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Neutrosophic and Plithogenic Operators: Negation, Intersection, Union, Implication, Equivalence

Operadores neutrosóficos y plitogénicos: Negación, intersección, unión, implicación, equivalencia

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Abstract. This short contribution presents the neutrosophic and plithogenic logical operators -- negation, intersection (conjunction), union (disjunction), implication, and equivalence -- expressed in terms of triangular norms (*t*-norms) and triangular conorms (*t*-conorms). For neutrosophic operators the truth component *T* employs the *t*-norm while the falsity component *F* employs the *t*-conorm (and vice versa), reflecting their opposed semantic roles; indeterminacy *I* is treated as predominantly of negative quality. Plithogenic operators differ only in the treatment of *I*, which receives the arithmetic mean of the *t*-norm and *t*-conorm applied to the *I* components. Several classical parametric families of *t*-norms are recalled as concrete instances.

Keywords: neutrosophic logic; plithogenic logic; triangular norm and conorm; neutrosophic operators; plithogenic operators.

Resumen. Esta breve contribución presenta los operadores lógicos neutrosóficos y plitogénicos —negación, intersección (conjunción), unión (disyunción), implicación y equivalencia— expresados en términos de normas triangulares (*t*-normas) y conormas triangulares (*t*-conormas). Para los operadores neutrosóficos, el componente de verdad *T* emplea la *t*-norma, mientras que el componente de falsedad *F* emplea la *t*-conorma (y viceversa), lo que refleja sus roles semánticos opuestos; la indeterminación *I* se trata como predominantemente de cualidad negativa. Los operadores plitogénicos difieren únicamente en el tratamiento de *I*, que recibe la media aritmética de la *t*-norma y la *t*-conorma aplicadas a los componentes *I*. Se recuerdan varias familias paramétricas clásicas de *t*-normas como ejemplos concretos.

Palabras clave: lógica neutrosófica; lógica plitogénica; norma y conorma triangular; operadores neutrosóficos y plitogénicos.

1 Introduction

First of all, we need to recall the *t*-norms (triangular norms) and *t*-conorms (triangular conforms) that the neutrosophic and plithogenic operators utilize.

Triangular norms and conorms are generalizations of the logical conjunction and logical disjunction to fuzzy logic [Hájek (1998)], and they are used to combine uncertain information.

The below *t*-norms and *t*-conorms are well-known in the literature. In this paper we took them from Mirko Navara (2007), Triangular norms and conforms, *Scholarpedia*, 2(3):2398.

2 Triangular Norms

2.1 Definition

A **triangular norm** (abbreviation ***t*-norm**) is a binary operation *T* on the interval [0,1] satisfying the following conditions:

- $T(x, y) = T(y, x)$ **(commutativity)**
- $T(x, T(y, z)) = T(T(x, y), z)$ **(associativity)**
- $y \leq z \Longrightarrow T(x, y) \leq T(x, z)$ (**monotonicity**)
- $T(x, 1) = x$ (**neutral element 1**)

2.2 Examples

$T_M(x,y) = min(x,y)$ (**minimum or Gödel *t*-norm**)
$T_P(x,y) = x \cdot y$ (**product *t*-norm**)
$T_L(x,y) = max(x+y-1,0)$ (**Lukasiewicz *t*-norm**)

No *t*-norm can attain greater values than T_M. There are many parametrized families of *t*-norms [Klement et al. (2000)]. The **Frank *t*-norms** are defined for all $r > 0, r \neq 1$ by

$$T_{F_r}(x,y) = \log_r\left(1 + \frac{(r^x-1)(r^y-1)}{r-1}\right)$$

The limit elements of this family are the above *t*-norms

$$T_{F_0} = T_M,$$
$$T_{F_1} = T_P,$$
$$T_{F_\infty} = T_L.$$

The only *t*-norms which are rational functions are the **Hamacher *t*-norms** defined for all $r > 0$ by

$$T_{H_r}(x,y) = \frac{xy}{r + (1-r)(x+y-xy)}$$

and for $r = 0$ by

$$T_{H_0}(x,y) = \frac{xy}{x+y-xy}$$

$$(T_{H_0}(0,0) = 0).$$

3 Triangular Conorms

3.1 Definition

The dual notion to a triangular norm is a **triangular conorm** (abbreviation ***t*-conorm**, also **s-norm**), S. Its neutral element is 0 instead of 1, all other conditions remain unchanged:

- $S(x,y) = S(y,x)$ (**commutativity**)
- $S(x,S(y,z)) = S(S(x,y),z)$ (**associativity**)
- $y \leq z \Longrightarrow S(x,y) \leq S(x,z)$ (**monotonicity**)
- $S(x,0) = x$ (**neutral element 0**)

3.2 Examples of *t*-conorms

- $S_M(x,y) = max(x,y)$ (**maximum or Gödel *t*-conorm**)
- $S_P(x,y) = x + y - x \cdot y$ (**product *t*-conorm, probabilistic sum**)
- $S_L(x,y) = min(x+y,1)$ (**Lukasiewicz *t*-conorm, bounded sum**)

No *t*-conorm can attain smaller values than S_M.

If T is a *t*-norm, then $S(x,y) = 1 - T(1-x, 1-y)$ is a *t*-conorm, and vice versa. We obtain a *dual pair* (T,S) of a *t*-norm and a *t*-conorm. (Instead of the *standard fuzzy negation*, $x \mapsto 1-x$, another strong fuzzy negation can be used in the duality formula.)

The classification and representations of *t*-conorms are dual to those of *t*-norms. Each continuous Archimedean *t*-conorm S has a (non-unique) additive generator, which is an increasing bijection $s: [0,1] \to [0,B]$ $(B \in]0,\infty])$ such that

$$S(x,y) = \begin{cases} s^{-1}(s(x)+s(y)) & \text{if } s(x)+s(y) \leq B, \\ 1 & \text{otherwise.} \end{cases}$$

4 Neutrosophic Operators employ the above *t*-norms and *t*-conorms

The neutrosophic triplets *T*, *I*, *F* are considered in the following way: the truth *T* of positive quality, the falsity *F* of negative quality (opposed to *T*), and the indeterminacy *I* of partially negative quality. That's why, when applying the *t*-norm on *T*'s, one needs to apply the opposite, i.e. the *t*-conorm on *F*'s, and vice versa. For most cases, since the indeterminacy *I* is being partially of negative quality, it is considered mostly closer to F than to T, that's why one also applies the same *t*-norm (or *t*-conorm) as on the F component.

Neutrosophic Negation

$$\neg\{\, N(T,I,F)\,\} = \neg N(F,I,T).$$

Neutrosophic Intersection (Conjunction)

$$N_1(T_1, I_1, F_1) \wedge N_2(T_2, I_2, F_2) = N(t_{norm}(T_1, T_2), t_{conorm}(I_1, I_2), t_{conorm}(F_1, F_2))$$

Neutrosophic Union (Disjunction)

$$N_1(T_1, I_1, F_1) \vee N_2(T_2, I_2, F_2) = N(t_{conorm}(T_1, T_2), t_{conorm}(I_1, I_2), t_{norm}(F_1, F_2))$$

Neutrosophic Implication

$$N_1(T_1, I_1, F_1) \rightarrow N_2(T_2, I_2, F_2) = \neg\{ N_1(T_1, I_1, F_1) \} \vee N_2(T_2, I_2, F_2) =$$
$$= (F_1, I_1, T_1) \vee (T_2, I_2, F_2) = N(t_{conorm}(F_1, T_2), t_{conorm}(I_1, I_2), t_{norm}(T_1, F_2))$$

Neutrosophic Equivalence

$$N_1(T_1, I_1, F_1) \leftrightarrow N_2(T_2, I_2, F_2) =$$
$$= \{N_1(T_1, I_1, F_1) \rightarrow N_2(T_2, I_2, F_2)\} \wedge \{ N_2(T_2, I_2, F_2) \rightarrow N_1(T_1, I_1, F_1)\}$$

5 Plithogenic Operators

They are similar to the Neutrosophic Operators, the only distinction being with respect to the indeterminacy *I*, which is considered between falsehood and truth hood, or partially false and partially true.

Therefore, instead of taking, either only $t_{conorm}(I_1, I_2)$, or only $t_{norm}(I_1, I_2)$, in plithogeny we take their averages:

$$0.5[t_{conorm}(I_1, I_2) + t_{norm}(I_1, I_2)].$$

Plithogenic Negation

$$\neg\{ P(T, I, F) \} =_P \neg P(F, I, T).$$

Plithogenic Intersection (Conjunction)

$$P_1(T_1, I_1, F_1) \wedge_P P_2(T_2, I_2, F_2) =_P P(t_{norm}(T_1, T_2), 0.5[t_{conorm}(I_1, I_2) + t_{norm}(I_1, I_2)], t_{conorm}(F_1, F_2))$$

Plithogenic Union (Disjunction)

$$P_1(T_1, I_1, F_1) \vee P_2(T_2, I_2, F_2) =_P P(t_{conorm}(T_1, T_2), 0.5[t_{conorm}(I_1, I_2) + t_{norm}(I_1, I_2)], t_{norm}(F_1, F_2))$$

Plithogenic Implication

$$P_1(T_1, I_1, F_1) \rightarrow P_2(T_2, I_2, F_2) =_P \neg\{ P_1(T_1, I_1, F_1) \} \vee P_2(T_2, I_2, F_2) =_P$$
$$=_P P(F_1, I_1, T_1) \vee P(T_2, I_2, F_2) =_P P(t_{conorm}(F_1, T_2), 0.5[t_{conorm}(I_1, I_2) + t_{norm}(I_1, I_2)], t_{norm}(T_1, F_2))$$

Plithogenic Equivalence

$$P_1(T_1, I_1, F_1) \leftrightarrow_P P_2(T_2, I_2, F_2) =_P$$
$$=_P \{P_1(T_1, I_1, F_1) \rightarrow_P P_2(T_2, I_2, F_2)\} \wedge_P \{ P_2(T_2, I_2, F_2) \rightarrow_P P_1(T_1, I_1, F_1) \}$$

Conclusion

This short note provides a unified reference for neutrosophic and plithogenic logical operators defined over an arbitrary *t*-norm/*t*-conorm pair. The central structural rule is that T and F always receive dual aggregation functions, while I is either treated as predominantly false (neutrosophic) or averaged between both aggregations (plithogenic). Substituting any specific *t*-norm — minimum, product, Lukasiewicz, Frank, or Hamacher — yields a complete, concrete family of neutrosophic or plithogenic connectives. These operators are foundational for neutrosophic decision-making, reasoning under indeterminacy, and plithogenic set theory.

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