cdot 6^\beta+2^\beta\cdot 6^\alpha)\\ &\quad +\frac{n-4}{2}\cdot(2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha)\\ &= \frac{n-4}{2}\cdot(2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha)+\frac{n-8}{2}\cdot 2^{\alpha+\beta}(3^\alpha+3^\beta)+4(3^\alpha+3^\beta)\\ &\quad +6^\alpha+6^\beta,\end{aligned}$$

$$\begin{aligned}LRZ_{\alpha,\beta,\gamma}(T_4^2) &= 4(1\cdot 3)^\alpha(1^\beta+3^\beta)^\gamma+(1\cdot 6)^\alpha(1^\beta+6^\beta)^\gamma\\ &\quad +\frac{n-8}{2}\cdot(2\cdot 6)^\alpha(2^\beta+6^\beta)^\gamma+\frac{n-4}{2}\cdot(2\cdot 3)^\alpha(2^\beta+3^\beta)^\gamma\\ &= 4\cdot 3^\alpha(1+3^\beta)^\gamma+6^\alpha(1+6^\beta)^\gamma+\frac{n-8}{2}\cdot 12^\alpha(2^\beta+6^\beta)^\gamma\\ &\quad +\frac{n-4}{2}\cdot 6^\alpha(2^\beta+3^\beta)^\gamma.\end{aligned}$$

This completes the proof. □

Theorem 2.7 *The general leap Zagreb-type indices of T_4^3, shown in Figure 2, are given by*

$$\begin{aligned}LZ_{\alpha,\beta}(T_4^3) &= \frac{n-11}{2}\cdot(2^\alpha\cdot 6^\beta+2^\beta\cdot 6^\alpha)+\frac{n-7}{2}\cdot(2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha)\\ &\quad +3^\alpha\cdot 6^\beta+3^\beta\cdot 6^\alpha+2\cdot 3^{1+\alpha+\beta}+3^{1+\alpha}+3^{\beta+1}+6^\alpha+6^\beta,\\ LRZ_{\alpha,\beta,\gamma}(T_4^3) &= 3^{1+\alpha}(1+3^\beta)^\gamma+6^\alpha(1+6^\beta)^\gamma+\frac{n-11}{2}\cdot 12^\alpha(2^\beta+6^\beta)^\gamma\\ &\quad +\frac{n-7}{2}\cdot 6^\alpha(2^\beta+3^\beta)^\gamma+2^\gamma\cdot 3^{1+2\alpha+\beta\gamma}+18^\alpha(3^\beta+6^\beta)^\gamma.\end{aligned}$$

Proof By the definition of T_4^3, we obtain the basic information on T_4^3 in the following table.

$m(1,3)$	$m(1,6)$	$m(2,6)$	$m(2,3)$	$m(3,3)$	$m(3,6)$
3	1	$\frac{n-11}{2}$	$\frac{n-7}{2}$	3	1

Thus, we have

$$\begin{aligned}
LZ_{\alpha,\beta}(T_4^3) &= 3(1^\alpha\cdot 3^\beta+1^\beta\cdot 3^\alpha)+(1^\alpha\cdot 6^\beta+1^\beta\cdot 6^\alpha)+\frac{n-11}{2}\cdot(2^\alpha\cdot 6^\beta+2^\beta\cdot 6^\alpha)\\
&\quad+\frac{n-7}{2}\cdot(2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha)+3(3^\alpha\cdot 3^\beta+3^\beta\cdot 3^\alpha)+(3^\alpha\cdot 6^\beta+3^\beta\cdot 6^\alpha)\\
&= \frac{n-11}{2}\cdot(2^\alpha\cdot 6^\beta+2^\beta\cdot 6^\alpha)+\frac{n-7}{2}\cdot(2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha)\\
&\quad+3^\alpha\cdot 6^\beta+3^\beta\cdot 6^\alpha+2\cdot 3^{1+\alpha+\beta}+3^{1+\alpha}+3^{\beta+1}+6^\alpha+6^\beta,\\
LRZ_{\alpha,\beta,\gamma}(T_4^3) &= 3(1\cdot 3)^\alpha(1^\beta+3^\beta)^\gamma+(1\cdot 6)^\alpha(1^\beta+6^\beta)^\gamma+\frac{n-11}{2}\cdot(2\cdot 6)^\alpha(2^\beta+6^\beta)^\gamma\\
&\quad+\frac{n-7}{2}\cdot(2\cdot 3)^\alpha(2^\beta+3^\beta)^\gamma+3(3\cdot 3)^\alpha(3^\beta+3^\beta)^\gamma+(3\cdot 6)^\alpha(3^\beta+6^\beta)^\gamma\\
&= 3^{1+\alpha}(1+3^\beta)^\gamma+6^\alpha(1+6^\beta)^\gamma+\frac{n-11}{2}\cdot 12^\alpha(2^\beta+6^\beta)^\gamma\\
&\quad+\frac{n-7}{2}\cdot 6^\alpha(2^\beta+3^\beta)^\gamma+2^\gamma\cdot 3^{1+2\alpha+\beta\gamma}+18^\alpha(3^\beta+6^\beta)^\gamma.
\end{aligned}$$

This completes the proof. □

§3. General Leap Zagreb-Type Indices of Some Chemical Chains

There are eight type chemical chains shown in Figure 3.

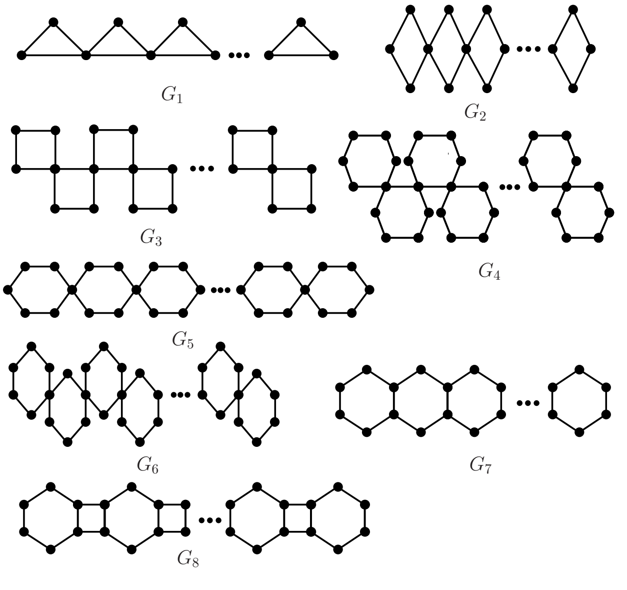


Figure 3 Eight types chemical chains.

Theorem 3.1 *Let t be the number of triangles in G_1, shown in Figure 3. Then the general leap Zagreb-type indices of G_1 are given by*

$$\begin{aligned} LZ_{\alpha,\beta}(G_1) &= (3t-10)\cdot 2^{1+2\alpha+2\beta}+6\cdot 2^{1+\alpha+\beta}+2^{\alpha+2\beta+2}+2^{2\alpha+\beta+2}, \\ LRZ_{\alpha,\beta,\gamma}(G_1) &= 6\cdot 2^{2\alpha+(\beta+1)\gamma}+2^{2+3\alpha+\beta\gamma}(1+2^{\beta})^{\gamma}+(3t-10)\cdot 2^{4\alpha+(2\beta+1)\gamma}. \end{aligned}$$

Proof By the definition of G_1, we obtain the basic information on G_1 in the following table.

$m(2,2)$	$m(2,4)$	$m(4,4)$
6	4	$3t-10$

Thus, we have

$$\begin{aligned} LZ_{\alpha,\beta}(G_1) &= 6(2^{\alpha}\cdot 2^{\beta}+2^{\beta}\cdot 2^{\alpha})+4(2^{\alpha}\cdot 4^{\beta}+2^{\beta}\cdot 4^{\alpha})+(3t-10)\cdot(4^{\alpha}\cdot 4^{\beta}+4^{\beta}\cdot 4^{\alpha}) \\ &= (3t-10)\cdot 2^{1+2\alpha+2\beta}+6\cdot 2^{1+\alpha+\beta}+2^{\alpha+2\beta+2}+2^{2\alpha+\beta+2}, \end{aligned}$$

$$\begin{aligned} LRZ_{\alpha,\beta,\gamma}(G_1) &= 6(2\cdot 2)^{\alpha}(2^{\beta}+2^{\beta})^{\gamma}+4(2\cdot 4)^{\alpha}(2^{\beta}+4^{\beta})^{\gamma}+(3t-10)(4\cdot 4)^{\alpha}(4^{\beta}+4^{\beta})^{\gamma} \\ &= 6\cdot 2^{2\alpha+(\beta+1)\gamma}+2^{2+3\alpha+\beta\gamma}(1+2^{\beta})^{\gamma}+(3t-10)\cdot 2^{4\alpha+(2\beta+1)\gamma}. \end{aligned}$$

This completes the proof. □

Theorem 3.2 *Let q be the number of quadrilaterals in G_2, shown in Figure 3. Then the general leap Zagreb-type indices of G_2 are given by*

$$\begin{aligned} LZ_{\alpha,\beta}(G_2) &= (4q-8)\cdot(2^{\alpha}\cdot 5^{\beta}+2^{\beta}\cdot 5^{\alpha})+4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+4(3^{\alpha}+3^{\beta}), \\ LRZ_{\alpha,\beta,\gamma}(G_2) &= 4\cdot 3^{\alpha}(1+3^{\beta})^{\gamma}+4\cdot 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+(4q-8)\cdot 10^{\alpha}(2^{\beta}+5^{\beta})^{\gamma}. \end{aligned}$$

Proof By the definition of G_2, we obtain the basic information on G_2 in the following table.

$m(1,3)$	$m(2,3)$	$m(2,5)$
4	4	$4q-8$

Thus, we have

$$\begin{aligned} LZ_{\alpha,\beta}(G_2) &= 4(1^{\alpha}\cdot 3^{\beta}+1^{\beta}\cdot 3^{\alpha})+4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+(4q-8)\cdot(2^{\alpha}\cdot 5^{\beta}+2^{\beta}\cdot 5^{\alpha}) \\ &= (4q-8)\cdot(2^{\alpha}\cdot 5^{\beta}+2^{\beta}\cdot 5^{\alpha})+4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+4(3^{\alpha}+3^{\beta}), \\ LRZ_{\alpha,\beta,\gamma}(G_2) &= 4(1\cdot 3)^{\alpha}(1^{\beta}+3^{\beta})^{\gamma}+4(2\cdot 3)^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+(4q-8)(2\cdot 5)^{\alpha}(2^{\beta}+5^{\beta})^{\gamma} \\ &= 4\cdot 3^{\alpha}(1+3^{\beta})^{\gamma}+4\cdot 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+(4q-8)\cdot 10^{\alpha}(2^{\beta}+5^{\beta})^{\gamma}. \end{aligned}$$

This completes the proof. □

Theorem 3.3 *Let q be the number of quadrilaterals in G_3, shown in Figure 3. Then the general leap Zagreb-type indices of G_3 are given by*

$$\begin{aligned}
LZ_{\alpha,\beta}(G_3) &= 3^{\alpha+\beta}(2q-6)\cdot(2^\alpha+2^\beta)+2(q-4)\cdot 6^{\alpha+\beta}+2(q-2)\cdot 3^{\alpha+\beta}\\
&\quad +6(3^\alpha\cdot 4^\beta+3^\beta\cdot 4^\alpha)+2^{1+\alpha+\beta}(2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha)+4(3^\alpha+3^\beta),\\
LRZ_{\alpha,\beta,\gamma}(G_3) &= 4\cdot 3^\alpha(1+3^\beta)^\gamma+(q-2)\cdot 2^\gamma\cdot 3^{2\alpha+\beta\gamma}+6\cdot 12^\alpha(3^\beta+4^\beta)^\gamma\\
&\quad +(2q-6)\cdot 18^\alpha(3^\beta+6^\beta)^\gamma+2\cdot 24^\alpha(4^\beta+6^\beta)^\gamma+(q-4)\cdot 2^\gamma\cdot 6^{2\alpha+\beta\gamma}.
\end{aligned}$$

Proof By the definition of G_3, we obtain the basic information on G_3 in the following table.

$m(1,3)$	$m(3,3)$	$m(3,4)$	$m(3,6)$	$m(4,6)$	$m(6,6)$
4	$q-2$	6	$2q-6$	2	$q-4$

Thus, we have

$$\begin{aligned}
LZ_{\alpha,\beta}(G_3) &= 4(1^\alpha\cdot 3^\beta+1^\beta\cdot 3^\alpha)+(q-2)\cdot(3^\alpha\cdot 3^\beta+3^\beta\cdot 3^\alpha)+6(3^\alpha\cdot 4^\beta+3^\beta\cdot 4^\alpha)\\
&\quad +(2q-6)\cdot(3^\alpha\cdot 6^\beta+3^\beta\cdot 6^\alpha)+2(4^\alpha\cdot 6^\beta+4^\beta\cdot 6^\alpha)+(q-4)\cdot(6^\alpha\cdot 6^\beta+6^\beta\cdot 6^\alpha)\\
&= 3^{\alpha+\beta}(2q-6)\cdot(2^\alpha+2^\beta)+2(q-4)\cdot 6^{\alpha+\beta}+2(q-2)\cdot 3^{\alpha+\beta}\\
&\quad +6(3^\alpha\cdot 4^\beta+3^\beta\cdot 4^\alpha)+2^{1+\alpha+\beta}(2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha)+4(3^\alpha+3^\beta),\\
LRZ_{\alpha,\beta,\gamma}(G_3) &= 4(1\cdot 3)^\alpha(1^\beta+3^\beta)^\gamma+(q-2)(3\cdot 3)^\alpha(3^\beta+3^\beta)^\gamma+6(3\cdot 4)^\alpha(3^\beta+4^\beta)^\gamma\\
&\quad +(2q-6)(3\cdot 6)^\alpha(3^\beta+6^\beta)^\gamma+2(4\cdot 6)^\alpha(4^\beta+6^\beta)^\gamma+(q-4)(6\cdot 6)^\alpha(6^\beta+6^\beta)^\gamma\\
&= 4\cdot 3^\alpha(1+3^\beta)^\gamma+(q-2)\cdot 2^\gamma\cdot 3^{2\alpha+\beta\gamma}+6\cdot 12^\alpha(3^\beta+4^\beta)^\gamma\\
&\quad +(2q-6)\cdot 18^\alpha(3^\beta+6^\beta)^\gamma+2\cdot 24^\alpha(4^\beta+6^\beta)^\gamma+(q-4)\cdot 2^\gamma\cdot 6^{2\alpha+\beta\gamma}.
\end{aligned}$$

This completes the proof. □

Theorem 3.4 *Let h be the number of hexagons in G_4, shown in Figure 3. Then the general leap Zagreb-type indices of G_4 are given by*

$$\begin{aligned}
LZ_{\alpha,\beta}(G_4) &= 2^{2\alpha+2\beta}(2h-6)\cdot(2^\alpha+2^\beta)+(h-4)\cdot 2^{1+3\alpha+3\beta}+(h+2)\cdot 2^{1+\alpha+\beta}\\
&\quad +2^{1+\alpha+\beta}(2^\alpha+2^\beta)h+3\cdot 2^{1+\alpha+\beta}(2^\alpha\cdot 3^\beta+2^\beta\cdot 3^\alpha)+2^{1+\alpha+\beta}(3^\alpha\cdot 4^\beta+3^\beta\cdot 4^\alpha),\\
LRZ_{\alpha,\beta,\gamma}(G_4) &= (h+2)\cdot 2^{2\alpha+(\beta+1)\gamma}+2^{1+3\alpha+\beta\gamma}(1+2^\beta)^\gamma\cdot h+6\cdot 24^\alpha(4^\beta+6^\beta)^\gamma\\
&\quad +(2h-6)\cdot 2^{5\alpha+2\beta\gamma}(1+2^\beta)^\gamma+2\cdot 48^\alpha(6^\beta+8^\beta)^\gamma+(h-4)\cdot 2^{6\alpha+(3\beta+1)\gamma}.
\end{aligned}$$

Proof By the definition of G_4, we obtain the basic information on G_4 in the following table.

$m(2,2)$	$m(2,4)$	$m(4,6)$	$m(4,8)$	$m(6,8)$	$m(8,8)$
$h+2$	$2h$	6	$2h-6$	2	$h-4$

Thus, we have

$$\begin{aligned} LZ_{\alpha,\beta}(G_4) &= (h+2)\cdot(2^{\alpha}\cdot 2^{\beta}+2^{\beta}\cdot 2^{\alpha})+2h(2^{\alpha}\cdot 4^{\beta}+2^{\beta}\cdot 4^{\alpha})+6(4^{\alpha}\cdot 6^{\beta}+4^{\beta}\cdot 6^{\alpha})\\ &\quad +(2h-6)\cdot(4^{\alpha}\cdot 8^{\beta}+4^{\beta}\cdot 8^{\alpha})+2(6^{\alpha}\cdot 8^{\beta}+6^{\beta}\cdot 8^{\alpha})+(h-4)\cdot(8^{\alpha}\cdot 8^{\beta}+8^{\beta}\cdot 8^{\alpha})\\ &= 2^{2\alpha+2\beta}(2h-6)\cdot(2^{\alpha}+2^{\beta})+(h-4)\cdot 2^{1+3\alpha+3\beta}+(h+2)\cdot 2^{1+\alpha+\beta}\\ &\quad +2^{1+\alpha+\beta}(2^{\alpha}+2^{\beta})h+3\cdot 2^{1+\alpha+\beta}(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+2^{1+\alpha+\beta}(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha}),\\ LRZ_{\alpha,\beta,\gamma}(G_4) &= (h+2)(2\cdot 2)^{\alpha}(2^{\beta}+2^{\beta})^{\gamma}+2h(2\cdot 4)^{\alpha}(2^{\beta}+4^{\beta})^{\gamma}+6(4\cdot 6)^{\alpha}(4^{\beta}+6^{\beta})^{\gamma}\\ &\quad +(2h-6)(4\cdot 8)^{\alpha}(4^{\beta}+8^{\beta})^{\gamma}+2(6\cdot 8)^{\alpha}(6^{\beta}+8^{\beta})^{\gamma}+(h-4)(8\cdot 8)^{\alpha}(8^{\beta}+8^{\beta})^{\gamma}\\ &= (h+2)\cdot 2^{2\alpha+(\beta+1)\gamma}+2^{1+3\alpha+\beta\gamma}(1+2^{\beta})^{\gamma}\cdot h+6\cdot 24^{\alpha}(4^{\beta}+6^{\beta})^{\gamma}\\ &\quad +(2h-6)\cdot 2^{5\alpha+2\beta\gamma}(1+2^{\beta})^{\gamma}+2\cdot 48^{\alpha}(6^{\beta}+8^{\beta})^{\gamma}+(h-4)\cdot 2^{6\alpha+(3\beta+1)\gamma}. \end{aligned}$$

This completes the proof. □

Theorem 3.5 *Let h be the number of hexagons in G_5, shown in Figure 3. Then the general leap Zagreb-type indices of G_5 are given by*

$$\begin{aligned} LZ_{\alpha,\beta}(G_5) &= (3h-4)\cdot 2^{2+2\alpha+2\beta}+2^{2+\alpha+\beta}(2^{\alpha}+2^{\beta})+2^{3+\alpha+\beta},\\ LRZ_{\alpha,\beta,\gamma}(G_5) &= 2^{2(\alpha+1)+(\beta+1)\gamma}+2^{2+3\alpha+\beta\gamma}(1+2^{\beta})^{\gamma}+(3h-4)\cdot 2^{1+4\alpha+(2\beta+1)\gamma}. \end{aligned}$$

Proof By the definition of G_5, we obtain the basic information on G_5 in the following table.

$m(2,2)$	$m(2,4)$	$m(4,4)$
4	4	$6h-8$

Thus, we have

$$\begin{aligned} LZ_{\alpha,\beta}(G_5) &= 4\cdot(2^{\alpha}\cdot 2^{\beta}+2^{\beta}\cdot 2^{\alpha})+4(2^{\alpha}\cdot 4^{\beta}+2^{\beta}\cdot 4^{\alpha})+(6h-8)\cdot(4^{\alpha}\cdot 4^{\beta}+4^{\beta}\cdot 4^{\alpha})\\ &= (3h-4)\cdot 2^{2+2\alpha+2\beta}+2^{2+\alpha+\beta}(2^{\alpha}+2^{\beta})+2^{3+\alpha+\beta}, \end{aligned}$$

$$\begin{aligned} LRZ_{\alpha,\beta,\gamma}(G_5) &= 4(2\cdot 2)^{\alpha}(2^{\beta}+2^{\beta})^{\gamma}+4(2\cdot 4)^{\alpha}(2^{\beta}+4^{\beta})^{\gamma}+(6h-8)(4\cdot 4)^{\alpha}(4^{\beta}+4^{\beta})^{\gamma}\\ &= 2^{2(\alpha+1)+(\beta+1)\gamma}+2^{2+3\alpha+\beta\gamma}(1+2^{\beta})^{\gamma}+(3h-4)\cdot 2^{1+4\alpha+(2\beta+1)\gamma}. \end{aligned}$$

This completes the proof. □

Theorem 3.6 *Let h be the number of hexagons in G_6, shown in Figure 3. Then the general leap Zagreb-type indices of G_6 are given by*

$$\begin{aligned} LZ_{\alpha,\beta}(G_6) &= 2^{1+\alpha+\beta}\cdot(h-2)\cdot(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+2^{1+\alpha+\beta}(2^{\alpha}+2^{\beta})\cdot h\\ &\quad +2^{2+2\alpha+2\beta}h+2^{3+\alpha+\beta}, \end{aligned}$$

$$\begin{aligned} LRZ_{\alpha,\beta,\gamma}(G_6) &= 2^{2(\alpha+1)+(\beta+1)\gamma}+2^{1+3\alpha+\beta\gamma}(1+2^{\beta})^{\gamma}\cdot h+2^{1+4\alpha+(2\beta+1)\gamma}\cdot h \\ &\quad +(2h-4)\cdot 24^{\alpha}\cdot(4^{\beta}+6^{\beta})^{\gamma}. \end{aligned}$$

Proof By the definition of G_6, we obtain the basic information on G_6 in the following table.

$m(2,2)$	$m(2,4)$	$m(4,4)$	$m(4,6)$
4	$2h$	$2h$	$2h-4$

Thus, we have

$$\begin{aligned} LZ_{\alpha,\beta}(G_6) &= 4\cdot(2^{\alpha}\cdot 2^{\beta}+2^{\beta}\cdot 2^{\alpha})+2h(2^{\alpha}\cdot 4^{\beta}+2^{\beta}\cdot 4^{\alpha})+2h(4^{\alpha}\cdot 4^{\beta}+4^{\beta}\cdot 4^{\alpha}) \\ &\quad +(2h-4)\cdot(4^{\alpha}\cdot 6^{\beta}+4^{\beta}\cdot 6^{\alpha}) \\ &= 2^{1+\alpha+\beta}\cdot(h-2)\cdot(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+2^{1+\alpha+\beta}(2^{\alpha}+2^{\beta})\cdot h \\ &\quad +2^{2+2\alpha+2\beta}h+2^{3+\alpha+\beta}, \end{aligned}$$

$$\begin{aligned} LRZ_{\alpha,\beta,\gamma}(G_6) &= 4(2\cdot 2)^{\alpha}(2^{\beta}+2^{\beta})^{\gamma}+2h(2\cdot 4)^{\alpha}(2^{\beta}+4^{\beta})^{\gamma} \\ &\quad +2h(4\cdot 4)^{\alpha}(4^{\beta}+4^{\beta})^{\gamma}+(2h-4)(4\cdot 6)^{\alpha}(4^{\beta}+6^{\beta})^{\gamma} \\ &= 2^{2(\alpha+1)+(\beta+1)\gamma}+2^{1+3\alpha+\beta\gamma}(1+2^{\beta})^{\gamma}\cdot h+2^{1+4\alpha+(2\beta+1)\gamma}\cdot h \\ &\quad +(2h-4)\cdot 24^{\alpha}\cdot(4^{\beta}+6^{\beta})^{\gamma}. \end{aligned}$$

This completes the proof. □

Theorem 3.7 *Let h be the number of hexagons in G_7, shown in Figure 3. Then the general leap Zagreb-type indices of G_7 are given by*

$$\begin{aligned} LZ_{\alpha,\beta}(G_7) &= 2^{1+2\alpha+2\beta}\cdot(5h-9)+4(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})+4(2^{\alpha}\cdot 3^{\beta} \\ &\quad +2^{\beta}\cdot 3^{\alpha})+2^{2+\alpha+\beta}, \\ LRZ_{\alpha,\beta,\gamma}(G_7) &= 2^{4\alpha+(2\beta+1)\gamma}\cdot(5h-9)+4\cdot 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+4\cdot 12^{\alpha}(3^{\beta}+4^{\beta})^{\gamma}+2^{2\alpha+1+(\beta+1)\gamma}. \end{aligned}$$

Proof By the definition of G_7, we obtain the basic information on G_7 in the following table.

$m(2,2)$	$m(2,3)$	$m(3,4)$	$m(4,4)$
2	4	4	$5h-9$

Thus, we have

$$\begin{aligned} LZ_{\alpha,\beta}(G_7) &= 2\cdot(2^{\alpha}\cdot 2^{\beta}+2^{\beta}\cdot 2^{\alpha})+4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+4(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha}) \\ &\quad +(5h-9)\cdot(4^{\alpha}\cdot 4^{\beta}+4^{\beta}\cdot 4^{\alpha}) \\ &= 2^{1+2\alpha+2\beta}\cdot(5h-9)+4(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})+4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+2^{2+\alpha+\beta}, \end{aligned}$$

$$\begin{aligned} LRZ_{\alpha,\beta,\gamma}(G_7) &= 2(2\cdot 2)^{\alpha}(2^{\beta}+2^{\beta})^{\gamma}+4(2\cdot 3)^{\alpha}(2^{\beta}+3^{\beta})^{\gamma} \\ &\quad +4(3\cdot 4)^{\alpha}(3^{\beta}+4^{\beta})^{\gamma}+(5h-9)(4\cdot 4)^{\alpha}(4^{\beta}+4^{\beta})^{\gamma} \\ &= 2^{4\alpha+(2\beta+1)\gamma}\cdot(5h-9)+4\cdot 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+4\cdot 12^{\alpha}(3^{\beta}+4^{\beta})^{\gamma} \\ &\quad +2^{2\alpha+1+(\beta+1)\gamma}. \end{aligned}$$

This completes the proof. □

Theorem 3.8 *Let q and h be the number of quadrilaterals and hexagons in G_8, shown in Fig. 3. Then the general leap Zagreb-type indices of G_8 are given by*

$$\begin{aligned} LZ_{\alpha,\beta}(G_8) &= (h-2)\cdot 2^{3+2\alpha+2\beta}+q\cdot 2^{3+2\alpha+2\beta}+4(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha}) \\ &\quad +4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+2^{2+\alpha+\beta}, \\ LRZ_{\alpha,\beta,\gamma}(G_8) &= 2^{2+4\alpha+(2\beta+1)\gamma}\cdot(h-2)+4\cdot 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+4\cdot 12^{\alpha}(3^{\beta}+4^{\beta})^{\gamma} \\ &\quad +q\cdot 2^{2+4\alpha+(2\beta+1)\gamma}+2^{2\alpha+1+(\beta+1)\gamma}. \end{aligned}$$

Proof By the definition of G_8, we obtain the basic information on G_8 in the following table.

$m(2,2)$	$m(2,3)$	$m(3,4)$	$m(4,4)$	$m(4,4)$
2	4	4	$4q$	$4h-8$

Thus, we have

$$\begin{aligned} LZ_{\alpha,\beta}(G_8) &= 2\cdot(2^{\alpha}\cdot 2^{\beta}+2^{\beta}\cdot 2^{\alpha})+4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+4(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha}) \\ &\quad +4q\cdot(4^{\alpha}\cdot 4^{\beta}+4^{\beta}\cdot 4^{\alpha})+(4h-8)\cdot(4^{\alpha}\cdot 4^{\beta}+4^{\beta}\cdot 4^{\alpha}) \\ &= (h-2)\cdot 2^{3+2\alpha+2\beta}+q\cdot 2^{3+2\alpha+2\beta}+4(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha}) \\ &\quad +4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+2^{2+\alpha+\beta}, \end{aligned}$$

$$\begin{aligned} LRZ_{\alpha,\beta,\gamma}(G_8) &= 2(2\cdot 2)^{\alpha}(2^{\beta}+2^{\beta})^{\gamma}+4(2\cdot 3)^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+4(3\cdot 4)^{\alpha}(3^{\beta}+4^{\beta})^{\gamma} \\ &\quad +4q(4\cdot 4)^{\alpha}(4^{\beta}+4^{\beta})^{\gamma}+(4h-8)(4\cdot 4)^{\alpha}(4^{\beta}+4^{\beta})^{\gamma} \\ &= 2^{2+4\alpha+(2\beta+1)\gamma}\cdot(h-2)+4\cdot 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+4\cdot 12^{\alpha}(3^{\beta}+4^{\beta})^{\gamma} \\ &\quad +q\cdot 2^{2+4\alpha+(2\beta+1)\gamma}+2^{2\alpha+1+(\beta+1)\gamma}. \end{aligned}$$

This completes the proof. □

§4. General Leap Zagreb-Type Indices of Some Benzenoid Systems

The structure of starphene $ST(r,s,t)$ for integers $r,s,t\geq 3$ is shown in Figure 4.

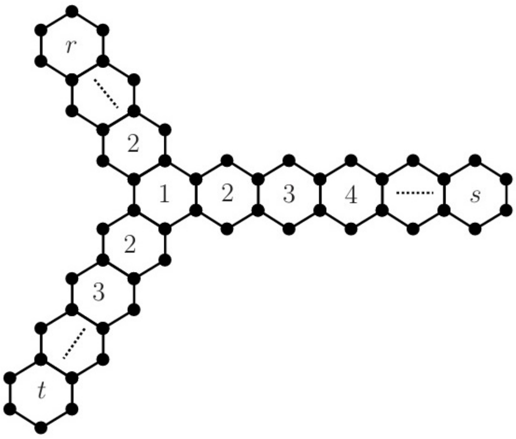


Figure 4 Starphene $ST(r,s,t)$ $(r,s,t\geq 3)$ structure.

Theorem 4.1 *The Leap connectivity indices of the starphene* $ST(r,s,t)$ $(r,s,t\geq 3)$ *structure, shown in Figure 4, are given by*

$$\begin{aligned}LZ_{\alpha,\beta}(ST(r,s,t)) &= 6(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+6(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})+6(4^{\alpha}\cdot 5^{\beta}+4^{\beta}\cdot 5^{\alpha})\\ &\quad +[5(r+s+t)-36]\cdot 2^{1+2\alpha+2\beta}+3\cdot 2^{1+\alpha+\beta}+12\cdot 5^{\alpha+\beta},\\ LRZ_{\alpha,\beta,\gamma}(ST(r,s,t)) &= 6^{1+\alpha}(2^{\beta}+3^{\beta})^{\gamma}+6\cdot 12^{\alpha}(3^{\beta}+4^{\beta})^{\gamma}+6\cdot 20^{\alpha}(4^{\beta}+5^{\beta})^{\gamma}\\ &\quad +[5(r+s+t)-36]\cdot 2^{4\alpha+(1+2\beta)\gamma}+3\cdot 2^{2\alpha+(1+\beta)\gamma}+6\cdot 2^{\gamma}\cdot 5^{2\alpha+\beta\gamma}.\end{aligned}$$

Proof By the definition of $ST(r,s,t)$, we obtain the basic information on $ST(r,s,t)$ in the following table.

$m(2,2)$	$m(2,3)$	$m(3,4)$	$m(4,4)$	$m(4,5)$	$m(5,5)$
3	6	6	$5(r+s+t)-36$	6	6

Thus, we have

$$\begin{aligned}LZ_{\alpha,\beta}(ST(r,s,t)) &= 3(2^{\alpha}\cdot 2^{\beta}+2^{\beta}\cdot 2^{\alpha})+6(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+6(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})\\ &\quad +[5(r+s+t)-36]\cdot(4^{\alpha}\cdot 4^{\beta}+4^{\beta}\cdot 4^{\alpha})+6(4^{\alpha}\cdot 5^{\beta}+4^{\beta}\cdot 5^{\alpha})\\ &\quad +6(5^{\alpha}\cdot 5^{\beta}+5^{\beta}\cdot 5^{\alpha})\\ &= 6(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+6(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})+6(4^{\alpha}\cdot 5^{\beta}+4^{\beta}\cdot 5^{\alpha})\\ &\quad +[5(r+s+t)-36]\cdot 2^{1+2\alpha+2\beta}+3\cdot 2^{1+\alpha+\beta}+12\cdot 5^{\alpha+\beta},\end{aligned}$$

$$\begin{aligned} LRZ_{\alpha,\beta,\gamma}(ST(r,s,t)) &= 3(2\cdot 2)^{\alpha}(2^{\beta}+2^{\beta})^{\gamma}+6(2\cdot 3)^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+6(3\cdot 4)^{\alpha}(3^{\beta}+4^{\beta})^{\gamma} \\ &\quad +[5(r+s+t)-36]\cdot(4\cdot 4)^{\alpha}(4^{\beta}+4^{\beta})^{\gamma}+6(4\cdot 5)^{\alpha}(4^{\beta}+5^{\beta})^{\gamma} \\ &\quad +6(5\cdot 5)^{\alpha}(5^{\beta}+5^{\beta})^{\gamma} \\ &= 6^{1+\alpha}(2^{\beta}+3^{\beta})^{\gamma}+6\cdot 12^{\alpha}(3^{\beta}+4^{\beta})^{\gamma}+6\cdot 20^{\alpha}(4^{\beta}+5^{\beta})^{\gamma} \\ &\quad +[5(r+s+t)-36]\cdot 2^{4\alpha+(1+2\beta)\gamma}+3\cdot 2^{2\alpha+(1+\beta)\gamma}+6\cdot 2^{\gamma}\cdot 5^{2\alpha+\beta\gamma}. \end{aligned}$$

This completes the proof. □

The structure of rhombic benzenoid system R_h for integer $h > 1$ is shown in Figure 5.

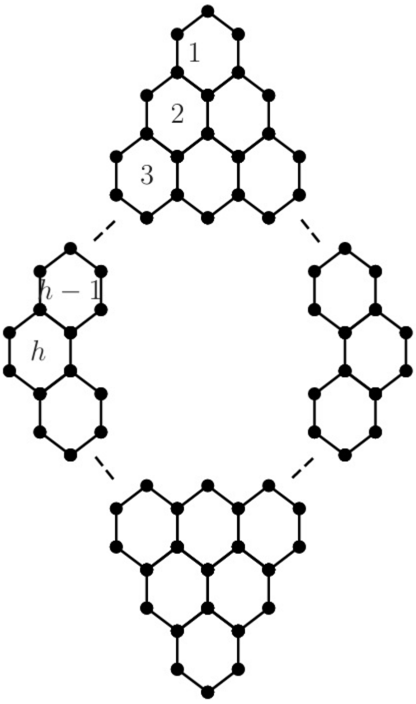


Figure 5 Rhombic benzenoid system R_h $(h > 1)$.

Theorem 4.2 *The Leap connectivity indices of the rhombic benzenoid system* R_h $(h > 1)$, *shown in Figure 5, are given by*

$$\begin{aligned} LZ_{\alpha,\beta}(R_h) &= 4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+8(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})+4(h-1)\cdot(4^{\alpha}\cdot 6^{\beta}+4^{\beta}\cdot 6^{\alpha}) \\ &\quad +2[(h-1)^2+2(h-1)(h-2)]\cdot 6^{\alpha+\beta}+(h-2)\cdot 2^{4+2\alpha+2\beta}+4\cdot 3^{\alpha+\beta}, \\ LRZ_{\alpha,\beta,\gamma}(R_h) &= 4\cdot 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+8\cdot 12^{\alpha}(3^{\beta}+4^{\beta})^{\gamma}+4(h-1)\cdot 24^{\alpha}(4^{\beta}+6^{\beta})^{\gamma} \\ &\quad +2^{\gamma}[(h-1)^2+2(h-1)(h-2)]\cdot 6^{2\alpha+\beta\gamma} \\ &\quad +(h-2)\cdot 2^{3+4\alpha+(1+2\beta)\gamma}+2^{1+\gamma}\cdot 3^{2\alpha+\beta\gamma}. \end{aligned}$$

Proof By the definition of R_h, we obtain the basic information on R_h in the following table.

$m(2,3)$	$m(3,3)$	$m(3,4)$	$m(4,4)$	$m(4,6)$	$m(6,6)$
4	2	8	$8(h-2)$	$4(h-1)$	$(h-1)^2+2(h-1)(h-2)$

Thus, we have

$$\begin{aligned}LZ_{\alpha,\beta}(R_h) &= 4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+2(3^{\alpha}\cdot 3^{\beta}+3^{\beta}\cdot 3^{\alpha})+8(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})\\ &\quad +8(h-2)(4^{\alpha}\cdot 4^{\beta}+4^{\beta}\cdot 4^{\alpha})+4(h-1)\cdot(4^{\alpha}\cdot 6^{\beta}+4^{\beta}\cdot 6^{\alpha})\\ &\quad +[(h-1)^2+2(h-1)(h-2)]\cdot(6^{\alpha}\cdot 6^{\beta}+6^{\beta}\cdot 6^{\alpha})\\ &= 4(2^{\alpha}\cdot 3^{\beta}+2^{\beta}\cdot 3^{\alpha})+8(3^{\alpha}\cdot 4^{\beta}+3^{\beta}\cdot 4^{\alpha})+4(h-1)\cdot(4^{\alpha}\cdot 6^{\beta}+4^{\beta}\cdot 6^{\alpha})\\ &\quad +2[(h-1)^2+2(h-1)(h-2)]\cdot 6^{\alpha+\beta}+(h-2)\cdot 2^{4+2\alpha+2\beta}+4\cdot 3^{\alpha+\beta},\end{aligned}$$

$$\begin{aligned}LRZ_{\alpha,\beta,\gamma}(R_h) &= 4(2\cdot 3)^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+2(3\cdot 3)^{\alpha}(3^{\beta}+3^{\beta})^{\gamma}+8(3\cdot 4)^{\alpha}(3^{\beta}+4^{\beta})^{\gamma}\\ &\quad +8(h-2)(4\cdot 4)^{\alpha}(4^{\beta}+4^{\beta})^{\gamma}+4(h-1)(4\cdot 6)^{\alpha}(4^{\beta}+6^{\beta})^{\gamma}\\ &\quad +[(h-1)^2+2(h-1)(h-2)](6\cdot 6)^{\alpha}(6^{\beta}+6^{\beta})^{\gamma}\\ &= 4\cdot 6^{\alpha}(2^{\beta}+3^{\beta})^{\gamma}+8\cdot 12^{\alpha}(3^{\beta}+4^{\beta})^{\gamma}+4(h-1)\cdot 24^{\alpha}(4^{\beta}+6^{\beta})^{\gamma}\\ &\quad +2^{\gamma}[(h-1)^2+2(h-1)(h-2)]\cdot 6^{2\alpha+\beta\gamma}+(h-2)\cdot 2^{3+4\alpha+(1+2\beta)\gamma}\\ &\quad +2^{1+\gamma}\cdot 3^{2\alpha+\beta\gamma}.\end{aligned}$$

This completes the proof. □

§5. General Leap Zagreb-Type Indices of Some Nanostructures

The structure of armchair polyhex nanotube $TUAC_6[2r;s]$ for integers $r>1, s>1$ is shown in Figure 6.

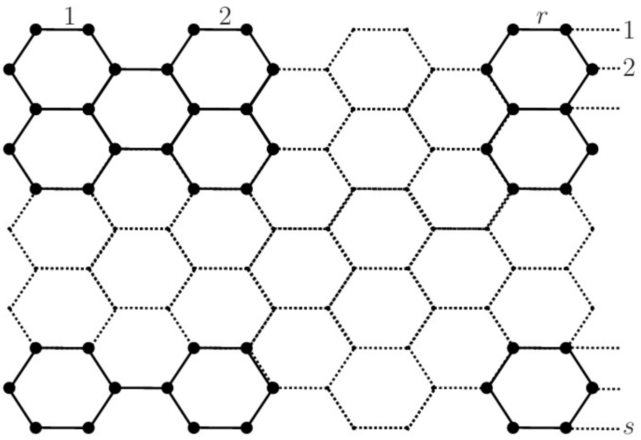


Figure 6 Armchair polyhex nanotube $TUAC_6[2r;s]$ $(r>1, s>1)$.

Theorem 5.1 *The leap connectivity indices of the armchair polyhex nanotube $TUAC_6[2r;s]$ $(r>1,s>1)$, shown in Figure 6, are given by*

$$\begin{aligned} LZ_{\alpha,\beta}(TUAC_6[2r;s]) &= 2(3rs-14r)\cdot 6^{\alpha+\beta}+4r(5^\alpha\cdot 6^\beta+5^\beta\cdot 6^\alpha)+4r(3^\alpha\cdot 5^\beta+3^\beta\cdot 5^\alpha)\\ &\quad +4r\cdot 5^{\alpha+\beta}+4r\cdot 3^{\alpha+\beta},\\ LRZ_{\alpha,\beta,\gamma}(TUAC_6[2r;s]) &= 2^\gamma(3rs-14r)\cdot 6^{2\alpha+\beta\gamma}+4r\cdot 30^\alpha(5^\beta+6^\beta)^\gamma+4r\cdot 15^\alpha(3^\beta+5^\beta)^\gamma\\ &\quad +2^{\gamma+1}r\cdot 5^{2\alpha+\beta\gamma}+2^{\gamma+1}r\cdot 3^{2\alpha+\beta\gamma}. \end{aligned}$$

Proof By the definition of $TUAC_6[2r;s]$, we obtain the basic information on $TUAC_6[2r;s]$ in the following table.

$m(3,3)$	$m(3,5)$	$m(5,5)$	$m(5,6)$	$m(6,6)$
$2r$	$4r$	$2r$	$4r$	$3rs-14r$

Thus, we have

$$\begin{aligned} LZ_{\alpha,\beta}(TUAC_6[2r;s]) &= 2r(3^\alpha\cdot 3^\beta+3^\beta\cdot 3^\alpha)+4r(3^\alpha\cdot 5^\beta+3^\beta\cdot 5^\alpha)+2r(5^\alpha\cdot 5^\beta+5^\beta\cdot 5^\alpha)\\ &\quad +4r(5^\alpha\cdot 6^\beta+5^\beta\cdot 6^\alpha)+(3rs-14r)\cdot(6^\alpha\cdot 6^\beta+6^\beta\cdot 6^\alpha)\\ &= 2(3rs-14r)\cdot 6^{\alpha+\beta}+4r(5^\alpha\cdot 6^\beta+5^\beta\cdot 6^\alpha)+4r(3^\alpha\cdot 5^\beta+3^\beta\cdot 5^\alpha)\\ &\quad +4r\cdot 5^{\alpha+\beta}+4r\cdot 3^{\alpha+\beta}, \end{aligned}$$

$$\begin{aligned} LRZ_{\alpha,\beta,\gamma}(TUAC_6[2r;s]) &= 2r(3\cdot 3)^\alpha(3^\beta+3^\beta)^\gamma+4r(3\cdot 5)^\alpha(3^\beta+5^\beta)^\gamma+2r(5\cdot 5)^\alpha(5^\beta+5^\beta)^\gamma\\ &\quad +4r(5\cdot 6)^\alpha(5^\beta+6^\beta)^\gamma+(3rs-14r)(6\cdot 6)^\alpha(6^\beta+6^\beta)^\gamma\\ &= 2^\gamma(3rs-14r)\cdot 6^{2\alpha+\beta\gamma}+4r\cdot 30^\alpha(5^\beta+6^\beta)^\gamma+4r\cdot 15^\alpha(3^\beta+5^\beta)^\gamma\\ &\quad +2^{\gamma+1}r\cdot 5^{2\alpha+\beta\gamma}+2^{\gamma+1}r\cdot 3^{2\alpha+\beta\gamma}. \end{aligned}$$

This completes the proof. □

Theorem 5.2 *The leap connectivity indices of a V-phenylenic nanotube $VPHX[r;s]$ $(r>1,s>1)$ shown in Figure 7, are given by*

$$\begin{aligned} LZ_{\alpha,\beta}(VPHX[r;s]) &= 4r(4^\alpha\cdot 5^\beta+4^\beta\cdot 5^\alpha)+4r(s-1)(5^\alpha\cdot 6^\beta+5^\beta\cdot 6^\alpha)+4r(2s-3)\cdot 5^{\alpha+\beta}\\ &\quad +2r(s-1)\cdot 6^{\alpha+\beta}+6r\cdot 2^{1+2\alpha+2\beta},\\ LRZ_{\alpha,\beta,\gamma}(VPHX[r;s]) &= 4r\cdot 20^\alpha(4^\beta+5^\beta)^\gamma+4r(s-1)\cdot 30^\alpha(5^\beta+6^\beta)^\gamma+2^{1+\gamma}(2s-3)r\cdot 5^{2\alpha+\beta\gamma}\\ &\quad +2^\gamma(s-1)r\cdot 6^{2\alpha+\beta\gamma}+6r\cdot 2^{4\alpha+(1+2\beta)\gamma}. \end{aligned}$$

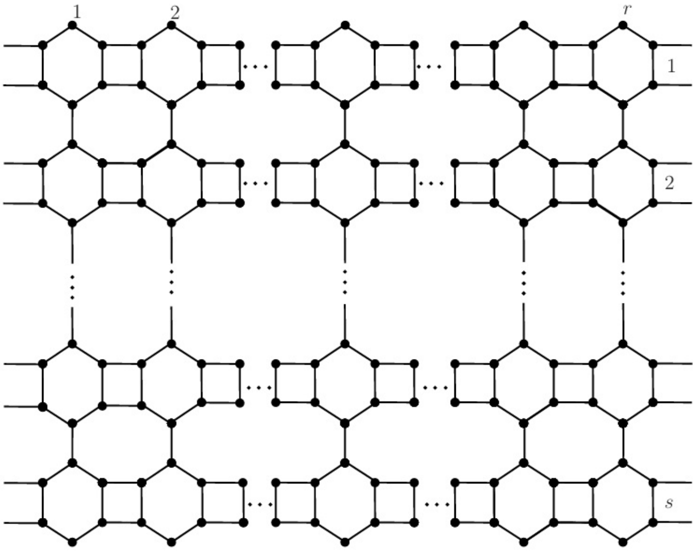


Figure 7 V-phenylenic nanotube $VPHX[r;s]$ $(r>1, s>1)$.

Proof By the definition of $VPHX[r;s]$, we obtain the basic information on $VPHX[r;s]$ in the following table.

$m(4,4)$	$m(4,5)$	$m(5,5)$	$m(5,6)$	$m(6,6)$
$6r$	$4r$	$2r(2s-3)$	$4r(s-1)$	$r(s-1)$

Thus, we have

$$\begin{aligned}LZ_{\alpha,\beta}(VPHX[r;s]) &= 6r(4^{\alpha}\cdot 4^{\beta}+4^{\beta}\cdot 4^{\alpha})+4r(4^{\alpha}\cdot 5^{\beta}+4^{\beta}\cdot 5^{\alpha})+2r(2s-3)(5^{\alpha}\cdot 5^{\beta}+5^{\beta}\cdot 5^{\alpha})\\ &\quad +4r(s-1)(5^{\alpha}\cdot 6^{\beta}+5^{\beta}\cdot 6^{\alpha})+r(s-1)\cdot(6^{\alpha}\cdot 6^{\beta}+6^{\beta}\cdot 6^{\alpha})\\ &= 4r(4^{\alpha}\cdot 5^{\beta}+4^{\beta}\cdot 5^{\alpha})+4r(s-1)(5^{\alpha}\cdot 6^{\beta}+5^{\beta}\cdot 6^{\alpha})+4r(2s-3)\cdot 5^{\alpha+\beta}\\ &\quad +2r(s-1)\cdot 6^{\alpha+\beta}+6r\cdot 2^{1+2\alpha+2\beta},\end{aligned}$$

$$\begin{aligned}LRZ_{\alpha,\beta,\gamma}(VPHX[r;s]) &= 6r(4\cdot 4)^{\alpha}(4^{\beta}+4^{\beta})^{\gamma}+4r(4\cdot 5)^{\alpha}(4^{\beta}+5^{\beta})^{\gamma}+2r(2s-3)(5\cdot 5)^{\alpha}(5^{\beta}+5^{\beta})^{\gamma}\\ &\quad +4r(s-1)(5\cdot 6)^{\alpha}(5^{\beta}+6^{\beta})^{\gamma}+r(s-1)(6\cdot 6)^{\alpha}(6^{\beta}+6^{\beta})^{\gamma}\\ &= 4r\cdot 20^{\alpha}(4^{\beta}+5^{\beta})^{\gamma}+4r(s-1)\cdot 30^{\alpha}(5^{\beta}+6^{\beta})^{\gamma}+2^{1+\gamma}(2s-3)r\cdot 5^{2\alpha+\beta\gamma}\\ &\quad +2^{\gamma}(s-1)r\cdot 6^{2\alpha+\beta\gamma}+6r\cdot 2^{4\alpha+(1+2\beta)\gamma}.\end{aligned}$$

This completes the proof. □

§6. Conclusion

In this paper, we propose the general leap-Zagreb-type indices, and obtain the calculation formulas of the general leap Zagreb-type indices for some chemical graphs. The general leap-Zagreb-type indices of some other graph structures can be computed for further study. Inspired by the multiplicative Zagreb-type indices, the general multiplicative leap-Zagreb-type indices, that is the multiplicative version of the leap-Zagreb-type indices, are naturally put forward as follows. It is worth mentioning that the research on extremal problem of the general (multiplicative) leap-Zagreb-type indices is probably an interesting subject.

$$\begin{aligned} LZM_{\alpha,\beta}(G) &= \prod_{uv\in E(G)} (\tau_u^{\alpha}\tau_v^{\beta} + \tau_u^{\beta}\tau_v^{\alpha}), \\ LRZM_{\alpha,\beta,\gamma}(G) &= \prod_{uv\in E(G)} (\tau_u\tau_v)^{\alpha}(\tau_u^{\beta} + \tau_v^{\beta})^{\gamma}. \end{aligned}$$

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