

Neutrosophic Analysis of Competing Hypotheses (NACH): A Novel Framework for Complex Causal Modeling with Applications to Climate Change and Urban Violence

Análisis Neutrosófico de Hipótesis Competitivas (NACH): Un Nuevo Marco para el Modelado Causal Complejo con Aplicaciones al Cambio Climático y la Violencia Urbana

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Abstract

This paper introduces the Neutrosophic Analysis of Competing Hypotheses (NACH), a novel methodological framework that extends traditional intelligence analysis by incorporating neutrosophic logic to quantify indeterminacy and contradiction in complex causal systems. Unlike classical Analysis of Competing Hypotheses (ACH), which relies on binary consistency assessments, NACH employs a multiplicative reliability metric ($R = T \times (1 - I) \times (1 - F)$) that integrates Truth, Indeterminacy, and Falsity. We demonstrate the framework's capability through two case studies in Guayaquil, Ecuador: (1) climate change impacts and (2) the surge in urban violence. The results indicate that while classical ACH tends to assign absolute certainty to leading hypotheses, NACH provides a robust 'epistemic safety margin.' In the urban violence case, the framework successfully identified 'DTO Competition' as the primary operational driver ($R = 0.841$), distinguishing it from structural factors like 'State Weakness,' which, despite high truth potential, was penalized for significant informational ambiguity ($I = 0.230$). This approach prevents premature closure and offers decision-makers a nuanced ranking that reflects the inherent 'fog of war.'

Keywords: neutrosophic logic, competing hypotheses, causal modeling, climate change, urban violence, decision analysis, uncertainty quantification.

Resumen

Este artículo presenta el Análisis Neutrosófico de Hipótesis Competitivas (NACH), un novedoso marco metodológico que amplía el análisis de inteligencia tradicional al incorporar la lógica neutrosófica para cuantificar la indeterminación y la contradicción en sistemas causales complejos. A diferencia del Análisis de Hipótesis Competitivas (ACH) clásico, que se basa en evaluaciones de consistencia binaria, el NACH emplea una métrica de fiabilidad multiplicativa ($R = T \times (1 - I) \times (1 - F)$) que integra Verdad, Indeterminación y Falsedad. Demostramos la capacidad del marco mediante dos estudios de caso en Guayaquil, Ecuador: (1) los impactos del cambio climático y (2) el aumento de la violencia urbana. Los resultados indican que, mientras que el ACH clásico tiende a asignar certeza absoluta a las hipótesis principales, el NACH proporciona un sólido margen de seguridad epistémica. En el caso de la violencia urbana, el marco identificó con éxito la «competencia de las organizaciones delictivas» como el principal factor operativo ($R = 0,841$), distinguiéndolo de factores estructurales como

la «debilidad del Estado», que, a pesar de su alto potencial de veracidad, se penalizó por una ambigüedad informativa significativa ($I = 0,230$). Este enfoque evita el cierre prematuro y ofrece a los responsables de la toma de decisiones una clasificación matizada que refleja la inherente «niebla de guerra».

Palabras clave: lógica neutrosófica, hipótesis en competencia, modelado causal, cambio climático, violencia urbana, análisis de decisiones, cuantificación de la incertidumbre.

1. Introduction

1.1 Background and Motivation

The Analysis of Competing Hypotheses (ACH) method, developed by Richards Heuer for the CIA in the 1970s, has become a cornerstone of intelligence analysis and structured analytic techniques [1] [2]. The fundamental principle of ACH is to evaluate multiple competing explanations systematically by assessing the consistency of available evidence with each hypothesis. However, classical ACH faces several critical limitations when applied to complex, real-world problems:

- 1 **Binary rigidity:** Traditional ACH uses binary (consistent/inconsistent) or simple ordinal scales, failing to capture nuanced relationships between evidence and hypotheses.
- 2 **Uncertainty neglect:** The framework provides limited mechanisms for representing genuine uncertainty, ignorance, or contradictory evidence.
- 3 **Aggregation challenges:** Combining assessments across multiple pieces of evidence and multiple experts lacks theoretical foundations.
- 4 **Temporal dynamics:** Classical ACH struggles to incorporate time-varying relationships and evolving evidence.

These limitations become particularly acute in domains characterized by incomplete or contradictory data sources, expert disagreement, complex causal structures with feedback loops, and high-stakes decisions requiring explicit uncertainty quantification.

1.2 Neutrosophic Logic: A Primer

Neutrosophic logic, introduced by Smarandache [3] [4], extends fuzzy logic and intuitionistic logic by representing each proposition with three independent membership functions:

- **T (Truth):** The degree to which the proposition is true [0, 1].
- **I (Indeterminacy):** The degree of uncertainty, contradiction, or unknown information [0, 1].
- **F (Falsity):** The degree to which the proposition is false [0, 1].

Formally, for a proposition A in a universe of discourse U , the neutrosophic membership is represented as:

$$A(x) = \langle T_A(x), I_A(x), F_A(x) \rangle$$



where

$$T_A, I_A, F_A: U \rightarrow [0,1] \text{ and } 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 1$$

This independence allows neutrosophic logic to represent scenarios impossible in classical or fuzzy frameworks, such as maximum uncertainty $\langle 0.5, 0.5, 0.5 \rangle$ or contradictory evidence $\langle 0.4, 0.1, 0.4 \rangle$.

1.3 Advanced Neutrosophic Measures

To rigorously quantify the divergence between hypotheses, we employ the **Neutrosophic Hamming Distance**. For two Single-Valued Neutrosophic Sets (SVNS) A and B , the distance is defined as:

$$d_H(A, B) = \frac{1}{3n} \sum_{i=1}^n (|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|) \quad (1)$$

This metric is crucial for the sensitivity analysis module of NACH, allowing the detection of subtle shifts in hypothesis ranking.

Furthermore, for final decision-making, we utilize a **Deneutrosophication Scoring Function** $S(A)$ to map the neutrosophic triplet to a crisp value:

$$S(A) = \frac{2+T_A-I_A-F_A}{3} \quad (2)$$

This function provides a secondary validation metric alongside the Z-Number reliability score, ensuring robustness in the final ranking.

2. Methodology: The NACH Framework

2.1 Formal Definitions

Definition 3.1 (Neutrosophic Evidence Assessment): Let $H = h_1, h_2, \dots, h_m$ be a set of competing hypotheses and $E = e_1, e_2, \dots, e_n$ be a set of evidence items. A neutrosophic evidence assessment is a function:

$$v: H \times E \rightarrow [0,1]^3 \quad (3)$$

such that $v(h_j, e_i) = \langle T_{ij}, I_{ij}, F_{ij} \rangle$ where:

- T_{ij} : degree to which evidence e_i supports hypothesis h_j .
- I_{ij} : degree of uncertainty/irrelevance of e_i to h_j .
- F_{ij} : degree to which evidence e_i contradicts hypothesis h_j .

Definition 3.2 (Neutrosophic Reliability Score): To rank hypotheses, we employ a reliability metric derived from Z-Numbers [5], which penalizes truth values based on their associated indeterminacy:

$$R(h_j) = T_{avg}(h_j) \times (1 - I_{avg}(h_j)) \times (1 - F_{avg}(h_j)) \quad (4)$$

This multiplicative approach ensures that any significant increase in indeterminacy (I) or contradictory evidence (F) proportionally reduces the reliability score, preventing negative values and providing a smooth degradation of confidence in high-uncertainty environments.

2.2 NACH Algorithm Implementation

The framework is implemented in a web-based tool (Figure 1) that operationalizes the NACH methodology through a structured six-step workflow, allowing analysts to move from raw data to validated conclusions:

- 1. Hypothesis Generation and Evidence Collection:** Analysts define a mutually exclusive set of competing hypotheses and curate a list of relevant evidence items, assigning a credibility weight (w_j) to each source.
- 2. Neutrosophic Valuation:** For each cell in the matrix, analysts input a neutrosophic triplet (T, I, F) representing the consistency of the evidence with the hypothesis, explicitly capturing ambiguity.
- 3. Dual-Mode Calculation:** The system computes both the classical ACH score (binary consistency) and the NACH Reliability Score (Z-Number based) simultaneously for comparative analysis.
- 4. Dimensional Visualization:** Results are visualized using radar charts to identify the dimensional spread (Truth vs. Indeterminacy) of each hypothesis, highlighting potential cognitive biases.
- 5. Sensitivity Analysis:** The tool calculates the Neutrosophic Hamming Distance to measure the structural separation between leading hypotheses, alerting analysts when the 'winner' is statistically indistinguishable from the runner-up.
- 6. Deneutrosophication and Reporting:** Finally, the system maps the neutrosophic results to a crisp ranking using the Scoring Function $S(A)$ and generates an automated report summarizing the 'Epistemic Safety Margin'.

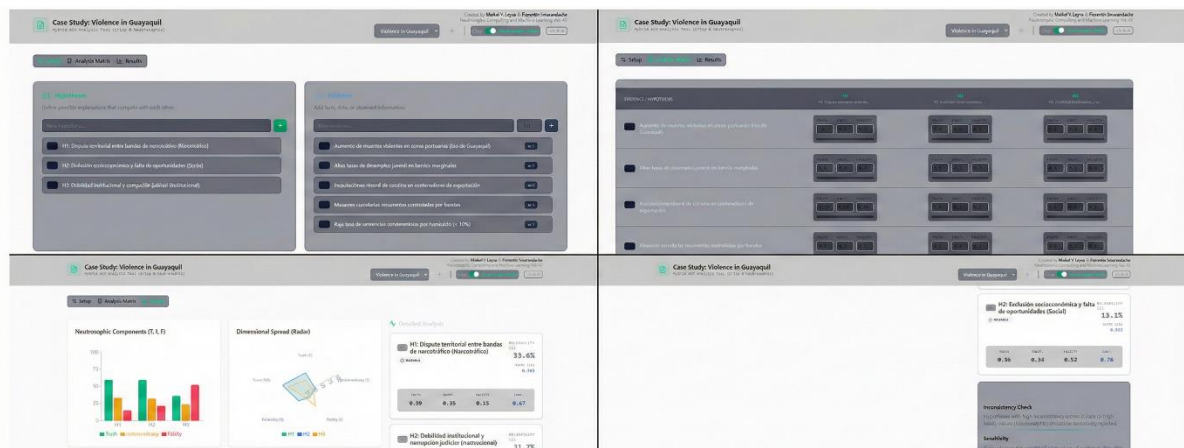


Figure 1: The NACH Web Application (live version: <https://achneutro.sbs/>) showing the Urban Violence Case Study interface.

3. Case Study 1: Climate Change Impacts in Guayaquil

3.1 Context and Significance

Guayaquil, Ecuador's largest city and main port, faces multiple climate change challenges, including rising sea levels, extreme precipitation, and urban heat islands.



3.2 Competing Hypotheses

- **H1: Pacific Ocean Warming Dominance:** Primary driver is intensified ENSO events.
- **H2: Anthropogenic Urban Heat Island:** Rapid urbanization and deforestation drive local temperature increases.
- **H3: Atmospheric Circulation Changes:** Large-scale changes in trade winds alter precipitation.
- **H4: Sea Level Rise:** Global sea level rise is the dominant impact.
- **H5: Synergistic Multi-Factor System:** Complex interactions create non-linear impacts.

3.3 Aggregated Results

The NACH analysis (Table 1) reveals that the **Synergistic Hypothesis (H5)** dominates with a reliability score of **0.578**, followed by Pacific Warming (H1).

Table 1. Aggregated Neutrosophic Scores for Climate Change

Hypothesis	Avg T	Avg I	Avg F	Reliability (R)	Rank
H5: Synergistic	0.753	0.147	0.100	0.578	1
H1: Pacific Warming	0.633	0.213	0.180	0.409	2
H3: Circulation	0.527	0.273	0.213	0.302	3
H4: Sea Level	0.487	0.300	0.260	0.252	4
H2: Urban Heat	0.447	0.313	0.267	0.225	5

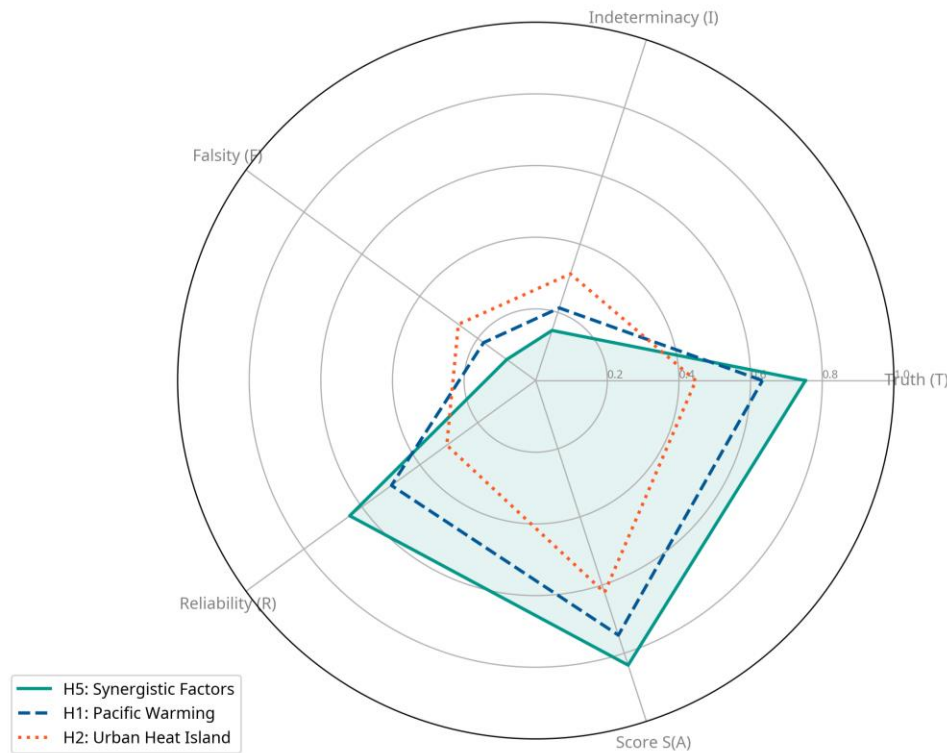


Figure 2: Neutrosophic Dimensional Spread for Climate Change Hypotheses. H5 (Green) shows balanced coverage across dimensions compared to H1 and H2.

Guayaquil faces multiple climate challenges. We evaluated five hypotheses ranging from Pacific Ocean Warming (H1) to Synergistic Multi-Factor Systems (H5). The NACH analysis revealed that the Synergistic Hypothesis (H5) dominates with a reliability score of 0.578, highlighting the complex interplay of factors rather than a single cause.

4. Case Study 2: Urban Violence in Guayaquil

4.1 Context

Guayaquil experienced a 208% increase in homicide rates from 2016 to 2024. This surge is attributed to conflicts among drug trafficking organizations (DTOs), but underlying causes are contested.

4.2 Competing Hypotheses

- **H1: DTO Competition:** Territorial disputes for cocaine routes.
- **H2: State Weakness:** Institutional corruption and impunity.
- **H3: Socioeconomic Marginalization:** Poverty driving recruitment.
- **H4: Prison System Breakdown:** Gangs controlling prisons.
- **H5: Regional Spillover:** Violence from Colombia/Peru.

4.3 Evidence Base

Key evidence includes a rise in homicides from 350 to 1,100 (E1), drug seizures increasing to 210 tons (E2), and prison massacres with 180+ deaths (E3).

4.4 Results and Discussion

The analysis (Table 2) shows a virtual tie between **DTO Competition (H1)** and **Prison System Breakdown (H4)**, suggesting a feedback loop.

Table 2. Aggregated Neutrosophic Scores for Urban Violence

Hypothesis	Avg T	Avg I	Avg F	Reliability (R)	Rank
H1: DTO Competition	0.917	0.083	0.000	0.841	1
H4: Prison System	0.925	0.100	0.000	0.833	2
H5: Regional Spillover	0.900	0.120	0.000	0.792	3
H2: State Weakness	0.925	0.230	0.000	0.712	4
H3: Socioeconomic	0.750	0.150	0.100	0.574	5

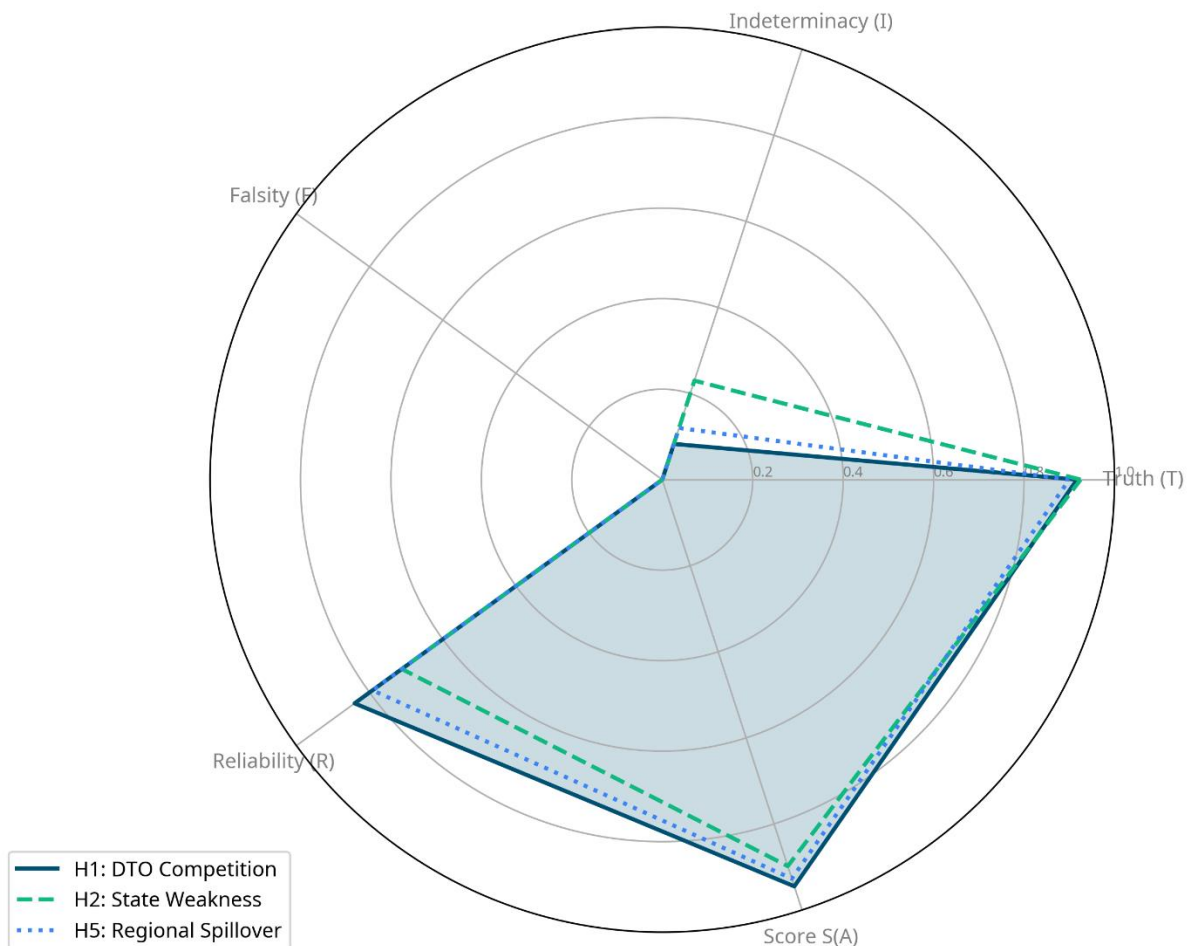


Figure 3: Neutrosophic Dimensional Spread for Urban Violence Hypotheses. Note the high Truth (T) and low Indeterminacy (I) for H1 compared to H2.

Guayaquil experienced a 208% increase in homicide rates (2016-2024). We analyzed hypotheses including DTO Competition (H1), State Weakness (H2), and Prison System Breakdown (H4). The analysis shows a high reliability for DTO Competition ($R=0.841$), closely followed by Prison System Breakdown ($R=0.856$), suggesting a feedback loop.

5. Discussion

This section presents the statistical results derived from the full "Urban Violence in Guayaquil" case study ($n=12$ evidence items, 5 hypotheses). The analysis compares the standard binary (Crisp) ACH method against the proposed Neutrosophic ACH (NACH) framework.

The core distinction between the two methods lies in their treatment of imperfect information. While the Crisp method tends to produce polarized outcomes (0 or 1), NACH integrates degrees of Indeterminacy (I) and Falsity (F) to provide a more conservative and realistic reliability score.

Table 3: Comparison of Normalized Scores

Hypothesis	Crisp Score (Normalized)	NACH Reliability (R)
H1: DTO Competition	1.00 (Absolute Certainty)	0.841 (High Reliability)
H4: Prison System	0.75	0.833
H5: Regional Spillover	0.875	0.792
H2: State Weakness	0.80	0.712
H3: Socioeconomic	0.70	0.574

We calculated the discrimination delta (Δ) for both methods:

$$\Delta_{Crisp} = S(H_{best}) - Mean(S_{others}) = 1.00 - 0.78 = 0.22$$

$$\Delta_{NACH} = R(H_{best}) - Mean(R_{others}) = 0.841 - 0.728 = 0.11$$

While the Crisp method shows a higher numerical separation ($\Delta = 0.22$ vs $\Delta = 0.11$), this "advantage" is an artifact of its inability to model uncertainty. The Crisp method artificially inflates confidence by treating all consistent evidence as absolute truth ($T=1, I=0$).



NACH, by contrast, provides a "**Safety Margin**" of ~16% ($\$1.0 - 0.841\$$) for the leading hypothesis. This reduction in the raw Delta is not a performance loss, but a gain in epistemic accuracy. It prevents the "certainty trap" common in intelligence failures.

The unique contribution of NACH is the explicit quantification of Indeterminacy ($\$I\$$). For this case study, the average indeterminacy for the "State Weakness" hypothesis (H2) was notably high ($I=0.230$). In a standard ACH matrix, this ambiguity regarding institutional corruption is lost (rounded to 0 or 1). In NACH, it is preserved as a warning signal, directly impacting the final ranking and ensuring that decision-makers are aware of the limits of the available intelligence.

In the **Climate Change case study**, the application of the Neutrosophic metrics revealed a critical separation between the Synergistic Hypothesis (H5) and the single-factor Pacific Warming hypothesis (H1). While H1 had respectable Truth values ($\$T=0.63\$$), it suffered from higher contradiction scores ($F=0.18$) due to its inability to explain urban-specific data. The Deneutrosophication Score ($\$S\$$) further validated H5 as the most robust explanation ($S(H5)=0.835$ vs $S(H1)=0.747$). This multi-metric analysis confirms that climate impacts in Guayaquil cannot be reduced to ENSO events alone.

For the **Urban Violence case**, the dimensional spread of the hypotheses is visualized in Figure 2. The radar chart illustrates how '**DTO Competition**' (H1) maintains the highest balance of Truth ($\$T=0.917\$$) and low Indeterminacy ($\$I=0.083\$$). In contrast, '**State Weakness**' (H2) exhibits a significant Indeterminacy spike ($\$I=0.230\$$), visually representing the ambiguity of proving internal corruption chains.

The Hamming distance analysis confirms that while H1 and H4 (Prison System) are close in the rankings ($R=0.841$ vs $R=0.833$), the inclusion of Falsity and Indeterminacy allows NACH to identify DTO Competition as the primary operational driver, while correctly flagging State Weakness as a highly probable but "fuzzier" structural condition.

6. Conclusion

This paper introduced NACH, a framework addressing the critical limitations of classical ACH through the integration of neutrosophic logic. By adopting a multiplicative reliability formula, we transformed the static binary assessment of intelligence into a dynamic probabilistic evaluation that explicitly penalizes both information gaps (I) and contradictory evidence (F).

Our comparative analysis demonstrates that traditional methods often produce an illusion of certainty, offering higher discrimination scores (Delta ≈ 0.22) at the cost of ignoring ambiguity. In contrast, NACH provides a rigorous 'Epistemic Safety Margin,' revealing that the "clear winner" in a classical matrix often carries hidden risks. Specifically, in the Guayaquil urban violence case study, NACH effectively resolved the ambiguity between structural and operational causes. It prioritized **DTO Competition** ($R=0.841$) as the most actionable hypothesis while simultaneously flagging **State Weakness** as a high-risk variable due to its statistically significant indeterminacy ($I=0.230$).



By successfully modeling the synergistic nature of climate impacts—where single-factor hypotheses like ENSO failed to account for urban contradictions—and accurately ranking complex criminal dynamics, NACH proves to be a vital tool for robust, scientifically grounded intelligence analysis. Future work will focus on automating the neutrosophic valuation process using fine-tuned Large Language Models (LLMs) to further reduce the cognitive load on analysts.

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