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A Study on the Effectiveness of Contradiction Values in Upside-Down Logic and De-Plithogenication within Plithogenic Sets

Takaaki Fujita[1] * *and Arif Mehmood* [2]
[1] Independent Researcher, Tokyo, Japan. Takaaki.fujita060@gmail.com
[2] Department of Mathematics, Institute of Numerical Sciences, Gomal University, Dera Ismail Khan 29050, KPK, Pakistan (mehdaniyal@gmail.com)
*Correspondence: Takaaki.fujita060@gmail.com

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Abstract. In the real world, many reversal phenomena occur, such as cases where a statement once regarded as false is later recognized as true. *Upside-Down Logic* is a framework designed to formalize such reversal phenomena as a logical system. It inverts the truth and falsity of propositions through contextual transformations, thereby capturing ambiguity and reversals within reasoning processes.

A *Plithogenic Set* models elements by means of attribute-based membership and contradiction functions, extending the classical frameworks of fuzzy, intuitionistic, and neutrosophic sets. However, the role and effectiveness of Upside-Down Logic within Plithogenic Fuzzy Sets, Plithogenic Intuitionistic Fuzzy Sets, and Plithogenic Neutrosophic Sets have not yet been sufficiently explored.

To address this gap, this paper investigates Upside-Down Logic in these three plithogenic settings. In particular, we analyze how contradiction values guide the application of Upside-Down Logic and provide illustrative examples of its use. Furthermore, we consider application cases of Upside-Down Logic in *Plithogenic Fuzzy Graphs*. Through these studies, we aim to demonstrate that Upside-Down Logic offers a formal means to model reversal phenomena arising from contradictions under uncertainty. We also define a method of resolving contradictions, referred to as *De-Plithogenication*. Through this process, for instance, a Plithogenic Fuzzy Set can be transformed into a structure resembling a classical Fuzzy Set.

Keywords: Upside-down-logic, Plithogenic Set, Plithogenic Fuzzy Set, Plithogenic Intuitionistic Fuzzy Set, Plithogenic Neutrosophic Set, Plithogenic Fuzzy Graph

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1. **Preliminaries**

This section presents the fundamental concepts and definitions that underpin the discussions in this paper. Throughout this paper, all structures and sets are assumed to be finite.

1.1. *Formal Definition of Upside-Down Logic*

This subsection presents the mathematical definition of Upside-Down Logic. In brief, Upside-Down Logic flips the truth and falsity of propositions under contextual transformations, thereby formalizing ambiguity and reversals in reasoning systems [1–6]. The related definitions and notations are given below.

Definition 1.1 (Logical System)**.** (cf. [7]) A *logical system* $\mathcal{M}$ is a mathematical structure that formalizes reasoning. It consists of

$$\mathcal{M} = (\mathcal{P}, \mathcal{V}, v),$$

where:

- $\mathcal{P}$ is the set of propositions (statements) in the formal language $\mathcal{L}$;
- $\mathcal{V}$ is the set of truth values (e.g., $\{\text{True}, \text{False}\}$ in classical logic);
- $v : \mathcal{P} \to \mathcal{V}$ is a *valuation* (interpretation) that assigns a truth value to each proposition in $\mathcal{P}$.

A logical system may additionally include:

- a set of *axioms* $\mathcal{A} \subseteq \mathcal{P}$ assumed true within the system;
- a set of *inference rules* $\mathcal{I}$ specifying valid transformations used to derive new truths.

Notation 1. *Let $\mathcal{P}$ be a set of propositions and $\mathcal{C}$ a set of contexts. Define the truth-valuation*

$$T : \mathcal{P} \times \mathcal{C} \longrightarrow \{\mathit{True}, \mathit{False}, \mathit{Indeterminate}\},$$

which assigns a truth value to each proposition–context pair.

Notation 2. *Let $\mathcal{L}$ be a formal language, and let $\mathcal{M}$ be a logical system with proposition set $\mathcal{P}$, truth-value set $\mathcal{V}$, and valuation $v : \mathcal{P} \to \mathcal{V}$.*

Definition 1.2 (Upside-Down Logic)**.** [2, 3] An *Upside-Down Logic* is a logical system $\mathcal{M}'$ obtained from $\mathcal{M}$ by introducing a transformation U on propositions and/or contexts such that:

(1) For any proposition $A \in \mathcal{P}$ with truth value $v(A)$ in context $\mathcal{C}$, there exists a transformed proposition $U(A)$ and/or a transformed context $U(\mathcal{C})$ for which:
   - *Falsification of the Truth*: If $v(A) = \text{True}$ in $\mathcal{C}$, then $v\big(U(A)\big) = \text{False}$ in $U(\mathcal{C})$.
   - *Truthification of the False*: If $v(A) = \text{False}$ in $\mathcal{C}$, then $v\big(U(A)\big) = \text{True}$ in $U(\mathcal{C})$.

(2) The transformation U is well defined and consistent within the resulting system $\mathcal{M}'$.

Definition 1.3 (Context)**.** [2, 3] A *context* $\mathcal{C}$ is a collection of parameters or conditions under which propositions are evaluated. These may include spatial, temporal, semantic, or interpretive settings.

Example 1.4 (Boiling point vs. pressure (physical context flip))**. Proposition.** A: "Water boils at 100°C."

Context. $\mathcal{C}$: standard atmospheric pressure $P_1 = 101.325$ kPa (sea level). Then $v(A, \mathcal{C}) =$ True.

Transformed context. $U(\mathcal{C})$: reduced pressure $P_2 = 70$ kPa (high altitude). Using Clausius–Clapeyron with $\Delta H_{\text{vap}} \approx 40.65$ kJ/mol and $R = 8.314$ J/(mol · K),

$$\ln \frac{P_2}{P_1} = -\frac{\Delta H_{\text{vap}}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right), \quad T_1 = 373.15 \text{ K } (100°\text{C})$$

gives $T_2 \approx 362.9$ K $= 89.8$°C at 70 kPa. Hence boiling occurs near 90°C, so

$$v(A, U(\mathcal{C})) = \text{False}.$$

Thus, the truth of A flips under the pressure-context transformation.

Example 1.5 (School-zone speed limit (temporal/regulatory context flip))**. Proposition.** A: "Driving at 40 km/h on Elm St is legal."

Context. $\mathcal{C}$: weekday 08:15 with school-zone limit 30 km/h active (posted: 30 km/h from 07:30–09:00 and 14:30–16:00; otherwise 50 km/h). Since $40 > 30$, it violates the active limit:

$$v(A, \mathcal{C}) = \text{False}.$$

Transformed context. $U(\mathcal{C})$: same road and driver at 11:00 (outside school-zone hours). Now the applicable limit is 50 km/h and $40 \leq 50$, hence

$$v(A, U(\mathcal{C})) = \text{True}.$$

The legality of the same act flips when the temporal/regulatory context changes.

Example 1.6 (Access control by role (organizational/policy context flip))**. Proposition.** A: "User Bob can access the `/admin` dashboard."

Context. $\mathcal{C}$: Bob's role = guest; access policy allows `/admin` for roles $\{\mathsf{analyst}, \mathsf{admin}\}$ only. Thus $\mathsf{guest} \notin \{\mathsf{analyst}, \mathsf{admin}\}$, and

$$v(A, \mathcal{C}) = \text{False}.$$

Transformed context. $U(\mathcal{C})$: incident response mode grants Bob temporary elevation (role $= \mathsf{admin}$) via break-glass policy (with audit/MFA). Now $\mathsf{admin} \in \{\mathsf{analyst}, \mathsf{admin}\}$, hence

$$v(A, U(\mathcal{C})) = \text{True}.$$

The same statement about access flips truth value when the organizational context (role) changes.

1.2. *Plithogenic Set*

A Plithogenic Set [8–12] models elements with attribute-based membership and contradiction functions, extending fuzzy [13, 14], intuitionistic [15, 16], and neutrosophic sets [17–20].

Definition 1.7 (Plithogenic Set)**.** [8, 21] Let S be a universal set and $P \subseteq S$ a nonempty subset. A *Plithogenic Set* is a quintuple

$$PS = (P,\ v,\ Pv,\ pdf,\ pCF),$$

where

- v is an attribute,
- Pv is the set of possible values of the attribute v,
- $pdf : P \times Pv \to [0,1]^s$ is the *Degree of Appurtenance Function (DAF)*,
- $pCF : Pv \times Pv \to [0,1]^t$ is the *Degree of Contradiction Function (DCF)*.

The DCF satisfies, for all $a, b \in Pv$,

$$\text{Reflexivity: } pCF(a,a) = 0, \qquad \text{Symmetry: } pCF(a,b) = pCF(b,a).$$

Here $s \in \mathbb{N}$ is the appurtenance dimension and $t \in \mathbb{N}$ the contradiction dimension.

Example 1.8 (Plithogenic Set in healthcare triage (instantiation with $s = 1,\ t = 1$))**. Universe and subset.** Let S be all patients arriving at an emergency department and choose $P = \{p_1, p_2, p_3\} \subseteq S$ (three representative patients).
Attribute and its values. Let v = "symptom severity," with $Pv = \{\text{mild}, \text{moderate}, \text{severe}\}$.
DAF interpretation. Define $pdf(x,a) = \mu_P(x \mid a) \in [0,1]$ as the degree to which patient x belongs to the *set of cases requiring immediate attention* when the attribute takes value a.
Contradiction function. Let $pCF(a,b) = c(a,b) \in [0,1]$ quantify how contradictory two severity values are for triage priority (symmetric, zero diagonal).

Concrete specification.

$\mu_P(x \mid a)$	mild	moderate	severe
p_1	0.15	0.55	0.90
p_2	0.10	0.60	0.85
p_3	0.25	0.50	0.80

$c(a,b)$	mild	moderate	severe
mild	0	0.40	0.90
moderate	0.40	0	0.60
severe	0.90	0.60	0

Interpretation. Higher μ_P values under *severe* indicate stronger membership in "requires immediate attention." The DCF reflects that *mild* vs *severe* is nearly maximally contradictory (0.90), while *moderate* is intermediate with both. Thus, $PS = (P, v, Pv, pdf, pCF)$

In the literature, DAF is defined in slightly different ways: some variants use powerset–valued constructions, others the simple cube $[0,1]^s$. We adopt the latter (classical) form here; cf. [22].

instantiates a Plithogenic Set that captures attribute-conditioned membership and inter-value contradictions for triage.

Example 1.9 (Plithogenic Set in smart-home scheduling (instantiation with $s = 1,\ t = 1$))**.**
Universe and subset. Let S be household electrical loads, and select $P = \{\mathrm{dw}, \mathrm{wm}, \mathrm{ev}\} \subseteq S$ where dw=dishwasher, wm=washing machine, ev=EV charger.
Attribute and its values. Let v = "time-of-use tariff period," with $Pv = \{\mathrm{offpeak}, \mathrm{shoulder}, \mathrm{peak}\}$.
DAF interpretation. Define $pdf(x, a) = \mu_P(x \mid a) \in [0, 1]$ as the degree to which it is *appropriate to operate* device x during tariff a (higher means more suitable given cost/comfort).
Contradiction function. Let $pCF(a, b) = c(a, b)$ measure the conflict between tariff periods from a scheduling perspective (e.g., *offpeak* vs *peak* is strongly contradictory).

Concrete specification.

$\mu_P(x \mid a)$	offpeak	shoulder	peak
dw	0.95	0.60	0.10
wm	0.90	0.50	0.20
ev	0.85	0.40	0.05

$c(a, b)$	offpeak	shoulder	peak
offpeak	0	0.50	0.90
shoulder	0.50	0	0.60
peak	0.90	0.60	0

Interpretation. Each device's suitability is highest in *offpeak*, moderate in *shoulder*, and low in *peak*. The DCF encodes that planning based on *offpeak* assumptions contradicts *peak* reality (0.90), while *shoulder* is less contradictory to either. Hence $PS = (P, v, Pv, pdf, pCF)$ neatly models attribute-conditioned scheduling with explicit inter-period contradictions.

2. Main Results

In this section, we examine the results of this paper, focusing on Upside-Down Logic within Plithogenic Fuzzy Sets, Plithogenic Intuitionistic Fuzzy Sets, and Plithogenic Neutrosophic Sets.

2.1. *Plithogenic Fuzzy Set*

A Plithogenic Fuzzy Set assigns fuzzy membership values influenced by contradiction functions, extending classical fuzzy sets with contextual contradictions [23–27]. We now examine the methods of applying Upside-Down Logic.

Definition 2.1 (Plithogenic Fuzzy Set ($s = 1,\ t = 1$))**.** [8] A *Plithogenic Fuzzy Set* is a Plithogenic Set $PS = (P, v, Pv, pdf, pCF)$ with

$$pdf : P \times Pv \longrightarrow [0, 1], \qquad pCF : Pv \times Pv \longrightarrow [0, 1].$$

For $x \in P$ and attribute value $a \in Pv$, write

$$\mu_P(x \mid a) \;:=\; pdf(x, a) \in [0, 1],$$

the (single) fuzzy membership degree of x w.r.t. a. The contradiction between two attribute values is the scalar $c(a, b) := pCF(a, b) \in [0, 1]$ with $c(a, a) = 0$ and $c(a, b) = c(b, a)$.

Example 2.2 (Plithogenic Fuzzy Set in real life: footwear suitability under weather)**.** Let the universe S be common footwear, and fix $P = \{s, n, b\}$ where s = sandals, n = sneakers, b = boots. Take the attribute v = "weather" with value set $Pv = \{\text{sunny}, \text{rainy}, \text{snowy}\}$. Interpret the DAF $\mu_P(x \mid a) \in [0, 1]$ as the *suitability* of footwear x in weather a.
Membership degrees (rows = $x \in P$, columns = $a \in Pv$):

$\mu_P(x \mid a)$	sunny	rainy	snowy
s (sandals)	0.90	0.10	0.00
n (sneakers)	0.60	0.70	0.50
b (boots)	0.20	0.80	0.95

Define the contradiction function $c(\cdot, \cdot) = pCF(\cdot, \cdot)$ on Pv (symmetric with zero diagonal):

$c(a, b)$	sunny	rainy	snowy
sunny	0	0.85	0.95
rainy	0.85	0	0.40
snowy	0.95	0.40	0

This (P, v, Pv, pdf, pCF) is a Plithogenic Fuzzy Set $(s = 1, t = 1)$: the DAF provides suitability scores, while pCF quantifies how contradictory two weather values are (e.g. sunny vs snowy is highly contradictory: 0.95).

Definition 2.3 (Upside-Down Logic in Plithogenic Fuzzy Set with contradiction reset)**.** Let $PS = (P, v, Pv, pdf, pCF)$ be a Plithogenic Fuzzy Set with

$$\mu_P(x \mid a) := pdf(x, a) \in [0, 1], \qquad c(a, b) := pCF(a, b) \in [0, 1],$$

and $c(a, a) = 0,\ c(a, b) = c(b, a)$. Fix a reference (anchor) attribute $b \in Pv$ and a threshold $\tau \in [0, 1]$. Define the *activation set*

$$A_\tau(b) \;:=\; \{\, a \in Pv \;:\; c(a, b) \geq \tau \,\}.$$

The *Upside-Down transform with contradiction reset* produces a new Plithogenic Fuzzy Set

$$PS^{U_{b,\tau}} \;=\; (P,\ v,\ Pv,\ pdf^{U_{b,\tau}},\ pCF^{U_{b,\tau}})$$

by

$$pdf^{U_{b,\tau}}(x, a) \;:=\; \begin{cases} 1 - \mu_P(x \mid a), & a \in A_\tau(b), \\ \mu_P(x \mid a), & a \notin A_\tau(b), \end{cases}$$

and the updated contradiction map $pCF^{U_{b,\tau}}$ defined for all $u, v \in Pv$ by

$$pCF^{U_{b,\tau}}(u,v) \;:=\; \begin{cases} 0, & \{u,v\} = \{a,b\} \text{ for some } a \in A_\tau(b), \\ pCF(u,v), & \text{otherwise.} \end{cases}$$

That is, whenever the flip is triggered for the pair (a, b) (i.e. $a \in A_\tau(b)$), the post-transform contradiction between a and b is *reset to zero*.

Example 2.4 (Upside-Down Logic in PFS with contradiction reset: reacting to adverse weather)**.** Use the same PS as in Example 2.2. Fix the *anchor* b = rainy and threshold $\tau = 0.8$. Activation set

$$A_\tau(b) \;=\; \{\, a \in Pv : \; c(a,b) \ge \tau \,\} = \{\text{sunny}\} \quad (\text{since } c(\text{sunny}, \text{rainy}) = 0.85,\; c(\text{snowy}, \text{rainy}) = 0.40).$$

The Upside-Down transform with contradiction reset (for $a \in A_\tau(b)$)

$$\mu_P^U(x \mid a) = 1 - \mu_P(x \mid a), \qquad pCF^U(a,b) = pCF^U(b,a) = 0,$$

and otherwise leaves values unchanged.

Transformed membership degrees:

$\mu_P^U(x \mid a)$	sunny	rainy	snowy
s (sandals)	$1 - 0.90 = 0.10$	0.10	0.00
n (sneakers)	$1 - 0.60 = 0.40$	0.70	0.50
b (boots)	$1 - 0.20 = 0.80$	0.80	0.95

Updated contradiction map (only the flipped pair is reset to zero; all other pairs unchanged):

$pCF^U(u,v)$	sunny	rainy	snowy
sunny	0	$\boxed{0}$	0.95
rainy	$\boxed{0}$	0	0.40
snowy	0.95	0.40	0

When planning for *rainy* conditions (anchor b), any footwear suitability learned under the highly contradictory context *sunny* is *flipped* (e.g. sandals drop from 0.90 to 0.10), and the very contradiction that triggered the flip is resolved by setting $pCF^U(\text{sunny}, \text{rainy}) = 0$. A second application is idempotent: no further flips can trigger because the post-reset contradiction is below τ.

Example 2.5 (Upside-Down Logic in a PFS for teaching methods vs. class size (with contradiction reset))**. Universe and subset.** Let S be teaching methods, and choose $P = \{\text{lec}, \text{grp}, \text{lab}\}$ where lec = lecture, grp = group discussion, lab = laboratory session.

Attribute and values. Let v = "class size" with $Pv = \{\text{small}, \text{medium}, \text{large}\}$.

DAF interpretation. $pdf(x, a) = \mu_P(x \mid a) \in [0, 1]$ is the *appropriateness* of method x under size a.

Contradiction function. $pCF(a,b) = c(a,b) \in [0,1]$ is symmetric with $c(a,a) = 0$.

Concrete specification (before flip).

$\mu_P(x \mid a)$	small	medium	large
lec	0.40	0.70	0.95
grp	0.90	0.60	0.15
lab	0.85	0.50	0.05

$c(a,b)$	small	medium	large
small	0	0.50	0.90
medium	0.50	0	0.60
large	0.90	0.60	0

Upside-Down setup. Fix the *anchor* $b =$ large and threshold $\tau = 0.8$. Activation set:

$$A_\tau(b) = \{\, a \in Pv : \; c(a,b) \geq \tau \,\} = \{\text{small}\}.$$

Transform and reset. For $a \in A_\tau(b)$ set $\mu_P^U(x \mid a) = 1 - \mu_P(x \mid a)$ and $pCF^U(a,b) = pCF^U(b,a) = 0$. Otherwise leave values unchanged.

Membership after flip.

$\mu_P^U(x \mid a)$	small	medium	large
lec	$1 - 0.40 = 0.60$	0.70	0.95
grp	$1 - 0.90 = 0.10$	0.60	0.15
lab	$1 - 0.85 = 0.15$	0.50	0.05

Updated contradiction map (only the flipped pair is reset).

$pCF^U(u,v)$	small	medium	large
small	0	0.50	$\boxed{0}$
medium	0.50	0	0.60
large	$\boxed{0}$	0.60	0

Planning for very *large* classes (anchor) renders assumptions learned in a highly contradictory *small* context unreliable, so they are flipped: methods that were excellent for *small* (e.g. group discussion 0.90) become poor (0.10) for the anchored perspective, and vice versa. The contradiction that triggered the flip is resolved locally by setting $c(\text{small}, \text{large})$ to 0 post-transform.

Example 2.6 (Upside-Down Logic in a PFS for notification channels vs. urgency (with contradiction reset))**. Universe and subset.** Let S be notification channels, and take $P = \{\text{em}, \text{im}, \text{ph}\}$ where em = email, im = instant messaging, ph = phone call.

Attribute and values. Let $v =$ "urgency level" with $Pv = \{\text{low}, \text{normal}, \text{critical}\}$.

DAF interpretation. $pdf(x,a) = \mu_P(x \mid a) \in [0,1]$ denotes the *appropriateness* of channel x at urgency a.

Contradiction function. $pCF(a,b) = c(a,b)$ is symmetric with $c(a,a) = 0$.

Concrete specification (before flip).

$\mu_P(x \mid a)$	low	normal	critical
em	0.95	0.75	0.20
im	0.60	0.80	0.70
ph	0.15	0.55	0.95

$c(a,b)$	low	normal	critical
low	0	0.45	0.95
normal	0.45	0	0.60
critical	0.95	0.60	0

Upside-Down setup. Fix the *anchor* $b =$ critical and threshold $\tau = 0.9$. Activation set:

$$A_\tau(b) = \{\, a \in Pv : \; c(a,b) \geq \tau \,\} = \{\text{low}\}.$$

Transform and reset. For $a \in A_\tau(b)$ set $\mu_P^U(x \mid a) = 1 - \mu_P(x \mid a)$ and $pCF^U(a,b) = pCF^U(b,a) = 0$. Otherwise leave values unchanged.

Membership after flip.

$\mu_P^U(x \mid a)$	low	normal	critical
em	$1 - 0.95 = 0.05$	0.75	0.20
im	$1 - 0.60 = 0.40$	0.80	0.70
ph	$1 - 0.15 = 0.85$	0.55	0.95

Updated contradiction map (only the flipped pair is reset).

$pCF^U(u,v)$	low	normal	critical
low	0	0.45	$\boxed{0}$
normal	0.45	0	0.60
critical	$\boxed{0}$	0.60	0

When the operational perspective is anchored at *critical* urgency, practices learned from the highly contradictory *low* context are inverted: asynchronous channels (email) drop sharply in appropriateness, while synchronous channels (phone) rise accordingly. The specific contradiction that caused the flip (*low* vs *critical*) is annulled after the transform to stabilize subsequent decisions.

Example 2.7 (Two consecutive Upside-Down Logic applications in a Plithogenic Fuzzy Set (distinct contradiction resets))**. Universe and subset.** Let S be household electrical loads, and choose $P = \{\text{dw}, \text{wm}, \text{ev}\}$ where dw = dishwasher, wm = washing machine, ev = EV charger.

Attribute and values. Let v = "time-of-use tariff period," with $Pv = \{\text{offpeak, shoulder, peak}\}$.

DAF (appropriateness to operate). For $x \in P$ and $a \in Pv$, $pdf(x,a) = \mu_P(x \mid a) \in [0,1]$ is the suitability of running device x during a. The *contradiction* $pCF(a,b) = c(a,b) \in [0,1]$ is symmetric with $c(a,a) = 0$.

Baseline (step 0).

$\mu_P^{(0)}(x \mid a)$	offpeak	shoulder	peak
dw	0.95	0.60	0.10
wm	0.90	0.50	0.20
ev	0.85	0.40	0.05

$c^{(0)}(a, b)$	offpeak	shoulder	peak
offpeak	0	0.50	0.90
shoulder	0.50	0	0.60
peak	0.90	0.60	0

Step 1 (apply U_{b_1,τ_1}). Anchor b_1 = peak, threshold $\tau_1 = 0.85$. Activation set $A_{\tau_1}(b_1) = \{a \in Pv : \ c^{(0)}(a, b_1) \geq 0.85\} = \{\text{offpeak}\}$. Flip offpeak memberships and reset the triggering contradiction:

$$\mu_P^{(1)}(x \mid a) = \begin{cases} 1 - \mu_P^{(0)}(x \mid a), & a = \text{offpeak}, \\ \mu_P^{(0)}(x \mid a), & a \in \{\text{shoulder}, \text{peak}\}, \end{cases}$$

$$c^{(1)}(\text{offpeak}, \text{peak}) = c^{(1)}(\text{peak}, \text{offpeak}) = 0,$$

all other $c^{(1)}(u, v) = c^{(0)}(u, v)$ unchanged.

$\mu_P^{(1)}(x \mid a)$	offpeak	shoulder	peak
dw	$1 - 0.95 = 0.05$	0.60	0.10
wm	$1 - 0.90 = 0.10$	0.50	0.20
ev	$1 - 0.85 = 0.15$	0.40	0.05

$c^{(1)}(a, b)$	offpeak	shoulder	peak
offpeak	0	0.50	$\boxed{0}$
shoulder	0.50	0	0.60
peak	$\boxed{0}$	0.60	0

Step 2 (apply U_{b_2,τ_2}). Use a *different* anchor and threshold: b_2 = shoulder, $\tau_2 = 0.60$. With $c^{(1)}$, we have $c^{(1)}(\text{peak}, \text{shoulder}) = 0.60 \geq 0.60$ (activates) and $c^{(1)}(\text{offpeak}, \text{shoulder}) = 0.50 < 0.60$ (does not activate). Thus $A_{\tau_2}(b_2) = \{\text{peak}\}$. Flip peak memberships (using the current $\mu_P^{(1)}$) and reset this *different* contradiction:

$$\mu_P^{(2)}(x \mid a) = \begin{cases} 1 - \mu_P^{(1)}(x \mid a), & a = \text{peak}, \\ \mu_P^{(1)}(x \mid a), & a \in \{\text{offpeak}, \text{shoulder}\}, \end{cases}$$

$$c^{(2)}(\text{peak}, \text{shoulder}) = c^{(2)}(\text{shoulder}, \text{peak}) = 0,$$

all other $c^{(2)}(u,v) = c^{(1)}(u,v)$ unchanged.

$\mu_P^{(2)}(x \mid a)$	offpeak	shoulder	peak
dw	0.05	0.60	$1 - 0.10 = 0.90$
wm	0.10	0.50	$1 - 0.20 = 0.80$
ev	0.15	0.40	$1 - 0.05 = 0.95$

$c^{(2)}(a,b)$	offpeak	shoulder	peak
offpeak	0	0.50	$\boxed{0}$
shoulder	0.50	0	$\boxed{0}$
peak	$\boxed{0}$	$\boxed{0}$	0

The two consecutive applications model a realistic, staged planning process: first, policies anchored at *peak* tariffs invert assumptions taken from the highly contradictory *offpeak* context; second, after peak-specific contradictions are neutralized, operations anchored at *shoulder* revisit and invert the still-contradictory *peak* assumptions. Each step resolves a *different* contradiction pair (here, offpeak–peak then peak–shoulder), yielding stable settings for subsequent decisions.

Example 2.8 (Two successive Upside-Down Logic applications in a Plithogenic Fuzzy Set (commuting under city conditions))**. Universe and subset.** Let S be commuting modes, and choose $P = \{\text{walk}, \text{bike}, \text{car}, \text{sub}\}$ where walk = walking, bike = bicycle, car = private car, sub = subway.

Attribute and values. Let v = "city condition" with $Pv = \{\text{clear}, \text{ heavy\_rain}, \text{ subway\_strike}\}$.

DAF (suitability). For $x \in P$ and $a \in Pv$, $pdf(x,a) = \mu_P(x \mid a) \in [0,1]$ quantifies how suitable mode x is under condition a.

Contradiction map. $pCF(a,b) = c(a,b) \in [0,1]$ is symmetric with $c(a,a) = 0$.

Baseline (step 0).

$\mu_P^{(0)}(x \mid a)$	clear	heavy_rain	subway_strike
walk	0.90	0.30	0.50
bike	0.95	0.10	0.40
car	0.60	0.80	0.90
sub	0.85	0.70	0.05

$c^{(0)}(a,b)$	clear	heavy_rain	subway_strike
clear	0	0.60	0.92
heavy_rain	0.60	0	0.84
subway_strike	0.92	0.84	0

Step 1: apply U_{b_1,τ_1} (anchor b_1 = subway_strike, threshold $\tau_1 = 0.90$). Activation set $A_{\tau_1}(b_1) = \{a : \ c^{(0)}(a,b_1) \geq 0.90\} = \{\text{clear}\}$. Flip μ for a = clear and reset the triggering contradiction:

$$\mu_P^{(1)}(x \mid a) = \begin{cases} 1 - \mu_P^{(0)}(x \mid a), & a = \text{clear}, \\ \mu_P^{(0)}(x \mid a), & a \in \{\text{heavy\_rain}, \text{subway\_strike}\}, \end{cases}$$

$$c^{(1)}(\text{clear}, \text{subway\_strike}) = c^{(1)}(\text{subway\_strike}, \text{clear}) = 0,$$

and all other $c^{(1)}(u, v) = c^{(0)}(u, v)$.

$\mu_P^{(1)}(x \mid a)$	clear	heavy_rain	subway_strike
walk	$1 - 0.90 = 0.10$	0.30	0.50
bike	$1 - 0.95 = 0.05$	0.10	0.40
car	$1 - 0.60 = 0.40$	0.80	0.90
sub	$1 - 0.85 = 0.15$	0.70	0.05

$c^{(1)}(a, b)$	clear	heavy_rain	subway_strike
clear	0	0.60	$\boxed{0}$
heavy_rain	0.60	0	0.84
subway_strike	$\boxed{0}$	0.84	0

Step 2: apply U_{b_2,τ_2} (anchor $b_2 =$ heavy_rain, threshold $\tau_2 = 0.80$). With $c^{(1)}$, we have $c^{(1)}(\text{subway\_strike}, \text{heavy\_rain}) = 0.84 \geq 0.80$ (activates), while $c^{(1)}(\text{clear}, \text{heavy\_rain}) = 0.60 < 0.80$ (inactive). Thus $A_{\tau_2}(b_2) = \{\text{subway\_strike}\}$. Flip μ for $a =$ subway_strike and reset this *different* contradiction:

$$\mu_P^{(2)}(x \mid a) = \begin{cases} 1 - \mu_P^{(1)}(x \mid a), & a = \text{subway\_strike}, \\ \mu_P^{(1)}(x \mid a), & a \in \{\text{clear}, \text{heavy\_rain}\}, \end{cases}$$

$$c^{(2)}(\text{subway\_strike}, \text{heavy\_rain}) = c^{(2)}(\text{heavy\_rain}, \text{subway\_strike}) = 0,$$

and all other $c^{(2)}(u, v) = c^{(1)}(u, v)$.

$\mu_P^{(2)}(x \mid a)$	clear	heavy_rain	subway_strike
walk	0.10	0.30	$1 - 0.50 = 0.50$
bike	0.05	0.10	$1 - 0.40 = 0.60$
car	0.40	0.80	$1 - 0.90 = 0.10$
sub	0.15	0.70	$1 - 0.05 = 0.95$

$c^{(2)}(a, b)$	clear	heavy_rain	subway_strike
clear	0	0.60	$\boxed{0}$
heavy_rain	0.60	0	$\boxed{0}$
subway_strike	$\boxed{0}$	$\boxed{0}$	0

The first application anchors decisions in a *subway strike* scenario and inverts assumptions learned under the highly contradictory *clear* condition, then neutralizes that contradiction. The second application, now anchored in *heavy rain*, inverts the still-contradictory *subway strike* assumptions and neutralizes that different contradiction. Thus, two distinct contradiction pairs are resolved in sequence, yielding a stable, context-aware planning basis.

Theorem 2.9 (Axioms preserved and bounds)**.** *For all* $x \in P$ *and* $a \in Pv$, $0 \le pdf^{U_{b,\tau}}(x,a) \le 1$. *Moreover,* $pCF^{U_{b,\tau}}$ *satisfies*

$$pCF^{U_{b,\tau}}(a,a) = 0, \qquad pCF^{U_{b,\tau}}(a,b) = pCF^{U_{b,\tau}}(b,a) \in [0,1].$$

Proof. Since $0 \le \mu_P \le 1$, also $0 \le 1 - \mu_P \le 1$, so the bound holds. For $pCF^{U_{b,\tau}}$, by construction diagonal values are 0. Symmetry is evident because the rule depends only on the unordered pair $\{u, v\}$, and the values are either 0 or inherited from $pCF \in [0,1]$. □

Theorem 2.10 (Idempotence under contradiction reset)**.** *With the dynamic update of pCF above,*

$$U_{b,\tau} \circ U_{b,\tau} \;=\; U_{b,\tau}$$

(pointwise on (pdf, pCF)*).*

Proof. After one application, for each $a \in A_\tau(b)$ we have $pCF^{U_{b,\tau}}(a,b) = 0 < \tau$, so the activation condition fails on the second application; hence no further flips occur and the second application is the identity on the updated pair $(pdf^{U_{b,\tau}}, pCF^{U_{b,\tau}})$. For $a \notin A_\tau(b)$ nothing changes at either application. □

2.2. *Plithogenic Intuitionistic Fuzzy Set*

A Plithogenic Intuitionistic Fuzzy Set models membership and non-membership with contradictions, extending intuitionistic fuzzy sets by adding contextual opposition handling [8,28–30]. We now examine the methods of applying Upside-Down Logic.

Definition 2.11 (Plithogenic Intuitionistic Fuzzy Set ($s = 2$, $t = 1$))**.** [8] A *Plithogenic Intuitionistic Fuzzy Set* is a Plithogenic Set $PS = (P, v, Pv, pdf, pCF)$ with

$$pdf : P \times Pv \longrightarrow [0,1]^2, \qquad pdf(x,a) = \big(\mu_P(x \mid a),\, \nu_P(x \mid a)\big),$$

and

$$pCF : Pv \times Pv \longrightarrow [0,1],$$

such that, for every $(x,a) \in P \times Pv$,

$$\mu_P(x \mid a) \in [0,1], \quad \nu_P(x \mid a) \in [0,1], \quad \mu_P(x \mid a) + \nu_P(x \mid a) \le 1.$$

Here μ_P is the membership degree and ν_P the non-membership degree (in the sense of Atanassov); the induced indeterminacy is $\pi_P(x \mid a) := 1 - \mu_P(x \mid a) - \nu_P(x \mid a) \in [0,1]$. The DCF is scalar and symmetric with $pCF(a,a) = 0$.

Example 2.12 (Plithogenic Intuitionistic Fuzzy Set in real life: credit risk under economic scenarios)**.** Let the universe S be bank customers applying for loans and take $P = \{A, B\}$ (two applicants). Choose the attribute $v =$ "macroeconomic scenario" with value set $Pv = \{\text{boom}, \text{normal}, \text{recession}\}$. For each applicant $x \in P$ and scenario $a \in Pv$, define

$$pdf(x, a) = \big(\mu_P(x \mid a), \nu_P(x \mid a)\big) \in [0,1]^2,$$

where μ_P is the degree of being *risky* (likely to default) and ν_P is the degree of being *safe* (not risky), with $\mu_P + \nu_P \leq 1$ and indeterminacy $\pi_P = 1 - \mu_P - \nu_P$.

Membership/non-membership (entries are (μ_P, ν_P)):

	boom	normal	recession
A	(0.30, 0.60)	(0.50, 0.40)	(0.80, 0.10)
B	(0.20, 0.70)	(0.45, 0.45)	(0.65, 0.20)

(All row–column sums satisfy $\mu_P + \nu_P \leq 1$; e.g. for A, boom: $0.30 + 0.60 = 0.90 \Rightarrow \pi_P = 0.10$.)

Define the contradiction function $c(\cdot, \cdot) = pCF(\cdot, \cdot)$ on Pv (symmetric, zero diagonal) as

$c(a, b)$	boom	normal	recession
boom	0	0.40	0.90
normal	0.40	0	0.60
recession	0.90	0.60	0

This yields a concrete Plithogenic Intuitionistic Fuzzy Set $PS = (P, v, Pv, pdf, pCF)$.

Definition 2.13 (Upside-Down Logic in Plithogenic Intuitionistic Fuzzy Set (truth–falsity swap))**.** Let $PS = (P, v, Pv, pdf, pCF)$ be a Plithogenic Intuitionistic Fuzzy Set with

$$pdf(x, a) = \big(\mu_P(x \mid a), \nu_P(x \mid a)\big) \in [0,1]^2, \quad \mu_P(x \mid a) + \nu_P(x \mid a) \leq 1,$$

and let $c(a, b) := pCF(a, b) \in [0, 1]$ with $c(a, a) = 0$ and $c(a, b) = c(b, a)$. Fix an *anchor* $b \in Pv$ and a *threshold* $\tau \in [0, 1]$. Declare the flip to *activate* when

$$\mathrm{Act}(a; b, \tau) \;:\Longleftrightarrow\; c(a, b) \;\geq\; \tau.$$

Define the Upside-Down transform $U_{b,\tau}$ on pdf by

$$pdf^{U_{b,\tau}}(x, a) \;:=\; U_{b,\tau}[pdf](x, a) \;:=\; \begin{cases} \big(\nu_P(x \mid a), \mu_P(x \mid a)\big), & \text{if } \mathrm{Act}(a; b, \tau), \\ \big(\mu_P(x \mid a), \nu_P(x \mid a)\big), & \text{otherwise.} \end{cases}$$

Thus, under high contradiction with the anchor b, the *truth* (membership) and *falsity* (non-membership) degrees are swapped.

Example 2.14 (Upside-Down Logic in PIFS: adapting to a recession anchor by truth–falsity swap)**.** Using PS from Example 2.12, fix the *anchor* $b =$ recession and threshold $\tau = 0.8$. Activation condition: $\mathrm{Act}(a; b, \tau) \iff c(a, b) \geq \tau$. From the table, $c(\text{boom}, \text{recession}) =$

$0.90 \geq 0.8$ activates, while $c(\text{normal}, \text{recession}) = 0.60 < 0.8$ does not. Thus the activation set is $A_\tau(b) = \{\text{boom}\}$.

Define the Upside-Down transform $U_{b,\tau}$ by swapping truth/falsity when activated:

$$pdf^{U_{b,\tau}}(x,a) = \begin{cases} \big(\nu_P(x \mid a),\, \mu_P(x \mid a)\big), & a = \text{boom}, \\ \big(\mu_P(x \mid a),\, \nu_P(x \mid a)\big), & a \in \{\text{normal}, \text{recession}\}. \end{cases}$$

Transformed (μ^U, ν^U) (indeterminacy π is preserved since $\mu^U + \nu^U = \mu_P + \nu_P$):

	boom	normal	recession
A	(0.60, 0.30)	(0.50, 0.40)	(0.80, 0.10)
B	(0.70, 0.20)	(0.45, 0.45)	(0.65, 0.20)

Numerical check (e.g. for A, boom): originally $(0.30, 0.60)$ with $\pi = 0.10$; after swap $(0.60, 0.30)$ with the same $\pi = 1 - 0.60 - 0.30 = 0.10$. Only the scenario highly contradictory to the anchor (*boom* vs *recession*) is flipped; *normal* remains unchanged.

Example 2.15 (Two consecutive Upside-Down Logic applications in a Plithogenic Intuitionistic Fuzzy Set (support channel vs. incident severity))**. Universe and subset.** Let S be customer-support channels, and choose $P = \{\text{chat}, \text{email}, \text{phone}\}$.

Attribute and values. Let $v =$ "incident severity," with $Pv = \{\text{low},\ \text{medium},\ \text{critical}\}$.

DAF (intuitionistic). For $x \in P$ and $a \in Pv$,

$$pdf(x,a) = \big(\mu_P(x \mid a),\, \nu_P(x \mid a)\big) \in [0,1]^2, \qquad \mu_P(x \mid a) + \nu_P(x \mid a) \leq 1,$$

where μ_P is the degree that channel x is *appropriate* under a, and ν_P is the degree that it is *inappropriate*. The indeterminacy is $\pi_P = 1 - \mu_P - \nu_P$.

Contradiction map. $pCF(a,b) = c(a,b) \in [0,1]$ is symmetric with $c(a,a) = 0$.

Baseline (step 0). Membership/non-membership pairs (μ, ν):

	low	medium	critical
chat	(0.85, 0.10)	(0.60, 0.30)	(0.15, 0.75)
email	(0.90, 0.07)	(0.65, 0.25)	(0.20, 0.70)
phone	(0.30, 0.60)	(0.65, 0.20)	(0.95, 0.03)

$c^{(0)}(a,b)$	low	medium	critical
low	0	0.82	0.92
medium	0.82	0	0.65
critical	0.92	0.65	0

Step 1: apply U_{b_1,τ_1} (anchor $b_1 =$ critical, threshold $\tau_1 = 0.90$). Activation set $A_{\tau_1}(b_1) = \{a :\ c^{(0)}(a, b_1) \geq 0.90\} = \{\text{low}\}$. Perform the truth–falsity swap on $a = \text{low}$:

$$pdf^{(1)}(x,a) = \begin{cases} \big(\nu_P^{(0)}(x \mid a),\, \mu_P^{(0)}(x \mid a)\big), & a = \text{low}, \\ pdf^{(0)}(x,a), & a \in \{\text{medium}, \text{critical}\}. \end{cases}$$

Contradiction reset (distinct pair low–critical). Set $c^{(1)}(\text{low}, \text{critical}) = c^{(1)}(\text{critical}, \text{low}) = 0$; all other entries unchanged. Resulting table:

	low	medium	critical
chat	(0.10, 0.85)	(0.60, 0.30)	(0.15, 0.75)
email	(0.07, 0.90)	(0.65, 0.25)	(0.20, 0.70)
phone	(0.60, 0.30)	(0.65, 0.20)	(0.95, 0.03)

$c^{(1)}(a,b)$	low	medium	critical
low	0	0.82	$\boxed{0}$
medium	0.82	0	0.65
critical	$\boxed{0}$	0.65	0

Step 2: apply U_{b_2,τ_2} (anchor $b_2 = \text{low}$, threshold $\tau_2 = 0.80$). Using $c^{(1)}$, we have $c^{(1)}(\text{medium}, \text{low}) = 0.82 \geq 0.80$ (activates), while $c^{(1)}(\text{critical}, \text{low}) = 0$ (inactive after reset). Thus $A_{\tau_2}(b_2) = \{\text{medium}\}$. Swap truth and falsity at $a = \text{medium}$:

$$pdf^{(2)}(x,a) = \begin{cases} \left(\nu_P^{(1)}(x \mid a),\, \mu_P^{(1)}(x \mid a)\right), & a = \text{medium}, \\ pdf^{(1)}(x,a), & a \in \{\text{low}, \text{critical}\}. \end{cases}$$

Second contradiction reset (distinct pair medium–low). Set $c^{(2)}(\text{medium}, \text{low}) = c^{(2)}(\text{low}, \text{medium}) = 0$; other entries inherit from $c^{(1)}$. Final table:

	low	medium	critical
chat	(0.10, 0.85)	(0.30, 0.60)	(0.15, 0.75)
email	(0.07, 0.90)	(0.25, 0.65)	(0.20, 0.70)
phone	(0.60, 0.30)	(0.20, 0.65)	(0.95, 0.03)

$c^{(2)}(a,b)$	low	medium	critical
low	0	$\boxed{0}$	$\boxed{0}$
medium	$\boxed{0}$	0	0.65
critical	$\boxed{0}$	0.65	0

Checks and interpretation. For every cell, $\mu + \nu$ is preserved by swaps (e.g., chat–medium: $0.60 + 0.30 = 0.90$ becomes $0.30 + 0.60 = 0.90$), hence $\pi = 1 - \mu - \nu$ is invariant and $\mu + \nu \leq 1$ holds. Operationally, phase 1 anchors on *critical* incidents: assumptions learned in the highly contradictory *low* context are inverted and the low–critical contradiction is neutralized. Phase 2 anchors on *low* severity for routine hours: assumptions from the still-contradictory *medium* context are inverted, and the medium–low contradiction is then neutralized. Thus two *distinct* contradiction pairs are resolved in sequence, yielding a stable, context-aware policy.

Theorem 2.16 (Well-definedness and involutivity)**.** *For all $x \in P$ and $a \in Pv$,*

$$0 \leq \mu^U(x \mid a),\, \nu^U(x \mid a) \leq 1, \qquad \mu^U(x \mid a) + \nu^U(x \mid a) \leq 1,$$

where $(\mu^U, \nu^U) = pdf^{U_{b,\tau}}(x,a)$. Moreover,

$$U_{b,\tau} \circ U_{b,\tau} \;=\; \mathrm{id}$$

(pointwise on pdf).

Proof. Each output coordinate is one of μ_P or ν_P, hence stays in $[0,1]$. The sum $\mu^U + \nu^U$ equals $\mu_P + \nu_P \leq 1$ whether swapped or not. For involutivity, the activation predicate depends only on $c(a,b)$, which is unchanged by $U_{b,\tau}$. If inactive, both applications are identities; if active, the first swap is undone by the second. □

Remark 2.17 (Graded flip (soft trigger))**.** Define the flip intensity

$$\alpha(a;b,\tau) \;:=\; \max\left\{0,\ \frac{c(a,b)-\tau}{1-\tau}\right\} \in [0,1] \quad (\text{with } \alpha \equiv 0 \text{ if } \tau = 1),$$

and set

$$\mu^U(x \mid a) := (1-\alpha)\,\mu_P(x \mid a) + \alpha\,\nu_P(x \mid a),$$
$$\nu^U(x \mid a) := (1-\alpha)\,\nu_P(x \mid a) + \alpha\,\mu_P(x \mid a).$$

Then $\mu^U, \nu^U \in [0,1]$ and $\mu^U + \nu^U = \mu_P + \nu_P \le 1$; full swap occurs at $c(a,b) = 1$.

Remark 2.18 (Optional contradiction reset after flip)**.** If one wishes to *annul* the contradiction that triggered the flip, define $pCF^{U_{b,\tau}}$ by

$$pCF^{U_{b,\tau}}(u,v) \;:=\; \begin{cases} 0, & \{u,v\} = \{a,b\} \text{ with } \mathrm{Act}(a;b,\tau), \\ pCF(u,v), & \text{otherwise.} \end{cases}$$

Then $U_{b,\tau}$ becomes idempotent on (pdf, pCF): $U_{b,\tau} \circ U_{b,\tau} = U_{b,\tau}$.

2.3. *Plithogenic Neutrosophic Set*

A Plithogenic Neutrosophic Set represents truth, indeterminacy, and falsity degrees under contradictions, extending neutrosophic sets with contextual contradiction-sensitive semantics [31–34]. We now examine the methods of applying Upside-Down Logic.

Definition 2.19 (Plithogenic Neutrosophic Set $(s = 3,\ t = 1)$)**.** [8] A *Plithogenic Neutrosophic Set* is a Plithogenic Set $PS = (P, v, Pv, pdf, pCF)$ with

$$pdf : P \times Pv \longrightarrow [0,1]^3, \qquad pdf(x,a) = \big(T_P(x \mid a),\, I_P(x \mid a),\, F_P(x \mid a)\big),$$

and

$$pCF : Pv \times Pv \longrightarrow [0,1],$$

where, for each $(x,a) \in P \times Pv$,

$$T_P(x \mid a),\ I_P(x \mid a),\ F_P(x \mid a) \in [0,1].$$

No global normalization is required; one may optionally impose $0 \le T_P + I_P + F_P \le 3$ (single-valued setting). Here T_P, I_P, F_P denote, respectively, the degrees of truth, indeterminacy, and falsity, each evaluated with respect to the attribute value a. The DCF is scalar, symmetric, and null on the diagonal.

Example 2.20 (Plithogenic Neutrosophic Set in real life: transit status from heterogeneous sources)**.** Let the universe S be claims about a city's public transit. Fix a single claim $P = \{c\}$ where $c =$ "Metro Line M is closed now." Choose the attribute $v =$ "information source type" with value set $Pv = \{\text{official},\ \text{eyewitness},\ \text{social\_media}\}$.

For each source $a \in Pv$, the neutrosophic DAF returns

$$pdf(c,a) = \big(T_P(c \mid a),\, I_P(c \mid a),\, F_P(c \mid a)\big) \in [0,1]^3,$$

interpreted as degrees of truth (supports closure), indeterminacy, and falsity (supports open). Concrete assessments (rows = source a, entries are (T, I, F)):

	T_P	I_P	F_P
official	0.85	0.10	0.05
eyewitness	0.65	0.20	0.15
social_media	0.40	0.25	0.35

Define the contradiction function $c(a,b) = pCF(a,b)$ (symmetric, zero diagonal) by

$c(a,b)$	official	eyewitness	social_media
official	0	0.50	0.90
eyewitness	0.50	0	0.70
social_media	0.90	0.70	0

Thus $PS = (P, v, Pv, pdf, pCF)$ forms a Plithogenic Neutrosophic Set capturing how each source type evaluates the same real-world claim with (T, I, F) degrees and how source types contradict each other.

Definition 2.21 (Upside-Down Logic in Plithogenic Neutrosophic Set (truth–falsity swap))**.** Let

$$PS = (P, v, Pv, pdf, pCF)$$

be a Plithogenic Neutrosophic Set with

$$pdf(x,a) = \big(T_P(x \mid a),\, I_P(x \mid a),\, F_P(x \mid a)\big) \in [0,1]^3, \qquad c(a,b) := pCF(a,b) \in [0,1],$$

where $c(a,a) = 0$ and $c(a,b) = c(b,a)$. Fix an *anchor* $b \in Pv$ and a *threshold* $\tau \in [0,1]$. Declare the flip to *activate* when

$$\mathrm{Act}(a; b, \tau) \;:\Longleftrightarrow\; c(a,b) \;\geq\; \tau.$$

Define the Upside-Down transform $U_{b,\tau}$ on *pdf* by

$$pdf^{U_{b,\tau}}(x,a) \;:=\; \begin{cases} \big(F_P(x \mid a),\; I_P(x \mid a),\; T_P(x \mid a)\big), & \text{if } \mathrm{Act}(a; b, \tau), \\ \big(T_P(x \mid a),\; I_P(x \mid a),\; F_P(x \mid a)\big), & \text{otherwise.} \end{cases}$$

That is, under high contradiction with the anchor b, the *truth* and *falsity* degrees are swapped, while the *indeterminacy* degree is preserved.

Example 2.22 (Upside-Down Logic in PNS: anchoring on official statements and swapping T and F)**.** Using PS from Example 2.20, fix the anchor b = official and threshold $\tau = 0.8$. Activation condition:

$$\mathrm{Act}(a; b, \tau) \iff c(a, b) \geq \tau.$$

From the table, $c(\text{social\_media}, \text{official}) = 0.90 \geq 0.8$ activates, while $c(\text{eyewitness}, \text{official}) = 0.50 < 0.8$ does not. Hence the activation set is $A_\tau(b) = \{\text{social\_media}\}$.

Apply the Upside-Down transform (truth–falsity swap with indeterminacy preserved):

$$pdf^{U_{b,\tau}}(c, a) = \begin{cases} \big(F_P(c \mid a),\, I_P(c \mid a),\, T_P(c \mid a)\big), & a \in A_\tau(b), \\ \big(T_P(c \mid a),\, I_P(c \mid a),\, F_P(c \mid a)\big), & a \notin A_\tau(b). \end{cases}$$

Numerical result (rows = source a, entries are (T^U, I^U, F^U)):

	T^U	I^U	F^U
official	0.85	0.10	0.05
eyewitness	0.65	0.20	0.15
social_media	$\boxed{0.35}$	$\boxed{0.25}$	$\boxed{0.40}$

Here the activated source social_media flips its truth and falsity:

$$(T, I, F) = (0.40,\, 0.25,\, 0.35) \;\mapsto\; (T^U, I^U, F^U) = (0.35,\, 0.25,\, 0.40),$$

with indeterminacy preserved ($I^U = I$) and the sum invariant:

$$T^U + I^U + F^U = 0.35 + 0.25 + 0.40 = 1.00 = 0.40 + 0.25 + 0.35 = T + I + F.$$

Sources not highly contradictory to the anchor remain unchanged.

Theorem 2.23 (Well-definedness and invariants)**.** *For all $x \in P$ and $a \in Pv$,*

$$0 \leq T^U(x \mid a),\, I^U(x \mid a),\, F^U(x \mid a) \leq 1,$$

and

$$T^U(x \mid a) + I^U(x \mid a) + F^U(x \mid a) \;=\; T_P(x \mid a) + I_P(x \mid a) + F_P(x \mid a).$$

Proof. Each output coordinate equals one of the input coordinates $T_P, I_P, F_P \in [0, 1]$; hence bounds hold. The sum is preserved because the transform is a permutation of the triple (T_P, I_P, F_P). □

Theorem 2.24 (Involutivity (with fixed contradiction map))**.** *If pCF is left unchanged by the transform, then*

$$U_{b,\tau} \circ U_{b,\tau} \;=\; \mathrm{id} \quad \textit{(pointwise on pdf)}.$$

Proof. The activation predicate depends only on $c(a, b)$, which is unchanged. If inactive, both applications are identities. If active, the first application swaps (T, F) and the second swaps them back. □

Remark 2.25 (Graded (soft-trigger) variant)**.** Define the flip intensity

$$\alpha(a; b, \tau) \;:=\; \max\left\{0,\; \frac{c(a,b)-\tau}{1-\tau}\right\} \in [0,1] \quad (\text{take } \alpha \equiv 0 \text{ when } \tau = 1).$$

Set, for each (x, a),

$$\begin{aligned} T^U(x \mid a) &:= (1-\alpha)\, T_P(x \mid a) + \alpha\, F_P(x \mid a), \\ I^U(x \mid a) &:= I_P(x \mid a), \\ F^U(x \mid a) &:= (1-\alpha)\, F_P(x \mid a) + \alpha\, T_P(x \mid a). \end{aligned}$$

Then $T^U, I^U, F^U \in [0,1]$ and $T^U + I^U + F^U = T_P + I_P + F_P$; full swap occurs at $c(a, b) = 1$.

2.4. *Upside-Down Logic in Plithogenic Fuzzy Graphs*

A plithogenic graph extends a classical (crisp) graph by assigning to vertices and edges (i) *degrees of appurtenance* (DAF) to attribute values and (ii) *degrees of contradiction* (DCF) between attribute values [8,10,35–38]. The DAF measures how well an element fits a value of an attribute, while the DCF measures how strongly two values oppose one another, enabling multi-attribute modeling under uncertainty.

Definition 2.26 (Plithogenic Graph)**.** (cf. [10,39]) Let $G = (V, E)$ be a crisp (simple, undirected) graph with $E \subseteq \{\{x, y\} : x, y \in V,\ x \neq y\}$. A *plithogenic graph* is a pair

$$PG = (PM, PN),$$

where the vertex and edge components are specified as follows.

Vertex component.

$$PM = (M,\ \ell,\ ML,\ \mathrm{adf},\ \mathrm{aCf}),$$

with

- $M \subseteq V$ a chosen vertex subset;
- ℓ an attribute attached to vertices;
- ML the set of possible values of ℓ;
- $\mathrm{adf} : M \times ML \to [0,1]^s$ the vertex DAF;
- $\mathrm{aCf} : ML \times ML \to [0,1]^t$ the vertex DCF.

Edge component.

$$PN = (N,\ m,\ ML',\ \mathrm{bdf},\ \mathrm{bCf}),$$

with

- $N \subseteq E$ a chosen edge subset;
- m an attribute attached to edges;
- ML' the set of possible values of m;
- $\mathrm{bdf} : N \times ML' \to [0,1]^s$ the edge DAF;
- $\mathrm{bCf} : ML' \times ML' \to [0,1]^t$ the edge DCF.

All inequalities in $[0,1]^k$ are interpreted *componentwise*. Fix $s,t \in \mathbb{N}$. The following axioms are required.

(A1) **Edge–vertex compatibility (appurtenance bound).** For all $\{x,y\} \in N$ and $a,b \in ML$,

$$\mathrm{bdf}\big(\{x,y\},(a,b)\big) \;\le\; \min\!\big\{\mathrm{adf}(x,a),\, \mathrm{adf}(y,b)\big\}. \tag{1}$$

(A2) **Contradiction consistency (edge vs. vertices).** For all $(a,b),(c,d) \in ML'$,

$$\mathrm{bCf}\big((a,b),(c,d)\big) \;\le\; \min\!\big\{\mathrm{aCf}(a,c),\, \mathrm{aCf}(b,d)\big\}. \tag{2}$$

(A3) **Reflexivity and symmetry of DCF.**

$$\mathrm{aCf}(u,u)=0, \quad \mathrm{aCf}(u,v)=\mathrm{aCf}(v,u) \qquad (\forall\, u,v \in ML),$$

$$\mathrm{bCf}(u,u)=0, \quad \mathrm{bCf}(u,v)=\mathrm{bCf}(v,u) \qquad (\forall\, u,v \in ML').$$

When $s=t=1$, all maps are scalar-valued in $[0,1]$ and (1)–(2) are scalar inequalities.

Definition 2.27 (Plithogenic Fuzzy Graph)**.** [23,25,40] A *plithogenic fuzzy graph* is the scalar ($s=t=1$) specialization of Definition 2.26 on a fixed attribute alphabet A for both vertices and edges:

$$\mathsf{PFG} = \big(M, N, \ell, A, \mu_V, c_V, \mu_E, c_E\big),$$

where

- $\mu_V : M \times A \to [0,1]$ is the vertex DAF;
- $c_V : A \times A \to [0,1]$ is the vertex DCF with $c_V(a,a)=0$ and $c_V(a,b)=c_V(b,a)$;
- $\mu_E : N \times (A \times A) \to [0,1]$ is the edge DAF (for undirected graphs, $\mu_E(\{x,y\},(a,b)) = \mu_E(\{x,y\},(b,a))$);
- $c_E : (A \times A) \times (A \times A) \to [0,1]$ is the edge DCF with $c_E(\theta,\theta)=0$ and $c_E(\theta,\theta')=c_E(\theta',\theta)$.

They satisfy

$$\mu_E\big(\{x,y\},(a,b)\big) \;\le\; \min\{\mu_V(x,a), \mu_V(y,b)\}, \tag{3}$$

$$c_E\big((a,b),(c,d)\big) \;\le\; \min\{c_V(a,c), c_V(b,d)\}. \tag{4}$$

Remark 2.28 (Reduction to classical fuzzy graphs)**.** If A is a singleton, then (3) reduces to the standard bound $\mu_E(\{x,y\}) \le \min\{\mu_V(x), \mu_V(y)\}$ and $c_V \equiv c_E \equiv 0$.

Example 2.29 (A concrete Plithogenic Fuzzy Graph)**.** We give a fully numerical instance of a plithogenic fuzzy graph $\mathsf{PFG} = (M, N, \ell, A, \mu_V, c_V, \mu_E, c_E)$ satisfying (3)–(4).

Underlying graph and attribute alphabet. Let $M = \{u, v, w\}$ and

$$N = \{\{u, v\},\ \{v, w\},\ \{u, w\}\}$$

(the undirected triangle). The common attribute alphabet is

$$A = \{\text{day}, \text{evening}, \text{night}\} = \{d, e, n\}.$$

Vertex DAF and DCF.. The vertex fuzzy appurtenance $\mu_V : M \times A \to [0, 1]$ is:

$\mu_V(x, \cdot)$	d	e	n
u	0.80	0.60	0.30
v	0.70	0.55	0.25
w	0.60	0.50	0.20

The vertex contradiction $c_V : A \times A \to [0, 1]$ is symmetric with zero diagonal:

$c_V(\cdot, \cdot)$	d	e	n
d	0	0.40	0.85
e	0.40	0	0.60
n	0.85	0.60	0

Edge DAF (before any transform). For each edge, μ_E is symmetric in (a, b) and satisfies $\mu_E(\{x, y\}, (a, b)) \le \min\{\mu_V(x, a), \mu_V(y, b)\}$.

Edge $\{u, v\}$.

$\mu_E(\{u, v\}, (a, b))$	$b = d$	$b = e$	$b = n$
$a = d$	0.63	0.50	0.22
$a = e$	0.50	0.50	0.22
$a = n$	0.22	0.22	0.22

Checks: $0.63 \le \min\{0.80, 0.70\} = 0.70,$
$0.50 \le \min\{0.60, 0.55\} = 0.55,$
$0.22 \le \min\{0.80, 0.25\} = 0.25,$ etc.

Edge $\{v, w\}$.

$\mu_E(\{v, w\}, (a, b))$	$b = d$	$b = e$	$b = n$
$a = d$	0.54	0.45	0.18
$a = e$	0.45	0.45	0.18
$a = n$	0.18	0.18	0.18

Checks: $0.54 \le \min\{0.70, 0.60\} = 0.60,$
$0.45 \le \min\{0.55, 0.50\} = 0.50,$ etc.

Edge $\{u, w\}$.

$\mu_E(\{u, w\}, (a, b))$	$b = d$	$b = e$	$b = n$
$a = d$	0.54	0.45	0.18
$a = e$	0.45	0.45	0.18
$a = n$	0.18	0.18	0.18

Checks: $0.54 \le \min\{0.80, 0.60\} = 0.60,$
$0.18 \le \min\{0.30, 0.20\} = 0.20,$ etc.

Edge DCF.. A convenient choice that *saturates* the consistency bound (4) is

$$c_E\big((a,b),(c,d)\big) \;:=\; \min\big\{\, c_V(a,c),\; c_V(b,d) \,\big\}, \qquad (a,b,c,d \in A).$$

Then c_E is symmetric with zero diagonal and automatically satisfies (4). (Other c_E bounded above by this choice are also valid.)
Verification. By inspection of each table entry, the edge bound (3) holds componentwise. By construction of c_E, the contradiction consistency (4) also holds. Hence the tuple above is a valid plithogenic fuzzy graph on the triangle (M, N) with attribute alphabet A.

We next define an *upside-down* transformation that flips (hard or soft) the vertex/edge appurtenances for attribute values highly contradictory to a chosen *anchor*. The construction preserves (3)–(4); with an optional reset of the contradiction map it becomes idempotent.

Definition 2.30 (Setting and activation)**.** Let $\mathsf{PFG} = (M, N, \ell, A, \mu_V, c_V, \mu_E, c_E)$ be a plithogenic fuzzy graph as in Definition 2.27. Fix an anchor $a_\star \in A$ and a threshold $\tau \in [0,1]$. Define the activation predicates

$$\mathsf{Act}_V(a) \;:\Longleftrightarrow\; c_V(a,a_\star) \ge \tau, \qquad \mathsf{Act}_E(a,b) \;:\Longleftrightarrow\; \max\{c_V(a,a_\star), c_V(b,a_\star)\} \ge \tau.$$

Thus an edge-attribute pair activates if at least one endpoint-attribute value is highly contradictory to the anchor.

Definition 2.31 (Upside-down transform in Plithogenic Fuzzy Graph)**.** The transform $U_{a_\star,\tau}$ produces

$$\mathsf{PFG}^U = (M, N, \ell, A, \mu_V^U, c_V^U, \mu_E^U, c_E^U)$$

as follows.

(U1) **Vertices (hard flip).**

$$\mu_V^U(x,a) = \begin{cases} 1-\mu_V(x,a), & \mathsf{Act}_V(a), \\ \mu_V(x,a), & \text{otherwise.} \end{cases}$$

(U2) **Edges (bounded hard flip).** Set

$$\widetilde{\mu}_E(\{x,y\},(a,b)) = \begin{cases} 1-\mu_E(\{x,y\},(a,b)), & \mathsf{Act}_E(a,b), \\ \mu_E(\{x,y\},(a,b)), & \text{otherwise,} \end{cases}$$

and then truncate to preserve (3):

$$\mu_E^U(\{x,y\},(a,b)) := \min\Big\{\widetilde{\mu}_E(\{x,y\},(a,b)),\; \mu_V^U(x,a),\; \mu_V^U(y,b)\Big\}. \tag{5}$$

(U3) **Contradiction maps.** Two standard choices:

- *No reset (involutive).* Keep $c_V^U = c_V$ and $c_E^U = c_E$.

- *Reset (idempotent).* Put

$$c_V^U(u,v)=\begin{cases}0, & \{u,v\}=\{a,a_\star\} \text{ with } \mathsf{Act}_V(a),\\ c_V(u,v), & \text{otherwise,}\end{cases}$$

and, for $\theta=(a,b)$, $\phi=(c,d)$,

$$c_E^U(\theta,\phi)=\begin{cases}0, & \{\theta,\phi\}=\{(a_\star,a_\star),(a,b)\} \text{ with } \mathsf{Act}_E(a,b),\\ \min\bigl\{c_E(\theta,\phi),\ c_V^U(a,c),\ c_V^U(b,d)\bigr\}, & \text{otherwise.}\end{cases}$$

The outer minimum ensures (4) holds after the update.

We write $\mathsf{PFG}^U=U_{a_\star,\tau}(\mathsf{PFG})$.

Example 2.32 (Numerical illustration of $U_{a_\star,\tau}$ on a Plithogenic Fuzzy Graph)**.** We work with a two-vertex graph to make every computation explicit.
Graph and attribute alphabet. Let $G=(V,E)$ with $V=\{u,v\}$ and $E=\bigl\{\{u,v\}\bigr\}$. Take the attribute alphabet

$$A=\{\text{low},\text{normal},\text{critical}\}=\{\ell,n,c\}.$$

The vertex DCF $c_V:A\times A\to[0,1]$ is specified by the symmetric table

$c_V(\cdot,\cdot)$	ℓ	n	c
ℓ	0	0.82	0.92
n	0.82	0	0.65
c	0.92	0.65	0

and the vertex DAF $\mu_V:M\times A\to[0,1]$ is chosen *equal on both vertices* (to preserve edge symmetry cleanly):

$$\mu_V(u,\ell)=\mu_V(v,\ell)=0.90,\quad \mu_V(u,n)=\mu_V(v,n)=0.60,\quad \mu_V(u,c)=\mu_V(v,c)=0.20.$$

The edge DAF $\mu_E:N\times(A\times A)\to[0,1]$ (for the unique edge $\{u,v\}$) is specified symmetrically in (a,b) and satisfies the bound $\mu_E(\{u,v\},(a,b))\le\min\{\mu_V(u,a),\mu_V(v,b)\}$ from (3):

$\mu_E(\{u,v\},(a,b))$	$b=\ell$	$b=n$	$b=c$
$a=\ell$	0.70	0.50	0.18
$a=n$	0.50	0.45	0.18
$a=c$	0.18	0.18	0.20

(For instance, $\mu_E(\{u,v\},(\ell,c))=0.18\le\min\{0.90,0.20\}=0.20$.)

Upside-down parameters and activation. Fix the anchor $a_\star = c$ and threshold $\tau = 0.90$. By Definition 2.30,

$$\mathsf{Act}_V(\ell) \iff c_V(\ell, c) = 0.92 \geq 0.90 \quad \text{(true)},$$

$$\mathsf{Act}_V(n) \iff 0.65 \geq 0.90 \quad \text{(false)},$$

$$\mathsf{Act}_V(c) \iff 0 \geq 0.90 \quad \text{(false)}.$$

Hence an edge pair (a, b) is activated (i.e. $\mathsf{Act}_E(a, b)$ holds) iff at least one of a, b equals ℓ.

Vertex transform (hard flip). By (U1) in Definition 2.31,

$$\mu_V^U(x, \ell) = 1 - \mu_V(x, \ell) = 1 - 0.90 = 0.10, \qquad \mu_V^U(x, n) = 0.60, \qquad \mu_V^U(x, c) = 0.20 \quad (x \in \{u, v\}).$$

Edge transform (bounded hard flip). For each $(a, b) \in A \times A$ we first complement on activation $\widetilde{\mu}_E = 1 - \mu_E$ and then truncate by (5):

$$\mu_E^U(\{u, v\}, (a, b)) \;=\; \min\Big\{\widetilde{\mu}_E(\{u, v\}, (a, b)),\; \mu_V^U(u, a),\; \mu_V^U(v, b)\Big\}.$$

Concretely:

$$\begin{aligned}
&(a,b) = (\ell,\ell): && \widetilde{\mu}_E = 1 - 0.70 = 0.30, && \mu_E^U = \min\{0.30,\ 0.10,\ 0.10\} = 0.10;\\
&(a,b) = (\ell,n): && \widetilde{\mu}_E = 1 - 0.50 = 0.50, && \mu_E^U = \min\{0.50,\ 0.10,\ 0.60\} = 0.10;\\
&(a,b) = (n,\ell): && \widetilde{\mu}_E = 1 - 0.50 = 0.50, && \mu_E^U = \min\{0.50,\ 0.60,\ 0.10\} = 0.10;\\
&(a,b) = (\ell,c): && \widetilde{\mu}_E = 1 - 0.18 = 0.82, && \mu_E^U = \min\{0.82,\ 0.10,\ 0.20\} = 0.10;\\
&(a,b) = (c,\ell): && \widetilde{\mu}_E = 1 - 0.18 = 0.82, && \mu_E^U = \min\{0.82,\ 0.20,\ 0.10\} = 0.10;\\
&(a,b) = (n,n): && \text{(not activated)} \Rightarrow \widetilde{\mu}_E = 0.45, && \mu_E^U = \min\{0.45,\ 0.60,\ 0.60\} = 0.45;\\
&(a,b) = (n,c): && \text{(not activated)} \Rightarrow \widetilde{\mu}_E = 0.18, && \mu_E^U = \min\{0.18,\ 0.60,\ 0.20\} = 0.18;\\
&(a,b) = (c,c): && \text{(not activated)} \Rightarrow \widetilde{\mu}_E = 0.20, && \mu_E^U = \min\{0.20,\ 0.20,\ 0.20\} = 0.20.
\end{aligned}$$

Tabular summary. Before (μ_V, μ_E) and after (μ_V^U, μ_E^U) the transform:

vertex x	$\mu_V(x,\ell)$	$\mu_V(x,n)$	$\mu_V(x,c)$
u (before)	0.90	0.60	0.20
v (before)	0.90	0.60	0.20
u (after)	**0.10**	0.60	0.20
v (after)	**0.10**	0.60	0.20

$\mu_E(\{u,v\},(a,b))$	$b=\ell$	$b=n$	$b=c$
$a=\ell$ (before)	0.70	0.50	0.18
$a=n$ (before)	0.50	0.45	0.18
$a=c$ (before)	0.18	0.18	0.20
$a=\ell$ (after)	**0.10**	**0.10**	**0.10**
$a=n$ (after)	**0.10**	0.45	0.18
$a=c$ (after)	**0.10**	0.18	0.20

Verification of axioms after the transform. For each (a, b),

$$\mu_E^U(\{u, v\}, (a, b)) \;\leq\; \min\{\mu_V^U(u, a), \mu_V^U(v, b)\}$$

by construction; e.g. for $(a, b) = (\ell, n)$ the right-hand side is $\min\{0.10, 0.60\} = 0.10$, matching the computed $\mu_E^U = 0.10$. Symmetry in (a, b) is preserved because μ_V was chosen equal on u, v and μ_E was symmetric before the transform.

Optional contradiction reset. If one adopts the *reset* option in Definition 2.31, then $c_V^U(\ell, c) = 0$ (and c_V^U coincides with c_V elsewhere), and for edge-attribute pairs with at least one ℓ,

$$c_E^U\big((\ell, \cdot), (c, c)\big) = 0.$$

Under this reset, a second application of $U_{a_\star,\tau}$ becomes the identity (idempotence); without reset, the second application restores the original memberships (involution).

Example 2.33 (Numerical example of the upside-down transform $U_{a_\star,\tau}$)**.** We illustrate $U_{a_\star,\tau}$ on a three-vertex plithogenic fuzzy graph with two edges.

Graph and attribute alphabet. Let $G = (V, E)$ with $V = \{u, v, w\}$ and $E = \{\{u, v\}, \{v, w\}\}$. Take the attribute alphabet

$$A = \{\text{offpeak}, \text{shoulder}, \text{peak}\} = \{o, s, p\}.$$

The vertex DCF $c_V : A \times A \to [0, 1]$ is (symmetric with zero diagonal)

$c_V(\cdot, \cdot)$	o	s	p
o	0	0.50	0.90
s	0.50	0	0.60
p	0.90	0.60	0

and the vertex DAF is *identical on every vertex* (so that edge appurtenance remains symmetric in (a, b)):

$$\mu_V(x, o) = 0.88, \qquad \mu_V(x, s) = 0.62, \qquad \mu_V(x, p) = 0.18 \quad (x \in \{u, v, w\}).$$

Edge DAF before the transform. For each edge, μ_E is symmetric in (a, b) and satisfies the bound $\mu_E(\{x, y\}, (a, b)) \le \min\{\mu_V(x, a), \mu_V(y, b)\}$ from (3).

Edge $\{u, v\}$:

$\mu_E(\{u, v\}, (a, b))$	$b = o$	$b = s$	$b = p$
$a = o$	0.78	0.52	0.16
$a = s$	0.52	0.48	0.16
$a = p$	0.16	0.16	0.17

Checks: $0.78 \le 0.88,\ 0.52 \le 0.62,$
$0.16 \le 0.18,\ 0.48 \le 0.62,\ 0.17 \le 0.18.$

Edge $\{v, w\}$:

$\mu_E(\{v, w\}, (a, b))$	$b = o$	$b = s$	$b = p$
$a = o$	0.74	0.50	0.15
$a = s$	0.50	0.46	0.15
$a = p$	0.15	0.15	0.16

Checks: $0.74 \le 0.88,\ 0.50 \le 0.62,$
$0.15 \le 0.18,\ 0.46 \le 0.62,\ 0.16 \le 0.18.$

Upside-down parameters and activation. Fix the anchor $a_\star = p$ and threshold $\tau = 0.85$. Then

$$\mathsf{Act}_V(o) \iff c_V(o,p) = 0.90 \geq 0.85 \text{ (true)},$$

$$\mathsf{Act}_V(s) \iff 0.60 < 0.85 \text{ (false)},$$

$$\mathsf{Act}_V(p) \iff 0 < 0.85 \text{ (false)}.$$

Hence an edge pair (a,b) is activated ($\mathsf{Act}_E(a,b)$ holds) iff at least one of a,b equals o.

Vertex transform (hard flip). By (U1),

$$\mu_V^U(x,o) = 1 - 0.88 = 0.12, \qquad \mu_V^U(x,s) = 0.62, \qquad \mu_V^U(x,p) = 0.18.$$

Edge transform (bounded hard flip). For each activated (a,b) we first complement $\widetilde{\mu}_E(\{x,y\},(a,b)) = 1 - \mu_E(\{x,y\},(a,b))$ and then truncate as in (5):

$$\mu_E^U(\{x,y\},(a,b)) = \min\Big\{\widetilde{\mu}_E(\{x,y\},(a,b)),\ \mu_V^U(x,a),\ \mu_V^U(y,b)\Big\}.$$

Non-activated pairs remain unchanged.

Edge $\{u,v\}$ after $U_{p,0.85}$:

$\mu_E^U(\{u,v\},(a,b))$	$b=o$	$b=s$	$b=p$
$a=o$	**0.12**	**0.12**	**0.12**
$a=s$	**0.12**	0.48	0.16
$a=p$	**0.12**	0.16	0.17

(Each bold entry comes from an activated case, where the minimum of the complemented value and updated vertex appurtenances was 0.12.)

Edge $\{v,w\}$ after $U_{p,0.85}$:

$\mu_E^U(\{v,w\},(a,b))$	$b=o$	$b=s$	$b=p$
$a=o$	**0.12**	**0.12**	**0.12**
$a=s$	**0.12**	0.46	0.15
$a=p$	**0.12**	0.15	0.16

(Here too, all bold entries are the results of activated pairs.)

Tabular vertex summary. For any $x \in \{u,v,w\}$:

	o	s	p
μ_V (before)	0.88	0.62	0.18
μ_V^U (after)	**0.12**	0.62	0.18

Post-transform verification. For every edge and (a, b),

$$\mu_E^U(\{x, y\}, (a, b)) \;\le\; \min\{\mu_V^U(x, a), \mu_V^U(y, b)\},$$

e.g. for $\{u, v\}$ and $(a, b) = (o, s)$,

$$\mu_E^U = \min\{0.48, 0.12, 0.62\} = 0.12 \;\le\; \min\{0.12, 0.62\} = 0.12.$$

Symmetry in (a, b) is preserved because μ_V (hence μ_V^U) is the same on both endpoints and μ_E was symmetric before the transform.

Optional reset (idempotence). If the reset option in Definition 2.31 is used, then $c_V^U(o, p) = 0$ (all other entries unchanged), and for edge-attribute pairs involving o one sets $c_E^U((o, \cdot), (p, p)) = 0$. A second application of $U_{p,0.85}$ performs no further flips (idempotent). Without reset, applying $U_{p,0.85}$ twice restores the original memberships (involution).

Theorem 2.34 (Well-definedness and axiom preservation)**.** *For all $x, y \in M$ with $\{x, y\} \in N$ and all $a, b, c, d \in A$:*

(a) $0 \le \mu_V^U(x, a) \le 1$ *and* $0 \le \mu_E^U(\{x, y\}, (a, b)) \le 1$.

(b) μ_E^U *satisfies the bound*

$$\mu_E^U(\{x, y\}, (a, b)) \;\le\; \min\{\mu_V^U(x, a), \mu_V^U(y, b)\}.$$

(c) c_V^U *and* c_E^U *are symmetric, have zero diagonal, take values in* $[0, 1]$*, and satisfy*

$$c_E^U\big((a, b), (c, d)\big) \;\le\; \min\{c_V^U(a, c),\, c_V^U(b, d)\}.$$

Proof. (a) Each vertex update is either μ_V or $1 - \mu_V$, hence remains in $[0, 1]$. For edges, (5) is a minimum of three numbers in $[0, 1]$. (b) The inequality is exactly (5). (c) Symmetry and zero diagonal hold by construction in both variants; the reset case enforces the edge–vertex contradiction inequality via the outer minimum. □

Theorem 2.35 (Involutive vs. idempotent behavior)**.** *For fixed $(a_\star, \tau)$:*

(i) No reset: $U_{a_\star,\tau} \circ U_{a_\star,\tau} = \mathrm{id}$ *on (μ_V, μ_E) (the transform is an involution).*

(ii) With reset: $U_{a_\star,\tau} \circ U_{a_\star,\tau} = U_{a_\star,\tau}$ *on the full tuple (μ_V, c_V, μ_E, c_E) (the transform is idempotent).*

Proof. (i) Activations depend only on c_V and τ, which are unchanged; vertices apply $a \mapsto 1 - a$ twice; edges apply complement twice and the final truncation recovers (3). (ii) After one application, every activated value a satisfies $c_V^U(a, a_\star) = 0$ (and similarly for c_E^U), so no further activations occur; the second application is the identity. □

Remark 2.36 (Soft (graded) upside-down)**.** Define the flip intensity

$$\alpha(a) := \max\Big\{0,\ \frac{c_V(a,a_\star)-\tau}{1-\tau}\Big\} \in [0,1] \quad (\alpha \equiv 0 \text{ if } \tau = 1), \qquad \alpha_E(a,b) := \max\{\alpha(a),\alpha(b)\}.$$

Set

$$\mu_V^U(x,a) = (1-\alpha(a))\,\mu_V(x,a) + \alpha(a)\,(1-\mu_V(x,a)),$$

$$\widehat{\mu}_E(\{x,y\},(a,b)) = (1-\alpha_E(a,b))\,\mu_E(\{x,y\},(a,b)) + \alpha_E(a,b)\,(1-\mu_E(\{x,y\},(a,b))),$$

and then apply the same truncation $\mu_E^U = \min\{\widehat{\mu}_E, \mu_V^U(x,a), \mu_V^U(y,b)\}$. All properties in Theorem 2.34 remain valid.

3. **De-Plithogenication: A Method for Resolving Contradictions**

We illustrate three consecutive applications of the *Upside-Down Logic with contradiction reset* on a Plithogenic Fuzzy Set so that, by the end, *all* pairwise contradiction degrees become zero. Related concepts include *De-Fuzzification* [41–44] and *De-Neutrosophication* [45–48].

Definition 3.1 (De-Plithogenication)**.** Let $PS = (P, v, Pv, pdf, pCF)$ be a Plithogenic Set and let $U_{b,\tau}$ denote an *Upside-Down transform with contradiction reset*: when applied with anchor $b \in Pv$ and threshold $\tau \in [0,1]$, it (i) flips the chosen appurtenance coordinates of $pdf(\cdot,a)$ for every $a \in Pv$ that *activates*, and (ii) sets the post-transform contradiction to zero on each activated pair $\{a,b\}$, i.e. $pCF^{U_{b,\tau}}(a,b) = 0$.
A *de-plithogenication* of PS is any finite composition

$$PS^{\text{dep}} \ :=\ U_{b_k,\tau_k} \circ \cdots \circ U_{b_2,\tau_2} \circ U_{b_1,\tau_1}(PS)$$

such that the resulting contradiction map is identically zero:

$$pCF^{\text{dep}}(u,w) = 0 \qquad \text{for all } u,w \in Pv.$$

We call PS^{dep} the *de-plithogenicated (contradiction-free) normal form* of PS, and say that PS is *de-plithogenizable* if such a finite sequence exists. Since $pCF^{\text{dep}} \equiv 0$, any further application of $U_{b,\tau}$ leaves PS^{dep} unchanged.

Example 3.2 (De-Plithogenication on a communication–urgency model)**.** Let the universe S be workplace communication channels and take $P = \{e,c,t\}$ where e = email, c = chat, t = telephone (call). Let the attribute be v = "message urgency" with value set $Pv = \{\text{low}, \text{medium}, \text{high}\}$. Interpret $\mu_P(x \mid a) \in [0,1]$ as the *suitability* of channel x for urgency level a.

Initial state (Step 0). Membership (rows: $x \in P$, columns: $a \in Pv$):

$\mu_P(x \mid a)$	low	medium	high
e (email)	0.90	0.60	0.20
c (chat)	0.50	0.80	0.60
t (call)	0.30	0.50	0.90

Contradiction (symmetric, zero diagonal):

$pCF(a,b)$	low	medium	high
low	0	0.60	0.90
medium	0.60	0	0.70
high	0.90	0.70	0

We apply three Upside-Down transforms with contradiction reset (cf. $U_{b,\tau}$), choosing anchors and thresholds so that each step eliminates one remaining nonzero contradiction pair. The flip rule is

$$\mu^{\text{new}}(x \mid a) = \begin{cases} 1 - \mu^{\text{old}}(x \mid a), & a \in A_\tau(b) := \{a : pCF(a,b) \geq \tau\}, \\ \mu^{\text{old}}(x \mid a), & \text{otherwise}, \end{cases}$$

$$\text{and} \quad pCF^{\text{new}}(a,b) = 0 \ \text{ for each activated } \{a,b\}.$$

Step 1 (anchor $b =$ high, threshold $\tau_1 = 0.80$). Activation: $A_{\tau_1}(\text{high}) = \{\text{low}\}$ (since $0.90 \geq 0.80$). Flip the *low* column; reset $pCF(\text{low}, \text{high})$ to 0:

$\mu^{(1)}(x \mid a)$	low	medium	high
e	$1 - 0.90 = 0.10$	0.60	0.20
c	$1 - 0.50 = 0.50$	0.80	0.60
t	$1 - 0.30 = 0.70$	0.50	0.90

$pCF^{(1)}$	low	medium	high
low	0	0.60	$\boxed{0}$
medium	0.60	0	0.70
high	$\boxed{0}$	0.70	0

Step 2 (anchor $b =$ medium, threshold $\tau_2 = 0.60$). Activation: $A_{\tau_2}(\text{medium}) = \{\text{low}\}$ (since $0.60 \geq 0.60$). Flip the *low* column again; reset $pCF(\text{low}, \text{medium})$ to 0:

$\mu^{(2)}(x \mid a)$	low	medium	high
e	$1 - 0.10 = 0.90$	0.60	0.20
c	$1 - 0.50 = 0.50$	0.80	0.60
t	$1 - 0.70 = 0.30$	0.50	0.90

$pCF^{(2)}$	low	medium	high
low	0	$\boxed{0}$	0
medium	$\boxed{0}$	0	0.70
high	0	0.70	0

Step 3 (anchor $b =$ high, threshold $\tau_3 = 0.70$). Activation: $A_{\tau_3}(\text{high}) = \{\text{medium}\}$ (since $0.70 \geq 0.70$). Flip the *medium* column; reset $pCF(\text{medium}, \text{high})$ to 0:

$\mu^{(3)}(x \mid a)$	low	medium	high
e	0.90	$1 - 0.60 = 0.40$	0.20
c	0.50	$1 - 0.80 = 0.20$	0.60
t	0.30	$1 - 0.50 = 0.50$	0.90

$pCF^{(3)}$	low	medium	high
low	0	0	0
medium	0	0	$\boxed{0}$
high	0	$\boxed{0}$	0

Result (contradiction-free normal form). After three applications,

$$pCF^{(3)}(u, w) = 0 \qquad \text{for all } u, w \in \{\text{low}, \text{medium}, \text{high}\},$$

so the system is fully *de-plithogenicated*. Any further $U_{b,\tau}$ has no effect. Note that the final suitability table differs only in the columns that were flipped an odd number of times (here, *medium*); columns flipped twice (here, *low*) return to their original values.

Example 3.3 (Three-step de-plithogenication for footwear vs. weather)**.** Let the universe S be common footwear, and take $P = \{s, n, b\}$ where $s =$sandals, $n =$sneakers, $b =$boots. Let the attribute be $v =$"weather" with value set $Pv = \{\text{sunny}, \text{rainy}, \text{snowy}\}$. Interpret $\mu_P(x \mid a) \in [0, 1]$ as the suitability of footwear x under weather a.

Initial data (Step 0). Membership degrees (rows: $x \in P$, columns: $a \in Pv$):

$\mu_P(x \mid a)$	sunny	rainy	snowy
s (sandals)	0.90	0.10	0.00
n (sneakers)	0.60	0.70	0.50
b (boots)	0.20	0.80	0.95

Contradiction map $c(\cdot, \cdot) = pCF(\cdot, \cdot)$ on Pv (symmetric, zero diagonal):

$c(a, b)$	sunny	rainy	snowy
sunny	0	0.85	0.95
rainy	0.85	0	0.40
snowy	0.95	0.40	0

Step 1 (anchor: rainy, threshold $\tau_1 = 0.80$). Activation set $A_{\tau_1}(\text{rainy}) = \{a \in Pv : c(a, \text{rainy}) \geq 0.80\} = \{\text{sunny}\}$. Apply the Upside-Down transform with contradiction reset:

$$\mu^{(1)}(x \mid a) = \begin{cases} 1 - \mu_P(x \mid a), & a = \text{sunny}, \\ \mu_P(x \mid a), & a \in \{\text{rainy}, \text{snowy}\}, \end{cases} \qquad pCF^{(1)}(\text{sunny}, \text{rainy}) = 0 \text{ (others as before)}.$$

Numerically,

$\mu^{(1)}(x \mid a)$	sunny	rainy	snowy
s	$1 - 0.90 = 0.10$	0.10	0.00
n	$1 - 0.60 = 0.40$	0.70	0.50
b	$1 - 0.20 = 0.80$	0.80	0.95

$pCF^{(1)}$	sunny	rainy	snowy
sunny	0	$\boxed{0}$	0.95
rainy	$\boxed{0}$	0	0.40
snowy	0.95	0.40	0

Step 2 (anchor: snowy, threshold $\tau_2 = 0.80$). Activation set $A_{\tau_2}(\text{snowy}) = \{a \in Pv : pCF^{(1)}(a, \text{snowy}) \geq 0.80\} = \{\text{sunny}\}$ (since $0.95 \geq 0.80$). Apply the transform with reset:

$$\mu^{(2)}(x \mid a) = \begin{cases} 1 - \mu^{(1)}(x \mid a), & a = \text{sunny}, \\ \mu^{(1)}(x \mid a), & a \in \{\text{rainy}, \text{snowy}\}, \end{cases}$$

$$pCF^{(2)}(\text{sunny}, \text{snowy}) = 0 \text{ (others as in } pCF^{(1)}).$$

Numerically, the *sunny* column flips back to its original values:

$\mu^{(2)}(x \mid a)$	sunny	rainy	snowy
s	$1 - 0.10 = 0.90$	0.10	0.00
n	$1 - 0.40 = 0.60$	0.70	0.50
b	$1 - 0.80 = 0.20$	0.80	0.95

$pCF^{(2)}$	sunny	rainy	snowy
sunny	0	0	$\boxed{0}$
rainy	0	0	0.40
snowy	$\boxed{0}$	0.40	0

Step 3 (anchor: rainy, threshold $\tau_3 = 0.40$). Activation set $A_{\tau_3}(\text{rainy}) = \{a \in Pv : pCF^{(2)}(a, \text{rainy}) \geq 0.40\} = \{\text{snowy}\}$. Apply the transform with reset:

$$\mu^{(3)}(x \mid a) = \begin{cases} 1 - \mu^{(2)}(x \mid a), & a = \text{snowy}, \\ \mu^{(2)}(x \mid a), & a \in \{\text{sunny}, \text{rainy}\}, \end{cases} \qquad pCF^{(3)}(\text{snowy}, \text{rainy}) = 0 \text{ (others as in } pCF^{(2)}\text{)}.$$

Numerically,

$\mu^{(3)}(x \mid a)$	sunny	rainy	snowy
s	0.90	0.10	$1 - 0.00 = 1.00$
n	0.60	0.70	$1 - 0.50 = 0.50$
b	0.20	0.80	$1 - 0.95 = 0.05$

$pCF^{(3)}$	sunny	rainy	snowy
sunny	0	0	0
rainy	0	0	$\boxed{0}$
snowy	0	$\boxed{0}$	0

Outcome. After the third application, *all* pairwise contradiction degrees are zero:

$$pCF^{(3)}(a, b) = 0 \quad \text{for all } a, b \in Pv.$$

In other words, the sequence of three context-anchored flips *de-plithogenizes* the attribute space by systematically eliminating contradictions: first (sunny, rainy), then (sunny, snowy), and finally (rainy, snowy). The membership columns affected at each step are flipped via $1-\mu$, while unaffected columns remain unchanged.

4. **Future Works**

In the future, we plan to explore the application of Upside-Down Logic to Neutrosophic Graphs [19, 49–51], as well as its potential use within SuperHyperPlithogenic Sets [52–54]. Moreover, we intend to further examine the effectiveness of Upside-Down Logic in the context of HyperGraphs [55–58] and SuperHyperGraphs [11, 59–63] derived from Plithogenic Fuzzy Graphs.

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Research Integrity

The authors hereby confirm that, to the best of their knowledge, this manuscript is their original work, has not been published in any other journal, and is not currently under consideration for publication elsewhere at this stage.

Use of Generative AI and AI-Assisted Tools

I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

Disclaimer (Note on Computational Tools)

No computer-assisted proof, symbolic computation, or automated theorem proving tools (e.g., Mathematica, SageMath, Coq, etc.) were used in the development or verification of the results presented in this paper. All proofs and derivations were carried out manually and analytically by the authors.

Code Availability

No code or software was developed for this study.

Clinical Trial

This study did not involve any clinical trials.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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