

Topological Study of Line Graph of Zanamivir and Oseltamivir Used in the Treatment of H1N1

H. M. Nagesh

(Department of Science & Humanities, PES University, Bangalore, India)

M. C. Mahesh Kumar

(Department of Mathematics, Government First Grade College, K. R. Puram, Bangalore, India)

E-mail: sachin.nagesh6@gmail.com, oftmahe15@gmail.com

Abstract: Topological indices are the molecular descriptors that characterize the formation of chemical compounds and predict certain physicochemical properties. Reverse degree-based topological indices play an important role in calculating topological descriptors. Line graphs illustrate the molecular graphs of chemical structures in another way and used to predict the physicochemical properties of chemical structures. In this article, we compute some reverse degree-based topological indices of the line graph of Zanamivir and Oseltamivir. This theoretical analysis may help chemists and people working in the pharmaceutical industry to predict the properties of H1N1 drugs without experimenting.

Key Words: Reverse degree, reverse topological indices, Zanamivir, Oseltamivir, line graph, H1N1.

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§1. Introduction

The study of causes and treatments of various infectious diseases has been one of the primary areas of study of modern scientific research in medicine. During one such study conducted in 2009, scientists discovered a new influenza A virus subtype, H1N1 (A/H1N1). It gained worldwide attention when it caused a global pandemic. The 2009 H1N1 pandemic resulted in a large number of infections worldwide, but most cases were mild and the overall mortality rate was relatively low. However, certain populations, such as young children, older adults, and individuals with underlying health conditions, were at higher risk of severe complications.

In response to the pandemic, public health organizations and governments implemented various measures to contain the spread of the virus. These measures included widespread vaccination campaigns, antiviral medications, public awareness campaigns, and enhanced surveillance and monitoring. Since the initial pandemic, H1N1 has continued to circulate as a seasonal flu virus. It is now included in the annual influenza vaccine to protect against this strain. Regular vaccination is recommended to prevent seasonal flu and its complications. Until today, various vaccines have been introduced and adopted by different countries worldwide to keep the

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virus under control.

Currently, two classes of antiviral drugs are globally approved to treat influenza in humans, namely, the adamantanes (amantadine and rimantadine) and the neuraminidase inhibitors (Zanamivir, Oseltamivir, and Peramivir). However, the widespread use of these antiviral drugs as monotherapies has resulted in the rise of influenza viruses resistant to both drug classes [10-12,15]. Due to this, only the neuraminidase inhibitors (Zanamivir and Oseltamivir) are presently prescribed for influenza. Combination therapy with Zanamivir and Oseltamivir is reasonable because these agents bind within the neuraminidase active site differently [17]. The likelihood of selecting viruses dually resistant to Zanamivir and Oseltamivir is low. As a result, combining these two drugs in therapy could be a manageable option for combating resistance.

Let $G = (V, E)$ be a graph, where $V(G)$ and $E(G)$ be vertex set and edge set of G , respectively. The *line graph* of G , written $L(G)$, is the graph whose vertices are the edges of G , with two vertices of $L(G)$ adjacent whenever the corresponding edges of G have a vertex in common.

One of the interesting methods in the studies of mathematical chemistry is to represent molecular graphs by means of parameters calculated for their derived structures. The line graph is a good illustration of derived structures of molecular graphs. Computational aspects of line graph of carbon nanocones are extensively studied in [7]. Zeroth-order general Randić index of line graphs of some chemical structures in drugs are studied in [16]. Topological indices for the line graph of k -subdivided linear $[n]$ tetracene, fullerene networks, tetracenic nanotori, and carbon nanotubes are discussed in [27]. Tomovic [24] and Estrada [3] presented the application of line graphs.

A molecular graph is a set of points denoting the atoms in the molecule and a collection of lines denoting the covalent bonds. The topological indices are helpful in the prediction of physicochemical properties and the biological activity of chemical compounds. Topological indices are obtained from the molecular structure. Topological indices have been used to demonstrate and enhance the statistical features of drugs. In recent times, topological indices of molecular graphs are extensively used for demonstrating correlations connecting the arrangement of a molecular compound and its physicochemical properties or biological activity.

Topological indices are important tools for investigating many physicochemical properties of molecules without performing any testing. They also used to study the Quantitative Structure-Activity Relationship (QSAR) of pharmaceuticals to determine their molecular characteristics by numerical computation.

Various types of topological indices of graphs are classified into distance-based topological indices, degree-based topological indices, and spectrum-based topological indices. Among these, degree-based topological indices play a vital role in theoretical chemistry and pharmacology. Some important degree-based indices are Randić index, Zagreb indices, Harmonic index, sum connectivity index, etc. For example, Randić index is one of the excellent molecular descriptors in QSAR studies and desirable for measuring the extent of branching of the carbon-atom skeleton of saturated hydrocarbons.

Kulli [14] introduced the notion of reverse vertex degree in a graph G . The *reverse vertex degree* in G is defined by $\mathcal{R}_\psi(G) = \Delta(G) - \psi(v) + 1$, where $\psi(v)$ and $\Delta(G)$ are the degree of

the vertex v and maximum degree of a vertex in G , respectively. Zhao et al. [28] computed some of the reverse degree-based topological indices, such as, reverse general Randić index, the reverse Balaban index, the reverse atom bond connectivity index, the reverse geometric index, the reverse Zagreb type indices, and the reverse augmented Zagreb index for metal-organic networks TM-TCNB.

Jung et al. [13] computed the first and second reverse Zagreb indices, first and second reverse hyper Zagreb indices, reverse atomic-bond connectivity index and reverse geometric-arithmetic index for TUC4[m,n]. Wei et al. [26] computed some reverse topological indices, namely, the reverse general Randić index, the reverse atom bond connectivity index, the reverse geometric arithmetic index, the reverse forgotten index, the reverse Balaban index, and the reverse Zagreb type indices for Remdesivir compound used in the treatment of Coronavirus (COVID 19). Ravi et al. [22] analyzed the QSPR of the reverse degree-based topological indices. Haoer and Virk [8] computed reverse Zagreb indices, reverse hyper indices, and their polynomials for metal-organic networks. Hashmi et al. [9] computed the reverse ABC index for different generations of dendrimers. Rosary [21] computed some reverse degree-based topological indices for the line graph Remdesivir. Recently, Prasanna Poojari et al. [18] computed some reverse degree-based topological indices for Zanamivir and Oseltamivir.

Motivated by this, we aim to compute the reverse topological indices for the line graph of molecular structures of Zanamivir and Oseltamivir.

Milan Randić [20] introduced the first degree-based index. Wei et al. [26] defined the reverse Randić index as

$$\mathcal{R}R_\alpha(G) = \sum_{mn \in E(G)} (R_{\psi(m)} \times R_{\psi(n)})^\alpha; \quad \alpha = 1, -1, \frac{1}{2}, -\frac{1}{2}. \quad (1.1)$$

If $\alpha = 1$, then it is called the *reverse second Zagreb index* $\mathcal{R}M_2(G)$.

Estrada et al. [2] presented the atom bond connectivity index. Wei et al. [26] defined the reverse atom bond connectivity index as

$$\mathcal{R}ABC(G) = \sum_{mn \in E(G)} \sqrt{\frac{R_{\psi(m)} + R_{\psi(n)} - 2}{R_{\psi(m)} \times R_{\psi(n)}}}. \quad (1.2)$$

Vukicevic et al. [25] proposed the geometric arithmetic index. Wei et al. [26] defined the reverse geometric arithmetic index as

$$\mathcal{R}GA(G) = \sum_{mn \in E(G)} 2 \frac{\sqrt{R_{\psi(m)} \times R_{\psi(n)}}}{R_{\psi(m)} + R_{\psi(n)}}. \quad (1.3)$$

In 1972, Gutman discussed the first and second Zagreb indices [6,4]. Wei et al. [26] defined the reverse first and reverse second Zagreb indices as

$$\mathcal{R}M_1(G) = \sum_{mn \in E(G)} (R_{\psi(m)} + R_{\psi(n)}), \quad (1.4)$$

$$\mathcal{RM}_2(G) = \sum_{mn \in E(G)} (R_{\psi(m)} \times R_{\psi(n)}). \quad (1.5)$$

In 2008, Doslic and Gutman [1,5] presented the first and second Zagreb co-indices. Wei et al. [26] defined the reverse first and reverse second Zagreb co-indices as

$$\mathcal{RM}_1(G) = 2|E(G)|(|V(G)| - 1) - \mathcal{RM}_1(G), \quad (1.6)$$

$$\mathcal{RM}_2(G) = 2|E(G)|^2 - \frac{1}{2}\mathcal{RM}_1(G) - \mathcal{RM}_2(G). \quad (1.7)$$

In 2013, Shirdel et al. [23] discussed the concept of hyper Zagreb index. Wei et al. [26] defined the reverse hyper Zagreb index as

$$\mathcal{RHM}(G) = \sum_{mn \in E(G)} (R_{\psi(m)} + R_{\psi(n)})^2. \quad (1.8)$$

Furtula and Gutman [6] introduced the Forgotten index. Wei et al. [26] defined the reverse forgotten index as

$$\mathcal{RF}(G) = \sum_{mn \in E(G)} ((R_{\psi(m)})^2 + (R_{\psi(n)})^2). \quad (1.9)$$

Wei et al. [26] defined the reverse Balaban index for a graph as: Let n and m be order and size of a graph G , respectively. Then,

$$\mathcal{RJ}(G) = \frac{m}{m-n+2} \sum_{mn \in E(G)} \frac{1}{\sqrt{R_{\psi(m)} \times R_{\psi(n)}}}. \quad (1.10)$$

Wei et al. [26] defined the reverse first multiple and the reverse second multiple Zagreb indices as

$$\mathcal{RPM}_1(G) = \prod_{mn \in E(G)} (R_{\psi(m)} + R_{\psi(n)}), \quad (1.11)$$

$$\mathcal{RPM}_2(G) = \prod_{mn \in E(G)} (R_{\psi(m)} \times R_{\psi(n)}). \quad (1.12)$$

Rajini et al. [19] defined the notion of redefined first, second and third Zagreb indices for a graph. Wei et al. [26] defined the reverse redefined first, second, and third Zagreb indices as

$$\mathcal{RReZG}_1(G) = \sum_{mn \in E(G)} \frac{R_{\psi(m)} + R_{\psi(n)}}{R_{\psi(m)} \times R_{\psi(n)}}, \quad (1.13)$$

$$\mathcal{RReZG}_2(G) = \sum_{mn \in E(G)} \frac{R_{\psi(m)} \times R_{\psi(n)}}{R_{\psi(m)} + R_{\psi(n)}}, \quad (1.14)$$

$$\mathcal{RReZG}_3(G) = \sum_{mn \in E(G)} (R_{\psi(m)} + R_{\psi(n)}) (R_{\psi(m)} \times R_{\psi(n)}). \quad (1.15)$$

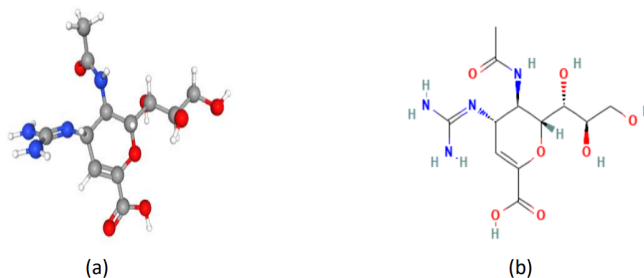
§2. Methods and Techniques

The methods used in this paper contain reverse vertex degree counting, division of vertices based on reverse degree, and partition of edges based on the reverse degree of end vertices. The topological indices as given in formulas (1.1)-(1.15) are calculated with the help of reverse vertex degree counting and partition of edges techniques. The chemical structures (both 2D and 3D) of Zanamivir and Oseltamivir are taken from PubChem, and molecular graphs of chemical structures are drawn using Microsoft Word. Graphical comparison of the reverse topological indices of the molecular structure of Zanamivir and Oseltamivir are plotted by using MATLAB.

§3. Main Results

2.1. Reverse Topological Indices of Line Graph of Zanamivir

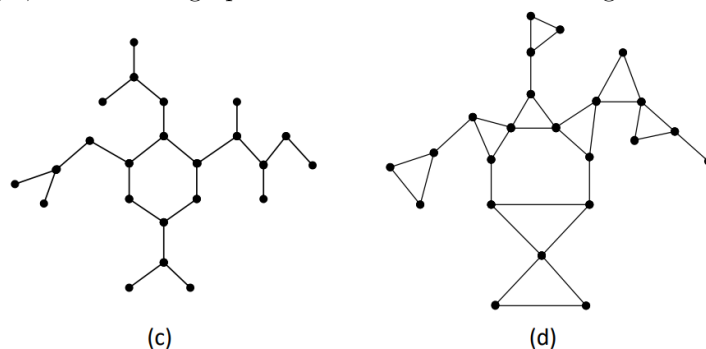
In this subsection, we find the reverse degree-based topological indices for the line graph of Zanamivir. The chemical structure and molecular graph of Zanamivir are shown in Figure 1.



(a) Chemical structure of Zanamivir, (b) Molecular graph of Zanamivir.

Figure 1

We consider hydrogen suppressed molecular graph of compounds since the vertices representing hydrogen atoms do not contribute graph isomorphism. The graph of Zanamivir with vertices and edges; and the line graph of Zanamivir are shown in Figure 2.



(c) Molecular graph of Zanamivir with vertices and edges, (d) Line graph of Zanamivir.

Figure 2

The number of vertices and edges of the line graph of molecular graph of Zanamivir are 23

and 32, respectively. The edge set is partitioned into seven sets based on the reverse degree of end vertices.

The first edge set of the partition includes 3 edges mn , where $R_{\psi(m)} = 1$ and $R_{\psi(n)} = 1$. The second edge set of the partition includes 9 edges mn , where $R_{\psi(m)} = 1$ and $R_{\psi(n)} = 2$. The third edge set of the partition includes 5 edges mn , where $R_{\psi(m)} = 1$ and $R_{\psi(n)} = 3$. The fourth edge set of the partition includes 6 edges mn , where $R_{\psi(m)} = 2$ and $R_{\psi(n)} = 2$. The fifth edge set of the partition includes 5 edges mn , where $R_{\psi(m)} = 2$ and $R_{\psi(n)} = 3$. The sixth edge set of the partition includes an edge mn , where $R_{\psi(m)} = 2$ and $R_{\psi(n)} = 4$. The seventh edge set of the partition includes 3 edges mn , where $R_{\psi(m)} = 3$ and $R_{\psi(n)} = 3$.

Clearly, the number of vertices and edges of the line graph of Zanamivir are 23 and 32, respectively.

The maximum degree of the vertex of the line graph of Zanamivir is 4. By using the definition of reverse vertex degree $R_{\psi}(L(G)) = \Delta(L(G)) - \psi(v) + 1$, the reverse degree-based edge partition of the line graph of Zanamivir is given in Table 1.

$(R_{\psi(m)}, R_{\psi(n)})$	(1,1)	(1,2)	(1,3)	(2,2)	(2,3)	(2,4)	(3,3)
Number of edges	3	9	5	6	5	1	3

Table 1. Edge partition of line graph of Zanamivir

We now find reverse degree-based topological indices for the line graph of Zanamivir.

- Reverse Randić index

$$\mathcal{RR}_{\alpha}(L(G)) = \sum_{mn \in E(L(G))} (R_{\psi(m)} \times R_{\psi(n)})^{\alpha}; \alpha = 1, -1, \frac{1}{2}, -\frac{1}{2}$$

For $\alpha = 1$,

$$\begin{aligned} \mathcal{RR}_1(L(G)) &= 3(1 \times 1) + 9(1 \times 2) + 5(1 \times 3) + 6(2 \times 2) + 5(2 \times 3) + 1(2 \times 4) + 3(3 \times 3) \\ &= 125. \end{aligned}$$

For $\alpha = -1$,

$$\begin{aligned} \mathcal{RR}_{-1}(L(G)) &= \frac{3}{1 \times 1} + \frac{9}{1 \times 2} + \frac{5}{1 \times 3} + \frac{6}{2 \times 2} + \frac{5}{2 \times 3} \\ &\quad + \frac{1}{2 \times 4} + \frac{3}{3 \times 3} = 11.96. \end{aligned}$$

For $\alpha = \frac{1}{2}$,

$$\begin{aligned} \mathcal{RR}_{\frac{1}{2}}(L(G)) &= 3\sqrt{1 \times 1} + 9\sqrt{1 \times 2} + 5\sqrt{1 \times 3} + 6\sqrt{2 \times 2} + 5\sqrt{2 \times 3} \\ &\quad + 1\sqrt{2 \times 4} + 3\sqrt{3 \times 3} \\ &= 60.46. \end{aligned}$$

For $\alpha = -\frac{1}{2}$,

$$\begin{aligned}\mathcal{RR}_{-\frac{1}{2}}(L(G)) &= \frac{3}{\sqrt{1 \times 1}} + \frac{9}{\sqrt{1 \times 2}} + \frac{5}{\sqrt{1 \times 3}} + \frac{6}{\sqrt{2 \times 2}} + \frac{5}{\sqrt{2 \times 3}} + \frac{1}{\sqrt{2 \times 4}} + \frac{3}{\sqrt{3 \times 3}} \\ &= 18.64.\end{aligned}$$

- Reverse atom bond connectivity index

$$\begin{aligned}\mathcal{RABC}(L(G)) &= \sum_{mn \in E(L(G))} \sqrt{\frac{R_{\psi(m)} + R_{\psi(n)} - 2}{R_{\psi(m)} \times R_{\psi(n)}} \\ &= 3\sqrt{\frac{1+1-2}{1 \times 1}} + 9\sqrt{\frac{1+2-2}{1 \times 2}} + 5\sqrt{\frac{1+3-2}{1 \times 3}} + 6\sqrt{\frac{2+2-2}{2 \times 2}} + 5\sqrt{\frac{2+3-2}{2 \times 3}} \\ &\quad + 1\sqrt{\frac{2+4-2}{2 \times 4}} + 3\sqrt{\frac{3+3-2}{3 \times 3}} \\ &= 20.93\end{aligned}$$

- Reverse geometric arithmetic index

$$\begin{aligned}\mathcal{RGA}(L(G)) &= \sum_{mn \in E(L(G))} 2 \frac{\sqrt{R_{\psi(m)} \times R_{\psi(n)}}}{R_{\psi(m)} + R_{\psi(n)}} \\ &= 3 \left(\frac{2\sqrt{1 \times 1}}{1+1} \right) + 9 \left(\frac{2\sqrt{1 \times 2}}{1+2} \right) + 5 \left(\frac{2\sqrt{1 \times 3}}{1+3} \right) + 6 \left(\frac{2\sqrt{2 \times 2}}{2+2} \right) \\ &\quad + 5 \left(\frac{2\sqrt{2 \times 3}}{2+3} \right) + 1 \left(\frac{2\sqrt{2 \times 4}}{2+4} \right) + 3 \left(\frac{2\sqrt{3 \times 3}}{3+3} \right) \\ &= 30.66\end{aligned}$$

- Reverse first Zagreb index

$$\begin{aligned}\mathcal{RM}_1(L(G)) &= \sum_{mn \in E(L(G))} (R_{\psi(m)} + R_{\psi(n)}) \\ &= 3(1+1) + 9(1+2) + 5(1+3) + 6(2+2) + 5(2+3) + 1(2+4) + 3(3+3) \\ &= 126\end{aligned}$$

- Reverse second Zagreb index

$$\begin{aligned}\mathcal{RM}_2(L(G)) &= \sum_{mn \in E(L(G))} (R_{\psi(m)} \times R_{\psi(n)}) \\ &= 3(1 \times 1) + 9(1 \times 2) + 5(1 \times 3) + 6(2 \times 2) + 5(2 \times 3) \\ &\quad + 1(2 \times 4) + 3(3 \times 3) \\ &= 125\end{aligned}$$

- Reverse first Zagreb co-index

$$\begin{aligned}\overline{\mathcal{RM}}_1(L(G)) &= 2|E(L(G))| (|V(L(G))| - 1) - \mathcal{RM}_1(L(G)) \\ &= 2(32)(23 - 1) - 126 \\ &= 1282\end{aligned}$$

- Reverse second Zagreb co-index

$$\begin{aligned}\overline{\mathcal{RM}}_2(L(G)) &= 2|E(L(G))|^2 - \frac{1}{2}\mathcal{RM}_1(L(G)) - \mathcal{RM}_2(L(G)) \\ &= 2(32)^2 - \frac{126}{2} - 125 \\ &= 1860\end{aligned}$$

- Reverse hyper Zagreb index

$$\begin{aligned}\mathcal{RHM}(L(G)) &= \sum_{mn \in E(L(G))} (R_{\psi(m)} + R_{\psi(n)})^2 \\ &= 3(1+1)^2 + 9(1+2)^2 + 5(1+3)^2 + 6(2+2)^2 + 5(2+3)^2 \\ &\quad + 1(2+4)^2 + 3(3+3)^2 \\ &= 538\end{aligned}$$

- Reverse forgotten index

$$\begin{aligned}\mathcal{RF}(L(G)) &= \sum_{mn \in E(L(G))} (R_{\psi(m)})^2 + (R_{\psi(n)})^2 \\ &= 3(1^2 + 1^2) + 9(1^2 + 2^2) + 5(1^2 + 3^2) + 6(2^2 + 2^2) + 5(2^2 + 3^2) \\ &\quad + 1(2^2 + 4^2) + 3(3^2 + 3^2) \\ &= 288\end{aligned}$$

- Reverse Balaban index: Let n and m be order and size of $L(G)$, respectively. Then

$$\begin{aligned}\mathcal{RJ}(L(G)) &= \frac{m}{m-n+2} \sum_{mn \in E(L(G))} \frac{1}{\sqrt{R_{\psi(m)} \times R_{\psi(n)}}} \\ &= \frac{32}{32-23+2} \left(\frac{3}{\sqrt{1 \times 1}} + \frac{9}{\sqrt{1 \times 2}} + \frac{5}{\sqrt{1 \times 3}} \right. \\ &\quad \left. + \frac{6}{\sqrt{2 \times 2}} + \frac{5}{\sqrt{2 \times 3}} + \frac{1}{\sqrt{2 \times 4}} \right) \\ &\quad + \frac{32}{32-23+2} \left(\frac{3}{\sqrt{3 \times 3}} \right) \\ &= 54.24\end{aligned}$$

- Reverse first multiple Zagreb index

$$\begin{aligned}\mathcal{RPM}_1(L(G)) &= \prod_{mn \in E(L(G))} (R_{\psi(m)} + R_{\psi(n)}) \\ &= 3(1+1) \times 9(1+2) \times 5(1+3) \times 6(2+2) \times 5(2+3) \\ &\quad \times 1(2+4) \times 3(3+3) = 209952000\end{aligned}$$

- Reverse second multiple Zagreb index

$$\begin{aligned}\mathcal{RPM}_2(L(G)) &= \prod_{mn \in E(L(G))} (R_{\psi(m)} \times R_{\psi(n)}) \\ &= 3(1 \times 1) \times 9(1 \times 2) \times 5(1 \times 3) \times 6(2 \times 2) \\ &\quad \times 5(2 \times 3) \times 1(2 \times 4) \times 3(3 \times 3) = 125971200\end{aligned}$$

- Reverse first refined Zagreb index

$$\begin{aligned}\mathcal{RReZG}_1(L(G)) &= \sum_{mn \in E(L(G))} \frac{R_{\psi(m)} + R_{\psi(n)}}{R_{\psi(m)} \times R_{\psi(n)}} \\ &= 3 \left(\frac{1+1}{1 \times 1} \right) + 9 \left(\frac{1+2}{1 \times 2} \right) + 5 \left(\frac{2+3}{2 \times 3} \right) + 6 \left(\frac{2+2}{2 \times 2} \right) \\ &\quad + 5 \left(\frac{2+3}{2 \times 3} \right) + 1 \left(\frac{2+4}{2 \times 4} \right) + 3 \left(\frac{3+3}{3 \times 3} \right) = 39.08\end{aligned}$$

- Reverse second refined Zagreb index

$$\begin{aligned}\mathcal{RReZG}_2(L(G)) &= \sum_{mn \in E(L(G))} \frac{R_{\psi(m)} \times R_{\psi(n)}}{R_{\psi(m)} + R_{\psi(n)}} \\ &= 3 \left(\frac{1 \times 1}{1+1} \right) + 9 \left(\frac{1 \times 2}{1+2} \right) + 5 \left(\frac{2 \times 3}{2+3} \right) + 6 \left(\frac{2 \times 2}{2+2} \right) \\ &\quad + 5 \left(\frac{2 \times 3}{2+3} \right) + 1 \left(\frac{2 \times 4}{2+4} \right) + 3 \left(\frac{3 \times 3}{3+3} \right) = 29.08\end{aligned}$$

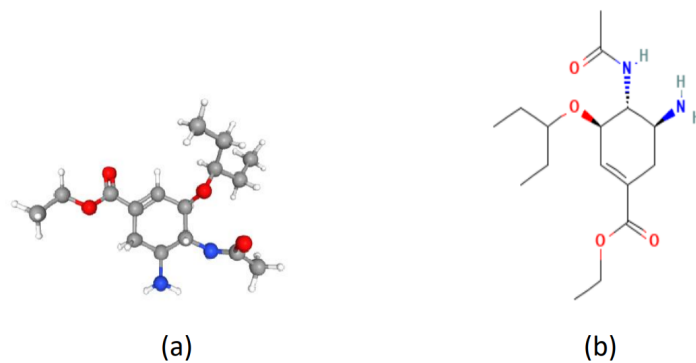
- Reverse third refined Zagreb index

$$\begin{aligned}\mathcal{RReZG}_3(L(G)) &= \sum_{mn \in E(L(G))} (R_{\psi(m)} + R_{\psi(n)}) (R_{\psi(m)} \times R_{\psi(n)}) \\ &= 3(2 \times 1) + 9(3 \times 2) + 5(4 \times 3) + 6(4 \times 4) + 5(5 \times 6) \\ &\quad + 1(6 \times 8) + 3(6 \times 9) = 576\end{aligned}$$

2.2. Reverse Topological Indices of Line Graph of Oseltamivir

In this subsection, we find the reverse degree-based topological indices for the line graph of Oseltamivir. Notice that the chemical structure and molecular graph of Oseltamivir are shown

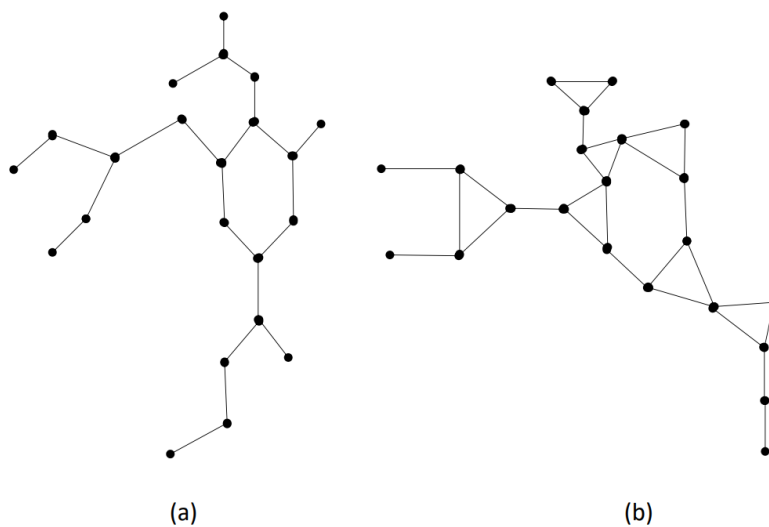
in Figure 3.



(a) Chemical structure of Oseltamivir. (b) Molecular graph of Oseltamivir.

Figure 3

The graph of Oseltamivir with vertices and edges; and the line graph of Oseltamivir are shown in Figure 4.



(a) Molecular graph of Oseltamivir with vertices and edges. (b) Line graph of Oseltamivir.

Figure 4

The number of vertices and edges of the line graph of molecular graph of Oseltamivir are 22 and 29, respectively. The edge set is partitioned into eight sets based on the reverse degree of end vertices.

The first edge set of the partition includes an edge mn , where $R_{\psi(m)} = 1$ and $R_{\psi(n)} = 1$. The second edge set of the partition includes 8 edges mn , where $R_{\psi(m)} = 1$ and $R_{\psi(n)} = 2$. The third edge set of the partition includes 2 edges mn , where $R_{\psi(m)} = 1$ and $R_{\psi(n)} = 3$. The fourth edge set of the partition includes 9 edges mn , where $R_{\psi(m)} = 2$ and $R_{\psi(n)} = 2$. The fifth edge set of the partition includes 5 edges mn , where $R_{\psi(m)} = 2$ and $R_{\psi(n)} = 3$. The sixth

edge set of the partition includes 2 edges mn , where $R_{\psi(m)} = 2$ and $R_{\psi(n)} = 4$. The seventh edge set of the partition includes an edge mn , where $R_{\psi(m)} = 3$ and $R_{\psi(n)} = 3$. The eight edge set of the partition includes an edge mn , where $R_{\psi(m)} = 3$ and $R_{\psi(n)} = 4$.

Clearly, the number of vertices and edges of the line graph of Oseltamivir are 22 and 29, respectively.

The maximum degree of the vertex of the line graph of Oseltamivir is 4. The reverse degree-based edge partition of the line graph of Oseltamivir is given in Table 2.

$(R_{\psi(m)}, R_{\psi(n)})$	(1,1)	(1,2)	(1,3)	(2,2)	(2,3)	(2,4)	(3,3)	(3,4)
Number of edges	1	8	2	9	5	2	1	1

Table 2. Edge partition of line graph of Oseltamivir

We now find obtain the reverse degree-based topological indices for the line graph of Oseltamivir.

- Reverse Randić index

$$\mathcal{RR}_\alpha(L(G)) = \sum_{mn \in E(L(G))} (R_{\psi(m)} \times R_{\psi(n)})^\alpha; \quad \alpha = 1, -1, \frac{1}{2}, -\frac{1}{2}$$

For $\alpha = 1$,

$$\begin{aligned} \mathcal{RR}_1(L(G)) &= 1(1 \times 1) + 8(1 \times 2) + 2(1 \times 3) + 9(2 \times 2) + 5(2 \times 3) \\ &\quad + 2(2 \times 4) + 1(3 \times 3) + 1(3 \times 4) = 126. \end{aligned}$$

For $\alpha = -1$,

$$\begin{aligned} \mathcal{RR}_{-1}(L(G)) &= \frac{1}{1 \times 1} + \frac{8}{1 \times 2} + \frac{2}{1 \times 3} + \frac{9}{2 \times 2} + \frac{5}{2 \times 3} + \frac{2}{2 \times 4} \\ &\quad + \frac{1}{3 \times 3} + \frac{1}{3 \times 4} = 9.19. \end{aligned}$$

For $\alpha = \frac{1}{2}$,

$$\begin{aligned} \mathcal{RR}_{\frac{1}{2}}(L(G)) &= 1\sqrt{1 \times 1} + 8\sqrt{1 \times 2} + 2\sqrt{1 \times 3} + 9\sqrt{2 \times 2} + 5\sqrt{2 \times 3} \\ &\quad + 2\sqrt{2 \times 4} + 1\sqrt{3 \times 3} + 1\sqrt{3 \times 4} \\ &= 58.15. \end{aligned}$$

For $\alpha = -\frac{1}{2}$,

$$\begin{aligned} \mathcal{RR}_{-\frac{1}{2}}(L(G)) &= \frac{1}{\sqrt{1 \times 1}} + \frac{8}{\sqrt{1 \times 2}} + \frac{2}{\sqrt{1 \times 3}} + \frac{9}{\sqrt{2 \times 2}} + \frac{5}{\sqrt{2 \times 3}} \\ &\quad + \frac{2}{\sqrt{2 \times 4}} + \frac{1}{\sqrt{3 \times 3}} + \frac{1}{\sqrt{3 \times 4}} = 15.68. \end{aligned}$$

- Reverse atom bond connectivity index

$$\begin{aligned}
 \mathcal{R}ABC(L(G)) &= \sum_{mn \in E(L(G))} \sqrt{\frac{R_{\psi(m)} + R_{\psi(n)} - 2}{R_{\psi(m)} \times R_{\psi(n)}} \\
 &= 1\sqrt{\frac{1+1-2}{1 \times 1}} + 8\sqrt{\frac{1+2-2}{1 \times 2}} + 2\sqrt{\frac{1+3-2}{1 \times 3}} + 9\sqrt{\frac{2+2-2}{2 \times 2}} \\
 &\quad + 5\sqrt{\frac{2+3-2}{2 \times 3}} + 2\sqrt{\frac{2+4-2}{2 \times 4}} + 1\sqrt{\frac{3+3-2}{3 \times 3}} + 1\sqrt{\frac{3+4-2}{3 \times 4}} \\
 &= 19.92
 \end{aligned}$$

- Reverse geometric arithmetic index

$$\begin{aligned}
 \mathcal{R}GA(L(G)) &= \sum_{mn \in E(L(G))} 2 \frac{\sqrt{R_{\psi(m)} \times R_{\psi(n)}}}{R_{\psi(m)} + R_{\psi(n)}} \\
 &= 1 \left(\frac{2\sqrt{1 \times 1}}{1+1} \right) + 8 \left(\frac{2\sqrt{1 \times 2}}{1+2} \right) + 2 \left(\frac{2\sqrt{1 \times 3}}{1+3} \right) + 9 \left(\frac{2\sqrt{2 \times 2}}{2+2} \right) \\
 &\quad + 5 \left(\frac{2\sqrt{2 \times 3}}{2+3} \right) + 2 \left(\frac{2\sqrt{2 \times 4}}{2+4} \right) + 1 \left(\frac{2\sqrt{3 \times 3}}{3+3} \right) + 1 \left(\frac{2\sqrt{3 \times 4}}{3+4} \right) \\
 &= 28.05
 \end{aligned}$$

- Reverse first Zagreb index

$$\begin{aligned}
 \mathcal{R}M_1(L(G)) &= \sum_{mn \in E(L(G))} (R_{\psi(m)} + R_{\psi(n)}) \\
 &= 1(1+1) + 8(1+2) + 2(1+3) + 9(2+2) + 5(2+3) + 2(2+4) \\
 &\quad + 1(3+3) + 1(3+4) = 120
 \end{aligned}$$

- Reverse second Zagreb index

$$\begin{aligned}
 \mathcal{R}M_2(L(G)) &= \sum_{mn \in E(L(G))} (R_{\psi(m)} \times R_{\psi(n)}) \\
 &= 1(1 \times 1) + 8(1 \times 2) + 2(1 \times 3) + 9(2 \times 2) + 5(2 \times 3) + 2(2 \times 4) \\
 &\quad + 1(3 \times 3) + 1(3 \times 4) \\
 &= 126
 \end{aligned}$$

- Reverse first Zagreb co-index

$$\begin{aligned}
 \overline{\mathcal{R}M}_1(L(G)) &= 2|E(L(G))| (|V(L(G))| - 1) - \mathcal{R}M_1(L(G)) \\
 &= 2(29)(22 - 1) - 120 \\
 &= 1098
 \end{aligned}$$

- Reverse second Zagreb co-index

$$\begin{aligned} \overline{\mathcal{RM}}_2(L(G)) &= 2|E(L(G))|^2 - \frac{1}{2}\mathcal{RM}_1(L(G)) - \mathcal{RM}_2(L(G)) \\ &= 2(29)^2 - \frac{120}{2} - 126 \\ &= 1496 \end{aligned}$$

- Reverse hyper Zagreb index

$$\begin{aligned} \mathcal{RHM}(L(G)) &= \sum_{mn \in E(L(G))} (R_{\psi(m)} + R_{\psi(n)})^2 \\ &= 1(1+1)^2 + 8(1+2)^2 + 2(1+3)^2 + 9(2+2)^2 + 5(2+3)^2 \\ &\quad + 2(2+4)^2 + 1(3+3)^2 + 1(3+4)^2 \\ &= 534 \end{aligned}$$

- Reverse forgotten index

$$\begin{aligned} \mathcal{RF}(L(G)) &= \sum_{mn \in E(L(G))} (R_{\psi(m)})^2 + (R_{\psi(n)})^2 \\ &= 1(1^2 + 1^2) + 8(1^2 + 2^2) + 2(1^2 + 3^2) + 9(2^2 + 2^2) + 5(2^2 + 3^2) + 2(2^2 + 4^2) \\ &\quad + 1(3^2 + 3^2) + 1(3^2 + 4^2) \\ &= 282 \end{aligned}$$

- Reverse Balaban index:

$$\begin{aligned} \mathcal{RJ}(L(G)) &= \frac{m}{m-n+2} \sum_{mn \in E(L(G))} \frac{1}{\sqrt{R_{\psi(m)} \times R_{\psi(n)}}} \\ &= \left(\frac{29}{29-22+2} \right) \left(\frac{1}{\sqrt{1 \times 1}} + \frac{8}{\sqrt{1 \times 2}} + \frac{2}{\sqrt{1 \times 3}} + \frac{9}{\sqrt{2 \times 2}} + \frac{5}{\sqrt{2 \times 3}} + \frac{2}{\sqrt{2 \times 4}} \right) \\ &\quad + \left(\frac{29}{29-22+2} \right) \left(\frac{1}{\sqrt{3 \times 3}} + \frac{1}{\sqrt{3 \times 4}} \right) \\ &= 50.53 \end{aligned}$$

- Reverse first multiple Zagreb index

$$\begin{aligned} \mathcal{RPM}_1(L(G)) &= \prod_{mn \in E(L(G))} (R_{\psi(m)} + R_{\psi(n)}) \\ &= 1(1+1) \times 8(1+2) \times 2(1+3) \times 9(2+2) \times 5(2+3) \times 2(2+4) \\ &\quad \times 1(3+3) \times 1(3+4) \\ &= 174182400 \end{aligned}$$

- Reverse second multiple Zagreb index

$$\begin{aligned}\mathcal{RPM}_2(L(G)) &= \prod_{mn \in E(L(G))} (R_{\psi(m)} + R_{\psi(n)}) \\ &= 1(1 \times 1) \times 8(1 \times 2) \times 2(1 \times 3) \times 9(2 \times 2) \times 5(2 \times 3) \times 2(2 \times 4) \\ &\quad \times 1(3 \times 3) \times 1(3 \times 4) \\ &= 179159040\end{aligned}$$

- Reverse first refined Zagreb index

$$\begin{aligned}\mathcal{RReZG}_1(L(G)) &= \sum_{mn \in E(L(G))} \frac{R_{\psi(m)} + R_{\psi(n)}}{R_{\psi(m)} \times R_{\psi(n)}} \\ &= 1 \left(\frac{1+1}{1 \times 1} \right) + 8 \left(\frac{1+2}{1 \times 2} \right) + 2 \left(\frac{2+3}{2 \times 3} \right) + 9 \left(\frac{2+2}{2 \times 2} \right) + 5 \left(\frac{2+3}{2 \times 3} \right) \\ &\quad + 2 \left(\frac{2+4}{2 \times 4} \right) + 1 \left(\frac{3+3}{3 \times 3} \right) + 1 \left(\frac{3+4}{3 \times 4} \right) \\ &= 32.58\end{aligned}$$

- Reverse second refined Zagreb index

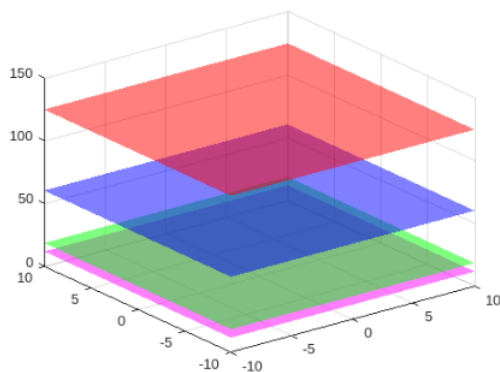
$$\begin{aligned}\mathcal{RReZG}_2(L(G)) &= \sum_{mn \in E(L(G))} \frac{R_{\psi(m)} \times R_{\psi(n)}}{R_{\psi(m)} + R_{\psi(n)}} \\ &= 1 \left(\frac{1 \times 1}{1+1} \right) + 8 \left(\frac{1 \times 2}{1+2} \right) + 2 \left(\frac{2 \times 3}{2+3} \right) + 9 \left(\frac{2 \times 2}{2+2} \right) + 5 \left(\frac{2 \times 3}{2+3} \right) \\ &\quad + 2 \left(\frac{2 \times 4}{2+4} \right) + 1 \left(\frac{3 \times 3}{3+3} \right) + 1 \left(\frac{3 \times 4}{3+4} \right) \\ &= 28.21\end{aligned}$$

- Reverse third refined Zagreb index

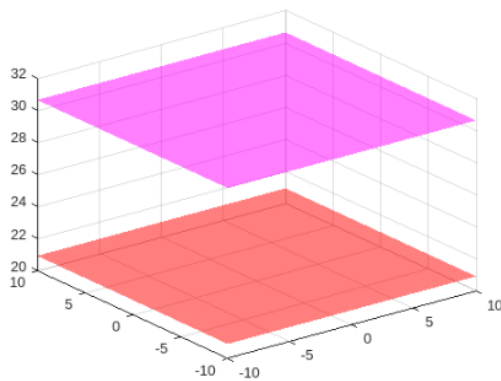
$$\begin{aligned}\mathcal{RReZG}_3(L(G)) &= \sum_{mn \in E(L(G))} (R_{\psi(m)} + R_{\psi(n)}) (R_{\psi(m)} \times R_{\psi(n)}) \\ &= 1(2 \times 1) + 8(3 \times 2) + 2(4 \times 3) + 9(4 \times 4) + 5(5 \times 6) \\ &\quad + 2(6 \times 8) + 1(6 \times 9) + 1(7 \times 12) \\ &= 602\end{aligned}$$

§4. Graphical Comparison of Line Graph of Zanamivir

In order to understand the similarities between the biological and statistical behavior of the line graph of Zanamivir, we have provided results for topological indices for the structure of line graph of Zanamivir.

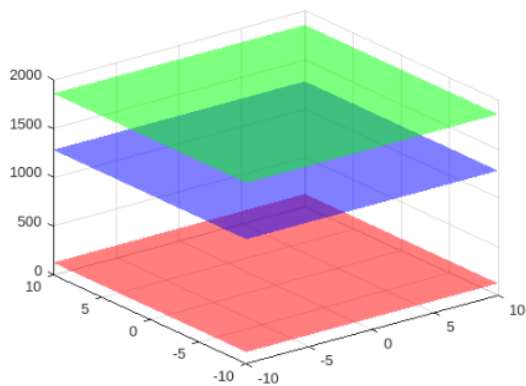


(a)

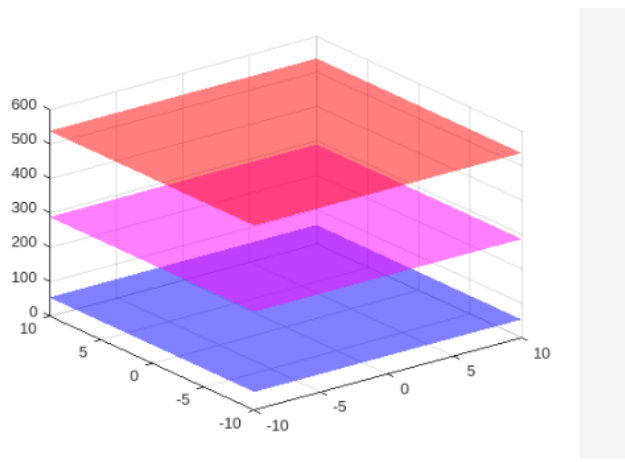


(b)

Figure 5. (a) Comparison of the reverse Randić index of the line graph of Zanamivir for $\alpha = 1, -1, \frac{1}{2}, -\frac{1}{2}$, where red, magenta, blue, and green denotes $\mathcal{RR}_1(L(G)), \mathcal{RR}_{-1}(L(G)), \mathcal{RR}_{\frac{1}{2}}(L(G))$, and $\mathcal{RR}_{-\frac{1}{2}}(L(G))$, respectively; (b) comparison of the reverse atom bond connectivity index and reverse geometric arithmetic index, where red and magenta denotes $\mathcal{RABC}(L(G))$ and $\mathcal{RGA}(L(G))$, respectively.

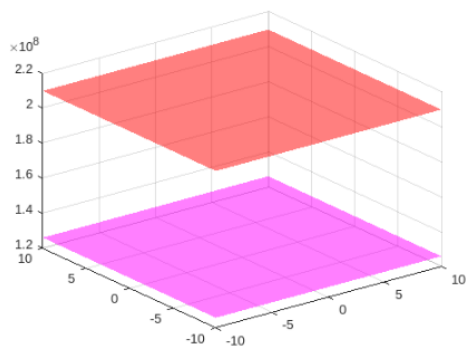


(a)

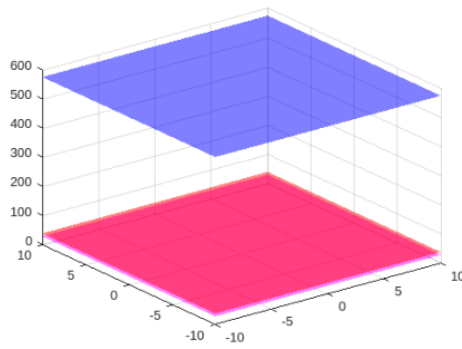


(b)

Figure 6. (a) Comparison of the reverse first Zagreb index, reverse second Zagreb index, reverse first Zagreb co-index, and reverse second Zagreb co-index of the line graph of Zanamivir, where red, magenta, blue, green denotes $\mathcal{RM}_1(L(G))$, $\mathcal{RM}_2(L(G))$, $\mathcal{RM}_1(L(G))$, and $\mathcal{RM}_2(L(G))$, respectively. (b) Comparison of the reverse hyper Zagreb index, reverse forgotten index, and reverse Balaban index, where all these of red, magenta and blue denotes $\mathcal{RHM}(L(G))$, $\mathcal{RF}(L(G))$, and $\mathcal{RJ}(L(G))$, respectively.



(a)



(b)

Figure 7. (a) Comparison of the reverse first multiple Zagreb index and reverse second multiple Zagreb index of the line graph of Zanamivir, where red and magenta denotes $\mathcal{RPM}_1(L(G))$ and $\mathcal{RPM}_2(L(G))$, respectively; (b) Comparison of the reverse first redefined Zagreb index, reverse second redefined Zagreb index, and reverse third redefined Zagreb index, where red, magenta, and blue denotes $\mathcal{RReZG}_1(L(G))$, $\mathcal{RReZG}_2(L(G))$, and $\mathcal{RReZG}_3(L(G))$, respectively.

§5. Graphical Comparison of Line Graph of Oseltamivir

The graphical results for topological indices for the structure of line graph of Oseltamivir.

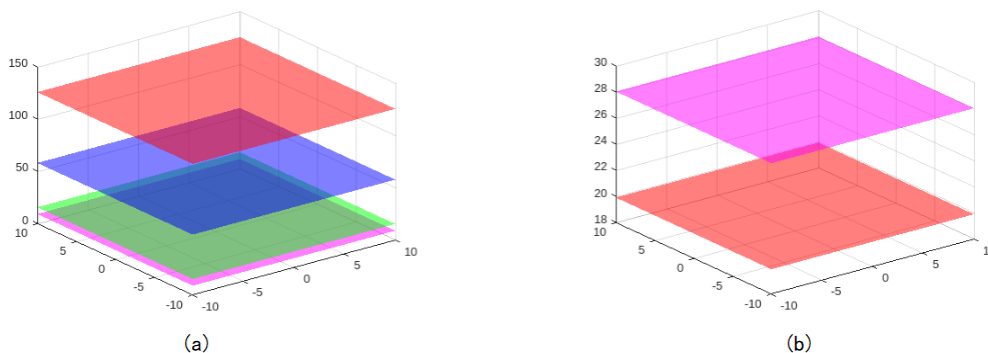


Figure 8. (a) Comparison of the reverse Randić index of the line graph of Oseltamivir for $\alpha = 1, -1, \frac{1}{2}, -\frac{1}{2}$, where red, magenta, blue, green denotes $\mathcal{RR}_1(L(G)), \mathcal{RR}_{-1}(L(G)), \mathcal{RR}_{\frac{1}{2}}(L(G))$ and $\mathcal{RR}_{-\frac{1}{2}}(L(G))$, respectively; (b) comparison of the reverse atom bond connectivity index and reverse geometric arithmetic index, where red, magenta denotes $\mathcal{RABC}(L(G)), \mathcal{RGA}(L(G))$, respectively.

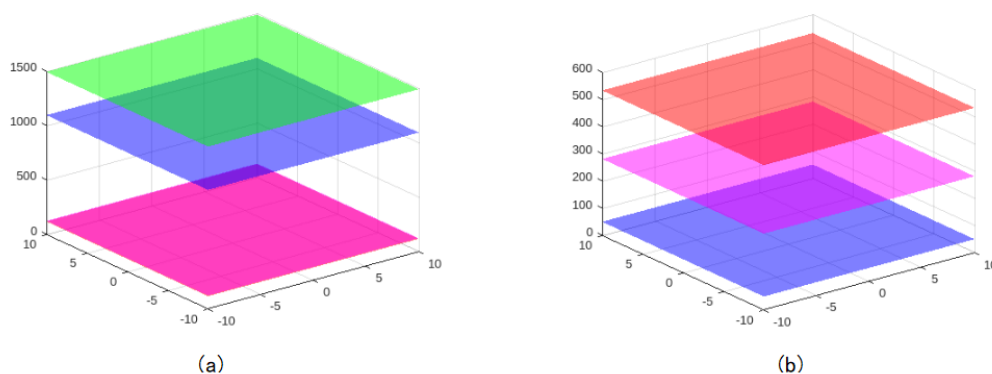


Figure 9. (a) Comparison of the reverse first Zagreb index, reverse second Zagreb index, reverse first Zagreb co-index, and reverse second Zagreb co-index of the line graph of Oseltamivir, where red, magenta, blue, and green denotes $\mathcal{RM}_1(L(G)), \mathcal{RM}_2(L(G)), \mathcal{RM}_1(L(G)),$ and $\mathcal{RM}_2(L(G))$, respectively; (b) Comparison of the reverse hyper Zagreb index, reverse forgotten index and reverse Balaban index, where these of red, magenta, blue denotes $\mathcal{RHM}(L(G)), \mathcal{RF}(L(G)), \mathcal{RJ}(L(G))$, respectively.

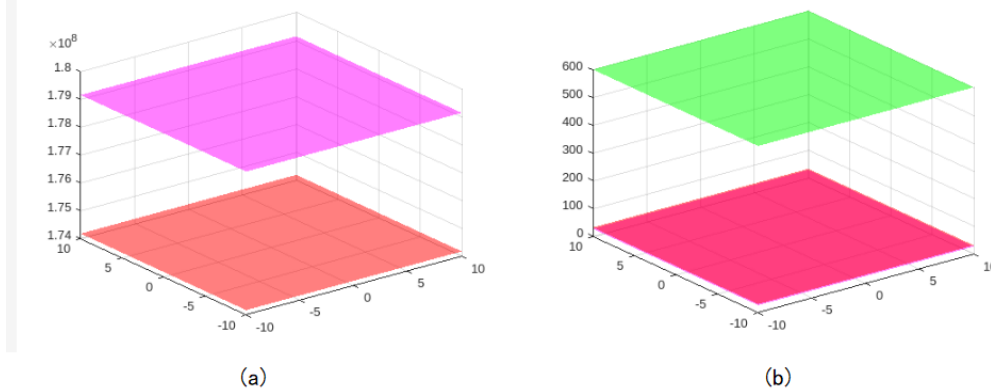


Figure 10. (a) Comparison of the reverse first multiple Zagreb index and reverse second multiple

Zagreb index of the line graph of Oseltamivir, where red and magenta denotes $\mathcal{RPM}_1(L(G))$ and $\mathcal{RPM}_2(L(G))$, respectively; (b) Comparison of the reverse first redefined Zagreb index, reverse second redefined Zagreb index, and reverse third redefined Zagreb index, where red, magenta, blue denotes $\mathcal{RReZG}_1(L(G))$, $\mathcal{RReZG}_2(L(G))$, $\mathcal{RReZG}_3(L(G))$, respectively.

§6. Conclusion

In this paper, the reverse degree-based topological indices, namely, reverse Randić index, reverse atomic bond connectivity index, reverse geometric arithmetic index, reverse first and second Zagreb indices, reverse first and second Zagreb co-indices, reverse hyper Zagreb index, reverse forgotten index, reverse Balaban index, and reverse first, second, and third refined Zagreb indices for the line graph of Zanamivir and Oseltamivir are computed. In addition, we have compared these topological indices graphically as shown in Figures 5-10. Using topological indices of the line graph of molecular graphs to predict various qualities and activities, such as entropy, critical pressure, boiling point, enthalpy, and more, can indeed be a valuable approach to drug discovery and design for H1N1 treatment.

References

- [1] T. Doslic, Vertex-weighted Wiener polynomials for composite graphs, *Ars Math. Conte.*, Vol.1, No. 1 (2008) 66-80.
- [2] E. Estrada, L. Torres, L. Rodriguez, I. Gutman, An atom-bond connectivity index: modelling the enthalpy of formation of Alkanes, *Indian J. Chem. Sect. A: Inorg. Phys. Theor. Anal.*, Vol.37, No. 10 (1998) 849-855.
- [3] E. Estrada, Edge adjacency relationships and a novel topological index related to molecular volume, *Journal of Chemical Information and Computer Sciences*, Vol.35, No. 1 (1995) 31-35.
- [4] B. Furtula, I. Gutman, A forgotten topological index, *J. Math. Chem.*, Vol.53, No. 4 (2015) 1184-1190.
- [5] I. Gutman, B. Furtula, Z. K. Vukicevic, G. Popivoda, On Zagreb indices and coindices, *Match Commun. Math. Comput. Chem.*, Vol.75, No. 5 (2015) 5-16.
- [6] I. Gutman, N. Trinajstic, Graph theory and molecular orbitals. Total pi-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.*, Vol.17, No. 4 (1972) 535-538.
- [7] Z. Hussain, M. Munir, W. Nazeer, M. S. Saleem, S. Min Kang, Y. Chel Kwun, Computational aspects of line graph of carbon nanocones, *Journal of the National Science Foundation of Sri Lanka*, Vol.47, No. 9 (2019) 435-443.
- [8] R. S. Haoer, A. U. R. Virk, Some reverse topological invariants for metal-organic networks, *J. Discret. Math. Sci.*, 24 (2021) 499-510.
- [9] M. K. Hashmi, F. Chaudhry, A. J. M. Khalaf, M. R. Farahani, Investigation of dendrimer structures by means of reverse atomic bond connectivity index, *J. Discret. Math. Sci. Cryptogr.*, 24 (2021) 473-485.

- [10] M. G. Ison, L. V. Gubareva, R. L. Atmar, J. Treanor, F. G. Hayden, Recovery of drug-resistant influenza virus from immuno compromised patients: a case series, *J. Infect Dis.*, Vol.193, (2006) 760-764.
- [11] M. G. Ison, V. P. Mishin, T. J. Braciale, F. G. Hayden, L. V. Gubareva, Comparative activities of Oseltamivir and A-322278 in immunocompetent and immunocompromised murine models of influenza virus infection, *J. Infect Dis.*, Vol.193, (2006) 765-772.
- [12] M. D. Jong, T. T. Tran, H. K. Trong, M. H. Vo, G. J. Smith, V. C. Nguyen, V. C. Bach, V. C. Phan, Q. H. Do, Y. Guan, J. S. Peiris, T. H. Tran, J. Farrar, Oseltamivir resistance during treatment of influenza A(H5N1) infection, *N. Engl. J. Med.*, Vol.353, (2005) 2667-2672.
- [13] C. Y. Jung, M. A. Gondal, N. Ahmad, S. M. Kang, Reverse degree based indices of some nanotubes, *J. Discret. Math. Sci. Cryptogr.*, 22 (2019) 1289-1294.
- [14] V R. Kulli, Reverse Zagreb and reverse hyper-Zagreb indices and their polynomials of Rhombus silicate networks, *Annals of Pure and Applied Mathematics*, Vol.16, No. 1 (2018) 47-51.
- [15] T. C. Li, M. C. Chan, N. Lee, Clinical implications of antiviral resistance in influenza, *Viruses*, Vol.7, (2015) 4929-4944.
- [16] G. E. Mehak, A. A. Bhatti, Zeroth-order general Randic index of line graphs of some chemical structures in drugs, *UPB Scientific Bulletin Series B Chemistry and Materials Science*, Vol.81, No. 7 (2019) 47-70.
- [17] A. Moscona, Oseltamivir resistance-disabling our influenza defenses, *N. Engl. J. Med.*, Vol.353, (2005) 2633-2636.
- [18] Prasanna Poojary, G. B. Gautham, N. Narahari, A. Raghavendra, B. Sooryanarayana, Nandini Poojary, Reverse topological indices of some molecules in drugs used in the treatment of H1N1, *Biointerface Research in Applied Chemistry*, Vol.1, No. 71 (2023) 1-13.
- [19] P. S. Ranjini, V. Loksha, A. Usha, Relation between phenylene and hexagonal squeeze using harmonic index, *Int. J. Graph Theory*, Vol.1, No. 4 (2013) 116-121.
- [20] M. Randić, Characterization of molecular branching, *J. Am. Chem. Soc.*, Vol.97, No. 23 (1975) 6609-6615.
- [21] M. S. Rosary, Topological study of line graph of Remdesivir compound used in the treatment of corona virus, *Polycycl.Aromat.Compdl.*, (2021) 1-17.
- [22] V. Ravi, M. K. Siddiqui, N. Chidambaram, K. Desikan, On topological descriptors and curvilinear regression analysis of antiviral drugs used in COVID-19 treatment, *Polycycl. Aromat. Compd.*, (2021), 1-14.
- [23] G. H. Shirdel, H. Rezapour, A. M. Sayadi, The hyper-Zagreb index of graph operations, *Iran. J. Math. Chem.*, Vol.4, No. 2 (2013) 213-220.
- [24] Z. Tomovic, I. Gutman, On the application of line graphs in quantitative structure-property studies, *Journal of the Serbian Chemical Society*, Vol.65, No. 8 (2000) 577-580.
- [25] D. Vukicevic, B. Furtula, Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges, *J. Math. Chem.*, Vol.46, No. 4 (2009) 1369-1376.
- [26] J. Wei, M. Cancan, A. Ur Rehman, M. K. Siddiqui, M. Nasir, M. T. Younas, M. F. Hanif,

- On topological indices of Remdesivir compound used in treatment of corona virus(COVID 19), *Polycycl. Aromat. Compd.*, (2021) 1-19.
- [27] X. Zuo, J.B. Liu, H. Iqbal, K. Ali, S. T. R. Rizvi, Topological indices of certain transformed chemical structures, *Journal of Chemistry*, Vol.2020, (2020) 1-7.
- [28] D. Zhao, Y. M. Chu, M. K. Siddiqui, K. Ali, M. Nasir, M. T. Younas, M. Cancan, On reverse degree based topological indices of polycyclic metal organic network, *Polycycl.Aromat.Comp.*, (2021) 1-18.