Neutrosophic Paraconsistent Logic: Evidence Degrees, Ontological Indeterminacy, and Scientific Evidence Synthesis

Lógica paraconsistente neutrosófica: Grados de evidencia, indeterminación ontológica y síntesis de evidencia científica

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Abstract

Classical logic prohibits contradiction as structural collapse: from a contradiction, anything follows (ex contradictione quodlibet). Two formal traditions have challenged this prohibition: da Costa's Annotated Paraconsistent Logic (LPA) [5,6], which tolerates contradictions without system collapse, and Smarandache's Neutrosophic Logic [1,2,3], which introduces genuine indeterminacy as an independent logical value. We show that LPA is algebraically a subset of neutrosophic logic via the embedding $\varphi(A:(\mu,\lambda)) = A:(T=\mu, I=0, F=\lambda)$, which preserves all LPA operations under neutrosophic connectives. This result supports Smarandache's claim that neutrosophic logic subsumes paraconsistent logic structurally. Building on this foundation, we propose Neutrosophic Paraconsistent Logic (NPL), a development within the neutrosophic framework that adds an evidential layer: μ (favorable evidence) and λ (contrary evidence) as evidence degrees alongside the ontological I, enabling a formal distinction between epistemic contradiction (resolvable by evidence) and ontological contradiction (structurally irreducible). Nine formal propositions establish NPL's properties, including bounded paraconsistency under a defined annotated consequence relation _NPL. We formalize an Evidence Synthesis algorithm (NPL-ES) and apply it to a clinical controversy: prenatal paracetamol and autism spectrum disorder (N=17 studies), demonstrating that quality-weighted NPL annotation produces materially different and more actionable recommendations than raw vote-counting.

Keywords: neutrosophic logic, paraconsistent logic, annotated logic, evidence synthesis, ontological indeterminacy, da Costa, Smarandache

Resumen

La lógica clásica prohíbe la contradicción como colapso estructural: de una contradicción se deduce cualquier cosa (ex againste quodlibet). Dos tradiciones formales han cuestionado esta prohibición: la Lógica Paraconsistente Anotada (LPA) de da Costa [5,6], que tolera las contradicciones sin colapso del sistema, y la Lógica Neutrosófica de Smarandache [1,2,3], que introduce la indeterminación genuina como un valor lógico independiente. Demostramos que la LPA es algebraicamente un subconjunto de la lógica neutrosófica mediante la incrustación $\varphi(A:(\mu,\lambda)) = A:(T=\mu, I=0, F=\lambda)$, que preserva todas las operaciones de la LPA bajo conectores neutrosóficos. Este resultado respalda la afirmación de Smarandache de que la lógica neutrosófica subsume estructuralmente la lógica paraconsistente. Partiendo de esta base, proponemos la Lógica Paraconsistente Neutrosófica (LPN), un desarrollo dentro del marco neutrosófico que añade una capa de evidencia: μ (evidencia favorable) y λ (evidencia contraria) como grados de evidencia junto con la indeterminación ontológica (I), lo que permite una distinción formal entre contradicción epistémica (resoluble mediante evidencia) y contradicción ontológica (estructuralmente irreductible). Nueve proposiciones formales establecen las propiedades de la LPN, incluyendo la paraconsistencia limitada bajo una relación de consecuencia anotada definida ⊨_LPN. Formalizamos un algoritmo de síntesis de evidencia (LPN-SE) y lo aplicamos a una controversia clínica: paracetamol prenatal y trastorno del espectro autista (N=17 estudios), demostrando que la anotación de LPN ponderada por calidad produce recomendaciones sustancialmente diferentes y más prácticas que el simple recuento de votos.

Palabras clave: lógica neutrosófica, lógica paraconsistente, lógica anotada, síntesis de evidencia, indeterminación ontológica, da Costa, Smarandache

1. Introduction

The principle of non-contradiction is among the oldest commitments of Western logic. Aristotle called it the most certain of all principles (Metaphysics Γ.3, 1005b17-18). Classical logic formalizes it through explosion: from {A, ¬A}, any B follows. Two traditions have challenged this with mathematical rigor. Da Costa's Annotated Paraconsistent Logic (LPA) [5,6,7] tolerates contradictions without system collapse. Smarandache's Neutrosophic Logic [1,2,3] introduces genuine indeterminacy I as a third logical value independent of truth T and falsity F, with the general condition $0 \leq T+I+F \leq 3$.

Smarandache has noted that neutrosophic logic already includes paraconsistency structurally: when $T+F>1$, a proposition simultaneously participates in truth and falsity beyond unity — the paraconsistent regime [1,2]. This paper formalizes this relationship precisely, shows that LPA neutrosophic logic algebraically, and proposes NPL as a development within the neutrosophic framework that adds an evidential layer enabling the epistemic/ontological distinction unavailable when all three dimensions carry the same semantic type.

The paper is organized as follows. Section 2 presents LPA and its embedding in neutrosophic logic. Section 3 presents neutrosophic logic and NPL's position within it. Section 4 develops the NPL formal system. Section 5 presents the NPL-ES evidence synthesis algorithm. Section 6 applies NPL-ES to a clinical case study. Section 7 concludes.

2. Annotated Paraconsistent Logic and its Embedding in Neutrosophic Logic

2.1 LPA: Structure and Canonical States

Da Costa's central insight is that the explosion principle is a design choice, not a logical necessity [5]. A logic is paraconsistent iff it does not validate $\{A, \neg A\}$ B for arbitrary B. In LPA [6,7], every proposition P carries annotation (μ, λ) $[0,1]^2$, where μ = degree of favorable evidence and λ = degree of contrary evidence.

State	Condition	Meaning
True	μ high, λ low	Well-supported
False	μ low, λ high	Well-refuted
Paraconsistent	$\mu+\lambda > 1$	Contradictory — strong evidence on both sides
Paracomplete	$\mu+\lambda < 1$, both low	Insufficient evidence
Trivial	$\mu = \lambda = 1$	Maximum contradiction — collapse

2.2 LPA is Algebraically a Subset of Neutrosophic Logic

We formalize Smarandache's claim that neutrosophic logic subsumes paraconsistent logic. Define the mapping φ: Form_LPA → Form_Neutrosophic by:

$$\varphi(A:(\mu, \lambda)) = A:(T = \mu, I = 0, F = \lambda) \quad (1)$$

All LPA operations are preserved under neutrosophic connectives:

Operation	LPA result	Neutrosophic under φ	Preserved?
Negation	$\neg A:(\lambda, \mu)$	$\neg A:(\lambda, 0, \mu)$ [neg(T,I,F)=(F,I,T)]	✓
Conjunction	$(\min(\mu_1,\mu_2), \max(\lambda_1,\lambda_2))$	$(\min(\mu_1,\mu_2), 0, \max(\lambda_1,\lambda_2))$	✓
Disjunction	$(\max(\mu_1,\mu_2), \min(\lambda_1,\lambda_2))$	$(\max(\mu_1,\mu_2), 0, \min(\lambda_1,\lambda_2))$	✓
Paraconsistent $\mu+\lambda>1$	valid in LPA	$T+F>1$ valid $(0 \leq T+I+F \leq 3)$	✓

Proposition 1 (LPA Neutrosophic Logic). The mapping φ is an algebraic embedding: it is injective and preserves all LPA connectives. Every LPA state has a valid neutrosophic representation; every LPA inference is valid under neutrosophic connectives. In this algebraic sense, LPA is a subset of neutrosophic logic with I=0. [1,2,5,6]

This result validates Smarandache's position [1,2,3]. However, the embedding φ changes the semantic type of the first two dimensions: μ (evidence degree — epistemic) becomes T (truth-value — semantic/ontological). The algebraic structure is preserved; the semantic interpretation is not. NPL addresses this by maintaining both simultaneously.

3. Neutrosophic Logic and NPL as its Evidential Extension

3.1 Neutrosophic Logic: Foundational Contribution

Smarandache's foundational insight is that indeterminacy I cannot be derived from truth T and falsity F — it is a third independent dimension of logical evaluation [1,2,3]. Every proposition

P is evaluated with (T, I, F) $[0,1]^3$ where $0 \leq T+I+F \leq 3$. The independence of I from T and F is definitional. This makes neutrosophic logic the broadest framework for multi-valued reasoning, subsuming classical logic, fuzzy logic [10], intuitionistic logic, and — as shown in Section 2 — annotated paraconsistent logic.

I covers at least four phenomena: (1) unknown unknowns; (2) ontological vagueness — genuine indeterminacy in the world; (3) structural undecidability; (4) states between dialectical extremes that are neither thesis nor antithesis and do not resolve into synthesis.

3.2 The Two-Level Semantic Structure of NPL

NPL is a development within the neutrosophic framework. It adds an evidential layer alongside the ontological I, keeping the two levels formally distinct:

Dimension	Origin	Semantic type	Answers
μ	LPA [5,6]	Epistemic — evidence degree	To what degree does the evidence support P?
λ	LPA [5,6]	Epistemic — evidence degree	To what degree does the evidence contradict P?
I	Neutrosophic Logic [1,2]	Ontological — truth-value	To what degree is P genuinely indeterminate in the world?

The embedding φ (Section 2.2) maps LPA into neutrosophic logic by treating μ as T and λ as F. NPL avoids this collapse: μ and λ retain their evidential meaning (epistemic — about what agents know), while I retains its neutrosophic ontological meaning (about what the world is). This combination enables NPL to distinguish contradictions that evidence can resolve from contradictions that are structurally irreducible — a distinction unavailable if all three dimensions carry the same semantic type.

3.3 Relationship to SVN Sets and Neutrosophic Statistics

NPL is structurally isomorphic to SVN sets [1]: both use $[0,1]^3$ with independent components and identical min/max operations. The contribution of NPL is semantic: it reinterprets the first two dimensions from truth-values (SVN: T=membership, F=non-membership) to evidence degrees (NPL: μ=favorable evidence, λ=contrary evidence). In neutrosophic statistics [4], I captures measurement imprecision in data. In NPL, I captures ontological indeterminacy of propositions — a different level of analysis. Crucially, $\mu+\lambda>1$ in NPL is a paraconsistent state (evidence base is contradictory); in SVN sets $T+F>1$ is merely a neutrosophic number with no paraconsistency interpretation.

4. NPL — Formal Development

4.1 Formal Language and Definitions

Definition 1 (NPL Annotated Formula). *The set $Form_{NPL}$ is defined inductively:*

1. If p Var and (μ,λ,I) $[0,1]^3$, then p:(μ,λ,I) $Form_{NPL}$

2. If φ:(μ_1,λ_1,I_1) $Form_{NPL}$, then ¬φ:(λ_1,μ_1,I_1) $Form_{NPL}$
3. If φ:(μ_1,λ_1,I_1), ψ:(μ_2,λ_2,I_2) Form_NPL then:
   - (φ∧ ψ):(min(μ_1,μ_2), max(λ_1,λ_2), max(I_1,I_2)) ∈ $Form_{NPL}$
   - (φ∨ ψ):(max(μ_1,μ_2), min(λ_1,λ_2), max(I_1,I_2)) ∈ $Form_{NPL}$
   - (φ→ψ):(min(1,1−μ_1+μ_2), max(0,λ_1−λ_2), max(I_1,I_2)) ∈ $Form_{NPL}$

Definition 2 (NPL Valuation). *V: Var → [0,1]³, extended to Form_NPL by Definition 1.*

Definition 3 (NPL States). *Given V(φ) = (μ,λ,I):*

State	Condition	Interpretation
NPL-V	μ>0.5, μ>λ	Predominantly supported
NPL-F	λ>0.5, λ>μ	Predominantly contradicted
NPL-Para	μ+λ>1	Paraconsistent — evidence contradictory
NPL-I	I>0.5, μ+λ≤1	Genuinely indeterminate
NPL-PC	μ<0.5, λ<0.5, I<0.5	Insufficient information
NPL-T	μ=λ=I=1	Maximal collapse

Definition 4 (Ontological vs. Epistemic Contradiction). *Let φ be NPL-Para (μ+λ>1). Let θ_{domain} (0,1) be the domain resolution threshold (Definition 6). φ is ontological if I> θ_{domain} (no domain evidence can reduce I); epistemic if I≤ θ_{domain} (conceivable evidence could reduce I toward zero).*

Definition 5 (Designated Values and _NPL). *An annotation (μ,λ,I) is designated if μ>λ and μ>0.5 (NPL-V). Γ $\vDash_{NPL}$ φ iff every valuation satisfying Γ with designated values also designates φ.*

4.2 Inference System and Nine Formal Propositions

Main inference rule (NPL-MP):

$$\frac{\varphi:(\mu_1,\lambda_1,I_1) \qquad (\varphi\to\psi):(\mu_2,\lambda_2,I_2)}{\psi:(\min(\mu,\mu),\ \max(\lambda,\lambda),\ \max(I,I))}$$

Proposition 2 (Proper Extension of LPA). *LPA is a proper subsystem of NPL.*

Proof. Embedding: φ(A:(μ,λ)) = A:(μ,λ,0). All LPA operations preserved (Section 2.2). Proper: P_1:(0.8,0.8,0.9) and P_2:(0.8,0.8,0.1) are identical in LPA but structurally distinct in NPL — NPL-IC applies to P_1 (ontological); P_2 admits I-reduction (epistemic). LPA has no representation for this distinction.

Proposition 3 (Proper Extension of Neutrosophic Logic). *Neutrosophic logic is a proper subsystem of NPL.*

Proof. Embedding: ψ(A:(T,I_n,F)) = A:(T,F,I_n). Connectives preserved under μ=T, λ=F, I=I_n. Proper: NPL adds an explicit annotated consequence relation $\vDash_{NPL}$ and Proposition 4 below, which had not been stated as a formal result within the neutrosophic literature [1,2,3]. NPL also accommodates annotations that the normalized variant (T+I+F=1) cannot express.

Proposition 4 (Bounded Paraconsistency). *Under $\vDash_{NPL}$, contradictions do not license arbitrary designated conclusions. Any ψ derivable from φ:(μ,λ,I) via NPL-MP satisfies μ(ψ)≤μ, λ(ψ)≥λ, I(ψ)≥I.*

Proof. By induction on derivation depth. Base: NPL-MP gives $\mu(\psi)=\min(\mu,\mu_2)\leq\mu$, $\lambda(\psi)=\max(\lambda,\lambda_2)\geq\lambda$, $I(\psi)=\max(I,I_2)\geq I$. Inductive step preserves bounds. For classical explosion (arbitrary designated conclusion): requires μ'=1, λ'=0. But λ(ψ)≥λ>0 for any paraconsistent premise. Only NPL-T=(1,1,1) allows all conclusions to inherit NPL-T.

Corollary 4.1. From φ:(1,1,0) — LPA's trivial state — NPL derives only conclusions with μ=1, λ=1 (NPL-Para). Conclusions are contradictory but never NPL-V. This is strictly weaker than LPA explosion.

Proposition 5 (Distinguishability of Ontological/Epistemic Contradiction). *P_1:(0.8,0.8,0.9) and P_2:(0.8,0.8,0.1) generate structurally distinct inference trees in NPL.*

Proof. By NPL-IC, all ψ from P_1 satisfy I(ψ)≥0.9 (I-closed). P_2 has I≤θ; NPL-IC does not apply; evidence can reduce I (I-open). In LPA, both produce identical trees. □

Proposition 6 (Monotonicity of I). *$I(\varphi\wedge\psi) = I(\varphi\vee\psi) = I(\varphi\to\psi) = \max(I_1,I_2) \geq I_1$ and $\geq I_2$.*

Proof. Immediate from Definition 1, clauses 2–3.

Proposition 7 (NPL Generalizes Classical Logic). *CL is a special case of NPL under μ∈{0,1}, λ=1−μ, I=0.*

Proof. Under these conditions: (1,0,0)=classical truth; (0,1,0)=classical falsity. Truth tables reproduced. NPL-MP reproduces classical modus ponens.

Corollary 7.1 (Subsumption): CL LPA NPL; Neutrosophic Logic NPL; Fuzzy [10] NPL (λ=1-μ, I=0); Belnap-4 [11] NPL (corners of $\{0,1\}^2\times\{0\}$).

Proposition 8 (Conservation of Ontological Contradiction). *If φ is ontological (μ+λ>1, I>θ) and I(φ→ψ)>θ, then ψ is ontological or NPL-I.*

Proof. By NPL-MP: I(ψ)=max(I(φ),I_{→})>θ. Result follows from Definition 3.

Proposition 9 (Full Expressiveness). *For any (a,b,c) $[0,1]^3$, there exists an NPL formula with exactly that annotation.*

Proof. NPL valuations assign values in $[0,1]^3$ with no sum constraint. Define atomic p with V(p)=(a,b,c). CL: 2 states; LPA: $[0,1]^2$; Neutrosophic normalized: 2-simplex; NPL: full $[0,1]^3$.

4.3 Reduction Conditions

Condition	NPL reduces to
μ∈ {0,1}, λ=1−μ, I=0	Classical logic
I=0 for all formulas	LPA (with LPA-trivial→NPL-Para)

λ=1−μ, I=0	Fuzzy logic [10]
{μ,λ}∈{0,1}², I=0	Belnap-4 [11] (corners)
T+I+F=1 imposed	Neutrosophic logic (normalized variant) [1]

4.4 Domain Resolution Threshold θ

Definition 6 (Domain Threshold). *θ_domain = sup{I₀ [0,1] : E domain-admissible such that r(E,A) > 1−ε}, where r(E,A) ∈ [0,1] is the resolution power of evidence E for A's indeterminacy. I ≤ θ_{domain}: epistemic (evidence can resolve). I>θ_{domain}: ontological (structurally irreducible).*

Domain	θ_domain	Resolvable by
Experimental sciences (RCT)	0.35	Randomized controlled trial
Observational epidemiology	0.45	Sibling-control cohort + systematic review
Quantitative social sciences	0.55	Quasi-experimental design
Law / jurisprudence	0.60	Constitutional interpretation or amendment
Philosophy / metaphysics	0.72	Formal argument within shared axioms

5. NPL Evidence Synthesis Algorithm (NPL-ES)

The distinction between μ, λ, and I enables principled multi-study evidence synthesis beyond vote-counting. We formalize NPL-ES, applicable to any structured literature corpus.

Definition 7 (NPL-ES). *Given research question P, N classified studies in C={Yes, Possibly, No, Indeterminate, Mixed}, and threshold θ_domain, NPL-ES produces annotation (μ,λ,I) and classification in four steps.*

Step 1 — Quality index Q_c

$$Q_c = (t_c + q_c + r_c)/3 + d_c$$

t_c = Tier 1 proportion in category c
$q_c = 1/Q_{rank_c}$ (journal quartile score)
r_c = recency score (normalized year)
d_c = design bonus ±0.20
Q_{max} = $max_c(Q_c)$

Steps 2–4 — μ, λ, I

$$\mu = [n_{Yes} \cdot Q_{Yes} \cdot 1.0 + n_{Pos} \cdot Q_{Pos} \cdot \alpha]/[N \cdot Q_{max}] \quad (\alpha=0.40 \text{ default})$$

$$\lambda = [n_{N0} \cdot Q_{No} \cdot 1.0 + n_{Pos} \cdot Q_{Pos} \cdot (1-\alpha)]/[N \cdot Q_{max}]$$

$$I = 0.50 \cdot I_{het} + 0.35 \cdot I_{gap} + 0.15 \cdot I_{amb}$$

I_{het} = methodologicalheterogeneitybetweenYes/Nogroups
$I_{gap} = (1 - causal_inference_strength)$ [high for observational evidence]
$I_amb = [n_{Ind} \cdot Q_{Ind} + n_{Mix} \cdot Q_{Mix}]/[N \cdot Q_{max}]$

Note on Indeterminate studies. Studies classified as "could not determine" contribute exclusively to I — not to μ or λ. In LPA, such studies have no formal representation and must be discarded or arbitrarily split. In NPL-ES, they are assigned to the indeterminacy dimension, preserving their epistemic content without distorting the evidential balance.

5.1 Minimal Example (N=6)

2 studies favor P, 3 contradict P, 1 could not determine. Uniform Q=1.0, α=0.40:

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μ = (2×1.0×1.0) / (6×1.0) = 0.333
λ = (3×1.0×1.0) / (6×1.0) = 0.500
I = 0.50×0.10 + 0.35×0.05 + 0.15×(1/6) = 0.093

Annotation: P:(0.333, 0.500, 0.093)
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Classification: $\mu+\lambda=0.833<1$ (not paraconsistent); NPL-F ($\lambda>\mu$, $\lambda>0.5$); $I=0.093 < \theta=0.45$ → epistemic. Rational response: commission better-designed studies. LPA alternative: the inconclusive study must be discarded (losing information) or split arbitrarily (μ+=0.083, λ+=0.083 without justification).

6. Application: Prenatal Paracetamol and Autism Spectrum Disorder

We apply NPL-ES to a live clinical controversy: whether prenatal paracetamol (acetaminophen) exposure causes autism spectrum disorder (ASD). Evidence corpus: Consensus.app, N=17 peer-reviewed studies with explicit quality indicators.

Proposition P: "Prenatal paracetamol exposure causes autism spectrum disorder."

Verdict	n (%)	Tier 1	Q-journal	Year	Design note
Yes	5 (29%)	3/5	Q1.2	2023	Meta-analyses; lack sibling controls; confounding risk
Possibly	4 (24%)	2/4	Q1.5	2016	Observational; design flaws; no causal inference
No	8 (47%)	3/8	Q1.6	2024	Sibling-control cohorts + systematic reviews

6.1 NPL-ES Computation

Category	t_c	q_c	r_c	d_c	Q_c
Yes	0.60	0.83	0.90	−0.20	0.57
Possibly	0.50	0.67	0.60	−0.20	0.38
No	0.38	0.63	1.00	+0.20	0.75

Q_max = 0.75 (No group: highest methodological quality — sibling controls, most recent, highest Q-journal).

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μ = [5×0.57×1.0  +  4×0.38×0.40] / [17×0.75]  =  3.46/12.75  =  0.27
λ = [8×0.75×1.0  +  4×0.38×0.60] / [17×0.75]  =  6.91/12.75  =  0.54

I_het = 0.25  (sibling-control vs. non-sibling-control: structural
methodological fault line)
I_gap = 0.245 (all evidence observational; 0.35×(1-0.30))
I_amb = 0.048 (4 Possibly papers)
I     = 0.50×0.25 + 0.35×0.245 + 0.15×0.048  =  0.22

Annotation:  P : (μ=0.27,  λ=0.54,  I=0.22)
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6.2 Classification and Interpretation

- $\mu+\lambda = 0.81 < 1$ → NOT paraconsistent: quality-weighted, no genuine evidential contradiction
- NPL-F ($\lambda>\mu$, $\lambda>0.5$): evidence leans clearly against P
- $I=0.22 < \theta\_epidemiology=0.45$ → Epistemic: resolvable with better-designed studies
- What drives I_het=0.25: sibling-control studies systematically differ from non-sibling-control — a structural methodological fault line, not random noise

Rational response: fund sibling-control longitudinal cohorts. Not "accept uncertainty" but "improve design."

6.3 Comparison with Consensus Meter (raw vote-counting)

Criterion	Consensus Meter (raw)	NPL-ES (quality-weighted)
Apparent result	53% Yes/Possibly vs 47% No — "mixed"	NPL-F: λ=0.54 > μ=0.27 — clear lean against
Quality information	Not incorporated	Sibling-control studies receive 32% higher weight
Indeterminacy	Undifferentiated "mixed"	I=0.22: epistemic, moderate, resolvable
Source of disagreement	Not identified	I_het: methodological incommensurability
Recommendation	Implicit: uncertain	Explicit: better causal designs will resolve it

7. Philosophical Note: Productive Contradiction

NPL's formal distinction between epistemic and ontological contradiction has direct philosophical import. The philosophical tradition — from Aristotle to Hegel — treated contradiction as a problem requiring resolution. Hegel's Aufhebung domesticates contradiction through dialectical synthesis. NPL contests this telos: some contradictions are stable, generative, and do not require resolution to be productive.

The Andean concept of tinku — opposing forces that generate energy without annihilating each other — and ubuntu philosophy's individual/collective tension both point to ontologically stable contradictions. NPL formalizes this: when $I>\theta_{domain}$, the contradiction is not a transitional state to be resolved but a structurally stable feature to be inhabited and reasoned within. The logical genealogy runs: da Costa (Brazil, 1963) [5] → Smarandache (Romania/USA, 1995) [1] → NPL (Ecuador/USA, 2026), connecting Latin American formal traditions with Global South philosophy of productive contradiction [15,16,17].

8. Conclusions

We have shown that LPA is algebraically a subset of neutrosophic logic via the embedding $\varphi(A:(\mu,\lambda)) = A:(T=\mu,\ I=0,\ F=\lambda)$, formally supporting Smarandache's claim that neutrosophic logic subsumes annotated paraconsistent logic [1,2]. Building on this foundation, we proposed NPL as a development within the neutrosophic framework that maintains the evidential semantics of μ and λ alongside the ontological I. Nine formal propositions establish that NPL:

4. Establishes LPA neutrosophic logic algebraically — all LPA operations preserved under φ
5. Properly extends LPA: NPL-T requires I=1 additionally; LPA-trivial ($\mu=\lambda=1$) is NPL-Para controlled
6. Properly extends neutrosophic logic: adds _NPL and Proposition 4 (bounded paraconsistency)
7. Formally distinguishes ontological from epistemic contradiction via θ_{domain}
8. Preserves I monotonically under all propositional connectives
9. Specializes to CL, LPA, fuzzy [10] and Belnap-4 [11] under explicit annotation constraints
10. Conserves ontological contradictions along inferential chains
11. Provides full expressiveness of $[0,1]^3$
12. Provides a graded generalization of LFI's [8,9] consistency operator: $I(A)\leq\theta$ corresponds to $\circ A$

The NPL-ES algorithm translates this formal system into a practical tool for evidence synthesis. Applied to the prenatal paracetamol/autism controversy (N=17), NPL-ES produces NPL-F with epistemic I=0.22 — a materially different and more actionable recommendation than the raw vote-count "research is mixed." Future work: model theory (Kripke semantics, completeness), full LFI comparison [8,9], and learned NPL annotations from text corpora.

References

[1] Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. American Research Press, 1995.

[2] Smarandache, F. Neutrosophy: Neutrosophic Probability, Set, and Logic. American Research Press, 1998.

[3] Smarandache, F. Definition of neutrosophic logic — a generalization of the intuitionistic fuzzy logic. Proceedings of the 3rd EUSFLAT Conference, 2003, pp. 141–146.

[4] Smarandache, F. The Score, Accuracy, and Certainty Functions determine a Total Order on the Set of Neutrosophic Triplets (T,I,F). Neutrosophic Sets and Systems, vol. 38, 2020, pp. 1–14.

[5] da Costa, N. C. A. Sistemas formais inconsistentes. Tese de Livre-Docência, Universidade Federal do Paraná, 1963.

[6] da Costa, N. C. A.; Subrahmanian, V. S. Paraconsistent logics as a formalism for reasoning about inconsistent knowledge bases. Artificial Intelligence in Medicine, vol. 1, no. 4, 1989, pp. 167–174.

[7] Abe, J. M. Fundamentos da Lógica Anotada. Universidade de São Paulo, 1992.

[8] Carnielli, W.; Coniglio, M. E.; Marcos, J. Logics of Formal Inconsistency. In Handbook of Philosophical Logic, vol. 14. Springer, 2007, pp. 1–93.

[9] Carnielli, W.; Coniglio, M. E. Paraconsistent Logic: Consistency, Contradiction and Negation. Springer, 2016.

[10] Zadeh, L. A. Fuzzy sets. Information and Control, vol. 8, no. 3, 1965, pp. 338–353.

[11] Belnap, N. D. A useful four-valued logic. In Modern Uses of Multiple-Valued Logic. Reidel, 1977, pp. 5–37.

[12] Priest, G. In Contradiction: A Study of the Transconsistent. Martinus Nijhoff, 1987.

[13] Priest, G. Doubt Truth to Be a Liar. Oxford University Press, 2006.

[14] Leyva-Vázquez, M.; Smarandache, F. Neutrosofía: Nuevos avances en el tratamiento de la incertidumbre. Pons, 2018.

[15] Dussel, E. América Latina: Dependencia y liberación. Fernando García Cambeiro, 1973.

[16] De Sousa Santos, B. Una epistemología del sur. CLACSO/Siglo XXI, 2009.

[17] Escobar, A. Designs for the Pluriverse. Duke University Press, 2018.

[18] Quijano, A. Coloniality of power and Eurocentrism in Latin America. International Sociology, vol. 15, no. 2, 2000, pp. 215–232.

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